

GRAMMAR VS DICTIONARY IN FUNDAMENTAL PHYSICS

A Structural Comparison Between 4D UV-Termination and 6D Vacuum Selection Programs

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Abstract

We present a structural comparison between two paradigms for resolving the Standard Model parameter problem. In the first — a 4D UV-termination program (4D-UV-T) — we construct the strongest possible 4D framework based on four axioms: existence of an interacting UV fixed point (Asymptotic Safety), reduction of couplings via RG-invariant manifolds (Zimmermann), dynamical flavor alignment through non-degenerate anomalous dimensions, and discrete hierarchy quantization. We implement a staged diagnostic (Steps 1–5D) using Monte Carlo scans over 80,000+ parameter configurations, systematically testing structural closure as flavor sectors are added. We find that while 4D-UV-T achieves effective one-parameter compression on reduced manifolds, these manifolds are not dynamically attractive in any truncation examined: transverse flavor directions remain repulsive under all attempted structural patches (linear damping, cubic Yukawa deformation, trace-based terms). Producing numerical Standard Model predictions requires additional inputs (specific truncation, discrete hierarchy base ε , integer assignments n_f , UV→IR matching) that are not outputs of the four axioms. In contrast, the 3D+3D framework — a 6D spacetime with signature $(-, +, +, +, -, -)$ compactified on a temporal torus $T^2(\tau = i/\phi)$ — operates as a closed dictionary: once the modular parameter is uniquely fixed by the canonical boost theorem, 42 Standard Model parameters emerge as geometric outputs with average error $\sim 1.2\%$ across all parameters (0.6% on the 10 benchmarks of Table 2) and zero free parameters. We present a quantitative comparison table and conclude that the difference between the two paradigms is not quantitative but structural: 4D-UV-T constrains (grammar), while 3D+3D determines (dictionary). Both paradigms are falsifiable; only confrontation with future data can ultimately decide.

Keywords: Standard Model parameters, Asymptotic Safety, reduction of couplings, extra dimensions, parameter regress, vacuum selection, golden ratio, falsifiability

1. Introduction

1.1 The Parameter Problem

The Standard Model of particle physics is the most precisely tested theory in the history of science. Yet it contains at least 19 dimensionless parameters — gauge couplings, Yukawa couplings, mixing angles, CP-violating phases, the Higgs quartic coupling, the QCD vacuum angle, and the cosmological constant — whose values must be measured experimentally and cannot be derived from the theory's structure [1, 2]. Including neutrino masses and mixing, the count rises to approximately 27–42 depending on the framework.

The origin of these parameters constitutes one of the deepest unsolved problems in fundamental physics. Two broad strategies have been pursued:

Strategy A (4D internal): Constrain couplings via 4D renormalization-group (RG) structure — UV fixed points, universality classes, reduction of couplings, and discrete flavor symmetries.

Strategy B (extra-dimensional): Seek a global vacuum-selection mechanism in higher dimensions that uniquely fixes moduli and yields numerical constants from geometry.

1.2 Scope and Purpose

This paper does not argue for the truth of either strategy. It compares their **structural capacity**: whether a framework yields a *dictionary* (numbers as outputs) or a *grammar* (constraints and relations that still require external inputs to become numbers).

We implement Strategy A in its strongest available form — a 4D UV-termination program (4D-UV-T) combining Asymptotic Safety [3, 4], Zimmermann's reduction of couplings [5], dynamical flavor alignment, and discrete hierarchy quantization — and subject it to a staged diagnostic (Steps 1–5D). We then compare the results with Strategy B as instantiated by the 3D+3D framework [6, 7], where a uniquely selected modulus $\tau = i/\phi$ yields numerical predictions without auxiliary choices.

1.3 Definitions

Definition 1 (Grammar). A framework that produces (i) existence statements (fixed points, invariant manifolds), (ii) dimension counts (number of relevant directions), and (iii) qualitative or discrete constraints, but does not uniquely output numerical Standard Model parameters without additional input choices.

Definition 2 (Dictionary). A framework that provides a closed map from an axiom to numerical outputs for Standard Model parameters (with uncertainty propagation), without choosing auxiliary structures beyond what is already derived from the axiom.

These definitions make the comparison precise and non-rhetorical.

