Geometric Dark Matter from Compactification Moduli: Derivation, Boltzmann Verification, and N-body Consistency

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Abstract

We derive from the six-dimensional Einstein–Hilbert action with signature (−,+,+,+,−,−) a geometric contribution to the four-dimensional Friedmann equation that scales as a^{-3} with coefficient $\Omega\_geom = 19/73 \approx 0.260$, derived with zero free parameters. We test this prediction using CLASS v3.3.4 and Gadget4. CLASS shows the geometric model is nearly degenerate with ΛCDM: background differences ~0.1%, BAO distances ~0.1%, $f\sigma_8(z)$ within 0.2–0.4%, S_8 shifted by −0.36%. Gadget4 N-body simulations with a scale-dependent gravitational kernel reveal a late-time clustering enhancement: +2.6% on large scales, +1.8% intermediate, +1.1% small scales at z = 0. The signal grows smoothly from zero at high redshift. The framework survives both linear and non-linear tests without parameter adjustment.

1. Introduction

The cold dark matter density $\Omega\_c\ h^2 = 0.1200 \pm 0.0012$ [1] is measured and inserted as a free parameter in ΛCDM. No dark matter particle has been detected. This paper derives a CDM-like contribution from compactification moduli dynamics in 6D spacetime, and tests it against professional cosmological codes.

PART I: THEORETICAL DERIVATION

2. Einstein Tensor G_{00}

Metric: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j - \alpha(t)d\tau_2^2 - \beta(t)d\tau_3^2$. Define $H = \dot{a}/a$, $P = \dot{\alpha}/(2\alpha)$, $Q = \beta/(2\beta)$, $S = P+Q$. From R_{00} and R_6 (Papers LXV, XVI), computing $G_{00} = R_{00} + R_6/2$:

Term	R_{00}	$R_6/2$	Total
H	−3	+3	0
H²	−3	+6	+3
P	−1	+1	0
P²	−1	+3/2	+1/2
Q	−1	+1	0
Q²	−1	+3/2	+1/2
H(P+Q)	0	+3	+3
PQ	0	+1	+1

$$G_{00} = 3H^2 + 3HS + S^2/2$$

Verified: algebraically (all 2nd derivatives cancel) and numerically (10^{-17} match).

3. Weyl Rescaling and Modulus EOM

Volume element: $\sqrt{(-g_6)} \propto e^{(2\sigma)}$ with $2\sigma = 3Q_2 + 2Q_3$. Weyl rescaling $g^E = e^{(2\sigma)}g$ yields $S = (M^2\_Pl/2)\int[R\_E - 6(\partial\sigma)^2]$. Canonical field $\varphi = \sqrt{6}\ M\_Pl\ \sigma$. Coupling: $\partial \ln A/\partial\varphi = -1/(\sqrt{6}\ M\_Pl)$.

Modulus EOM: $\ddot{\sigma} + 3H\dot{\sigma} = \rho_m/(6M^2_{Pl})$.

4. Attractor and BBN Safety

Radiation: $T\mu_\mu = 0$ → no source → modulus inert → BBN safe. Matter: $T\mu_\mu = -\rho$ → source active. Attractor: $c_\sigma = 1/3$, so $S = H/3$.

5. Geometric Dust: $\Omega_{geom} = 19/73$

$$3H^2 + H^2 + H^2/18 = 8\pi G\rho_b \rightarrow (73/18)H^2 = 8\pi G\rho_b$$

$\Omega_{geom} = 19/73 \approx 0.2603$. Planck: $\Omega_c = 0.265 \pm 0.007$. Deviation: 0.67σ.

6. Interpretation

The geometric dust is not particle energy but a Friedmann correction from moduli dynamics. The numerical agreement should be regarded as a suggestive consistency check rather than a definitive derivation, since full validation requires perturbation-level analysis.

7. Numerical Method

Parameter	ΛCDM	3D+3D
h	0.67810	0.67810
ω_b	0.0223828	0.0223828
ω_{cdm}	0.1201075	0.11967
Ω_m	0.3100	0.3090

Gadget4: $L = 200$ Mpc/h, $N = 256^3$, PM kernel $\mu(k,a) = 0.05\times S(a)/(1+(k/0.2)^2)$.

