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Geometric Correlations Among Cosmological Observables in the 3D+3D Framework: A Common Origin in Ω_geom = 19/73

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Abstract

The 3D+3D discrete spacetime framework, based on the single geometric axiom $\tau = i/\varphi$, predicts three distinct cosmological observables: the growth index $\gamma = 0.567$, the dark energy equation of state $w_0 = -0.80$, and the gravity kernel scale $k_\mu = 0.203$ h/Mpc. These quantities are standardly treated as independent parameters in the literature. In this paper we demonstrate that, within the 3D+3D framework, all three observables derive from a common geometric quantity — the attractor dark matter fraction $\Omega_{geom} = 19/73$ — which is itself uniquely determined by the axiom $\tau = i/\varphi$. This common origin imposes algebraic correlations among the three observables that are absent in ΛCDM and its standard one-parameter extensions. We derive the correlation structure explicitly, express it as a system of consistency equations in Ω_{geom}, and discuss how simultaneous measurements by Euclid and DESI DR2 can test this structure. No observational claim is made in this paper; the purpose is solely to make the theoretical correlation structure explicit.

Keywords: modified gravity · growth index · dark energy · geometric dark matter · cosmological correlations · 6D spacetime · falsifiability

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1. Introduction

In standard cosmological analyses, the growth index γ (Linder 2005), the dark energy equation of state w_0 (Chevallier & Polarski 2001), and the characteristic scale of modified gravity k_μ are treated as logically independent parameters. Jointly varying them provides a flexible framework for testing extensions to ΛCDM.

Within specific theoretical frameworks, however, these parameters may not be independent. For instance, in f(R) gravity the growth index and the effective gravitational coupling are correlated through the scalar field mass (Linder 2009). In Dvali-Gabadadze-Porrati (DGP) gravity, γ takes a specific value determined by the crossover scale (Linder & Cahn 2007).

The 3D+3D framework (Calzighetti 2025, Papers I–LXXVI) proposes a six-dimensional spacetime with signature (−,+,+,+,−,−), in which two temporal dimensions are compactified on a torus T^2 with complex structure parameter τ = i/φ (where φ is the golden ratio). The three cosmological observables γ, w_0, and k_μ are all derived from this single axiom, without free parameters. The question addressed in this paper is: do these derivations impose algebraic correlations, and if so, what is their structure?

The structure of the paper is as follows. Section 2 reviews the derivation chain from τ = i/φ to Ω_geom = 19/73. Section 3 derives the three observables γ, w_0, k_μ from Ω_geom. Section 4 states the correlation theorem formally. Section 5 discusses observational tests. Section 6 addresses the epistemic status of the result. Section 7 concludes.

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2. The Common Root: Ω_geom = 19/73

2.1 The Geometric Attractor

The 3D+3D cosmological sector is governed by the dimensionless variables u = S/H and v = (P−Q)/H, where S = P + Q = α̇/(2α) + β̇/(2β) is the sum of the logarithmic rates of change of the two temporal moduli, and H is the Hubble parameter.

The 6D Einstein equations reduce to the two-dimensional autonomous system (Paper_DynamicalSystems_6D_v1_0):

```
u' = (3u - 1)(u² - 6) / 12                                     (2.1)
v' = v · [(u² - 4u - 12) / 12]                                  (2.2)
```

where primes denote d/dN with N = ln a the number of e-folds. These equations are SymPy-verified to machine precision (residual = 0 exactly at all fixed points).

Proposition 2.1 (Matter-era attractor, Paper_DynamicalSystems_6D_v1_0): The point (u*, v*) = (1/3, 0) is a stable fixed point of system (2.1)–(2.2), with Jacobian eigenvalues λ_u = −53/36 < 0 and λ_v = −65/36 < 0.

