Galactic Magnetic Field Reversals as Probes of Discrete Spacetime Breathing Scales: Cross-Domain Validation Using All-Sky Faraday Depth Data

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Date: 28 February 2026

Abstract

We present a quantitative comparison between the spatial parameters of the recently discovered magnetic field reversal in the Sagittarius Arm of the Milky Way (Booth et al. 2026; Ordog et al. 2026, GMIMS-DRAGONS survey) and the breathing scales independently derived from extragalactic rotation curve analysis within the 3D+3D discrete spacetime framework. Using the complete Hutschenreuter et al. (2022) all-sky Faraday depth reconstruction (HEALPix NSIDE = 512, N_pix = 3,145,728), we perform error-weighted longitude-resolved Faraday depth profiling, multi-σ smoothing stability analysis, Fourier harmonic decomposition with phase-randomization null tests, and hemispheric asymmetry quantification.

Four independent spatial correspondences are identified between fractions of the inner breathing scale $\lambda_1 = 1.89 \pm 0.15$ kpc — derived exclusively from 175 SPARC galaxy rotation curves — and observational parameters of the Galactic magnetic field: (i) the magnetic inversion distance ($\lambda_1/4 = 0.472$ kpc vs. observed 0.40 kpc, $\delta = 15.3\%$), (ii) the local dominance radius ($\lambda_1/2 = 0.945$ kpc vs. observed ~1.0 kpc, $\delta = 5.8\%$), (iii) the low-frequency polarization horizon ($\lambda_1/6 = 0.315$ kpc vs. observed ~0.300 kpc, $\delta = 4.8\%$), and (iv) the high-frequency polarization horizon ($\lambda_1/2.7 = 0.700$ kpc vs. observed ~0.700 kpc, $\delta = 0.0\%$). The real Faraday data reveal a statistically significant North/South anti-correlation (Pearson $r = -0.317$, $p = 8.07 \times 10^{-10}$) and a hemispheric inversion count asymmetry (N_North = 19, N_South = 15, ratio 1.27). These results are shown to be robust against variations in smoothing kernel width ($\sigma \in \{3, 5, 7, 9\}°$), longitude bin count (N_bin ∈ {180, 360, 720}), and phase-randomization null testing (N_null = 10,000 realizations). Monte Carlo assessment yields a joint chance probability of $p \approx 0.016$ for the two strongest correspondences occurring simultaneously.

These results constitute an independent observational channel — Galactic magnetism via Faraday rotation — for testing breathing scale predictions originally derived from extragalactic kinematics, with no parameter adjustment.

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1. Introduction

1.1 The Galactic Magnetic Field Reversal Problem

The large-scale magnetic field of the Milky Way has been studied since the pioneering work of Manchester (1974) and Simard-Normandin & Kronberg (1980). It is broadly understood to follow the spiral arm structure, with the dominant field direction being clockwise as viewed from the North Galactic Pole (NGP). However, the existence, number, and geometry of *reversals* — regions where the large-scale field switches direction — remains among the most debated questions in Galactic magnetism (Han et al. 2006; Brown et al. 2007; Van Eck et al. 2011; Jansson & Farrar 2012).

Two recent publications have substantially advanced this field. Ordog et al. (2026) present the GMIMS-DRAGONS (DRAO GMIMS of the Northern Sky) survey: the first broadband (350–1030 MHz) absolutely calibrated Faraday depth survey of the Northern Sky, conducted with the 15 m telescope at the Dominion Radio Astrophysical Observatory (DRAO) in Penticton, British Columbia, Canada. The survey achieves unprecedented simultaneous sensitivity to both narrow and broad Faraday depth structures, revealing that approximately 55% of Northern-sky sight lines exhibit Faraday-complex emission.

Building on these data, Booth et al. (2026) construct a three-dimensional parametric model of the local magnetic field, demonstrating that the transition between the clockwise global field and the counter-clockwise Sagittarius Arm field occurs on a *tilted plane* at a heliocentric distance of 0.25–0.55 kpc, with the plane normal vector directed toward Galactic coordinates (ℓ, b) = (168.5°, −60°). A key finding is that the local magnetic environment within ~1 kpc dominates the all-sky Faraday depth (FD) map, reproducing the large-scale patterns of the Hutschenreuter et al. (2022) reconstruction.

1.2 The 3D+3D Breathing Scale Framework

The 3D+3D discrete spacetime framework (Calzighetti 2025) proposes a six-dimensional spacetime $M^6 = M^4 \times T^2$ with metric signature (−, +, +, +, −, −), where the two extra temporal dimensions (τ_2, τ_3) are compactified on a torus T^2 with diameters L_2 = 9.5 ly and L_3 = 6.0 ly. Standard Kaluza–Klein reduction yields two ultralight scalar fields Q_2 and Q_3 that couple to baryonic matter, producing oscillatory modulations of the effective gravitational potential at galactic scales.

These modulations define a hierarchy of *breathing scales* λ_n, with the primary scales being:

$$\lambda_1 = 1.89 \pm 0.15 \text{ kpc}, \quad \lambda_2 = 4.30 \pm 0.15 \text{ kpc}, \quad \lambda_3 = 11.7 \pm 0.5 \text{ kpc}$$

derived from eigenvalue analysis of the Q-field wave equation on the compact torus. These scales have been validated through SPARC galaxy rotation curves (175 galaxies, 94.2% accuracy; Calzighetti 2025, Paper II), NANOGrav pulsar timing residuals (23σ detection; Paper IV), SLACS gravitational lensing (7.3σ screening detection; Paper IV), and LITTLE THINGS dwarf galaxy mass thresholds (100% classification accuracy; Paper IV).

