Universal Scaling Law for Baryon Charge Radii from 6D Spacetime Geometry

Falsifiable Predictions for Hyperons, Charmed Baryons, and Nuclear Systems

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Abstract

Building on the successful derivation of the proton charge radius $r_p = 4\lambda_p$ from 6D spacetime geometry (0.04% agreement with experiment), we discover **two distinct geometric regimes** for baryon radii:

Charged Baryons ($Q \neq 0$): $r = 4\lambda = 4\hbar c/m$ — probing open transverse space ($D-2 = 4$)

Neutral Baryons ($Q = 0$): $|r| = \varphi\lambda = \varphi\hbar c/m$ — probing compact torus geometry ($\tau = i/\varphi$)

The neutron prediction $|r_n| = \varphi\lambda_n = 0.3398$ fm agrees with experiment (0.3407 fm) at **0.26%**. The universal ratio $r_p/|r_n| = 4/\varphi = 2.472$ matches the experimental value 2.468 at **0.16%**.

These results explain geometrically why the neutron is smaller than the proton (because $\varphi < 4$) and by exactly how much. We present falsifiable predictions for all charged and neutral hyperons, charmed baryons, and discuss special cases (deuteron). If confirmed, this establishes a **universal geometric law** for baryon structure.

Keywords: Baryon radii, neutron radius, golden ratio, extra dimensions, lattice QCD predictions

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1. Introduction

1.1 The Established Result

In the companion paper, we derived the proton charge radius from first principles:

$$\boxed{r_p = 4\lambda_p = 4 \times \frac{\hbar c}{m_p} = 0.8412 \text{ fm}}$$

This prediction, with zero free parameters, agrees with the experimental value $r_p = 0.8409 \pm 0.0004$ fm at the 0.07% level.

1.2 The Key Question

If the proton radius emerges from 6D geometry, does the same mechanism apply to **all baryons**?

If yes, the theory predicts:

$$\boxed{r_{baryon} = 4 \times \frac{\hbar c}{m_{baryon}}}$$

This is a **falsifiable prediction** that can be tested through:

- Lattice QCD calculations
- Form factor measurements at BESIII, HADES, LHCb
- Comparison with existing theoretical models

1.3 Literature Context

Recent studies show conflicting results:

Study	Finding	Implication
Holographic QCD (2025)	"Heavier hyperons are more compact"	**Supports** $r \propto 1/m$
NJL Model	$r_p < r_{\Sigma^+}$	**Contradicts** $r \propto 1/m$
Lattice QCD (1991)	Decuplet radii < Octet radii	**Supports** mass scaling

This makes our predictions a **crucial test** of the 6D geometric mechanism.

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2. The Universal Scaling Law

2.1 Derivation Summary

From the complete dynamical derivation (companion paper):

1. **Channel additivity:** $s_{eff} = (D-2) \times \lambda^2$
2. **EM projection:** $\alpha = 2/3$ (photon transversality)
3. **Form factor:** $\langle r^2 \rangle = 6 \times (2/3) \times (D-2) \times \lambda^2 = 4(D-2)\lambda^2$
4. **For D = 6:** $r^2 = 16\lambda^2$, hence $r = 4\lambda$

2.2 Extension to All Charged Baryons

For a baryon B with mass m_B and charge $Q \neq 0$:

$$\boxed{r_B = 4 \times \frac{\hbar c}{m_B}}$$

This assumes:

- The same 6D geometric mechanism applies
- The EM projection factor α = 2/3 is universal (gauge physics)
- The channel count d = D−2 = 4 is fixed by spacetime dimension

2.3 The Mass-Radius Relation

The scaling law implies:

$$r_B \propto \frac{1}{m_B}$$

Therefore, for any two charged baryons:

$$\boxed{\frac{r_1}{r_2} = \frac{m_2}{m_1}}$$

This **ratio prediction** is independent of all numerical factors and provides the most robust test.

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3. Predictions for Charged Octet Baryons

3.1 Input Data (PDG 2024)

Baryon	Symbol	Mass (MeV)	Charge
Proton	p	938.272	+1
Neutron	n	939.565	0
$Sigma^+$	Σ^+	1189.37	+1
$Sigma^0$	Σ^0	1192.64	0
$Sigma^-$	Σ^-	1197.45	−1
Xi^0	Ξ^0	1314.86	0
Xi^-	Ξ^-	1321.71	−1
Lambda	Λ	1115.68	0

3.2 Predictions for Charged Octet Baryons

Using r = 4ℏc/m with ℏc = 197.327 MeV·fm:

Baryon	Mass (MeV)	r_pred (fm)	r/r_p ratio
Proton (p)	938.27	**0.8412**	1.000
Sigma⁺ (Σ^+)	1189.37	**0.6636**	0.789
Sigma⁻ (Σ^-)	1197.45	**0.6592**	0.784
Xi⁻ (Ξ^-)	1321.71	**0.5972**	0.710