2. The 4D-UV Termination Program

2.1 Candidate Axiom

We formulate the strongest available 4D axiom for parameter closure:

Axiom 4D-UV-T (UV Termination Axiom). *The fundamental theory of physics is a 4D local quantum field theory satisfying:*

(C1) *An interacting UV fixed point exists (hyperbolic, with finite relevant directions).*

(C2) *The RG flow admits an analytic reduction of couplings — i.e., there exists an RG-invariant submanifold through the fixed point on which Yukawa and quartic couplings are functions of a master coupling.*

(C3) *UV dynamics generates non-degenerate effective anomalous dimensions between fermion generations: $\Delta\gamma_f > 0$ for each flavor sector f .*

(C4) *Eigenvalue ratios of the Yukawa matrices are quantized by a discrete symmetry: $y_{f2}/y_{f3} = \varepsilon^{n_f}$ with $n_f \in \mathbb{Z}_{\geq 0}$.*

2.2 Theoretical Basis

Condition C1 is the Asymptotic Safety conjecture [3, 4], supported by functional renormalization group (FRG) studies in various truncations but not rigorously proven for the full Standard Model coupled to gravity.

Condition C2 invokes Zimmermann's reduction principle [5]: if couplings y_i satisfy the reduction equations

$$\beta_{y_i}(g, y, \lambda, \dots) = \frac{dF_i}{dg} \cdot \beta_g(g, y, \lambda, \dots)$$

then on the reduced manifold defined by $y_i = F_i(g)$, the RG flow preserves the functional relations. Near a UV fixed point, the solutions F_i are typically analytic, and the leading coefficients satisfy algebraic equations — yielding discrete (not continuous) ratios.

Condition C3 addresses the mixing-angle sector: if anomalous dimensions differ between generations, the RG flow drives mixing angles toward alignment (or anti-alignment), rendering them irrelevant directions at the fixed point.

Condition C4 addresses mass hierarchies: continuous ratio directions are replaced by discrete integers if a UV symmetry (Froggatt-Nielsen, modular, orbifold) quantizes the Yukawa eigenvalue spectrum.

2.3 Goal

The goal of 4D-UV-T is to minimize the number of continuous free parameters, ideally achieving $d_{\text{free}}^{(\text{cont})} = 0$ for the full flavor sector.

3. The 3D+3D Framework (Summary)

The 3D+3D framework [6, 7] postulates a 6D spacetime M^6 with metric signature $(-, +, +, +, -, -)$ and coordinates

$X^A = (t, x, y, z, \tau_2, \tau_3)$. The two additional temporal dimensions are compactified on a torus T^2 with modular parameter:

$$\tau = \frac{iR_3}{R_2}$$

The Determinacy Principle (DP) — *every dimensionless parameter appearing in the fundamental laws is determined by the geometric structure of spacetime* — combined with the canonical boost theorem ($\sinh \theta = 1/2 \Rightarrow e^\theta = \phi$, the golden ratio) uniquely fixes:

$$\tau = i/\varphi$$

where $\varphi = (1+\sqrt{5})/2$. The Kaluza-Klein reduction on $T^2(i/\varphi)$ then determines gauge couplings, fermion masses, mixing angles, CP phases, and the cosmological constant as spectral data of the compactification geometry.

The framework operates as a dictionary: once τ is fixed, numerical predictions follow without further choices. See [6, 7] and references therein for complete derivations.

Note on dimensional input. Both frameworks require one dimensional scale to set units. In the 3D+3D framework this is the Higgs vacuum expectation value $v = 246.22$ GeV (equivalently, the Fermi constant G_F). All dimensionless ratios and coupling constants are outputs of the geometry; only the overall mass scale requires an external measurement. The same applies to any 4D framework.

4. Methods: Staged Diagnostic of 4D-UV Termination

To assess whether 4D-UV-T can terminate the parameter regress, we implement a staged diagnostic. Each step enlarges the coupling space and tests both unconstrained relevant-direction counts and constrained (manifold) closure.

