6D QFT with Signature (-,+,+,+,-,-): Mathematical Exploration

Pure Research Mode - No Paper Goals

Date: November 21, 2025
Philosophy: Discover what works and what doesn't - EDISON MODE

🎯 OBJECTIVE:

Understand quantum field theory in 6D spacetime with signature (-,+,+,+,-,-) through explicit calculations.

NOT worrying about: Papers, publications, deadlines, community opinion
ONLY caring about: Does the math work? What happens physically?

📐 CALCULATION 1: FREE FIELD PROPAGATOR

Setup
6D flat spacetime:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 - d\tau_2^2 - d\tau_3^2$$

Compactification:

$$\tau_2 \sim \tau_2 + 2\pi L_2 \quad (L_2 = 9.5 \text{ ly})$$
$$\tau_3 \sim \tau_3 + 2\pi L_3 \quad (L_3 = 6.0 \text{ ly})$$

Free scalar field action:

```
S = ∫ d⁴x dτ₂ dτ₃ [
    1/2 ∂_M φ η^MN ∂_N φ - 1/2 m² φ²
]

where η^MN = diag(-1, +1, +1, +1, -1, -1)
```

Explicitly:

$$
\begin{aligned}
S = \int d^4x\, d\tau_2\, d\tau_3 \, [\\
&-1/2\, (\partial\_t\, \varphi)(-\partial\_t\, \varphi) && [= +1/2\, (\partial\_t\, \varphi)^2] \\
&+1/2\, (\partial\_x\, \varphi)(+\partial\_x\, \varphi) && [= +1/2\, (\partial\_x\, \varphi)^2] \\
&+1/2\, (\partial\_y\, \varphi)(+\partial\_y\, \varphi) && [= +1/2\, (\partial\_y\, \varphi)^2] \\
&+1/2\, (\partial\_z\, \varphi)(+\partial\_z\, \varphi) && [= +1/2\, (\partial\_z\, \varphi)^2] \\
&-1/2\, (\partial\_\tau_2\, \varphi)(-\partial\_\tau_2\, \varphi) && [= +1/2\, (\partial\_\tau_2\, \varphi)^2] \\
&-1/2\, (\partial\_\tau_3\, \varphi)(-\partial\_\tau_3\, \varphi) && [= +1/2\, (\partial\_\tau_3\, \varphi)^2] \\
&- 1/2\, m^2\, \varphi^2 \\
]
\end{aligned}
$$

So:

$$
\begin{aligned}
S = \int d^4x\, d\tau_2\, d\tau_3 \, [\\
&+1/2\, (\partial\_t\, \varphi)^2 - 1/2\, (\nabla\varphi)^2 \\
&+1/2\, (\partial\_\tau_2\, \varphi)^2 + 1/2\, (\partial\_\tau_3\, \varphi)^2 \\
&- 1/2\, m^2\, \varphi^2 \\
]
\end{aligned}
$$

Fourier Decomposition

Because τ_2, τ_3 are compactified:

$$
\varphi(x^\mu, \tau_2, \tau_3) = \Sigma\_\{n_2,n_3\}\ \varphi\_\{n_2,n_3\}(x^\mu)\ \exp[i(n_2\tau_2/L_2 + n_3\tau_3/L_3)]
$$

where $n_2, n_3 \in \mathbb{Z}$.

Kinetic terms:

$$
\begin{aligned}
&\partial\_\tau_2\, \varphi = \Sigma\ i(n_2/L_2)\ \varphi\_\{n_2,n_3\}\ \exp[...] \\
&\partial\_\tau_3\, \varphi = \Sigma\ i(n_3/L_3)\ \varphi\_\{n_2,n_3\}\ \exp[...]
\end{aligned}
$$

Integral over τ_2, τ_3:

$$
\begin{aligned}
&\int_0\^\{2\pi L_2\}\ d\tau_2 \int_0\^\{2\pi L_3\}\ d\tau_3\ \exp[i((n_2-n_2')\tau_2/L_2 + (n_3-n_3')\tau_3/L_3)] \\
&\quad = (2\pi L_2)(2\pi L_3)\ \delta\_\{n_2,n_2'\}\ \delta\_\{n_3,n_3'\}
\end{aligned}
$$

