Single-Minus Gluon Amplitudes and Klein Space: A Natural Explanation from 6D Spacetime with Signature (3,3)

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Abstract

A recent preprint by Guevara, Lupsasca, Skinner, Strominger, and Weil (arXiv:2602.12176) demonstrates that single-minus gluon tree-level scattering amplitudes, long presumed to vanish, are in fact nonzero for half-collinear configurations existing in Klein space $\mathbb{R}^{2,2}$ or for complexified momenta. We show that the 3D+3D discrete spacetime framework — a six-dimensional theory with metric signature $(-,+,+,+,-,-)$ — provides a natural geometric explanation for this result. Specifically, we demonstrate three key connections: (i) Klein space $\mathbb{R}^{2,2}$ arises as a canonical subspace of the full (3,3)-signature spacetime upon partial dimensional reduction; (ii) the compactification of two temporal dimensions on a torus T^2 automatically generates the complexified momentum configurations required for non-vanishing amplitudes; (iii) the piecewise-constant chamber structure of the amplitude mirrors the discrete topology of the internal torus. We derive the effective 4D gluon scattering amplitude including Kaluza-Klein contributions from the temporal tower, showing that the half-collinear regime corresponds precisely to resonant modes of the compactified temporal dimensions.

Keywords: *gluon scattering amplitudes, Klein space, split signature, extra temporal dimensions, 3D+3D framework, half-collinear regime, Kaluza-Klein compactification*

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1. Introduction

1.1 The Guevara—Strominger Result

In a striking recent development, Guevara et al. [1] have demonstrated that single-minus tree-level n-gluon scattering amplitudes — configurations where one gluon carries negative helicity and the remaining $n-1$ carry positive helicity — do not universally vanish as textbook arguments suggest. The standard proof of vanishing relies on a specific choice of reference spinors that forces all polarization contractions to zero. However, this argument assumes *generic* particle momenta.

The key finding of [1] is that in the **half-collinear regime** — a precisely defined slice of momentum space where gluon momenta obey a special alignment condition — the standard argument breaks down. The amplitude is not merely nonzero but admits a closed-form piecewise-constant expression satisfying multiple consistency conditions including the Berends—Giele recursion relation and Weinberg's soft theorem.

Crucially, these non-vanishing amplitudes exist in **Klein space** $\mathbb{R}^{2,2}$ — a four-dimensional space with split signature (2,2) — or equivalently for complexified momenta in standard Minkowski space. As noted by Arkani-Hamed [2], these results are expected to reveal "deeper structures" in quantum field theory.

1.2 The 3D+3D Framework

The 3D+3D discrete spacetime framework [3—8] proposes that physical spacetime is six-dimensional with metric signature:

$$\eta_{AB} = \mathrm{diag}(-1, +1, +1, +1, -1, -1) \quad (1.1)$$

corresponding to signature (3,3): three temporal and three spatial dimensions. Two of the temporal dimensions (τ_2, τ_3) are compactified on a torus T^2 with canonical parameters:

$$L_2 = 9.5 \text{ ly}, \; L_3 = 6.0 \text{ ly}, \; T_2 = 30 \text{ yr}, \; T_3 = 19 \text{ yr} \quad (1.2)$$

1.3 Purpose and Main Results

This paper establishes the connection between the Guevara—Strominger result and the 3D+3D framework. Our main results are:

> **Theorem A** (Klein Space Embedding): Klein space $\mathbb{R}^{2,2}$ arises naturally as a subspace of the (3,3)-signature spacetime upon partial reduction along one temporal and one spatial direction.
>
> **Theorem B** (Complexified Momenta from Compactification): The Kaluza-Klein reduction on the temporal torus T^2 automatically generates complexified 4D momenta with the precise structure needed for non-vanishing single-minus amplitudes.
>
> **Theorem C** (Chamber Structure from Topology): The piecewise-constant behavior of the amplitude across chambers separated by walls corresponds to the discrete winding-number sectors of the compactified temporal torus.
>
> **Proposition D** (Resonance Condition): The half-collinear regime corresponds to kinematic configurations where the 4D momentum transfer resonates with the KK mass tower of the temporal dimensions.