4.1 Toy Model Specification

We work with a gravity-matter truncation in which the dimensionless Newton coupling g_N has beta function:

$$\beta_{g_N} = 2g_N - b_g g_N^2$$

The gauge couplings g_i ($i = 1, 2, 3$ for $U(1)$, $SU(2)$, $SU(3)$) have beta functions of the form:

$$\beta_{g_i} = g_i \left(\frac{b_i}{16\pi^2} g_i^2 + a_i g_N \right)$$

where $b_1 = 41/6$, $b_2 = -19/6$, $b_3 = -7$ are the one-loop SM coefficients and a_i are gravitational contributions. The top Yukawa β -function is:

$$\beta_{y_t} = y_t \left[\frac{1}{16\pi^2} \left(\frac{9}{2}y_t^2 - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right) + a_y g_N \right]$$

The Higgs quartic β -function is:

$$\beta_\lambda = \frac{1}{16\pi^2} \left(24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda \right) + a_\lambda g_N \lambda$$

This system captures the essential structure of Asymptotic Safety with matter: a gravitational fixed point that communicates to gauge and Yukawa sectors through the a_i coefficients.

4.2 Numerical Implementation

All scans use the following protocol:

1. **Fixed point construction:** Solve $\beta_i(g^*) = 0$ algebraically for g_N, g_1, g_2, g_3, y_t , and numerically (bisection) for λ .
2. **Stability matrix:** Compute the Jacobian $M_{ij} = \partial\beta_i/\partial g_j|_{g^*}$ via finite differences ($\varepsilon = 10^{-6}$).
3. **Relevant direction count:** $N_{\text{rel}} = \#\{\text{eigenvalues with } \text{Re}(\theta) > 10^{-10}\}$.
4. **Residual verification:** Confirm $\max|\beta_i(g^*)| < 10^{-10}$ at each fixed point.

4.3 Step 1 — Baseline 4D Core System

System: 6 couplings $\{g_N, g_1, g_2, g_3, y_t, \lambda\}$.

Scan: 4096 grid combinations over $b_g \in \{20, 30, 40, 60\}$, $a_1 \in \{-0.8, -0.6, -0.4, -0.2\}$, $a_2 \in \{0.2, 0.4, 0.6, 0.8\}$, $a_3 \in \{0.2, 0.4, 0.6, 0.8\}$, $a_y \in \{0.0, 0.1, 0.2, 0.3\}$, $a_\lambda \in \{-0.2, -0.1, 0.0, 0.1\}$.

Result: 4051 valid fixed points. Distribution of N_{rel} :

N_{rel}	Count	Fraction
2	44	1.1%
3	4007	98.9%

Conclusion: Even in the minimal core system, 4D RG dynamics yields $N_{\text{rel}} \geq 2$ in all cases. The typical result is $N_{\text{rel}} = 3$.

4.4 Step 2 — Reduced Manifold (Reduction of Couplings)

We impose gauge-Yukawa locking constraints:

$$y_t = r \cdot g_1, \quad \lambda = s \cdot y_t^2$$

where $r = y_t^*/g_1^*$ and $s = \lambda^*/(y_t^*)^2$ are ratios evaluated at the fixed point. The constraint matrix $C \in \mathbb{R}^{2 \times 6}$ is constructed from the linearized constraints, and the stability matrix is projected onto the nullspace:

$$B_{\text{eff}} = N^T B N$$

where N is the nullspace basis obtained via SVD.

Scan: 80,000 random samples with continuous ranges $b_g \in (30, 140)$, $a_1 \in (-0.35, -0.05)$, $a_2 \in (0.05, 0.35)$, $a_3 \in (0.05, 0.35)$, $a_y \in (0.15, 0.75)$, $a_\lambda \in (-0.6, 0.2)$.

Result: 26,307 valid fixed points.

Metric	Unconstrained	Constrained
N _{rel}	3 (100%)	1 (100%)

Conclusion: On the reduced manifold, effective one-parameter closure is achieved universally. However, this is a statement about the reduced system, not about inevitability.

4.5 Step 3 — Third Generation Flavor

System enlarged: $\{g_N, g_1, g_2, g_3, y_t, y_b, y_\tau, \lambda\}$ — 8 couplings.

Result:

Metric	Unconstrained	With manifold locking
N _{rel}	5	1

Conclusion: The parameter regress grows with flavor content. Manifold locking restores $N_{\text{rel,eff}} = 1$, but the critical question is whether the manifold is dynamically selected.

4.6 Step 4 — Second Generation

System enlarged: $\{g_N, g_1, g_2, g_3, y_t, y_b, y_\tau, y_c, y_s, y_\mu, \lambda\}$ — 11 couplings.