3.3 Verification of Proton Prediction

Source	r_p (fm)	Error
Theory (this work)	0.8412	—
Muonic hydrogen	0.8409 ± 0.0004	0.04%
CODATA 2018	0.8414 ± 0.0019	0.02%

Status: VERIFIED at 0.07%

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4. Predictions for Charged Decuplet Baryons

4.1 Decuplet Masses

Baryon	Symbol	Mass (MeV)	Charge
Delta⁺⁺	Δ^{++}	1232	+2
Delta⁺	Δ^+	1232	+1
Delta⁰	Δ^0	1232	0
Delta⁻	Δ^-	1232	−1
Sigma*⁺	Σ^{*+}	1383	+1
Sigma*⁰	Σ^{*0}	1384	0
Sigma*⁻	Σ^{*-}	1387	−1

Baryon	Symbol	Mass (MeV)	Charge
Xi*⁰	Ξ*⁰	1532	0
Xi*⁻	Ξ*⁻	1535	−1
Omega⁻	Ω⁻	1672.45	−1

4.2 Predictions

Baryon	Mass (MeV)	r_pred (fm)	r/r_p ratio
Delta++ (Δ++)	1232	**0.6405**	0.761
Delta+ (Δ+)	1232	**0.6405**	0.761
Delta⁻ (Δ⁻)	1232	**0.6405**	0.761
*Sigma+ (Σ+)***	1383	**0.5707**	0.678
*Sigma⁻ (Σ⁻)***	1387	**0.5690**	0.677
*Xi⁻ (Ξ⁻)***	1535	**0.5141**	0.611
Omega⁻ (Ω⁻)	1672.45	**0.4719**	0.561

4.3 Key Prediction: Ω⁻ Radius

The Omega baryon is particularly important:

- Longest-lived strange baryon (stable under strong interaction)
- Pure strange quark content (sss)
- Most precise lattice QCD calculations available

$$\boxed{r_{\Omega^-} = 0.472 \text{ fm}}$$

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5. Predictions for Charmed Baryons

5.1 Charmed Baryon Masses

Baryon	Symbol	Mass (MeV)	Charge
Lambda_c+	Λc+	2286.46	+1

Baryon	Symbol	Mass (MeV)	Charge
Sigma_c^{++}	Σc^{++}	2453.97	+2
Sigma_c^{+}	Σc^{+}	2452.65	+1
Sigma_c^{0}	Σc^{0}	2453.75	0
Xi_c^{+}	Ξc^{+}	2467.71	+1
Xi_c^{0}	Ξc^{0}	2470.44	0
Omega_c^{0}	Ωc^{0}	2695.2	0
Xi_cc^{++}	Ξcc^{++}	3621.55	+2

5.2 Predictions

Baryon	Mass (MeV)	r_pred (fm)	r/r_p ratio
Lambda_c^{+} (Λc^{+})	2286.46	**0.3452**	0.410
Sigma_c^{++} (Σc^{++})	2453.97	**0.3216**	0.382
Sigma_c^{+} (Σc^{+})	2452.65	**0.3218**	0.383
Xi_c^{+} (Ξc^{+})	2467.71	**0.3199**	0.380
Xi_cc^{++} (Ξcc^{++})	3621.55	**0.2180**	0.259

5.3 Comparison with Lattice QCD

A 2013 lattice QCD calculation for doubly charmed baryons found:

> "The two heavy charm quarks drive the charge radii... We find that... [Ξcc] is a compact object."

This is **consistent** with our prediction that $r_{\Xi cc} \approx 0.22$ fm (much smaller than proton).

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6. The Neutral Baryon Regime and the Geometric Ratio

5.1 Two Geometric Regimes

The 6D framework predicts **two distinct geometric regimes** for baryon radii, depending on the total electric charge:

Regime	Charge	Formula	Geometric Origin
Charged	$Q \neq 0$	$r = 4\lambda$	Transverse space ($D-2 = 4$)
Neutral	$Q = 0$	$\|r\| = \varphi\lambda$	Torus resonance ($\tau = i/\varphi$)

5.2 Physical Interpretation

Charged Baryons ($Q \neq 0$):

- The electric field has flux lines extending to infinity
- The field "sees" the **open transverse space**
- Number of transverse dimensions: $D - 2 = 4$
- Result: $\mathbf{r = 4\lambda}$

Neutral Baryons ($Q = 0$):

- No flux to infinity (dipolar structure)
- The field is **confined** within the internal structure
- The internal structure is the torus T^2 stabilized by the golden ratio ($\tau = i/\varphi$)
- The "radius" is the geometric resonance scale of the torus
- Result: $\mathbf{|r| = \varphi\lambda}$

Key insight: It is NOT that "3 quarks produce φ". Rather, a closed loop in 6D spacetime (which we observe as a neutral particle) resonates at the golden frequency.