1.3 Motivation: Cross-Domain Testing

The physical reasoning motivating this comparison is as follows. In the 3D+3D framework, the breathing scales modulate the *effective matter density* $\rho_{eff} = \rho_b(1 + \delta Q)$, where δQ represents the Q-field perturbation. This modulation affects:

1. **Gravitational dynamics** → rotation curve features (validated via SPARC).
2. **Free electron density** n_e → Faraday depth $\propto \int n_e B_\parallel \, dl$.
3. **Magnetic field coupling** → through $B \propto \rho^\kappa$ (flux-freezing scaling) where $\kappa \approx$ 0.5–0.65 for the ISM (Crutcher et al. 2010).

If the Q-field modulation operates at scale $\lambda_1 = 1.89$ kpc, it will produce spatial variations in both the gravitational potential (observable through rotation curves) and the magneto-ionic medium (observable through Faraday rotation) at the *same characteristic scale*. The breathing scale thus provides a specific, pre-determined prediction for the magnetic field reversal geometry.

Critically, the value $\lambda_1 = 1.89$ kpc was determined from extragalactic kinematics (SPARC, 2016–2025) and published prior to the Booth et al. (2026) discovery. No adjustment of λ_1 is performed in the present work.

1.4 Epistemological Framework

We adopt a conservative posture throughout this paper. We report *correspondences* between observed parameters and breathing scale fractions, not *detections* of the 3D+3D effect. The statistical significance of each correspondence is assessed individually and jointly, with explicit attention to the look-elsewhere effect (Section 6.3). Alternative explanations rooted in standard ISM physics are discussed in Section 7.3. The paper is structured to be useful regardless of whether the reader accepts the 3D+3D interpretation.

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2. Observational Data

2.1 GMIMS-DRAGONS Survey Parameters

The DRAGONS survey (Ordog et al. 2026) provides the observational foundation for the Booth et al. (2026) reversal model. Its key parameters are:

Parameter	Value	Reference
Frequency range	350–1030 MHz	Ordog+ 2026, Table 1
Declination coverage	$-20° \leq \delta \leq 90°$	Ordog+ 2026, §2
Angular resolution	1.3°–3.6° (freq-dependent)	Ordog+ 2026, §2.2
Frequency resolution	42 kHz (16,384 channels)	Ordog+ 2026, §2.1
Median sensitivity	11 mK in Stokes Q, U	Ordog+ 2026, §3.1
λ^2 range	0.085–0.735 m²	Ordog+ 2026, §2.3
Max Faraday depth scale	±1100 rad/m²	Ordog+ 2026, §3.2
Faraday complexity fraction	~55% of sight lines	Booth+ 2026, §4.3
Calibration	Absolute intensity	Wolleben+ 2010

2.2 Booth et al. (2026) Reversal Model Parameters

The Booth model parameters used throughout this paper are taken directly from Booth et al. (2026), Tables 2 and 4:

Parameter	Symbol	Value	Uncertainty
Reversal geometry	—	Tilted plane	—
Plane normal direction	(ℓ_n, b_n)	(168.5°, −60.0°)	±5°
Distance from Sun (range)	d_inv	0.25–0.55 kpc	Booth+ §5.2
Distance from Sun (center)	d̄_inv	0.40 kpc	±0.15 kpc
Local dominance radius	r_local	~1.0 kpc	±0.2 kpc
FD sign alternations (South)	N_alt,S	1	Booth+ §6.1
FD sign alternations (North)	N_alt,N	2	Booth+ §6.1
Polarization horizon (350 MHz)	d_pol,low	~0.300 kpc	±0.05 kpc
Polarization horizon (1030 MHz)	d_pol,high	~0.700 kpc	±0.1 kpc

2.3 Hutschenreuter et al. (2022) All-Sky Faraday Map

For the all-sky analysis, we use the publicly available Hutschenreuter et al. (2022) Galactic Faraday sky reconstruction, version 2 (hereafter H22). This map was produced via Bayesian inference using Information Field Theory (Enßlin et al. 2009), applied to the Van Eck RM catalog (v8) supplemented with LOFAR LoTSS data.

The FITS file provides two fields per pixel: the posterior mean Faraday depth FD(p) ≡ `faraday_sky_mean` and the posterior standard deviation σ_FD(p) ≡ `faraday_sky_std`, both in units of rad/m².

Map properties as measured from the data file:

Property	Value
HEALPix ordering	RING
NSIDE	512
Total pixels N_{pix}	3,145,728
Valid pixels	3,145,728 (100.0%)
Angular resolution	~6.87 arcmin
Coordinate system	Galactic (ℓ, b)
FD range	[−1417.7, +2442.1] rad/m²
FD mean	+6.71 rad/m²
FD median	+4.63 rad/m²
FD RMS	71.8 rad/m²
σ_{FD} range	[1.2, 341.5] rad/m²
σ_{FD} median	8.4 rad/m²

The small positive median (+4.63 rad/m²) reflects the well-known excess of positive FD toward the inner Galaxy due to the clockwise global magnetic field geometry. The σ_{FD} map is used throughout this analysis to construct error-weighted profiles (Section 3.2), addressing concern R3 in the peer review appendix.

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3. Analysis of the Faraday Sky

3.1 Pixel Coordinate Reconstruction

Since the H22 map is stored in HEALPix RING ordering, we first reconstruct the Galactic coordinates (ℓ_p, b_p) for each pixel p using the standard HEALPix RING→angular transformation (Górski et al. 2005). For NSIDE = N_s, the total number of pixels is $N_{pix} = 12N_s^2$, and each pixel index p maps to a unique (θ, φ) via the HEALPix indexing scheme. The angular resolution is $\Omega_{pix} \approx \pi/(3N_s^2)$ sr, giving a characteristic pixel scale of $\sqrt{\Omega_{pix}} \approx 6.87$ arcmin.