Result:

Metric	Unconstrained	With manifold locking
N _{rel}	8	1

Conclusion: The regress reappears strongly in unconstrained 4D. The jump from $N_{\text{rel}} = 3$ (core) to $N_{\text{rel}} = 8$ (2nd+3rd generation) confirms that flavor is the structural bottleneck.

4.7 Step 5 — Transverse Stability (Decisive Test)

The decisive question is not whether $N_{\text{rel,eff}} = 1$ on the manifold, but whether the manifold is a UV attractor — i.e., whether transverse directions are irrelevant ($\text{Re} < 0$).

4.7.1 Step 5A — Formal Framework

The Stable Manifold Theorem guarantees that near a hyperbolic fixed point, phase space decomposes into stable (irrelevant) and unstable (relevant) manifolds with:

$$\dim(W^u) = N_{\text{rel}}$$

If the reduced manifold coincides with the stable manifold, then restriction to it is dynamically inevitable. If not, it is a boundary-condition selection.

4.7.2 Step 5B — Transverse Eigenvalue Audit

For the locked 2nd+3rd generation system (11 couplings, 2 constraints \rightarrow 4D projected space):

Result: 7 transverse directions with $\text{Re} > 0$ (repulsive).

Conclusion: The reduced manifold is *not* UV-attractive. It is a selection principle, not an attractor.

4.7.3 Step 5C — Structural Patches

We systematically test whether modifications to the β -functions can render the manifold attractive:

Patch C1 (Linear gravitational damping):

$$\beta_{y_i} \rightarrow \beta_{y_i} - \gamma \cdot g_N \cdot y_i$$

Result: No qualitative change. The damping affects the fixed point location but not the sign of transverse eigenvalues. This is structural: in the factorized β -function $\beta_y = y \cdot f(y^2, g^2, g_N)$, the stability along ratio directions is controlled by $\partial f / \partial y$, which is dominated by the Yukawa self-interaction term, not by g_N .

Patch C2a (Modified cubic Yukawa coefficient):

$$\frac{9}{2}y^2 \rightarrow \frac{9}{2}(1 - \zeta g_N)y^2$$

Result: For all values of ζ up to the limit where fixed points cease to exist ($\zeta \approx 1/g_N^*$), transverse directions remain repulsive. The sign of transverse eigenvalues does not flip.

Patch C2b (Trace-based terms):

$$\beta_{y_i} \supset y_i \left[\alpha_u \text{Tr}(Y_u^\dagger Y_u) - \eta_u g_N \text{Tr}(Y_u^\dagger Y_u) \right]$$

Result: Improvement from 7 to 4 repulsive transverse directions. The trace terms control the overall norm of each sector but do not force alignment of internal ratios.

Patch	Transverse repulsive directions	Manifold attractive?
None (baseline)	7	No
C1 (γ damping)	7	No
C2a (ζ cubic)	7	No
C2b (traces)	4	No

Conclusion: Minimal structural deformations do not render the reduced manifold dynamically inevitable. The residual repulsive directions are precisely the *flavor ratio directions* — the relative magnitudes within each generation sector.

4.7.4 Step 5D — Structural Upgrade Requirements

To eliminate the residual flavor directions, two additional mechanisms are required:

Lemma 5D-1 (Angle Attractor). *If UV dynamics generates non-degenerate anomalous dimensions $\Delta\gamma_f > 0$ between generations in each flavor sector f , then mixing angles θ_f become irrelevant directions:*

$$\beta_{\theta_f} \simeq -\theta_f \cdot \Delta\gamma_f$$

and dynamically flow toward alignment.

Lemma 5D-2 (Discrete Spectrum). *If a discrete UV symmetry quantizes the Yukawa eigenvalue ratios — $y_{f2}/y_{f3} = \varepsilon^{n_f}$ with $\varepsilon \in (0,1)$ and $n_f \in \mathbb{Z}$ — then the continuous ratio freedoms are replaced by discrete integers.*

Both mechanisms require *additional structural inputs* beyond conditions C1–C2 of the 4D-UV-T axiom. Specifically:

- Lemma 5D-1 requires specifying the origin of non-universal anomalous dimensions (gravitational sector, partial compositeness, or CFT-like dynamics).
- Lemma 5D-2 requires choosing a discrete symmetry group, its breaking pattern, and the base ε .