Action becomes:

$$
\begin{aligned}
S = (2\pi L_2)(2\pi L_3)\ \Sigma\_\{n_2,n_3\} \int d^4x \, [\\
&+1/2\, (\partial\_t\, \varphi\_\{n_2,n_3\})^2 - 1/2\, (\nabla\varphi\_\{n_2,n_3\})^2 \\
&- 1/2\, (n_2/L_2)^2\, |\varphi\_\{n_2,n_3\}|^2 \\
&- 1/2\, (n_3/L_3)^2\, |\varphi\_\{n_2,n_3\}|^2 \\
&- 1/2\, m^2\, |\varphi\_\{n_2,n_3\}|^2 \\
]
\end{aligned}
$$

Effective 4D action per KK mode (n_2, n_3):

$$\begin{aligned}
&S_{n_2,n_3} = \int d^4x \, [\\
&\quad +1/2\, (\partial_\mu \varphi)(\eta^{\mu\nu} \partial_\nu \varphi) - 1/2\, M^2_{n_2,n_3}\, \varphi^2 \\
&]
\end{aligned}$$

where:

$$M^2_{n_2,n_3} = m^2 + (n_2/L_2)^2 + (n_3/L_3)^2$$

WAIT! Sign is POSITIVE on $(n/L)^2$ terms!

Let me recalculate carefully...

CAREFUL RECALCULATION:

Original action with signature:

$$S = \int \sqrt{(-g)}\, [-1/2\, g^{MN} \partial_M \varphi\, \partial_N \varphi - 1/2\, m^2 \varphi^2]$$

In flat space $g^{MN} = \eta^{MN}$:

$$S = \int [-1/2\, \eta^{MN} \partial_M \varphi\, \partial_N \varphi - 1/2\, m^2 \varphi^2]$$

With $\eta^{MN} = \mathrm{diag}(-1,+1,+1,+1,-1,-1)$:

$$\begin{aligned}
&-1/2\, \eta^{MN} \partial_M \varphi\, \partial_N \varphi = \\
&\quad -1/2\, [\\
&\qquad -1\, (\partial_t \varphi)^2 + 1\, (\partial_x \varphi)^2 + 1\, (\partial_y \varphi)^2 + 1\, (\partial_z \varphi)^2 \\
&\qquad -1\, (\partial_{\tau_2} \varphi)^2 - 1\, (\partial_{\tau_3} \varphi)^2 \\
&\quad] \\
&\quad = +1/2\, (\partial_t \varphi)^2 - 1/2\, (\nabla\varphi)^2 + 1/2\, (\partial_{\tau_2} \varphi)^2 + 1/2\, (\partial_{\tau_3} \varphi)^2
\end{aligned}$$

OK, so action is:

$$\begin{aligned}
&S = \int [\\
&\quad +1/2\, (\partial_t \varphi)^2 - 1/2\, (\nabla\varphi)^2 \\
&\quad +1/2\, (\partial_{\tau_2} \varphi)^2 + 1/2\, (\partial_{\tau_3} \varphi)^2 \\
&\quad - 1/2\, m^2 \varphi^2 \\
&]
\end{aligned}$$

After Fourier expansion and integration:

$$\begin{aligned}
&\int d\tau_2\, d\tau_3\, (\partial_{\tau_2} \varphi)^2 \rightarrow \int d\tau_2\, d\tau_3\, [i(n_2/L_2)\, \varphi_{n_2,n_3}]^2 \\
&\quad = -(n_2/L_2)^2 \int d\tau_2\, d\tau_3\, |\varphi_{n_2,n_3}|^2 \\
&\quad = -(n_2/L_2)^2\, (2\pi L_2)(2\pi L_3)\, |\varphi_{n_2,n_3}|^2
\end{aligned}$$

So the action term is:

$$+1/2 \int (\partial\_\tau_2 \varphi)^2 d\tau_2 d\tau_3 = -1/2 (n_2/L_2)^2 (2\pi L_2)(2\pi L_3) |\varphi\_\{n_2,n_3\}|^2$$

AH! Negative sign from $(i)^2 = -1$!

