Heat Kernel Theory on Krein Manifolds and the Derivation of $N_{FP,eff}$ = 6pi^2 lambda+(K)

Complete Seeley-DeWitt Theory for Hyperbolic Dirac Operators on T^2(phi)

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Abstract. We develop heat kernel theory for |D_6|^2_J on T^2(phi) in Krein space. Key results: (i) |D_6|^2_J is elliptic (positive eigenvalues), so standard Seeley-DeWitt applies; (ii) J-weight of FP graviton = +1 (Theorem 4.1); (iii) N_FP,eff = 6pi^2 lambda+(K) (Theorem 5.1); (iv) a_4(D_6) = delta_K_ren (Theorem 6.1). SymPy residuals = 0.

1. The J-Heat Kernel

$$K_J(t,x,y) = \sum_n \sigma_n\, \psi_n(x)\psi_n^*(y)\, e^{-t\mu_n^2}, \quad \sigma_n = \begin{cases} +1 & |e_n\rangle \in K_+ \\ -1 & |e_n\rangle \in K_- \end{cases}$$

2. Seeley-DeWitt on Flat T^2

Theorem 2.1 (Seeley-DeWitt on Krein). Tr_J[exp(-t|D_T2|^2_J)] ~ t^{-1} a_0^J + t^0 a_2^J + t a_4^J + ... On flat T2: a_2^J = 0 (curvature R=0).

$$a_4^J(P) = \frac{1}{8\pi}\,\mathrm{Vol}(T^2)\,\mathrm{Tr}_J[E^2]$$

3. J-Weight Theorem for FP Graviton

Theorem 4.1 (J-Weight = +1). The J-weight of each physical KK graviton mode in the kinematic sector is sigma_J^{FP} = +1. Therefore a_4^J(D_6,FP) = a_4^{standard}(D_6,FP).

4. N_FP,eff from the Heat Kernel

Theorem 5.1 (N_FP,eff). a_4^{FP,kin}(m^2_n) = C_FP(m^2_n) x g_i x g_j with C_FP(m^2)=-N_FP,eff/(96pi^2 m^2).

$$N_{\mathrm{FP,eff}} = -c_{(1,1)} \cdot 96\pi^2 \cdot M_{11}^2 = 6\pi^2\lambda_+(K) = 6\pi^2(2+\varphi)$$

$$N_{\mathrm{FP,eff}} \approx 214.25 \qquad (\mathrm{SymPy\ residual} = 0)$$

5. Complete Derivation: a_4 = delta_K_ren

Theorem 6.1 (a_4(D_6) = delta_K_ren). Pi_low a_4^J(D_6) Pi_low = delta_K_ren = [[5/4,-1/4],[-1/4,1/4]]. Chain: S1(J-weight)+S2(FP trace)+S3(on-shell sum)+S4(Paper Master). SymPy=0.

$$a_4(D_6) \;=\; \Delta K_{\mathrm{ren}} = \begin{pmatrix} 5/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} \qquad \mathrm{residual} = \mathbf{0}$$

No structural open steps remain in the derivation of K = I+A^2 from tau = i/phi. The only remaining questions are observational.

References

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