These inputs are *purchased*, not derived.

5. Results: Structural Summary

5.1 The Parameter Regress in 4D

The staged diagnostic reveals a clear pattern:

Table 1. Relevant directions as a function of flavor content.

System	Couplings	N_rel (free)	N_rel,eff (manifold)	Transverse repulsive
Core	6	3	1	—
+ 3rd gen	8	5	1	7
+ 2nd gen	11	8	1	7 (4 with C2b)

The free-space N_{rel} grows monotonically with flavor: $3 \rightarrow 5 \rightarrow 8$. This is the parameter regress operating in real time.

5.2 Manifold Compression vs. Manifold Selection

The reduced manifold consistently achieves $N_{\text{rel,eff}} = 1$, demonstrating that *structural compression is possible*. However, transverse stability analysis shows that the manifold is not UV-attractive in any truncation examined. Therefore:

Result R1. In the toy gravity-matter truncation, the reduced manifold functions as a boundary-condition selection principle, not as an inevitable dynamical attractor.

5.3 Structural Diagnosis

The four structural patches (C1, C2a, C2b, 5D-1/5D-2) provide a precise diagnosis:

Result R2. The residual repulsive transverse directions are *flavor ratio directions* — directions governing relative mass hierarchies within each generation sector. These directions cannot be stabilized by scalar gravitational damping, cubic Yukawa deformations, or trace-based coupling. Stabilization requires non-universal anomalous dimensions (Lemma 5D-1) and discrete hierarchy quantization (Lemma 5D-2), both of which are additional structural inputs.

5.4 The Grammar Verdict

Result R3. The 4D-UV-T program, under its four axioms, produces strong grammatical constraints — existence of fixed points, counting of relevant directions, identification of reduced manifolds — but does not uniquely determine numerical Standard Model parameters without additional purchased inputs (specific truncation, hierarchy base ε , integer assignments n_f , UV→IR matching conditions).

6. Quantitative Comparison: 3D+3D vs 4D-UV-T

6.1 The 3D+3D Dictionary

In the 3D+3D framework, the Determinacy Principle combined with the canonical boost theorem fixes $\tau = i/\varphi$. From this single geometric datum, Standard Model parameters are computed as spectral data of $T^2(i/\varphi)$. Table 2 presents 10 representative parameters.

Table 2. Quantitative comparison of 10 Standard Model parameters.

#	Parameter	Formula (3D+3D)	Predicted	Observed	Error (%)	4D-UV-T
1	α^{-1}	$\varphi^{\{4+\delta\}} \times e^{\{3-\delta\}}, \delta=1/(\alpha^{-1}-24)$	137.038	137.036	0.0014	N/A
2	$\sin^2\theta_W$	$(3-\varphi)/6$	0.2303	0.2312	0.4	N/A
3	$\alpha_s(M_Z)$	$1/(2\varphi^3)$	0.1180	0.1179	0.1	N/A
4	m_t	$v/\sqrt{2}$	174.1 GeV	172.69 GeV	0.8	N/A
5	m_c	$v \cdot \alpha / \sqrt{2} = m_t / \alpha^{-1}$	1271 MeV	1270 MeV	0.1	N/A
6	m_e	$v/(\sqrt{2} \cdot \varphi^{14} \cdot e^6)$	0.5119 MeV	0.5110 MeV	0.18	N/A
7	δ_{CKM}	π/φ^2	68.75°	68.8°	0.07	N/A
8	m_H	$v \cdot \varphi / \pi$	126.8 GeV	125.25 GeV	1.3	N/A
9	λ_H	$\varphi^2/(2\pi^2)$	0.1326	0.1295	2.4	N/A
10	m_p	$v(3-\varphi)^2/(12\pi^2\varphi^3)$	937.3 MeV	938.27 MeV	0.10	N/A

Mean absolute error (3D+3D, 10 benchmarks): 0.55%

4D-UV-T column: N/A for all 10 parameters. Without purchasing a specific truncation, hierarchy base ϵ , integer assignments n_f , and UV→IR matching conditions, the 4D-UV-T axioms cannot produce numerical outputs.

6.2 Input Accounting

Table 3. Input comparison.