Effective 4D action:

$$\begin{aligned}
&S_\{n_2,n_3\} = \int d^4x \, [\\
&\quad +1/2 (\partial_t \varphi)^2 - 1/2 (\nabla\varphi)^2 \\
&\quad - 1/2 [(n_2/L_2)^2 + (n_3/L_3)^2] \varphi^2 \\
&\quad - 1/2 m^2 \varphi^2 \\
&]
\end{aligned}$$

Effective mass-squared:

$$M^2\_eff = m^2 - (n_2/L_2)^2 - (n_3/L_3)^2$$

CRITICAL:

For $(n_2, n_3) = (0, 0)$: $M^2\_eff = m^2$ ✅ (OK)

For large $|n_2|$, $|n_3|$: $M^2\_eff < 0$ ⚠️ (TACHYON!)

🔴 TACHYON ANALYSIS

What is a tachyon?

Particle with $M^2 < 0 \rightarrow$ imaginary mass

Klein-Gordon equation:

$$(\square + M^2) \varphi = 0$$

If $M^2 < 0$:

$$(\square - |M^2|) \varphi = 0$$

Plane wave solution:

$$\varphi \sim \exp(-i\omega t + ik\cdot x)$$

Dispersion relation:

$$\begin{aligned}
&-\omega^2 + k^2 = -|M^2| \\
&\omega^2 = k^2 + |M^2|
\end{aligned}$$

Looks OK? But in Minkowski signature:

$$k^2 = -\omega^2 + k\square^2 \quad \text{(4-momentum squared)}$$

If on-shell:

$$k^2 = -M^2\_eff = +|M^2|$$

$$-\omega^2 + k\square^2 = +|M^2|$$

$$\omega^2 = k\square^2 - |M^2|$$

If $|M^2| > k\square^2$: $\omega^2 < 0 \rightarrow \omega$ imaginary!

Solution grows exponentially:

$$\varphi \sim \exp(|\omega|t) + \exp(-|\omega|t)$$

INSTABILITY! 💥

Quantum interpretation

Vacuum not at $\varphi = 0$!

Tachyon field wants to "roll" to minimum of potential.

Classical potential:

$$V(\varphi) = +1/2\ |M^2|\ \varphi^2 \quad \text{(upside-down parabola!)}$$

No minimum at $\varphi = 0$. Vacuum unstable!

Field will evolve to:

- Develop VEV (vacuum expectation value)
- Spontaneous symmetry breaking?
- Or run away to infinity?

🤔 POSSIBLE RESOLUTION PATHS

Path A: Interactions stabilize vacuum

Idea: Add φ^4 interaction

$$V(\varphi) = -1/2\ |M^2|\ \varphi^2 + \lambda\ \varphi^4$$

Minima at:

$$\varphi\_min = \pm\sqrt{(|M^2|/4\lambda)}$$

Vacuum at $\varphi \neq 0$!

Question: What does this mean physically?

For each KK mode (n_2, n_3) with $M^2<0$, field develops VEV!

Infinite tower of VEVs? 🤯

Path B: Compactification radius constraint

Idea: Maybe L_2, L_3 must be large enough that NO modes are tachyonic?

Condition:

$$M^2_{n_2,n_3} = m^2 - (n_2/L_2)^2 - (n_3/L_3)^2 \geq 0 \quad \forall\, n_2,n_3$$

Most dangerous mode: $n_2 = \pm1$, $n_3 = 0$ (or $n_2=0$, $n_3=\pm1$)

Require:

$$m^2 - (1/L_2)^2 \geq 0$$
$$L_2 \geq 1/m$$

For m ~ 10^{-24} eV (our Q-field masses):

$L_2 \geq \hbar/(mc) \sim 10$ ly ✅ (We have $L_2 = 9.5$ ly - close!)

INTERESTING: Our values are RIGHT at the boundary!

Path C: Ghost decay (Donoghue mechanism)

Idea: Tachyon modes are not stable - they DECAY!

Add interactions with ordinary matter:

$$L_{int} = g\, \varphi_{n_2,n_3}\, \chi^\dagger\chi$$

where χ = ordinary matter field.

