

Spectral Lemma: Rayleigh Structure of the Kinetic Matrix K

and the Near-Alignment of the Galactic Coherent Mode with the Golden Eigenvector

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Abstract

We prove the Spectral Lemma for the kinetic matrix $K = [[3,1],[1,2]]$ of the 3D+3D moduli sector. The Rayleigh quotient of the galactic coherent mode $u = (1,1)/\sqrt{2}$ equals $R_c = 7/2$ exactly, while the global maximum over the unit sphere equals $R_{\max} = (5+\sqrt{5})/2 = 2+\phi$, achieved by the golden eigenvector v_+ proportional to $(1, 1/\phi)$. The fractional gap between the two is $(\sqrt{5}-2)/7 \sim 3.37\%$, and the angular separation between u and v_+ is 13.28 degrees. These are exact mathematical results.

The physical question of why the galactic scale selects $u = (1,1)$ rather than v_+ remains open; we show that the gap is smaller than current observational uncertainties on λ_2 , so no observational distinction is currently possible. The "in-between" position of the observed SPARC value (4.300 kpc) relative to the coherent prediction (4.255 kpc) and the golden optimum (4.399 kpc) is stated as a structural curiosity, not a physical claim.

All results are verified at machine precision (residuals $< 1e-14$).

1. Setup

The kinetic matrix of the Q-moduli sector in the 3D+3D framework (Papers VII, XVI, LXV; Paper_eta_geom_Lemma_v1_1) is:

$$K = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \quad (1.1)$$

Its spectral properties, established in Paper_eta_geom_Lemma_v1_1 §A, are:

$$\begin{aligned} \det(K) &= \text{tr}(K) = 5 \\ \text{eigenvalues: } \lambda_+ &= (5+\sqrt{5})/2 = 2+\phi = 3.6180... \\ \lambda_- &= (5-\sqrt{5})/2 = 3-\phi = 1.3820... \\ \lambda_+/\lambda_- &= \phi^2 \\ \text{eigenvectors: } v_+ &\sim (1, 1/\phi), \quad v_- \sim (1, -\phi) \end{aligned} \quad (1.2)$$

where $\phi = (1+\sqrt{5})/2 = 1.6180339...$ is the golden ratio.

The galactic coherent mode — the direction in moduli space corresponding to the symmetric excitation $Q_2 = Q_3$ — is:

$$\begin{aligned} u_c &= (1, 1) \quad [\text{un-normalized}] \\ u_{\hat{c}} &= (1, 1)/\sqrt{2} \quad [\text{unit vector}] \end{aligned} \quad (1.3)$$

2. The Spectral Lemma

Lemma (Spectral Structure of K):

For $K = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ and $u_c = (1, 1)/\sqrt{2}$:

- (i) $R(u_c) = u_{\hat{c}}^T K u_{\hat{c}} = 7/2$ [exact]
- (ii) $\max_{\{||v||=1\}} R(v) = R_{\max} = (5+\sqrt{5})/2 = 2+\phi$ [exact, at $v = v_+$]
- (iii) $(R_{\max} - R_c)/R_c = (\sqrt{5} - 2)/7 \sim 3.37\%$ [exact]
- (iv) $R_{\max}/R_c = (5+\sqrt{5})/7$ [exact]
- (v) $\text{angle}(u_c, v_+) = \arccos(0.9732) = 13.28 \text{ degrees}$ [exact]

Corollary: The coherent mode u_c is 13.28 degrees from the golden optimum v_+ , with 94.7% of its power concentrated in v_+ .

3. Proof

3.1 Part (i): $R(u_c) = 7/2$

Normalize u_c :

$$u_{\hat{c}} = (1, 1)/\sqrt{2}$$

Compute $K * (1,1)^T$:

$$K * (1,1)^T = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} * (1,1)^T = (3+1, 1+2)^T = (4, 3)^T$$

Rayleigh quotient:

$$\begin{aligned} R(u_c) &= u_c^T * K * u_c \\ &= (1/\sqrt{2}, 1/\sqrt{2}) * (4, 3)^T \\ &= (4 + 3) / 2 = 7/2 \quad \text{QED} \end{aligned} \quad (3.1)$$

Equivalently: $W = u_c^T K u_c = (1,1)(4,3)^T = 7$, and $R = W/\|u\|^2 = 7/2$.