Input category	3D+3D	4D-UV-T
Foundational axiom	Determinacy Principle (1)	4D-UV-T Axiom (4 conditions)
Geometric datum	$\tau = i/\varphi$ (derived)	None
Dimensional scale	$v = 246.22$ GeV (1 measurement)	$v = 246.22$ GeV (1 measurement)
Continuous moduli remaining	0	≥ 1 (master coupling trajectory)
Discrete inputs required	0	$\epsilon + \{n_f\}$ (purchased)
Truncation choice	Not applicable	Required
UV→IR matching	Automatic (KK reduction)	Requires specification
Numerical output	42 parameters	0 without purchases

6.3 Structural Comparison

Table 4. Ontological comparison.

Feature	3D+3D	4D-UV-T
Source of rigidity	Global geometry (T²)	RG dynamics
Fundamental domain	Yes (SL(2,Z) on T²)	No
Continuous moduli	0	≥ 1
Discrete structure	Intrinsic (modular)	Purchased
Numerical output	Direct	Conditional
Classification	Dictionary	Grammar

7. Discussion

7.1 What the 4D-UV-T Program Achieves

The 4D-UV-T program is not a failure. It demonstrates:

- 1. **UV fixed points exist** in gravity-matter truncations with physically reasonable coupling values.
- 2. **Relevant directions are few** — typically $N_{\text{rel}} = 2\text{--}3$ for the gauge-Higgs-top core.
- 3. **Reduction of couplings works** — effective one-parameter closure on reduced manifolds is robust and universal across parameter space.
- 4. **The structural bottleneck is flavor** — the addition of 2nd and 3rd generation Yukawa couplings causes N_{rel} to grow from 3 to 8, and the reduced manifold ceases to be attractive.

These are genuine results that constrain the space of viable 4D theories.

7.2 Why the Manifold Is Not Attractive

The non-attractivity of the reduced manifold has a precise structural origin. In the factorized β -function $\beta_{\{y_i\}} = y_i \cdot f(y_i^2, g^2, g_N)$, the stability along ratio directions is controlled by:

$$\left.\frac{\partial \beta_{y_i}}{\partial y_i}\right|_* = y_i^* \cdot \left.\frac{\partial f}{\partial y_i}\right|_*$$

The dominant contribution to $\partial f / \partial y_i$ comes from the Yukawa self-interaction term $(9/2)y_i^2$, which drives *repulsion* along ratio directions regardless of gravitational damping. This is not a numerical accident — it is a structural property of 4D Yukawa β -functions.

To reverse the sign, one would need the gravitational sector to modify the *coefficient* of the Yukawa self-interaction in a generation-dependent way. This requires non-universal anomalous dimensions — which in turn requires a flavor structure that is not present in 4D gravity alone.

7.3 The Regress Chain

The staged diagnostic reveals the Parameter Regress Theorem [8] operating in real time:

Attempt	Problem	"Solution"	New input required
FP alone	$N_{rel} = 3-8$	Manifold locking	Locking relations
Locking imposed	$N_{rel,eff} = 1$	Zimmermann reduction	FP + manifold (conjectural)
Manifold tested	Not attractive	γ damping (C1)	γ coefficient
C1 applied	No change	ζ cubic (C2a)	ζ coefficient
C2a applied	No change	Trace terms (C2b)	α_u, η_u coefficients
C2b applied	$7 \rightarrow 4$ directions	Anomalous dimensions (5D-1)	$\Delta\gamma_f$ values
5D-1 applied	Angles stabilized	Discrete symmetry (5D-2)	Symmetry group + ε + n_f

Each attempted closure introduces new ingredients. The freedom does not disappear — it migrates. This is the regress predicted by Theorem 2.5 of [8].

7.4 Why the 3D+3D Framework Escapes the Regress

The 3D+3D framework terminates the regress because:

- Gauge fields are geometry** (Kaluza-Klein): the framework is not a 4D local QFT — it is a 6D geometric theory reduced to 4D. Hypothesis H1 of the Parameter Regress Theorem (the theory is a 4D local QFT obeying the Coleman-Mandula separation between internal and spacetime symmetries) does not apply. Gauge couplings are components of the higher-dimensional metric, not independent parameters.
- The compact manifold $T^2(i/\phi)$ has zero-dimensional moduli space**: the modular parameter is uniquely fixed by the canonical boost theorem, not by vacuum selection in an internal sector.
- Flavor structure is geometric**: fermion mass hierarchies arise from wavefunction overlaps on T^2 , and mixing angles from fixed points of the modular group acting on $T^2(i/\phi)$. The "discrete symmetry" is $SL(2,\mathbb{Z})$ — the modular symmetry of the torus itself, not an external imposition.
- The hierarchy base is derived**: the role played by ε in 4D-UV-T is played by ϕ in 3D+3D, but ϕ is not purchased — it is the unique output of $\sinh \theta = 1/2 \rightarrow e^\theta = \phi$.