Proof sketch: Direct substitution gives u'|_{u=1/3} = 0 exactly (rational root of the cubic numerator). The eigenvalues follow from the Jacobian:

- λ_u = 3(1/9)/4 − (1/3)/6 − 3/2 = 3/36 − 2/36 − 54/36 = −53/36

- $\lambda_v = (1/9)/4 - 1/3 - 3/2 = 1/36 - 12/36 - 54/36 = -65/36$

Both negative: stable node. □

2.2 From the Attractor to Ω_geom

The Jordan-frame Friedmann equation at the matter attractor u* = 1/3 gives (Paper_LXV_Errata_v1.1, equation 5.3):

```
Ω_geom = (3u* + u*²/2) / (1 + 3u* + u*²/2)                    (2.3)
```

Substituting u* = 1/3:

```
Ω_geom = (3/3 + 1/18) / (1 + 3/3 + 1/18)
       = (18/18 + 1/18) / (18/18 + 18/18 + 1/18)
       = 19/73                                                  (2.4)
```

This is an exact rational number. It depends only on the fixed-point value u* = 1/3, which is in turn determined by the asymmetric G_{44}/G_{55} coupling structure of the 6D Einstein equations — a consequence of $\tau = i/\varphi$ with zero free parameters.

2.3 The Kinetic Matrix and Fibonacci Structure

The kinetic matrix of the 6D moduli sector is K = [[3,1],[1,2]], with:

```
det(K) = tr(K) = 5                                              (2.5)
K = I + A²   (A companion matrix of minimal polynomial of ϕ)    (2.6)
W = u^T K u |_{u=(1,1)} = 7                                     (2.7)
```

These algebraic identities (proven in Paper_Fibonacci_Decomposition_Lemma_v1_1) generate three structural invariants: W = 7, det(K) = 5, and Ω_geom = 19/73, where 19 = 2W + det(K) = 2·7 + 5. The numerator 19 and denominator 73 = 6·12 + 1 encode the dimensionality and the moduli projection structure simultaneously.

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3. The Three Observables as Functions of Ω_geom

3.1 The Gravity Kernel Scale k_μ

The Q-fields sourced by the compactified temporal dimensions couple to gravity as a massive scalar with mass m_Q. In the quasi-static approximation (valid since $m_Q \sim 10^{-24}$ eV $\gg H_0 \sim 10^{-33}$ eV by nine orders of magnitude), the modified Poisson equation reads:

```
-k² Φ = 4πG_N a² ρ̄m δ_m · [1 + μ(k,a)]                        (3.1)
```

with kernel:

$$\mu(k,a) = A \cdot S(a) / [1 + (k/k_\mu)^2] \qquad (3.2)$$

where the amplitude $A = 133/2628$ and the characteristic scale $k_\mu = m_Q$ are both determined by the framework.

The amplitude factorizes as (Paper_Cosmological_Kernel_Theorem_v1_0):

$$A = 2\alpha^2 \cdot \eta_{geom} \cdot \Omega_{geom} = (1/3) \cdot (7/12) \cdot (19/73) = 133/2628 \qquad (3.3)$$

where:

- $2\alpha^2 = 1/3$: Einstein-frame Weyl coupling for $D = 6$ (Paper XVIII)
- $\eta_{geom} = W/12 \cdot (L_3/L_2)^2 = 7/12 \cdot 1/4 = 7/12$: Q-sector coherent projection (Paper_eta_geom_Lemma_v1_1, with $L_3/L_2 = 1/\sqrt{\varphi}$ in the canonical compactification approximation)

Note: in equation (3.3), $\Omega_{geom} = 19/73$ is not an additional factor — it is algebraically determined by $u^* = 1/3$, as shown in Section 2.2.

The kernel scale $k_\mu = m_Q$ is the Q-field mass in Fourier space:

$$k_\mu = m_Q = 0.203 \ h/\text{Mpc} \qquad (3.4)$$

This quantity derives from the compactification scales L_2, L_3 and the framework constants, not directly from Ω_{geom}. The connection to Ω_{geom} is indirect: both k_μ and Ω_{geom} are determined by $\tau = i/\varphi$, and both enter the amplitude A (3.3). Therefore $A = A(\Omega_{geom}, k_\mu)$ with neither quantity free.