3.2 Part (ii): $R_{\max} = 2+\phi$

By the spectral theorem for symmetric matrices, the maximum of the Rayleigh quotient over the unit sphere equals the largest eigenvalue:

$$\max_{\|v\|=1} v^T K v = \lambda_+ = (5+\sqrt{5})/2 = 2+\phi$$

achieved uniquely at $v = v_+/\|v_+\|$. QED

3.3 Part (iii): Fractional gap

$$\begin{aligned} (R_{\max} - R_c)/R_c &= ((5+\sqrt{5})/2 - 7/2) / (7/2) \\ &= (\sqrt{5} - 2) / 7 \\ &= 0.23607... / 7 = 0.033724... = 3.372\% \end{aligned} \quad (3.2)$$

QED. Exact algebraic form: $(\sqrt{5}-2)/7$.

3.4 Part (iv): Ratio

$$R_{\max}/R_c = ((5+\sqrt{5})/2) / (7/2) = (5+\sqrt{5})/7 \quad (3.3)$$

3.5 Part (v): Angular separation

The eigenvector $v_+ = (1, 1/\phi)$. Norm: $\|v_+\|^2 = 1 + 1/\phi^2 = 1 + (2-\phi) = 3-\phi$ (using $1/\phi^2 = 2-\phi$). Therefore $\|v_+\| = \sqrt{3-\phi}$.

Inner product with $u_c = (1,1)/\sqrt{2}$:

$$u_c \cdot v_+ = (1/\sqrt{2}) * (1 + 1/\phi) / \sqrt{3-\phi}$$

Using $1 + 1/\phi = \phi$ (from $\phi - 1 = 1/\phi$, so $1 + 1/\phi = \phi$):

$$= (1/\sqrt{2}) * \phi / \sqrt{3-\phi}$$

$$= \phi / \sqrt{2*(3-\phi)}$$

Numerically: $\phi = 1.6180\dots$, $3-\phi = 1.3820\dots$, $2*(3-\phi) = 2.7639\dots$:

$$\cos(\theta) = 1.6180 / \sqrt{2.7639} = 1.6180 / 1.6625 = 0.9732\dots$$

$$\theta = \arccos(0.9732) = 13.28 \text{ degrees} \quad \text{QED} \quad (3.4)$$

Power decomposition:

$$\text{Power in } v_+: (u_c \cdot v_+)^2 = 0.9732^2 = 0.9471 = 94.7\%$$

$$\text{Power in } v_-: 1 - 0.9471 = 0.0529 = 5.3\% \quad (3.5)$$

All residuals at machine precision ($< 1e-14$). See Appendix.

4. Physical Interpretation

4.1 What is Solidly Established

The mode $u_c = (1,1)$ corresponds to the symmetric excitation $Q_2 = Q_3$ — the lowest-order approximation to a coherent ground-state oscillation of the two coupled moduli. Its Rayleigh quotient $R_c = 7/2$ is exactly half the total kinetic rigidity $W = 7$ used in the η_{geom} derivation (Paper_ η_{geom} Lemma_v1_1).

The near-alignment with $v_+ = (1, 1/\phi)$ at 13.28 degrees means that 94.7% of the power of u_c projects onto the dominant, most stable normal mode of K . This explains structurally why the galactic scale inherits the golden signature of the spectrum while producing the integer rigidity $W = 7$: the coherent mode is geometrically close to the golden eigenvector but is constrained to the symmetric direction by the ground state selection.

4.2 The Open Question: Why $u_c = (1,1)$?

Why does the physical galactic scale select the symmetric direction $u_c = (1,1)$ rather than the exact golden optimum $v_+ = (1, 1/\phi)$? The most natural answer is that $u_c = (1,1)$ is the symmetric ground state in the nearly-degenerate limit of the two moduli Q_2 and Q_3 , but a derivation from the field equations is not provided in this paper and constitutes an open theoretical question.

4.3 The "In-Between" Observation — Honest Assessment

FLAG V6 (Vega): The claim that the observed $\lambda_2 = 4.30$ kpc lies "between" the coherent (4.255 kpc) and golden-optimal (4.399 kpc) predictions requires careful scrutiny.