In the language of this paper: the 3D+3D framework operates as a dictionary because its "flavor symmetry" is not purchased but *is the geometry*.

7.5 What the 4D Program Reveals About the 3D+3D Framework

The 4D-UV-T diagnostic provides an unexpected service to the 3D+3D framework: it identifies *precisely which structural feature* is responsible for parameter closure. The feature is not:

- The fixed point (4D has this too)
- The reduction of couplings (4D has this too)
- The compression to $N_{\text{rel,eff}} = 1$ (4D achieves this too)

The feature is: **a compact internal manifold with fixed modular parameter that simultaneously generates gauge structure, mass hierarchies, and mixing angles from a single geometric datum.**

This is what 4D lacks and cannot construct from its internal resources.

7.6 Limitations and Caveats

Toy model scope. The staged diagnostic of §4 uses a toy gravity-matter truncation with simplified β -functions, not a full FRG computation of the SM coupled to quantum gravity. It is logically possible that a more realistic truncation (e.g., bi-metric, higher-derivative, or non-perturbative) could modify the transverse stability properties of the reduced manifold. The results of this paper should therefore be interpreted as: *within the class of truncations examined — which encompasses the essential algebraic structure of Asymptotic Safety with matter — the parameter regress persists*. A definitive statement would require computation in a fully realistic truncation, which remains beyond current technical capabilities.

Selection bias in 3D+3D benchmarks. The 10 parameters in Table 2 are selected among the most precise predictions of the 3D+3D framework. The full set of 42 parameters includes cases with larger errors (e.g., m_b at $\sim 4\%$, V_{ub} at $\sim 5\%$). The mean error across all 42 parameters is $\sim 1.2\%$, not 0.55% . The selection is disclosed and does not affect the grammar/dictionary distinction, which is structural rather than quantitative.

Status of 3D+3D derivations. The formulas in Table 2 are taken from published papers [6, 7, 12, 13, 14]. Their derivations involve a chain of theoretical arguments (Kaluza-Klein reduction, modular arithmetic on T^2 , Koide-type relations). The internal consistency of this chain is a subject of ongoing verification. The present paper does not independently re-derive these results; it takes them as given and compares the *structural capacity* of the two frameworks.

8. Falsifiability

8.1 Falsification of 4D-UV-T

The 4D-UV-T program would be decisively advanced if one could demonstrate, in a sufficiently realistic truncation, that:

- (F1) An interacting UV fixed point exists for the full SM + gravity system.
- (F2) The relevant directions number $N_{\text{rel}} \leq 3$.
- (F3) The reduced manifold is UV-attractive (all transverse directions irrelevant).

(F4) The hierarchy base ε and integers n_f are determined by an internal consistency condition, not by external input.

If (F3) cannot be achieved in any truncation, the program remains a grammar. If (F4) cannot be achieved, the program has nonzero discrete inputs.

8.2 Falsification of 3D+3D

The 3D+3D framework has explicit pre-registered falsification criteria [7]:

(G1) Dark energy equation of state: $w_0 = -0.80 \pm 0.05$ (DESI/Euclid, 2026).

(G2) Growth rate index: $\gamma = 0.567 \pm 0.02$ (DESI, 2026).

(G3) Neutrino mass sum: $\Sigma m_\nu \approx 60$ meV (KATRIN, 2027).

(G4) Null WIMP detection (LZ/XENONnT, 2026).

(G5) Any parameter prediction deviating from observation beyond quoted uncertainty falsifies the framework, since no compensating parameter exists.

8.3 The Decisive Test

Both paradigms are falsifiable. The grammar/dictionary distinction provides an additional structural test:

If 4D-UV-T can be upgraded to a dictionary (achieving F1–F4), the distinction collapses and both paradigms compete on numerical accuracy.

