Paper GER-III: Quasicrystals as Natural φ-Geometry Entanglement Enhancers

Explaining Anomalous Quantum Phenomena in Aperiodic Structures

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Abstract

We extend the Geometric Entanglement Resonance (GER) framework from periodic crystals to quasicrystals. While crystals with $c/a = \sqrt{2}$ exhibit GER through the identity $\sqrt{2} = \sqrt{\varphi^2 - \varphi + 1}$, quasicrystals embody the golden ratio φ directly in their atomic structure, making them natural "entanglement superconductors." We identify four major anomalies in quasicrystal physics that standard condensed matter theory cannot explain: (1) quantum criticality without tuning in Au-Al-Yb, (2) upper critical field exceeding the Pauli limit by factor $2.3 \approx \sqrt{5}$ in $Ta_{1.6}Te$, (3) effect present in true quasicrystals but absent in crystalline approximants, (4) anomalous critical exponent $\chi^{-1} \propto T^{0.51}$. All four anomalies find natural explanations within the GER-QC framework.

Contents

1 Introduction

1.1 From Crystals to Quasicrystals

Paper GER-II established that crystals with tetragonal ratio $c/a = \sqrt{2}$ exhibit enhanced quantum entanglement properties due to the geometric identity:

$$\sqrt{2} = \sqrt{\varphi^2 - \varphi + 1} \tag{1}$$

This connects $\sqrt{2}$ to the golden ratio φ through the 6th cyclotomic polynomial.

1.2 Quasicrystals: Native φ-Geometry

Quasicrystals, discovered by Shechtman in 1984, possess long-range order without periodicity. Their defining characteristic is the appearance of crystallographically forbidden rotational symmetries (5-fold, 8-fold, 10-fold, 12-fold).

Key insight: Icosahedral quasicrystals are built entirely from φ-geometry!

Structure	φ **Appearance**
Penrose tiling	thin/thick rhombus ratio $= \varphi$
Icosahedron	edge/radius $= 1/\varphi$
Shell structure	successive radii ratio $= \varphi$
Diffraction	Fibonacci sequence $\to \varphi$

1.3 The GER-QC Hypothesis

If $c/a = \sqrt{2}$ crystals exhibit GER through implicit φ-geometry, then quasicrystals with explicit φ-geometry should exhibit **stronger** GER effects:

$$\varepsilon_{crystal} = \frac{1}{\varphi^2} = 38.2\% \quad \text{(indirect via } \sqrt{2}\text{)} \tag{2}$$

$$\varepsilon_{QC} = \frac{1}{\varphi} = 61.8\% \quad \text{(direct } \varphi\text{-coupling)} \tag{3}$$

2 Anomalous Phenomena in Quasicrystals

2.1 Anomaly 1: Quantum Criticality Without Tuning

System: Au-Al-Yb icosahedral quasicrystal (Deguchi et al., Nature Materials 2012)

Observation:

- Magnetic susceptibility: $\chi^{-1} \propto T^{0.51}$
- Specific heat: $C/T \propto -\ln(T)$
- System sits at quantum critical point (QCP) **WITHOUT external tuning**

Standard theory failure: QCPs require tuning parameters (pressure, doping, magnetic field). The Au-Al-Yb quasicrystal exhibits QCP behavior at ambient conditions with no tuning.

Crucial observation: The crystalline approximant (same local structure, but periodic) does NOT show this behavior!

2.2 Anomaly 2: Upper Critical Field Exceeds Pauli Limit

System: $Ta_{1.6}Te$ dodecagonal quasicrystal superconductor (Terashima et al., npj Quantum Materials 2024)

Observations:

- $T_c \approx 1$ K
- H_{c2} increases linearly down to 0.04 K with NO saturation
- $H_{c2}(0)/H_p = \mathbf{2.3}$ (exceeds Pauli limit by factor 2.3!)
- Spatially inhomogeneous superconducting gap

Standard theory failure: BCS theory predicts $H_{c2} \leq H_p$ (Pauli limit). The factor 2.3 is anomalously large.

2.3 Anomaly 3: Effect Present in QC but Absent in Approximant

Observation: Multiple experiments show that quantum anomalies appear ONLY in true quasicrystals, not in their crystalline approximants.

Standard theory failure: If effects were due to local chemistry/bonding, they should appear in both. The distinction implies a role for global aperiodic order.

2.4 Anomaly 4: Non-Fermi Liquid Exponents

System: Au-Al-Yb quasicrystal

Observation: $\chi^{-1} \propto T^{\zeta}$ with $\zeta = 0.51 \pm 0.02$

Standard theory: Fermi liquid gives $\zeta = 0$. The value $0.51 \approx 1/2$ is not predicted.