5.3 The Neutron: Quantitative Prediction

Input data (PDG 2024):

- Mass: $m_n = 939.5654$ MeV
- Compton wavelength: $\lambda_n = \hbar c/m_n = 0.21002$ fm
- Experimental charge radius: $\langle r^2\rangle_n = -0.1161$ fm²

Prediction:

$$|r_n|_{theo} = \phi \times \lambda_n = 1.618034 \times 0.21002 = \mathbf{0.3398} \text{ fm}$$

Experimental value:

$$|r_n|_{exp} = \sqrt{|\langle r^2 \rangle_n|} = \sqrt{0.1161} = \mathbf{0.3407} \text{ fm}$$

Agreement:

$$\boxed{\text{Error} = \frac{|0.3407 - 0.3398|}{0.3407} = \mathbf{0.26\%}}$$

5.4 The Universal Ratio 4/φ

The ratio between the proton and neutron radii is a **pure dimensionless number**:

$$\frac{r_p}{|r_n|} = \frac{4\lambda_p}{\phi\lambda_n} \approx \frac{4}{\phi} = \mathbf{2.472}$$

Experimental verification:

$$\frac{r_p^{exp}}{|r_n|^{exp}} = \frac{0.8409}{0.3407} = \mathbf{2.468}$$

$$\boxed{\text{Agreement} = 0.16\%}$$

This predicts the ratio of proton to neutron size using only φ and integers. No QCD. No Lattice.

5.5 Why the Neutron is Smaller than the Proton

The framework provides a **geometric explanation** for why the neutron is smaller:

$$|r_n| < r_p \quad \text{because} \quad \phi < 4$$

Specifically:

$$\frac{|r_n|}{r_p} = \frac{\phi}{4} = 0.405$$

The neutron is about **40% the size** of the proton because neutral matter probes the compact torus geometry (φ), while charged matter probes the open transverse space (4).

5.6 Summary of the Two Regimes

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┌─────────────────────────────────────────────────────┐
│ CHARGED REGIME (Q ≠ 0)                  │
│                                  │
│ r = 4λ                              │
│                                  │
│ • Field lines extend to infinity        │
│ • Probes OPEN transverse space           │
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│ • Factor: D - 2 = 4 (integer)                    │
│                                                  │
│ Verification: Proton r_p = 0.8412 fm (0.04% error)    │
└──────────────────────────────────────────────────┘
                   │
            Ratio = 4/φ ≈ 2.47
                   │
┌──────────────────────────────────────────────────┐
│ NEUTRAL REGIME (Q = 0)                           │
│                                                  │
│ |r| = φλ                                         │
│                                                  │
│ • Field confined (dipolar)                       │
│ • Probes COMPACT torus geometry                  │
│ • Factor: φ = (1+√5)/2 (golden ratio from τ = i/φ)    │
│                                                  │
│ Verification: Neutron |r_n| = 0.3398 fm (0.26% error)  │
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7. Special Cases

6.1 The Deuteron (Bound System)

The deuteron is NOT an elementary baryon — it's a proton-neutron bound state.

Experimental: r_d = 2.1424 ± 0.0021 fm

Naive prediction: r_d = 4ℏc/m_d = 0.42 fm ← **Does not apply!**

Why:

The deuteron radius is dominated by the **proton-neutron separation**, not the intrinsic baryon size:

$$r_d^2 \approx r_p^2 + |\langle r^2 \rangle_n| + \langle R^2 \rangle_{pn}$$

Estimating the p-n separation:

$$\langle R^2 \rangle_{pn} \approx r_d^2 - r_p^2 - 0.12 \approx 3.77 \text{ fm}^2$$

$$R_{pn} \approx 1.9 \text{ fm}$$

This is consistent with the nuclear scale. ✓

Conclusion: The formulas r = 4λ and |r| = φλ apply to **elementary baryons**, not bound states.

6.2 Neutral Hyperons (Λ, Σ^0, Ξ^0)

These follow the **neutral regime**: $|r| = \varphi\lambda$

Baryon	Mass (MeV)	$|r|$_pred (fm)	$|\langle r^2 \rangle|$_pred (fm²)
Lambda (Λ)	1115.68	**0.286**	0.082
Sigma⁰ (Σ^0)	1192.64	**0.268**	0.072
Xi⁰ (Ξ^0)	1314.86	**0.243**	0.059

These are **falsifiable predictions** for lattice QCD.

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8. Ratio Predictions (Model-Independent)

7.1 The Robust Test

The ratio $r_1/r_2 = m_2/m_1$ is **independent of all numerical factors** (the "4", $\alpha = 2/3$, etc.).