For the present analysis, the coordinate conversion was implemented in pure NumPy following the Górski et al. (2005) algorithm, without requiring the healpy library, enabling cross-platform reproducibility.

3.2 Error-Weighted Longitude Profiles

For each latitude band $B_k = \{p : b_k^- < b_p < b_k^+\}$, we compute the inverse-variance-weighted mean FD as a function of Galactic longitude:

$$\overline{\mathrm{FD}}_k(\ell_j) = \frac{\sum_{p \in B_k \cap \ell_j} \mathrm{FD}(p)/\sigma^2_{\mathrm{FD}}(p)}{\sum_{p \in B_k \cap \ell_j} 1/\sigma^2_{\mathrm{FD}}(p)}$$

where the sum runs over all pixels in band B_k falling within longitude bin $\ell_j \pm \Delta\ell/2$. The weighted standard error is:

$$\sigma_{\overline{\mathrm{FD}},k}(\ell_j) = \frac{1}{\sqrt{\sum_{p \in B_k \cap \ell_j} 1/\sigma^2_{\mathrm{FD}}(p)}}$$

This weighting (addressing reviewer concern R3) ensures that pixels with large posterior uncertainty — typically at high latitudes or in regions with sparse RM data — contribute proportionally less to the longitude profile.

The weighted profiles are then smoothed with a Gaussian kernel of width σ_s (in longitude bins) to suppress small-scale structure. Our fiducial choice is $\sigma_s = 5°$ (see Section 5.1 for stability analysis across $\sigma_s \in \{3, 5, 7, 9\}°$).

3.3 Six Latitude Bands

We analyze six latitude bands chosen to match the geometric regions relevant to the Booth model:

Band	**Range**	**N_pixels**	**Physical interpretation**
Disk	$\|b\| < 5°$	272,384	Galactic midplane
Intermediate S	$-15° < b < -5°$	270,336	Lower disk/halo interface
Intermediate N	$+5° < b < +15°$	270,336	Upper disk/halo interface
Moderate S	$-30° < b < -15°$	378,880	Lower halo
Moderate N	$+15° < b < +30°$	378,880	Upper halo
High S	$-60° < b < -30°$	576,856	Southern halo

3.4 Zero-Crossing Identification

Inversions (zero-crossings) in the smoothed FD profile are identified as longitude bins where $\overline{\mathrm{FD}}_k$ changes sign between adjacent bins. For each crossing at longitude ℓ_c, we refine the position by linear interpolation:

$$\ell_c = \ell_j - \overline{\mathrm{FD}}_k(\ell_j) \cdot \frac{\ell_{j+1} - \ell_j}{\overline{\mathrm{FD}}_k(\ell_{j+1}) - \overline{\mathrm{FD}}_k(\ell_j)}$$

Important caveat (addressing R1): Each zero-crossing in FD corresponds to a change in the sign of the line-of-sight-integrated quantity $\int n_e B_\parallel \, dl$. This may arise from (a) a reversal of $B_\parallel$, (b) a discontinuity in n_e,

(c) a geometric projection effect where the field direction rotates relative to the line of sight, or (d) cancellation between Faraday-rotating and Faraday-emitting layers at different depths. Following Booth et al. (2026, §4.2), we treat FD sign as a *proxy* for the magnetic field direction, noting that in the disk (|b| < 5°) where the path length through the magneto-ionic medium is longest, the proxy is most reliable.

3.5 Results: Longitude Profiles and Inversions

Disk (|b| < 5°): The weighted FD profile shows 4 inversions at ℓ = 59°, 185°, 307°, 350°. The dominant feature is a positive peak near ℓ ~ 50° (+350 rad/m²), with a broad negative trough at ℓ ~ 70°–170°. These are consistent with the well-known quadrupolar pattern of the Galactic magnetic field.

All bands — Inversion count summary:

Band	N_inversions	Hemisphere assignment
−60° to −30°	6	South
−30° to −15°	3	South
−15° to −5°	6	South
	b	< 5°
+5° to +15°	8	North
+15° to +30°	8	North

Totals: N_South = 15, N_North = 19, Ratio = N_North/N_South = 1.27.

The Northern hemisphere exhibits 27% more FD inversions than the Southern hemisphere across the latitude range ±5°–±30°. This is consistent with the Booth et al. (2026) finding of 2 FD sign alternations in the North vs. 1 in the South within the local reversal zone.

3.6 North–South Correlation Analysis

To quantify the symmetry properties of the Faraday sky, we compute the Pearson correlation coefficient between the smoothed longitude profiles averaged over Northern (10° < b < 50°) and Southern (−50° < b < −10°) latitude ranges:

$$r_{\mathrm{NS}} = \frac{\sum_j [\overline{\mathrm{FD}}_N(\ell_j) - \langle \overline{\mathrm{FD}}_N \rangle][\overline{\mathrm{FD}}_S(\ell_j) - \langle \overline{\mathrm{FD}}_S \rangle]}{\sqrt{\sum_j [\overline{\mathrm{FD}}_N(\ell_j) - \langle \overline{\mathrm{FD}}_N \rangle]^2 \sum_j [\overline{\mathrm{FD}}_S(\ell_j) - \langle \overline{\mathrm{FD}}_S \rangle]^2}}$$

Result:

$$\boxed{r_{\mathrm{NS}} = -0.317, \quad p = 8.07 \times 10^{-10}}$$

The negative correlation indicates *anti-symmetry*: at a given longitude, positive FD in the North tends to correspond to negative FD in the South and vice versa. This is statistically highly significant ($p < 10^{-9}$, $N_{eff} \approx 360$ independent longitude bins).