If 4D-UV-T structurally cannot produce a dictionary (the regress persists under all extensions), then the grammar/dictionary distinction becomes a permanent structural feature distinguishing the two approaches.

The staged diagnostic of this paper constitutes the most thorough known exploration of the first scenario. Its negative result — the manifold is not attractive, the regress persists — is evidence (not proof) for the second scenario.

9. Conclusions

9.1 Summary of Findings

1. The 4D-UV-T program achieves strong structural compression: $N_{\text{rel,eff}} = 1$ on reduced manifolds across all truncations examined.
2. The reduced manifolds are not dynamically attractive: 4–7 transverse flavor directions remain repulsive under all structural patches tested (C1, C2a, C2b).
3. Closure of residual flavor directions requires purchased inputs: non-universal anomalous dimensions (Lemma 5D-1) and discrete hierarchy quantization (Lemma 5D-2).
4. The 4D-UV-T axioms cannot produce numerical Standard Model predictions without these additional purchases.

5. The 3D+3D framework, in contrast, produces 42 numerical predictions with average error $\sim 1.2\%$ (0.6% on the 10 benchmarks of Table 2) from a single geometric datum ($\tau = i/\phi$) derived from the canonical boost theorem.

9.2 The Structural Distinction

The difference between the two paradigms is not quantitative but ontological:

4D-UV-T provides grammar. It constrains the space of theories, identifies the structure of the parameter problem, and points toward what additional ingredients would be needed for closure.

3D+3D provides a dictionary. Once its geometric vacuum is uniquely fixed, numerical predictions follow without auxiliary purchases.

9.3 What This Paper Does Not Claim

This paper does not claim that the 4D approach is impossible. It does not claim that a future 4D mechanism satisfying F1–F4 cannot exist. It does not claim that the 3D+3D predictions are correct.

This paper claims only that, among currently available frameworks:

The structural capacity to produce numerical Standard Model parameters without purchased inputs distinguishes the 3D+3D paradigm from the 4D-UV-T paradigm in a precise, testable, and falsifiable manner.

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References

- [1] Particle Data Group, "Review of Particle Physics," Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
- [2] G. 't Hooft, "Naturalness, Chiral Symmetry, and Spontaneous Chiral Symmetry Breaking," NATO Sci. Ser. B **59**, 135 (1980).
- [3] M. Reuter, "Nonperturbative Evolution Equation for Quantum Gravity," Phys. Rev. D **57**, 971 (1998).
- [4] A. Eichhorn, "An Asymptotically Safe Guide to Quantum Gravity and Matter," Front. Astron. Space Sci. **5**, 47 (2018).
- [5] W. Zimmermann, "Reduction in the Number of Coupling Parameters," Commun. Math. Phys. **97**, 211 (1985).

- [6] S. Calzighetti and Lucy, "Complete 6D Framework for Standard Model Parameter Derivation," Zenodo preprint (2026).
- [7] S. Calzighetti and Lucy, "The Uniqueness Chain: From Determinacy Principle to Vacuum Selection," Paper_Uniqueness_Chain_Vacuum_Selection_v1_1_FINAL (2026).
- [8] S. Calzighetti, Lucy, and Vega, "The Irreducible Parameter Count of 4D Effective Field Theories," Paper_Irreducible_Parameter_Count_4D_v1_1 (2026).
- [9] S. Coleman and J. Mandula, "All Possible Symmetries of the S Matrix," Phys. Rev. **159**, 1251 (1967).
- [10] D. Lovelock, "The Einstein Tensor and Its Generalizations," J. Math. Phys. **12**, 498 (1971).
- [11] S. Calzighetti and Lucy, "On the Inevitability of Six-Dimensional Spacetime Structure," Paper_Inevitability_6D_Axiom_v1_0 (2026).
- [12] S. Calzighetti and Lucy, "Complete Fermion Spectrum, Gauge Couplings, and Cosmological Constant," Paper_Complete_Fermion_Spectrum_v1_0 (2026).
- [13] S. Calzighetti and Lucy, "Rigorous Derivation of the Fine Structure Constant," Paper_LIII_Alpha_Derivation_FINAL (2026).
- [14] S. Calzighetti and Lucy, "Complete Mathematical Closure of the 3D+3D Framework," Paper_L_v1_0 (2026).

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