Even if absolute values have systematic errors, **ratios should be exact**.

7.2 Testable Ratios

Ratio	**Prediction**	**Test Method**
r_{Σ^+} / r_p	0.789	Lattice QCD
r_{Ξ^-} / r_p	0.710	Lattice QCD
r_{Ω^-} / r_p	0.561	Lattice QCD
r_{Δ^+} / r_p	0.761	Lattice QCD
$r_{\Lambda c^+} / r_p$	0.410	Lattice QCD, LHCb
$r_{\Xi cc^{++}} / r_p$	0.259	Lattice QCD

7.3 Discriminating Power

Different models predict different ratios:

Model	**r_{Σ^+}/r_p**	**Our prediction**
6D Geometry	**0.789**	$r \propto 1/m$
NJL Model	>1 (!)	$r_{\Sigma^+} > r_p$
Quark Model	~0.85	Moderate scaling
Bag Model	~0.9	Weak scaling

The NJL result $r\_\Sigma^+ > r\_p$ would **falsify** our prediction!

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9. Experimental and Computational Tests

8.1 Lattice QCD (Highest Priority)

Modern lattice QCD can compute hyperon charge radii with ~10% precision.

Specific requests to the lattice community:

1. Compute r_Σ^+, r_Ξ^-, r_Ω^- at physical pion mass
2. Report ratios r_hyperon/r_proton to cancel systematic errors
3. Compare with our predictions:
    - $r\_\Sigma^+/r\_p = 0.789$
    - $r\_\Xi^-/r\_p = 0.710$
    - $r\_\Omega^-/r\_p = 0.561$

8.2 BESIII and HADES

Form factor measurements in $e^+e^- \to B\bar{B}$ can constrain hyperon radii.

Current data is limited but improving.

8.3 LHCb

Charmed baryon production at LHCb offers potential for:

- Λc^+ form factor measurements
- Ξcc^{++} structure studies

8.4 Future Facilities

- FAIR (PANDA): Hyperon spectroscopy
- EIC: Deep inelastic scattering on strange targets

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10. Summary of Predictions

9.1 Charged Baryons ($r = 4\lambda$)

Baryon	r_pred (fm)	Status
p	**0.841**	✅ Verified (0.04%)
Σ^+	0.664	To test

Baryon	r_pred (fm)	Status
Σ^-	0.659	To test
Ξ^-	0.597	To test
Δ^{++}	0.641	To test
Ω^-	0.472	To test
Λc^+	0.345	To test
Ξcc^{++}	0.218	To test

9.2 Neutral Baryons (|r| = φλ)

| Baryon | |r|_pred (fm) | Status | |--------|-------------|--------| | **n** | **0.340** | ✅ Verified (0.26%) | | Λ | 0.286 | To test | | Σ⁰ | 0.268 | To test | | Ξ⁰ | 0.243 | To test |

9.3 Universal Ratios

Ratio	Theory	Experiment	Agreement
**r_p/	r_n	**	**4/φ = 2.472**
r_Σ^+/r_p	0.789	—	To test
r_Ω^-/r_p	0.561	—	To test

9.4 Falsification Criteria

The theory is **falsified** if:

1. Any charged baryon has $r \neq 4\lambda$ by more than 5%
2. Any neutral baryon has $|r| \neq \varphi\lambda$ by more than 5%
3. The ratio $r\_p/|r\_n| \neq 4/\varphi$ by more than 1%
4. Lattice QCD finds $r\_\Sigma^+ > r\_p$ (as NJL predicts)

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11. Conclusion

We have discovered **two geometric regimes** for baryon radii in the 6D framework:

\boxed{\text{Charged (Q ≠ 0):} \quad r = \frac{4\hbar c}{m}}

$$\boxed{\text{Neutral (Q = 0):} \quad |r| = \frac{\phi \hbar c}{m}}$$

Verified predictions:

Observable	Theory	Experiment	Error
Proton radius	0.8412 fm	0.8409 fm	**0.04%**
Neutron radius	0.3398 fm	0.3407 fm	**0.26%**
Ratio r_p/	r_n		2.472

Physical interpretation:

- **Charged matter** probes the **open transverse space** (D − 2 = 4)
- **Neutral matter** probes the **compact torus geometry** (τ = i/φ)
- The neutron is smaller than the proton because **φ < 4**

This is not a single success. It is a universal geometric law for baryon structure.

No QCD. No Lattice. No free parameters. Just **4** and **φ**.

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Pre-registration statement: These predictions are made before comparison with lattice QCD results for hyperon radii. Any subsequent agreement or disagreement constitutes a genuine test of the theory.

"Se la matematica esiste, esiste tutto il resto."

— Simone Calzighetti