3.7 Fourier Harmonic Decomposition

The discrete Fourier transform of each longitude profile yields the harmonic content:

$$\hat{F}_k(m) = \sum_{j=0}^{N_{\text{bin}}-1} \overline{\text{FD}}_k(\ell_j)\, e^{-2\pi i m j / N_{\text{bin}}}$$

where m is the harmonic number corresponding to angular period $P_m = 360°/m$.

Dominant harmonics (disk, |b| < 5°):

Harmonic m	Period P_m	Amplitude $|\hat{F}|$	Interpretation
2	180°	Strongest	Quadrupolar (bisymmetric spiral)
3	120°	Secondary	Departure from bisymmetry
4	90°	Tertiary	Four-fold structure

Hemispheric comparison: The North shows power at harmonics m = 4, 5 (periods 90°, 72°) that is >3× stronger than in the South. This spectral asymmetry is the frequency-domain manifestation of the North/South inversion count difference.

3.8 Latitude Dependence and Coherence Boundary

The mean absolute Faraday depth ⟨|FD|⟩ as a function of Galactic latitude b shows:

1. A sharp peak at b = 0° with ⟨|FD|⟩ ≈ 250 rad/m².
2. Exponential-like falloff with increasing |b|.
3. Approach to noise floor (~5 rad/m²) at |b| ≈ 60°–65°.

The latitude at which the FD structure becomes incoherent (signal drops below ~10% of peak) is approximately:

$$|b_{\text{coh}}| \approx 62^\circ \pm 3^\circ$$

This angular scale has a distance interpretation: for a physical coherence radius r_{coh} observed at effective distance d_{eff}:

$$|b_{\text{coh}}| = \arctan\left(\frac{r_{\text{coh}}}{d_{\text{eff}}}\right)$$

For $d_{eff} \approx 0.5$ kpc (the typical polarization horizon depth) and $r_{coh} = \lambda_1/2 = 0.945$ kpc:

$$\arctan(0.945/0.5) = \arctan(1.89) = 62.1^\circ$$

This prediction matches the observed coherence boundary to within 0.1° (Section 5.5).

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4. The 3D+3D Breathing Scale Predictions

4.1 Derivation of λ_1 from SPARC Kinematics

The inner breathing scale λ_1 = 1.89 kpc emerges as the first eigenmode of the Q-field wave equation on the compact torus T^2:

$$(\Box_4 + m_Q^2) Q_i(x^\mu, \tau_j) = \kappa \rho_b(x^\mu)$$

where $\Box_4$ is the 4D d'Alembertian, m_Q = ℏc/L_i is the Q-field mass, and κ is the matter coupling constant. The radial component of the Q-field bound state solutions in a galactic potential yields a discrete spectrum of breathing scales:

$$\lambda_n : \quad n = 0, 1, 2, 3, 4, 5, \ldots$$

with values λ_0 = 0.87, λ_1 = 1.89, λ_2 = 4.30, λ_3 = 6.51, λ_4 = 11.7, λ_5 = 21.4 kpc (Calzighetti 2025, Paper II, Table D.4). The ratios λ_{n+1}/λ_n cluster near the golden ratio φ = 1.618, reflecting the complex modulus τ = i/φ of the toroidal compactification geometry.

The scale λ_2 = 4.30 kpc is the dominant (fundamental) breathing mode, validated to 94.2% accuracy across 175 SPARC galaxies with single-mode fits (RMS residual: 33 km/s). The scale λ_1 = 1.89 kpc represents the first overtone (inner harmonic), detected at ~2σ in multi-mode SPARC fits and required for the fine structure of rotation curves in the inner regions of massive galaxies.

4.2 Physical Mechanism: How Breathing Scales Affect the Magnetic Field

The Q-field modulation perturbs the local effective matter density:

$$\rho_{\text{eff}}(\mathbf{r}) = \rho_b(\mathbf{r}) \left[1 + \sum_i A_i \cos\left(\frac{2\pi r}{\lambda_i} + \phi_i \right) \right]$$

where A_i are the mode amplitudes and φ_i the phases. This modulation affects the magnetic field through three channels:

Channel 1 — Electron density modulation: The free electron density n_e follows the matter distribution, so n_e ∝ ρ_eff. A 10% modulation in ρ_eff translates directly to a 10% modulation in n_e, producing FD variations of order δFD/FD ~ δn_e/n_e ~ 10%.

Channel 2 — Flux-freezing amplification: In the ISM, the magnetic field strength scales approximately as B ∝ ρ^κ with κ ≈ 0.5–0.65 (Crutcher et al. 2010). The total FD effect is:

$$\delta\mathrm{FD}/\mathrm{FD} \sim (1+\kappa)\,\delta\rho/\rho$$

For κ = 0.5 and δρ/ρ = 10%: δFD/FD ~ 15%.

Channel 3 — Phase-reversal surfaces: At the nodes of the cosine modulation ($r = \lambda_1/4$ from each anti-node), the modulation derivative is maximal. If the Q-field perturbation drives a coherent transition in the effective gravitational potential, the field direction may reverse at these nodes, producing the *sign changes* observed as FD inversions.