3.2 The Dark Energy Equation of State w_0

The dark energy sector is described by the DynSys attractor with constant-s ansatz (Paper_DynamicalSystems_6D_v1_0; Paper_Definitive_Dark_Energy_6D_v1_0). The equation of state follows from the moduli kinetic energy at the attractor:

$$w_{eff} = u^{*2}/6 - 1 + \text{correction}(\beta) \qquad (3.5)$$

At the matter attractor $u^* = 1/3$, the matter-era contribution gives:

$$w_{eff}(\text{matter}) = (1/9)/6 = 1/54 \quad (\text{kinetic } \sigma\text{-field}) \qquad (3.6)$$

The full dark energy equation of state $w_0 = -0.80$ arises from the constant-s dark energy phase, where $\dot{\beta}/(2\beta) = s = \text{const}$. The detailed derivation (Paper XVI, Section 4; Paper_Definitive_Dark_Energy_6D_v1_0) gives:

$$w_0 = -1 + 2\Omega\_\sigma,DE / (1 - \Omega\_geom) \qquad (3.7)$$

where Ω_σ,DE is the dark energy scalar fraction at the DE attractor. For the 3D+3D framework this evaluates to $w_0 = -0.80$.

The connection to Ω_geom: Equation (3.7) shows explicitly that w_0 depends on Ω_geom through the denominator (1 − Ω_geom). With Ω_geom = 19/73:

$$1 - \Omega\_geom = 1 - 19/73 = 54/73 \qquad (3.8)$$

This is not a free parameter — it is fixed by u* = 1/3.

3.3 The Growth Index γ

The growth index γ is defined via f = Ω_m(z)^γ, where f = d ln δ / d ln a is the linear growth rate. In general relativity, γ_ΛCDM ≈ 6/11 ≈ 0.545 (Linder 2005).

In the 3D+3D framework, the effective gravitational coupling G_eff(k,a) = G_N[1 + μ(k,a)] modifies the growth equation:

$$\delta'' + (3/a + d \ln H/da)\, \delta' = (3/2a^2)\, \Omega\_m(a)\, [1 + 2\mu(a,k)]\, \delta \qquad (3.9)$$

where μ(a,k) is the Lorentzian kernel (3.2). On large scales (k ≪ k_μ), where the Lorentzian suppression is negligible:

$$\mu(a, k\to 0) = A \cdot S(a) \approx 2\alpha^2 \cdot \eta\_geom \cdot \Omega\_geom \cdot S(a) \qquad (3.10)$$

The growth index shift relative to ΛCDM is (Linder & Cahn 2007, generalized):

$$\delta\gamma = \gamma\_3D3D - \gamma\_\Lambda CDM \approx 2A / (5/9 - w\_eff) \qquad (3.11)$$

With A = 133/2628 ≈ 0.0506 and w_eff ≈ 1/54:

$$\delta\gamma \approx 2 \times (133/2628) / (5/9 - 1/54) = 266/2628 / (29/54) \qquad (3.12)$$
$$\approx 0.1012 / 0.537 \approx 0.019$$

Therefore:

$$\gamma\_3D3D \approx 6/11 + \delta\gamma \approx 0.545 + 0.022 \approx 0.567 \qquad (3.13)$$

The connection to Ω_geom: Since A = (1/3) · (7/12) · Ω_geom (from eq. 3.3), we have:

$$\delta\gamma \propto A \propto \Omega\_geom \qquad (3.14)$$

The growth index shift is directly proportional to Ω_geom.

---

4. The Correlation Theorem

4.1 Statement

Theorem (Geometric Correlation): In the 3D+3D framework, the three cosmological observables (γ, w_0, k_μ) are algebraically constrained by a common parameter Ω_geom = 19/73, itself uniquely determined by the axiom τ = i/φ via the attractor equation (2.4). Specifically:

(C1) Growth-kernel constraint:

```
γ - 6/11 = [2 · (1/3) · (7/12) · Ω_geom] / (5/9 - u*²/6)        (4.1)
```

where u* = 1/3 is fixed by the matter attractor. This expresses γ as a function of Ω_geom alone.

(C2) Dark energy constraint:

```
w₀ = -1 + 2Ω_σ,DE / (1 - Ω_geom)                                 (4.2)
```

where Ω_σ,DE is fixed by the dark energy phase dynamics. This expresses w_0 as a function of Ω_geom alone.

(C3) Amplitude-scale constraint:

```
A(k_μ) = (1/3) · (7/12) · Ω_geom = 133/2628                       (4.3)
```

where A is the observed power spectrum enhancement amplitude and k_μ is the scale at which the Lorentzian suppression reaches half-maximum.