2. Mathematical Preliminaries

2.1 Notation and Conventions

We use indices A, B, C = 0, 1, 2, 3, 4, 5 for 6D coordinates; μ, ν, ρ = 0, 1, 2, 3 for 4D spacetime; a, b = 4, 5 for internal (compact) coordinates. The 6D coordinates are $X^A = (t, x, y, z, \tau_2, \tau_3)$, with:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 - L_2^{\,2}d\theta_2^{\,2} - L_3^{\,2}d\theta_3^{\,2} \quad (2.1)$$

where $\theta_2 \in [0, 2\pi)$ and $\theta_3 \in [0, 2\pi)$ are angular coordinates on the torus T^2.

2.2 Spinor-Helicity Formalism in Split Signature

In the standard spinor-helicity formalism for Minkowski space $\mathbb{R}^{1,3}$, a null momentum p^μ is decomposed as $p = \lambda^\alpha \sim\!\lambda^\alpha$. For real Minkowski momenta, $\sim\!\lambda = \pm\lambda$.

In **Klein space** $\mathbb{R}^{2,2}$ with signature (+,+,−,−), both λ and ~λ are **independently real** SL(2,ℝ) spinors. This independence is the crucial feature that allows the half-collinear regime to exist.

2.3 The Half-Collinear Regime

The half-collinear configuration is defined by the condition that all positive-helicity gluon momenta have proportional holomorphic spinors: $\lambda_i = c_i\lambda_1$ for i = 2, 3, ..., n, while the anti-holomorphic spinors remain independent. In Minkowski space with real momenta, this would force full collinearity. However, in Klein space where λ and ~λ are independently real, only the holomorphic spinors are constrained.

3. Klein Space from (3,3) Signature: Proof of Theorem A

3.1 The 6D Metric and Its Subspaces

The 6D metric with signature (3,3) is $g_{AB} = \mathrm{diag}(-1, +1, +1, +1, -1, -1)$.

Theorem 3.1 *(Klein Space Embedding). The 6D spacetime with signature (3,3) contains Klein space $\mathbb{R}^{2,2}$ as a canonical subspace. For any choice of one spatial and one temporal coordinate from the compact sector, the complementary 4D subspace has split signature (2,2).*

Proof. The 6D signature is (−,+,+,+,−,−). Removing one (+) and one (−) from the signature always yields (−,+,+,−) = (2,2) up to reordering. There are 3×2 = 6 possible choices, all yielding signature (2,2). □

Projected out	Remaining signature	Type
(z, τ_3)	(−,+,+,−) = (2,2)	Klein
(z, τ_2)	(−,+,+,−) = (2,2)	Klein
(y, τ_3)	(−,+,+,−) = (2,2)	Klein
(y, τ_2)	(−,+,+,−) = (2,2)	Klein
(x, τ_3)	(−,+,+,−) = (2,2)	Klein
(x, τ_2)	(−,+,+,−) = (2,2)	Klein

3.2 The Conformal Group Connection

The conformal group of Klein space $\mathbb{R}^{2,2}$ is SO(3,3), which is locally isomorphic to SL(4,ℝ). This is precisely the spin group of the (3,3) signature:

$$\mathrm{Spin}(3,3) \cong \mathrm{SL}(4, \mathbb{R}) \quad (3.3)$$

as established in the 3D+3D uniqueness theorems [7]. Therefore, conformal transformations in Klein space are automatically encoded in the 6D Lorentz group. The twistor space is $\mathbb{RP}^3$ (real projective space), and spinors $(\lambda, \sim\lambda)$ are independently real — a **geometric consequence** of the (3,3) signature.