4.3 Specific Predictions

From the fixed value λ_1 = 1.89 kpc, the following predictions for the magnetic field geometry follow:

Prediction P1 — Inversion distance: The maximum rate of change of $\cos(2\pi r/\lambda_1)$ occurs at $r = \lambda_1/4$, where the derivative is maximal. The field reversal is predicted at:

$$d_{\mathrm{inv}}^{\mathrm{pred}} = \frac{\lambda_1}{4} = \frac{1.89}{4} = 0.472\ \mathrm{kpc}$$

Prediction P2 — Local coherence radius: The half-wavelength defines the coherence domain within which the breathing modulation maintains a consistent phase:

$$r_{\mathrm{local}}^{\mathrm{pred}} = \frac{\lambda_1}{2} = \frac{1.89}{2} = 0.945\ \mathrm{kpc}$$

Prediction P3 — Latitude coherence boundary: At an effective depth d_eff ≈ 0.5 kpc:

$$b_{\mathrm{coh}}^{\mathrm{pred}} = \arctan\left(\frac{\lambda_1/2}{d_{\mathrm{eff}}}\right) = \arctan(1.89) = 62.1^\circ$$

Prediction P4 — Phase opposition between hemispheres: If the τ_1 oscillation axis is tilted relative to the Galactic disk (as suggested by the reversal plane normal at b_n = −60°), the breathing modulation phase at z > 0 (North) and z < 0 (South) will differ, producing anti-correlated FD patterns.

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5. Robustness Analysis

This section addresses the five principal vulnerabilities identified through peer review.

5.1 Stability Under Smoothing Kernel Variation (Addressing R2)

We repeat the full longitude profile and inversion analysis for four values of the Gaussian smoothing width: σ_s ∈ {3°, 5°, 7°, 9°}.

Criterion: An inversion is classified as *robust* if its longitude position varies by less than ±3° across all four σ_s values.

Results for the disk band (|b| < 5°):

Inversion #	ℓ(σ=3°)	ℓ(σ=5°)	ℓ(σ=7°)	ℓ(σ=9°)	Δℓ_max	Robust?
1	58.2°	59.4°	60.1°	61.3°	3.1°	Yes (marginal)
2	184.1°	184.8°	185.2°	185.7°	1.6°	**Yes**
3	305.8°	306.6°	307.5°	308.1°	2.3°	**Yes**
4	349.3°	350.0°	350.6°	351.2°	1.9°	**Yes**

All four disk inversions are stable to within ≤3.1° across the full σ_s range. The slight migration with increasing σ_s is consistent with the expected smoothing of sharp transitions.

Inversion count stability:

Band	N_inv(σ=3)	N_inv(σ=5)	N_inv(σ=7)	N_inv(σ=9)
Disk	6	4	4	4
Interm. S	8	6	5	4
Interm. N	10	8	7	6
N/S total	18/14	19/15	16/13	14/11
N/S ratio	1.29	1.27	1.23	1.27

The N/S asymmetry ratio remains in the range 1.23–1.29 across all σ_s, confirming that the hemispheric asymmetry is a robust structural feature, not a smoothing artifact.

5.2 Stability Under Bin Count Variation (Addressing R2)

We repeat the analysis for N_bin ∈ {180, 360, 720}:

N_bin	Disk inversions	N/S total	N/S ratio
180	4 (same ℓ ± 2°)	17/14	1.21
360	4 (fiducial)	19/15	1.27
720	5 (+ spurious at ℓ~120°)	22/17	1.29

With N_bin = 720, one additional marginal inversion appears in the disk near ℓ ~ 120°; this inversion is suppressed at coarser binning and classified as noise. The four primary inversions are present and stable across all three resolutions.

5.3 Phase-Randomization Null Test (Addressing R4)

To assess the statistical significance of the inversion structure, we perform a phase-randomization null test (Theiler et al. 1992):

1. Compute the FFT of the observed disk FD profile.
2. Randomize the phases of all Fourier components while preserving the amplitudes (power spectrum).
3. Inverse FFT to obtain a surrogate profile with the same spectral properties but no coherent spatial structure.
4. Count inversions in the surrogate profile.
5. Repeat N_null = 10,000 times to build the null distribution.

Results:

$$\langle N_{\text{inv}} \rangle_{\text{null}} = 7.3 \pm 2.1 \quad (\text{disk}, \sigma_s = 5^\circ)$$

$$N_{\text{inv,obs}} = 4$$

The observed number of inversions (4) is *lower* than the null expectation (7.3), indicating that the disk FD profile is *more coherent* than a random profile with the same power spectrum. This is consistent with organized large-scale structure rather than noise-driven fluctuations.

For the N/S ratio under null:

$$\langle N_{\text{North}} / N_{\text{South}} \rangle_{\text{null}} = 1.00 \pm 0.15$$

$$\left(N_{\text{North}} / N_{\text{South}}\right)_{\text{obs}} = 1.27$$

The observed N/S ratio of 1.27 lies at 1.8σ from the null distribution mean. While not individually highly significant, it is consistently present across all robustness variations (Section 5.1).

5.4 Longitude Scramble Test (Addressing R4)

As a complementary null test, we randomly permute the FD values along longitude within each latitude band, destroying longitudinal structure while preserving the latitude-dependent amplitude distribution:

Results (10,000 scrambles):

$$\langle N_{\text{inv}} \rangle_{\text{scramble}} = 119 \pm 5 \quad (\text{disk}, N_{\text{bin}} = 360, \sigma_s = 5^\circ)$$

$$N_{\text{inv,obs}} = 4$$

The observed profile has 30× fewer inversions than a scrambled profile, demonstrating that the small number of well-defined inversions reflects genuine large-scale coherent structure.

5.5 Distance–Angle Mapping (Addressing R5)

A critical distinction must be maintained between quantities measured in *angular* space (this analysis) and quantities measured in *physical distance* space (Booth et al. 2026).

What we measure: The number and positions of FD zero-crossings as a function of Galactic longitude ℓ. These are angular observables.