Tachyon can decay:

$\varphi_{tachyon} \to \chi + \chi^\dagger$ (if kinematically allowed)

Decay width:

$\Gamma \sim g^2/M_{tachyon}$ (rough estimate)

Modified propagator:

$$\Delta(k^2) = 1 / [k^2 - M^2 + iM\Gamma]$$

Pole shifted off real axis!

Physical consequence:

- Tachyon not in asymptotic spectrum
- Exists only as virtual particle
- Lifetime $\tau \sim \hbar/\Gamma$

Question: What is g in our theory? How large is Γ?

📊 PATH INTEGRAL ANALYSIS

Euclidean path integral

Wick rotate: $t \to -i\tau_E$

Action becomes:

```
S_E = ∫ d⁴x_E dτ₂ dτ₃ [
   +1/2 (∂_τ_E φ)² + 1/2 (∇φ)²
   +1/2 (∂_τ₂ φ)² + 1/2 (∂_τ₃ φ)²
   + 1/2 m² φ²
]
```

All positive! ✅

But wait - what about KK expansion?

After Fourier on τ_2, τ_3:

```
S_E = ∫ d⁴x_E Σ [
   +1/2 (∂_τ_E φ_{n₂,n₃})² + 1/2 (∇φ_{n₂,n₃})²
   - 1/2 (n₂/L₂)² φ² - 1/2 (n₃/L₃)² φ²
   + 1/2 m² φ²
]
```

Effective Euclidean mass:

```
M²_E = m² - (n₂/L₂)² - (n₃/L₃)²
```

Still can be negative!

Path integral:

```
Z_E = ∫ Dφ exp[-S_E]
```

For $M^2_E < 0$:

```
exp[-∫ (+1/2 |M²_E| φ²)] = exp[+∫ 1/2 |M²_E| φ²]
```

Gaussian with WRONG SIGN! Integral diverges! 💥

Euclidean path integral NOT well-defined for tachyon modes!

💡 KEY INSIGHT FROM CALCULATIONS

The core problem:
With signature (-,+,+,+,-,-) and compactification:

Unavoidable consequence:

```
KK modes have: M²_eff = m² - (n/L)²
```

For large enough n: $M^2\_eff < 0$ (tachyon!)

This causes:

1. **Classically:** Hamiltonian unbounded (Ostrogradsky)
2. **Quantum - Minkowski:** Exponentially growing solutions
3. **Quantum - Euclidean:** Path integral diverges

Possible escapes:
Option 1: Interactions stabilize (φ^4 term)

- Vacuum at $\varphi \neq 0$
- Each tachyon mode gets VEV
- Infinite tower of VEVs?? 🤔

Option 2: L large enough that NO tachyon modes

- Requires L > 1/m_min
- Our L_2, L_3 ~ 10 ly, m_2,m_3 ~ 10^{-24} eV
- $\hbar/(m_2c)$ ~ 10 ly ✅ (marginal!)
- Maybe ONLY (0,0) mode is physical?

Option 3: Ghost decay mechanism

- Coupling to matter → decay width Γ
- Pole off real axis
- Not in asymptotic spectrum

🔬 NEXT CALCULATION:

What should we compute next?

A) Explicit φ^4 potential minimum analysis

- Find VEV as function of (n_2, n_3)
- Check if makes sense

B) Decay width calculation

- Model interaction with matter
- Compute Γ for tachyon modes
- See if large enough to decouple

C) Effective theory with cutoff

- Keep only $|n_2|, |n_3| \leq N\_max$
- Choose N such that all modes safe
- See what physics remains

D) Fundamental reconsideration

- Maybe signature should be different?
- Or compactification is wrong?
- Question basic assumptions?

🤔 SIMONE - WHAT DO YOU WANT TO EXPLORE?

I can calculate any of these in detail!

Which direction feels most interesting to you?

CURRENT STATUS:

✅ Understood problem precisely
✅ Multiple escape routes identified
⚠️ Need detailed calculations to see which (if any) works
🔬 Pure exploration mode - no pressure!

Let's discover together what the math tells us! 🔥