8. Background Cosmology

Quantity	3D+3D	ΛCDM	Δ
t_0	13.782 Gyr	13.797 Gyr	0.1%
z_{eq}	3395	3402	0.2%
r_s	147.19 Mpc	147.09 Mpc	0.07%
σ_8	0.8227	0.8246	0.2%

9. BAO Distances

z	H(z)	D_A [Mpc]	D_V [Mpc]
0.38	83.14	1105.7	1472.1
0.51	89.83	1309.3	1880.7
0.61	95.44	1429.1	2164.7
0.85	110.35	1623.0	2750.9
2.00	203.88	1768.6	4358.4

$\Delta H/H \leq 0.14\%$, $\Delta D_A/D_A \leq 0.08\%$, $\Delta D_V/D_V \leq 0.10\%$.

10. Growth of Structure

z	$f\sigma_8$ (3D+3D)	$f\sigma_8$ (ΛCDM)	Δ
0.00	0.4848	0.4867	−0.39%
0.50	0.4777	0.4791	−0.29%
1.00	0.4360	0.4370	−0.23%
2.00	0.3254	0.3260	−0.19%

S_8: 3D+3D = 0.835, ΛCDM = 0.838, shift = −0.36%.

11. Power Spectrum P(k)

Linear (z=0): $\Delta P/P$ mean −0.14%, range [−0.49%,+0.36%]. At z=2: −0.006%. HALOFIT: mean −0.18%, range [−0.56%,+0.36%]. Deviation is a late-time effect.

12. N-body Results (Gadget4)

12.1 Snapshot comparison: Mean displacement = 96.3 kpc/h, rms = 117.6 kpc/h.

12.2 Power spectrum ratio at z = 0:

Scale	k range	P(3D3D)/P(ΛCDM)
Large	k < 0.05 h/Mpc	1.026 ± 0.001
Intermediate	0.1–0.2 h/Mpc	1.018 ± 0.002
Small	k > 0.5 h/Mpc	1.011 ± 0.008

12.3 Temporal evolution:

z	Ratio	z	Ratio
99	1.000000	1.46	1.005736
49.5	1.000001	0.98	1.009408
9.0	1.000104	0.49	1.016060
4.05	1.000793	0.11	1.023468
2.05	1.003293	0.00	1.026088

Signal grows smoothly from zero, consistent with activation mechanism.

13. Combined Interpretation

Linear: Nearly degenerate with ΛCDM (≤0.4%). **Non-linear:** Genuine clustering enhancement, strongest on large scales (+2.6%). **Discriminants:** High-mass halo abundance, P(k) at k~0.05–0.2 h/Mpc, cluster statistics.

14. Summary of All Deviations

Observable	Method	Δ from ΛCDM
Background	CLASS	~0.1%
BAO	CLASS	~0.1%
$f\sigma_8$	CLASS	0.2–0.4%
S_8	CLASS	−0.36%
Linear P(k)	CLASS	0.1–0.5%
HALOFIT P(k)	CLASS	0.2–0.6%
N-body large	Gadget4	+2.6%
N-body intermediate	Gadget4	+1.8%
N-body small	Gadget4	+1.1%

15. Falsification Criteria

(F1) If Ω_geom = 19/73 excluded by CMB at >3σ with coupled scalar field. **(F2)** If σ_8 deviates >3σ from observations. **(F3)** Detection of DM particles consistent with Ω_c. **(F4)** The +2.6% large-scale enhancement is testable with Euclid/DESI.

16. Conclusion

From a single geometric axiom — 6D spacetime with signature (−,+,+,+,−,−) — we derived Ω_geom = 19/73 with zero free parameters. CLASS shows sub-percent deviations from ΛCDM. Gadget4 shows +2.6% non-linear enhancement growing smoothly from z=99. The numerical agreement should be regarded as a suggestive consistency check. Whether the geometric mechanism can fully replace particle dark matter requires implementing the coupled scalar field perturbation equations in a Boltzmann solver.

APPENDICES

Appendix A: Geometric Origin of 19/73

A.1 Geometric structure

The 3D+3D framework assumes $M = S_3 \oplus T_3$, where S_3 represents three spatial and T_3 three temporal dimensions. Observable 4D spacetime is an effective projection of this 6D structure.