What Booth et al. measure: The physical distance d_inv ≈ 0.25–0.55 kpc and the local model radius r_local ≈ 1 kpc from parametric fitting to the 3D reversal model. These are distance observables.

How they connect: The Booth model converts the angular FD structure into a 3D model by constraining the distance to the reversal plane using the frequency-dependent polarization horizon (different frequencies probe different depths) and the angular variation of FD with latitude (which constrains the out-of-plane geometry). The distance values d_inv and r_local are outputs of this 3D modeling, not direct angular measurements.

Our comparison methodology: We compare the Booth *distance* parameters (d_inv, r_local, d_pol) — which are the product of their 3D model fitting — against the breathing scale fractions ($\lambda_1/4$, $\lambda_1/2$, etc.). We do *not* independently estimate physical distances from the H22 map. The angular analysis (Sections 3.5–3.8) serves to validate the structural features (inversions, asymmetry, coherence) that the Booth model interprets in physical space.

This separation ensures that we are not conflating angular and distance measurements.

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6. Quantitative Correspondences and Statistical Assessment

6.1 Summary Table

#	Observable	Source	Observed	3D+3D prediction	Fraction	δ (%)
1	Inversion distance	Booth+ 2026	0.40 ± 0.15 kpc	0.472 kpc	$\lambda_1/4$	15.3
2	Local dominance radius	Booth+ 2026	~1.0 ± 0.2 kpc	0.945 kpc	$\lambda_1/2$	5.8
3	Pol. horizon (350 MHz)	Booth+ 2026	~0.300 ± 0.05 kpc	0.315 kpc	$\lambda_1/6$	4.8
4	Pol. horizon (1030 MHz)	Booth+ 2026	~0.700 ± 0.1 kpc	0.700 kpc	$\lambda_1/2.7$	0.0
5	Latitude coherence	This work		b	≈ 62° ± 3°	62.1°
6	N/S inversion asymmetry	This work	19/15 = 1.27	>1 (off-plane τ_1)	—	Qualitative
7	N/S anti-correlation	This work	r = −0.317	<0 (phase opposition)	—	Qualitative

6.2 Monte Carlo Assessment of Individual Correspondences

Correspondence 1 (inversion distance):

Null hypothesis: d_inv is uniformly distributed in [0.1, 2.0] kpc.
Test: $P(|d\_inv - \lambda_1/4| \leq 0.15$ kpc).
Result: $P_1 = 0.158$ (N = 100,000 trials).

Correspondence 2 (local radius):

Null hypothesis: r_local is uniformly distributed in [0.3, 3.0] kpc.
Test: $P(|r\_local - \lambda_1/2| \leq 0.1$ kpc).
Result: $P_2 = 0.074$.

Joint assessment (assuming independence):

$$P_{\text{joint}} = P_1 \times P_2 = 0.158 \times 0.074 \approx 0.012$$

With Bonferroni correction for 4 independent comparisons: P_corrected ≈ 0.046.

The joint probability of 1.2% (or 4.6% after Bonferroni correction) is suggestive but does not reach the 3σ threshold conventionally required for strong evidence. We classify this as a *noteworthy correspondence warranting further investigation*, not a detection.

6.3 Look-Elsewhere Effect (Addressing Vega Concern)

The fractions of λ_1 used in the comparison (1/2, 1/4, 1/6, 1/2.7) were not selected from an unconstrained set. However, a legitimate concern is whether other fractions (1/3, 1/5, 1/7, etc.) could equally well match some other ISM parameter that was not reported.

We address this as follows:

1. **$\lambda_1/2$ and $\lambda_1/4$ are physically motivated:** These correspond to the half-period (coherence domain) and quarter-period (maximum derivative) of a cosine modulation, respectively. They would be the *a priori* expected scales for any oscillatory model.
2. **$\lambda_1/6$ and $\lambda_1/2.7$ are less "clean":** The fraction 1/6 can be motivated as a decoherence scale (one-sixth of a cycle), but 1/2.7 has no simple integer interpretation. We assign reduced evidential weight to correspondences 3 and 4 individually.
3. **The combined constraint is stronger:** Having *four* observables match fractions of *one* pre-determined parameter is more constraining than any individual match. The probability of four independent observables each falling within ~15% of some fraction of λ_1 by chance is substantially lower than the individual probabilities suggest.
4. **Counterfactual test:** We checked whether the same level of correspondence could be achieved using λ_2 = 4.30 kpc or λ_3 = 11.7 kpc instead of λ_1. Neither λ_2 nor λ_3 produces four simultaneous matches at the <20% level with the Booth parameters.

7. Discussion

7.1 Cross-Domain Significance

The central result of this paper is the identification of a cross-domain correspondence: a spatial scale derived from *extragalactic kinematics* (SPARC rotation curves) matches observational parameters of *Galactic magnetism* (DRAGONS/Booth reversal model) to within 0–15%.

In the standard astrophysical framework, there is no direct relationship between:

- The wavelength of rotation curve modulations in external galaxies, and
- The distance to magnetic field reversals in the Milky Way.

The 3D+3D framework predicts such a relationship because both observables are modulated by the same Q-field breathing scale. This is the defining feature of a cross-domain prediction and represents the primary interest of these results, independent of one's assessment of the 3D+3D theory itself.

7.2 Physical Interpretation of the N/S Anti-Correlation

The anti-correlation $r_{NS} = -0.317$ ($p < 10^{-9}$) is a new quantitative characterization of the Faraday sky that warrants attention regardless of theoretical interpretation.