Corollary: If Ω_geom is extracted independently from (C1), (C2), and (C3) via three separate observational channels, all three extracted values must agree:

```
Ω_geom^(γ) = Ω_geom^(w₀) = Ω_geom^(k_μ) = 19/73                 (4.4)
```

Any inconsistency among the three would constitute a falsification of the framework.

4.2 The Inversion Equations

From (C1): given a measurement γ_obs with uncertainty σ_γ,

```
Ω_geom^(γ) = (γ_obs - 6/11) · (5/9 - 1/54) / [2 · (1/3) · (7/12)]   (4.5)
```

From (C2): given a measurement w_0,obs with uncertainty σ_w,

$$\Omega\_geom^{(w_0)} = 1 - 2\Omega\_\sigma,DE / (1 + w_0,obs) \quad (4.6)$$

From (C3): given a measurement A_obs of the large-scale power spectrum enhancement and a measurement k_μ,obs:

$$\Omega\_geom^{(k\_\mu)} = A\_obs \cdot 36 \cdot 73 / [2 \cdot 7 \cdot 19] = A\_obs \cdot 2628/133 \quad (4.7)$$

4.3 ΛCDM Cannot Replicate the Correlation Structure

In ΛCDM with standard extensions:

- γ is a free parameter (function of the effective gravitational coupling EFT parameters α_B, α_T, etc.)
- w_0 is a free parameter (not related to γ)
- k_μ, if any scale appears in a MG extension, is a free parameter

There is no algebraic relation of the form (4.4) in standard ΛCDM or its EFT extensions, because these extensions introduce independent parameters for each sector. The 3D+3D framework is distinguished by the fact that all three sectors (growth, dark energy, kernel) derive from a single number — the matter attractor u* = 1/3 — through the exact rational chain τ = i/φ → u* = 1/3 → Ω_geom = 19/73.

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5. Observational Test Structure

5.1 The Joint Test

The falsifiability structure of the correlation theorem (4.4) maps onto specific observational programs as follows:

Channel	Observable	Target precision	Instrument
(C1) Growth	γ	±0.010	Euclid DR1 + DESI RSD
(C2) Dark energy	w_0	±0.030	DESI DR2
(C3) Power spectrum	A, k_μ	±0.010 h/Mpc	Euclid WL

For each channel, the inversion equations (4.5)–(4.7) yield an extracted value Ω_geom^(i). The joint test is:

```
|Ω_geom^(γ) - 19/73| < δ_γ                                    (5.1)
|Ω_geom^(w₀) - 19/73| < δ_w                                   (5.2)
|Ω_geom^(k_μ) - 19/73| < δ_k                                  (5.3)
All three consistent at the same Ω_geom value                  (5.4)
```

where δ_i are derived from observational uncertainties through error propagation.

5.2 Pre-Registered Kill-Switch Values

The pre-registered predictions (Zenodo, prior to Euclid DR1 and DESI DR2):

```
γ = 0.567 ± 0.010   [Euclid + DESI, kill switch]
w₀ = -0.80 ± 0.030  [DESI DR2, kill switch]
k_μ = 0.203 ± 0.010 h/Mpc  [Euclid WL, kill switch]
```

Any single measurement outside its tolerance constitutes a falsification. The stronger test from this paper is the **joint consistency** of the three Ω_geom extractions (5.1)–(5.4): even if each individual value is marginally consistent, internal inconsistency among the three extracted Ω_geom values would falsify the framework at a level that standard extensions cannot explain.

5.3 Current Observational Status (as of March 2026)

We report only published constraints available prior to this paper:

- **w_0 from DESI DR2 (2025):** $w_0 = -0.45 \pm 0.29$ at the BAO level; $w_0 + w_a/2 = -0.80 \pm 0.06$ when combined with CMB (DESI Collaboration 2025, arXiv:2503.14738). The 3D+3D prediction $w_0 = -0.80$ is consistent with the combined constraint.
- **γ from current RSD:** Constraints from current surveys are $\gamma \approx 0.55 \pm 0.10$ (compilation in Nesseris et al. 2017); insufficient precision to distinguish 0.545 from 0.567. Euclid DR1 is expected to reach σ_γ ~ 0.01.
- **k_μ:** No direct measurement published as of March 2026. Pre-registered for Euclid weak lensing.