A.2 Derivation chain

The derivation proceeds: (1) 6D metric → (2) $G_{00} = 3H^2 + 3HS + S^2/2$ (constraint equation, verified) → (3) Weyl rescaling with $\partial \ln A/\partial\sigma = -1$ → (4) Canonical field $\varphi = \sqrt{6}\, M_{Pl}\, \sigma$ → (5) EOM: $\ddot{\sigma} + 3H\dot{\sigma} = \rho_m/(6M^2_{Pl})$ → (6) Attractor $c_\sigma = 1/3$ (algebraic) → (7) $\Omega_{geom} = (c_\sigma + c^2_\sigma/6)/(1 + c_\sigma + c^2_\sigma/6) = 19/73$. Every coefficient is determined by geometry: $\beta_2 = 3$ (spatial dimensions), $\beta_3 = 2$ (compact temporal dimensions).

A.3 Cosmological mapping

$\Omega_m = \Omega_b + \Omega_{geom}$. With $\omega_b = 0.0223828$ and $h = 0.67810$: $\Omega_m \approx 0.309$, within the observationally allowed region.

Appendix B: CLASS Implementation

B.1 Parameter mapping

$\omega_{cdm} = \Omega_{geom} \times h^2 = (19/73) \times 0.45982 = 0.11967$. Baryon density fixed at $\omega_b = 0.0223828$.

B.2 Configuration

CLASS v3.3.4, synchronous gauge, scalar modes. Power spectra computed for $k = 10^{-4}$ to 1 h/Mpc at z = 0, 0.25, 0.5, 0.75, 1, 1.5, 2. Outputs: matter power spectra, background distances, thermodynamics, transfer functions.

B.3 σ_8 calculation

$\sigma^2_R = (1/2\pi^2) \int P(k)\, W^2(kR)\, k^2\, dk$, with top-hat filter $W(x) = 3(\sin x - x \cos x)/x^3$ and $R = 8\ h^{-1}$ Mpc.

Appendix C: Gadget4 Implementation

C.1 Setup

Box: L = 200 Mpc/h, $N = 256^3 = 16{,}777{,}216$ particles, periodic boundaries, final snapshot at $z \approx 0$.

C.2 Modified gravitational kernel

$G_{eff}(k,a) = G[1 + \mu(k,a)]$ with $\mu(k,a) = \mu_0\, S(a) / (1 + (k/k_\mu)^2)$. Parameters: $\mu_0 = 0.05$, $k_\mu = 0.20$ h/Mpc. Activation: $S(a) = a^3/(a^3 + a_t^3)$ with $a_t \approx 0.45$.

Appendix D: Power Spectrum Reconstruction

D.1 Mass assignment: Cloud-In-Cell (CIC) scheme on regular grid.

D.2 Fourier transform: $\delta(x) = \rho(x)/\bar{\rho} - 1$, then 3D FFT. Estimator: $P(k) = \langle|\delta(k)|^2\rangle$.

D.3 Window correction: $P_{corr}(k) = P_{meas}(k)/|W(k)|^2$.

D.4 Shot noise: $P_{shot} = V/N$ subtracted from measured spectrum.

Appendix E: Direct Snapshot Comparison

Final snapshots compared by matching particle IDs:

Quantity	Value
Mean displacement	96.3 kpc/h
RMS displacement	117.6 kpc/h

Max displacement	4388 kpc/h
Median displacement	84.1 kpc/h
95th percentile	193.1 kpc/h
99.9th percentile	740.0 kpc/h
Velocity RMS difference	194.8
Velocity max difference	5065.6

This confirms the gravitational modification produces a genuine dynamical effect.

Appendix F: Reproducibility

All calculations reproducible with: CLASS v3.3.4, Gadget4 (TreePM), Python 3, NumPy, h5py, SciPy. Key scripts: analyze_pk_v3.py (CIC power spectrum estimator), growth reconstruction, background distance extraction. Initial conditions: identical for both models (matched random seed).

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Edison Mode: Errors found, documented, corrected. G_{00} coefficient corrected from $(P+Q)^2$ to $(P+Q)^2/2$. Python solver instability identified; CLASS/Gadget4 verification performed independently.