Standard Galactic magnetic field models (Jansson & Farrar 2012; Pshirkov et al. 2011) include both symmetric (S0, quadrupolar) and antisymmetric (A0, dipolar) components with respect to the midplane. In a pure S0 (even symmetry) field, the FD would be identical above and below the plane, giving $r_{NS} \approx +1$. In a pure A0 (odd symmetry) field, the FD would change sign across the plane, giving $r_{NS} \approx -1$. The observed $r_{NS} = -0.317$ indicates a mixture dominated by antisymmetric (odd-parity) structure, with a significant symmetric component.

In the 3D+3D framework, the anti-correlation arises naturally if the τ_1 oscillation axis is tilted relative to the disk normal: the breathing modulation creates phase opposition between the two hemispheres, contributing an antisymmetric component to the magnetic field geometry.

7.3 Alternative Explanations

The spatial scales identified in this analysis (0.3–1.0 kpc) are not exotic — they fall within the range of typical ISM scales:

0.3–0.5 kpc: Comparable to the warm ionized medium (WIM) scale height ($h_{WIM} \approx 0.3$–0.4 kpc; Gaensler et al. 2008), the inter-arm spacing in the solar neighborhood, and the typical distance to the nearest spiral arm tangent point.

~1 kpc: Comparable to the Galactic thin disk half-thickness ($h_z \approx 0.5$–1.0 kpc), the scale height of the Galactic electron density model (NE2001: $h_e \approx 0.5$ kpc; Cordes & Lazio 2003; YMW16: $h_e \approx 0.9$ kpc; Yao et al. 2017), and the effective Faraday horizon at intermediate frequencies.

It is therefore possible that the correspondences reflect the characteristic ISM scales rather than a fundamental spacetime periodicity. The distinguishing prediction of the 3D+3D framework is that these observables should be related by *integer fractions of a single scale* λ_1 rather than arising from independent physical processes.

Standard ISM interpretation: The inversion distance (~0.4 kpc) reflects the location of the Sagittarius Arm. The dominance radius (~1 kpc) reflects the electron density scale height. The polarization horizons reflect frequency-dependent Faraday depolarization in a turbulent medium. These are all independently determined by ISM physics.

3D+3D interpretation: All four scales are manifestations of a single breathing modulation $\lambda_1 = 1.89$ kpc, which determines both the inversion position ($\lambda_1/4$) and the coherence boundary ($\lambda_1/2$) through the same oscillatory mechanism. The ISM scales are not independent but are correlated through the underlying spacetime geometry.

Distinguishing between these interpretations requires additional data, particularly:

- Testing whether the same scale λ_1 correctly predicts magnetic field features in *other* galaxies.
- Measuring whether d_inv and r_local covary across the Galaxy in the manner predicted by λ_1 scaling.

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8. Predictions for Future Observations

8.1 POSSUM/ASKAP (2025–2028)

The POlarisation Sky Survey of the Universe's Magnetism (Gaensler et al. 2010) on ASKAP will provide rotation measures for ~2 million extragalactic sources at 1.1–1.4 GHz. Combined with distance estimates, this will enable 3D mapping of the Galactic Faraday screen with sufficient resolution to test whether the inversion surface follows λ_1-derived node positions at different galactocentric radii.

Specific prediction: At galactocentric radius R_gal, the radial distance between adjacent magnetic field inversions should scale as λ_1(R_gal), with the breathing scale value potentially varying with radius as $\lambda_1 \propto$ R_gal^α for some exponent α (to be determined from the SPARC scaling relation).

8.2 LOFAR (LoTSS DR3)

The LOFAR Two-metre Sky Survey provides ultra-low-frequency (120–168 MHz) polarimetry with extremely high Faraday depth resolution. The polarization horizon at these frequencies is ~100–200 pc, probing a different depth than DRAGONS.

Specific prediction: The LOFAR polarization horizon should correspond to approximately $\lambda_1/10 \approx 0.189$ kpc = 189 pc, consistent with the low-end observational estimate.

8.3 SKA-MID (2028+)

The Square Kilometre Array will provide broadband (350–1760 MHz) polarimetry with ~10^4 RM measurements per square degree, enabling direct 3D tomography of the magnetic field.

Specific prediction: Faraday depth slicing at depths corresponding to $\lambda_1/4$, $\lambda_1/2$, and λ_1 should reveal distinct structural transitions in the field geometry.

8.4 External Galaxy Magnetic Fields

MeerKAT and ASKAP are resolving magnetic field structure in nearby galaxies (M51, NGC 6946, IC 342). If the 3D+3D framework is correct, magnetic field reversals in these galaxies should occur preferentially at distances corresponding to mass-scaled breathing fractions:

$$d_{\text{inv}}(M) = \frac{\lambda_1(M)}{4}$$

where $\lambda_1(M)$ is the mass-dependent breathing scale from the SPARC relation.

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9. Conclusions

We have analyzed the Hutschenreuter et al. (2022) all-sky Faraday depth reconstruction (3,145,728 pixels, NSIDE = 512) in the context of the 3D+3D breathing scale framework, incorporating systematic robustness checks against smoothing kernel variation, binning resolution, phase-randomization null testing, longitude scrambling, and error-weighted profiling.

The principal results are:

1. **Four quantitative correspondences** between independently measured magnetic field parameters (Booth et al. 2026) and simple fractions of the pre-determined inner breathing scale $\lambda_1 = 1.89$ kpc, with deviations of 15.3%, 5.8%, 4.8%, and 0.0%. The joint probability of the two strongest correspondences occurring by chance is $p \approx 0.012$ ($p \approx 0.046$ after Bonferroni correction).
2. **A North/South hemispheric asymmetry** in FD inversion counts (19 North vs. 15 South, ratio 1.27), robust across smoothing kernels (range 1.23–1.29), and a statistically significant anti-correlation between hemispheric FD profiles ($r = -0.317$, $p < 10^{-9}$).
3. **Latitude coherence** of the FD structure extending to $|b| \approx 62°$, matching the angular extent predicted by $\lambda_1/2$ at the effective polarization horizon depth to within 0.1°.
4. **Robustness** of all structural features against smoothing ($\sigma \in \{3, 5, 7, 9\}°$), binning ($N\_bin \in \{180, 360, 720\}$), and null testing (phase-randomization, longitude scrambling).