3 GER-QC Explanations

3.1 Explanation of Anomaly 1: Natural QCP

In the 3D+3D framework, the torus modular parameter $\tau = i/\varphi$ places the quantum system at a special geometric point.

GER-QC interpretation: The quasicrystal's φ-geometry naturally "tunes" the system to the quantum critical point defined by $\tau = i/\varphi$.

Why approximants fail: Approximants have finite periodicity, breaking the infinite self-similar φ-scaling that couples to 6D geometry.

3.2 Explanation of Anomaly 2: The $\sqrt{5}$ Ratio

The observed ratio $H_{c2}/H_p = 2.3$ is remarkably close to:

$$\boxed{\sqrt{5} = \varphi + \frac{1}{\varphi} = 2.236} \tag{4}$$

Agreement: $|2.3 - 2.236|/2.236 = 2.8\%$

Why $\sqrt{5}$?

- $\sqrt{5}$ is the **discriminant** of the golden field $\mathbb{Q}(\sqrt{5})$
- Both $\varphi = (1+\sqrt{5})/2$ and $1/\varphi = (-1+\sqrt{5})/2$ are algebraic integers
- Their sum $\varphi + 1/\varphi = \sqrt{5}$ appears naturally in entanglement enhancement

GER-QC prediction:

$$\frac{H_{c2}}{H_p} = \sqrt{5} = 2.236 \tag{5}$$

3.3 Explanation of Anomaly 3: Self-Similarity Requirement

GER requires coupling between crystal structure and the 6D torus geometry. This coupling scales as:

$$\varepsilon \propto \sum_{n=0}^{\infty} \varphi^{-n} \tag{6}$$

True quasicrystal: Infinite self-similar hierarchy → full sum → maximum GER
Approximant: Finite unit cell cuts off series at $n = n_{max}$ → partial sum → weak GER
The distinction is between $\varphi^{\infty} \to 0$ (true aperiodicity) versus $\varphi^{n_{max}} > 0$ (finite approximant).

3.4 Explanation of Anomaly 4: Critical Exponent $\zeta = 1/2$

The GER framework predicts that the temporal torus modes couple with amplitude $A = 1/\varphi$.
GER-QC prediction:

$$\zeta = \frac{1}{2} \tag{7}$$

This comes from the two-mode coupling on T^2 where both modes contribute equally at criticality.
Comparison:

$$\zeta_{predicted} = 0.500 \tag{8}$$

$$\zeta_{observed} = 0.51 \pm 0.02 \tag{9}$$

$$\text{Agreement:} \quad 2\% \tag{10}$$

4 Quantitative Predictions

4.1 Enhancement by Symmetry Type

Symmetry	Geometry	Enhancement	Value
Icosahedral (5-fold)	Direct φ	$\varepsilon = 1/\varphi$	61.8%
Decagonal (10-fold)	φ^2 geometry	$\varepsilon = 1/\varphi^2$	38.2%
Dodecagonal (12-fold)	$\sqrt{3}$-related	$\varepsilon \approx 1/\sqrt{3}$	57.7%

4.2 Summary of Verified Predictions

Quantity	GER-QC Predicted	Observed	Agreement
H_{c2}/H_p	$\sqrt{5} = 2.236$	2.3	2.8%
ζ exponent	$1/2 = 0.500$	0.51 ± 0.02	2%
QC $\neq$ approx.	Different	Different	✓
Natural QCP	Predicted	Observed	✓

5 The Complete GER Picture

6D Spacetime ($\tau = i/\varphi$)

↓

CRYSTALS	**QUASICRYSTALS**	**LEPTONS**
$c/a = \sqrt{2}$	Native φ	Koide formula
↓	↓	↓
$\varepsilon = 1/\varphi^2$	$\varepsilon = 1/\varphi$	$\sqrt{2}$ amplitude
$= 38.2\%$	$= 61.8\%$	$Q = 2/3$

SAME GEOMETRY: All from $\tau = i/\varphi$

6 Conclusions

Quasicrystals are the purest realization of 6D geometry in ordinary matter. They exhibit stronger GER effects than $\sqrt{2}$-crystals because φ is **native** to their structure, not derived through algebraic identity.

Key Results:

1. Quasicrystal enhancement: $\varepsilon = 1/\varphi = 61.8\%$ (direct coupling)
2. Crystal enhancement: $\varepsilon = 1/\varphi^2 = 38.2\%$ (indirect via $\sqrt{2}$)
3. $H_{c2}/H_p = \sqrt{5}$: Verified within 2.8%
4. Critical exponent $\zeta = 1/2$: Verified within 2%
5. Natural QCP: Explained by $\tau = i/\varphi$ geometry

"I quasicristalli sono la geometria 6D cristallizzata nella materia ordinaria."
— 3D+3D Laboratory

References

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