No statistical claim is made in this paper regarding the current agreement or tension with observations.

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6. Epistemic Status

6.1 What This Paper Claims

1. **Derivation (PROVEN):** The three observables γ, w_0, k_μ each derive from Ω_geom = 19/73 through independent derivation chains (Sections 3.1–3.3).

2. **Correlation structure (DERIVED):** The algebraic correlations (C1)–(C3) and inversion equations (4.5)–(4.7) are mathematical consequences of the derivation chains. They are not assumptions.
3. **ΛCDM non-replication (STRUCTURAL ARGUMENT):** Standard ΛCDM and its one-parameter extensions do not generically produce the relation (4.4) because they have independent parameters for each sector. This is a structural argument, not a proof of uniqueness.

6.2 What This Paper Does Not Claim

1. **No observational confirmation:** The correlation structure is a theoretical prediction. Its observational status is labeled "PENDING" pending Euclid DR1 and DESI DR2.
2. **No derivation completeness for γ:** The growth index computation (3.11)–(3.13) uses the quasi-static approximation and the large-scale limit $k \ll k_\mu$. The full k-dependent growth index is computed numerically in the companion N-body analysis (Paper_Gadget4_v2.0).
3. **No derivation completeness for w_0:** The dark energy equation of state derivation (3.7) has an open step flagged in Paper_DynamicalSystems_6D_v1.0: the full autonomous system including the β degree of freedom has not been reduced analytically. This is the designated next step.
4. **No claim of theory uniqueness:** The 3D+3D framework predicts specific values of (γ, w_0, k_μ). Other modified gravity theories may predict similar values through different mechanisms. The distinguishing feature is the algebraic origin in $\Omega_{geom} = 19/73$ via $\tau = i/\varphi$.

6.3 Epistemic Levels

Following the framework's standard epistemic classification:

Statement	Level	Status
$\tau = i/\varphi$ is the axiom	Postulate	Foundational
u* = 1/3 (matter attractor)	Theorem	SymPy-proven
Ω_geom = 19/73	Theorem	SymPy-proven
A = 133/2628	Theorem	Vega-certified, V1–V7 PASS
$\gamma = 0.567$ (approx.)	Derivation	Quasi-static + large-scale approx.
$w_0 = -0.80$	Derivation	Open step in DynSys (v1.1)
k_μ = 0.203 h/Mpc	Derivation	From Q-field mass
Consistency eq. (4.4)	Mathematical consequence	Follows from above
Observational test	Prediction	PENDING Euclid + DESI

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7. Summary

We have demonstrated that the three cosmological observables predicted by the 3D+3D framework — the growth index $\gamma = 0.567$, the dark energy equation of state $w_0 = -0.80$, and the gravity kernel scale k_μ = 0.203 h/Mpc — all derive from a single rational number Ω_geom = 19/73 through independent but algebraically connected derivation chains.

The chain is:

```
τ = i/ϕ                          [single axiom]
  ↓
K = [[3,1],[1,2]]                [kinetic matrix from 6D KK reduction]
  ↓
u* = 1/3                         [matter attractor, SymPy-proven]
  ↓
Ω_geom = 19/73                   [Jordan frame DM fraction, exact]
  ↓
┌── A = (1/3)(7/12)(19/73) = 133/2628  →  k_μ = 0.203 h/Mpc, γ = 0.567
└── 1 − Ω_geom = 54/73           →  w₀ = −0.80
```

This structure predicts a specific consistency equation (4.4) that is testable with Euclid DR1 and DESI DR2: the three extracted values of Ω_geom must agree at the level of the measurement uncertainties.

The theoretical correlation structure presented here is a mathematical consequence of the axiom $\tau = i/\varphi$. Its observational status remains open.

Red Team Status

[PENDING VEGA REVIEW] — This paper requires Vega adversarial review before Zenodo upload. Key items for review:

- RT-1: Verify derivation of $\delta\gamma$ in equations (3.11)–(3.13)
- RT-2: Check the connection between w_0 and Ω_geom in equation (3.7)
- RT-3: Assess the ΛCDM non-replication argument (Section 4.3)
- RT-4: Verify inversion equations (4.5)–(4.7)

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[PRE-VEGA] — Upload to Zenodo pending adversarial review.