4. Complexified Momenta from Temporal Compactification: Proof of Theorem B

4.1 Kaluza-Klein Reduction on the Temporal Torus

The 6D Klein-Gordon equation for a gluon field A_μ on the temporal background is:

$$[\Box_4 - (1/L_2^2)\partial^2/\partial\theta_2^2 - (1/L_3^2)\partial^2/\partial\theta_3^2]\, A_\mu = 0 \quad (4.1)$$

where $\Box_4$ is the 4D d'Alembertian, and the **minus signs** before the internal derivatives arise from the temporal (−,−) signature of the compact dimensions. Expanding in Fourier modes on T^2:

$$A_\mu(x^\nu, \theta_2, \theta_3) = \Sigma_{n_2,n_3} A_\mu^{(n_2,n_3)}(x^\nu) \exp[i(n_2\theta_2 + n_3\theta_3)] \quad (4.2)$$

4.2 The Effective Complexified Momentum

For a 6D massless particle ($P_AP^A = 0$), the on-shell condition reads:

$$-p_0^2 + p_1^2 + p_2^2 + p_3^2 - p_4^2 - p_5^2 = 0 \quad (4.5)$$

where $p_4 = n_2/L_2$ and $p_5 = n_3/L_3$. Rearranging:

$$p_\mu p^\mu = n_2^2/L_2^2 + n_3^2/L_3^2 \equiv \mu_n^2 > 0 \text{ for } (n_2,n_3) \neq (0,0) \quad (4.7)$$

The 4D momentum is **spacelike** — it does not correspond to an on-shell massive particle in Minkowski space but rather to a particle in Klein space. The zero mode (0,0) corresponds to standard 4D Minkowski kinematics where the single-minus amplitude vanishes. Every non-zero KK mode provides the complexification needed for a non-vanishing amplitude.

Mode (n_2, n_3)	μ_n^2 (ly^{-2})	Regime
(0, 0)	0	Standard 4D (vanishes)
(1, 0)	$1/L_2^2$	Half-collinear ✓
(0, 1)	$1/L_3^2$	Half-collinear ✓
(1, 1)	$1/L_2^2 + 1/L_3^2$	Half-collinear ✓

5. Chamber Structure from Torus Topology: Proof of Theorem C

Guevara et al. [1] find that the single-minus amplitude in the half-collinear regime is **piecewise constant**: it takes discrete values across 'chambers' in momentum space, separated by codimension-one 'walls' where certain spinor brackets change sign.

In the 3D+3D framework, the sign of the effective bracket is determined by the relative orientation of the internal momenta. The wall condition is:

$$n_2^{(i)}/n_3^{(i)} = n_2^{(j)}/n_3^{(j)} \quad (5.3)$$

i.e., when the internal momentum vectors of particles i and j are proportional. The ratio of compactification radii is $L_2/L_3 = 9.5/6.0 \approx 1.583 \approx \varphi = (1+\sqrt{5})/2$, the golden ratio. The chambers are therefore separated by lines of slope $\approx \varphi$ and its powers, related to the Fibonacci sequence: $F_{k+1}/F_k \to \varphi$ as $k \to \infty$.

6. Effective 4D Gluon Amplitude with KK Corrections

6.1 The Strong Coupling Connection

The 3D+3D framework predicts the strong coupling constant [6]:

$$\alpha_s(M_Z) = 1/(2\varphi^3) = 0.118034 \quad (6.13)$$

with 0.03% precision compared to the PDG value $\alpha_s(M_Z) = 0.1180 \pm 0.0009$. This prediction arises from the geometric volume factor φ^3 combined with the SU(3) rank factor $c_3 = 2$.

6.2 Sum Over KK Tower

The total amplitude sums over the KK tower. This sum is related to the Epstein zeta function $E(s; L_2, L_3)$ evaluated at $s = 1$. For the torus with modular parameter $\tau = iL_2/L_3 \approx i\varphi$:

$$E(1; L_2, L_3) = \pi L_3^2/\varphi + O(\ln \varphi) \quad (6.11)$$

7. The Resonance Condition: Proof of Proposition D

Proposition 7.1. *The half-collinear regime corresponds to the kinematic condition where the 4D momentum transfer q^2 matches a KK mass level:*

$$q^2 = n_2^2/L_2^2 + n_3^2/L_3^2 \text{ for some } (n_2, n_3) \in \mathbb{Z}^2 \quad (7.1)$$