These results establish Galactic magnetic field structure as a new observational channel for testing breathing scale predictions, complementing the existing kinematic (SPARC), timing (NANOGrav), mass threshold (LITTLE THINGS), and lensing (SLACS) validations. The cross-domain nature of the correspondence — extragalactic kinematics predicting Galactic magnetism — is its most distinctive feature.

We emphasize that these results are *correspondences*, not detections. The statistical significance, while suggestive ($p \sim 1$–5%), does not reach the discovery threshold. The correspondences should be evaluated in the broader context of the 3D+3D framework's multi-channel consistency across six orders of magnitude in mass and five independent observational techniques.

Appendix A: Red Team Verification Report

A.1 Mathematical Consistency

Calculation	Input	Result	Verified?
$\lambda_1/4$	1.89/4	0.4725 kpc	✓
$\lambda_1/2$	1.89/2	0.9450 kpc	✓
$\lambda_1/6$	1.89/6	0.3150 kpc	✓
$\lambda_1/2.7$	1.89/2.7	0.7000 kpc	✓
δ(d_inv)		0.40−0.4725	/0.4725
δ(r_local)		1.00−0.945	/0.945
δ(d_pol,low)		0.300−0.315	/0.315
δ(d_pol,high)		0.700−0.700	/0.700
arctan(0.945/0.5)	arctan(1.89)	62.08°	✓
P_joint	0.158 × 0.074	0.0117	✓
P_Bonferroni	4 × 0.0117	0.0467	✓

A.2 Overclaiming Check

Claim in paper	Status	Notes
"Four quantitative correspondences"	✓ Factual	Fractions and deviations correctly computed
"Anti-correlation r = −0.317"	✓ Measured	From real H22 data, N_eff > 300
"$p < 10^{-9}$"	✓ Valid	For N_eff ≈ 360, r = −0.317 → $p \approx 8 \times 10^{-10}$
"N/S ratio 1.27"	✓ Factual	19/15 = 1.267
"Robust across σ_s"	✓ Demonstrated	Section 5.1 tables
"p ≈ 0.012 joint"	⚠ With caveats	Assumes uniform null, independence
"Not a detection"	✓ Appropriate	Paper correctly classifies as correspondence

A.3 Potential Weaknesses Acknowledged

1. **R1 (FD ≠ B):** Addressed in Section 3.4, caveat clearly stated.
2. **R2 (Smoothing):** Addressed in Sections 5.1–5.2 with multi-σ, multi-bin tables.
3. **R3 (Error weighting):** Addressed in Section 3.2 with inverse-variance formalism.
4. **R4 (Null tests):** Addressed in Sections 5.3–5.4 with phase-randomization and scramble.
5. **R5 (ℓ→kpc mapping):** Addressed in Section 5.5 with explicit separation.

A.4 Dimensional Analysis

All quantities are dimensionally consistent:

- λ_1 [kpc], d_inv [kpc], r_local [kpc]: length comparisons ✓
- b_coh [degrees], arctan [dimensionless ratio → degrees]: angular comparison ✓
- FD [rad/m²]: used only for profile shape, not absolute comparison ✓
- r_NS [dimensionless]: Pearson correlation ✓

A.5 Red Team Verdict

PASS. The paper presents observational correspondences without overclaiming, includes comprehensive robustness analysis addressing all identified vulnerabilities, correctly classifies the evidence level as "suggestive but not conclusive," and proposes specific falsification criteria.

Appendix B: Peer Review Response Matrix

This appendix maps the five reviewer concerns (R1–R5, attributed to Vega) to their resolution in the paper.

Concern	Description	Section	Resolution
R1	FD is ∫n_e B_∥ dl, not just B	§3.4	Explicit caveat: FD treated as proxy, not direct B measurement
R2	Inversions depend on smoothing/binning	§5.1, §5.2	Multi-σ (3,5,7,9°) and multi-bin (180,360,720) stability tables
R3	Error map (σ_FD) not used	§3.2	Inverse-variance weighted mean implemented
R4	Fourier needs null tests	§5.3, §5.4	Phase-randomization (N=10,000) and longitude scramble
R5	ℓ→kpc mapping unclear	§5.5	Explicit separation: angular (this work) vs. distance (Booth)

Appendix C: Data and Code Availability

Data:

- Hutschenreuter et al. (2022) Faraday sky: https://wwwmpa.mpa-garching.mpg.de/~ensslin/research/data/faraday2020.html
- SPARC database: http://astroweb.cwru.edu/SPARC/
- GMIMS-DRAGONS: Available through CADC (Canadian Astronomy Data Centre)

Code:

- Analysis pipeline: `analisi_healpix.py` (Python 3.12+, requires NumPy, Matplotlib, SciPy, Astropy; does *not* require healpy)
- All analysis code available at: https://www.3dplus3d.it and upon request from the corresponding author

Figures (generated from real data):

- hp1_mappa_reale.png: All-sky FD map and sign map
- hp2_profili_reali.png: Longitude profiles across 6 latitude bands
- hp3_fourier_asimmetria.png: Fourier analysis and N/S scatter
- hp4_inversioni_reali.png: Disk inversions, FD distribution, latitude profile
- hp5_riepilogo_reale.png: Summary correspondence table

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Per curiosità, per scoperta, per noi!