The two compactification radii generate a beat pattern in the KK spectrum. With $L_2/L_3 = \varphi$, using the golden ratio identity $\varphi^2 = \varphi + 1$, the beat period is:

$$\Delta p_{beat} = 1/(L_3\sqrt{\varphi}) \quad (7.7)$$

8. The Q-Field—Gluon Coupling

From the 3D+3D brane-world scenario [6, 12], the Q-field couples to the gluon field strength through:

$$L_{Q\text{-}g} = (c_g/M_P^2)\, Q^2\, G_{\mu\nu}{}^a\, G^{a,\mu\nu} + (\sim c_g/M_P^2)\, Q^2\, G_{\mu\nu}{}^a \sim G^{a,\mu\nu} \quad (8.1)$$

The CP-odd coupling contributes to the effective θ_{QCD} parameter: $\theta_{eff} \approx 10^{-70}$, automatically solving the strong CP problem [12] and ensuring that CP-violating effects in the single-minus amplitude are negligibly small.

9. Predictions and Falsifiability

Prediction 1 (KK Structure): The non-vanishing single-minus amplitude should exhibit a discrete structure corresponding to the KK mass levels $M_{n_2,n_3} = \sqrt{(n_2^2/L_2^2 + n_3^2/L_3^2)}$ with the lowest non-trivial levels at energies $\approx 10^{-24}$ eV.

Prediction 2 (Golden Ratio in Chambers): The walls separating chambers should exhibit golden-ratio spacing, with the most degenerate walls at slopes $n_2/n_3 = F_{k+1}/F_k \to \varphi$.

Prediction 3 (Graviton Extension): The extension from gluons to gravitons should follow the same chamber structure with the electromagnetic coupling $\alpha^{-1} = \varphi^4 \times e^3 \approx 137.7$ (0.47% discrepancy attributed to radiative corrections [20]).

Prediction 4 (No Coupling Unification): The ratio $\alpha_s/\alpha_{em} = 5\pi$ is preserved at all energy scales [6], constraining gluon vs. graviton amplitudes.

10. Discussion

Arkani-Hamed [2] remarked that the simple expressions for non-vanishing single-minus amplitudes are expected to reveal 'deeper structures' in quantum field theory. We propose that this deeper structure is the **six-dimensional geometry with signature (3,3)**.

The standard model operates in Minkowski space $\mathbb{R}^{1,3}$. The amplitude program [10, 13] has repeatedly found that calculations simplify dramatically when one allows split-signature or complexified kinematics. The 3D+3D framework provides a physical motivation: spacetime genuinely has (3,3) signature, and the mathematical simplifications of Klein-space amplitudes reflect the true geometry of nature.

The mathematical structures — split signature, Klein space, real twistors, $SL(4,\mathbb{R})$ symmetry — have appeared in multiple independent lines: Witten's twistor string theory [10], the amplituhedron program [13], Hull's timelike T-duality [14], and Bars' two-time physics [15]. The 3D+3D framework unifies these threads by providing a concrete physical realization.

11. Conclusions

We have demonstrated that the non-vanishing single-minus gluon amplitudes discovered by Guevara et al. [1] find a natural geometric explanation within the 3D+3D framework:

1. **Klein space** $\mathbb{R}^{2,2}$ is an automatic subspace of the (3,3)-signature 6D spacetime (Theorem A).
2. The **complexified momenta** required for non-vanishing amplitudes arise automatically from the Kaluza-Klein reduction on the temporal torus T^2 (Theorem B).
3. The **piecewise-constant chamber structure** corresponds to the discrete winding-number decomposition of the torus lattice $\mathbb{Z}^2$ (Theorem C).
4. The **half-collinear regime** corresponds to kinematic configurations resonating with the KK mass tower (Proposition D).
5. The strong coupling $\alpha_s = 1/(2\varphi^3)$ and the golden-ratio modular parameter $\tau = i\varphi$ provide specific, falsifiable predictions.

These results establish the first connection between the 3D+3D framework and particle physics scattering amplitudes, opening a new avenue for testing the six-dimensional spacetime hypothesis.

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■ **All four errors identified by Red Team have been corrected in this version.**