The numerical situation is:

lambda_2 (coherent pure, this work) = 4.255 kpc

lambda_2 (golden optimum, R_max/R_c) = 4.399 kpc [= 4.255 * (5+sqrt(5))/7]

lambda_2 (SPARC observed) = 4.300 kpc

lambda_2(obs) - lambda_2(coh) = +0.045 kpc [31% of the full gap]

Full gap coherent to golden: = +0.144 kpc

Observational uncertainty: ~ 5-10% ~ 0.21-0.43 kpc

Since the gap (0.144 kpc) is well within the observational uncertainty of the SPARC phi-ladder calibration (sigma ~ 0.21-0.43 kpc), no observational distinction is currently possible between:

- The pure coherent prediction (4.255 kpc, 1.05% error vs SPARC)
- Any linear combination of coherent and golden modes

The statement "the observed value lies between the two predictions" is mathematically true but physically non-informative at current observational precision. It is documented here as a structural curiosity, not as physical evidence for a mixed-mode interpretation.

5. Summary Table

All results exact. Numerical residuals < 1e-14.

Quantity	Exact form	Numerical
R(u_c) — coherent mode	7/2	3.500000
R_max — golden optimum	(5+sqrt(5))/2 = 2+phi	3.618034
Fractional gap	(sqrt(5)-2)/7	0.033724 = 3.37%
Ratio R_max/R_c	(5+sqrt(5))/7	1.033724
Angle u_c vs v_+	arccos(phi/sqrt(2(3-phi)))	13.28 degrees
Power of u_c in v_+	(u.v_+)^2	94.7%
lambda_2 (coherent)	(7/12) * a_0^grav	4.255 kpc
lambda_2 (golden optimum)	(R_max/R_c) * above	4.399 kpc
lambda_2 (SPARC observed)	calibrated	4.300 kpc

6. Red Team Vega Certification

- V1 [PASS] — $R_c = 7/2$: direct computation $K \cdot (1,1) = (4,3)$, dot with $(1,1) = 7$, divide by $\|u\|^2 = 2$. Residual $8.88e-16$.
- V2 [PASS] — $R_{\max} = 2+\phi$: equals λ_+ by spectral theorem. Residual $0.00e+00$.
- V3 [PASS] — Fractional gap $= (\sqrt{5}-2)/7 = 0.033724$: exact algebra verified.
- V4 [PASS] — Angle $= 13.28$ degrees: $\cos = 0.9732...$ verified numerically.
- V5 [PASS] — Power decomposition: 94.7% in v_+ , 5.3% in v_- . Sum = 1.000. Check.
- V6 [FLAG] — The "observation lies between" claim: mathematically true but physically non-informative. Gap (0.045 kpc) is well within observational sigma (0.21-0.43 kpc). Stated as structural curiosity only — NOT as physical evidence for a mixed mode. This flag is retained in all versions for scientific transparency.
- V7 [PASS] — Physical question (why $u_c = (1,1)$?) correctly flagged as open. No false claim of derivation from field equations.

VEGA VERDICT: CERTIFIED with FLAG V6.
The Spectral Lemma is mathematically exact.
The physical interpretation of the 'gap' is correctly
bounded as a curiosity, not a prediction.

7. Relation to Paper_Fibonacci_Decomposition_Lemma_v1_1

The results of this paper are now fully explained by the Fibonacci decomposition
 $K = I + A^2$ (Paper_Fibonacci_Decomposition_Lemma_v1_1, March 10, 2026):

- $R_{\max} = \lambda_+ = 2+\phi$ arises because $K = I + A^2$ and A has eigenvalue ϕ .
- $R_c = 7/2$ arises because $W = u^T K u = 7 = 2 + 5$ (Euclidean + Fibonacci), and $R_c = W/\|u\|^2 = 7/2$.
- The gap $(\sqrt{5}-2)/7$ is a consequence of the companion matrix structure.
- The eigenvector $v_+ \sim (1, 1/\phi)$ is the golden eigenvector of the Fibonacci generator A .

Reading order: Paper_eta_geom_Lemma_v1_1 -> Paper_Spectral_Lemma_v1_0 ->
Paper_Fibonacci_Decomposition_Lemma_v1_1.

Appendix: Self-Contained Python Verification

python

```

import numpy as np

phi = (1 + np.sqrt(5)) / 2

K = np.array([[3, 1], [1, 2]], dtype=float)
u = np.array([1, 1]) / np.sqrt(2)
v_p = np.array([1, 1/phi]); v_p /= np.linalg.norm(v_p)

# Rayleigh quotient of coherent mode
R_c = u @ K @ u          # = 3.5 = 7/2
assert abs(R_c - 7/2) < 1e-14

# Maximum eigenvalue (golden optimum)
R_max = np.linalg.eigvalsh(K)[-1]    # = 3.618... = 2+phi
assert abs(R_max - (2+phi)) < 1e-14

# Fractional gap
gap = (R_max - R_c) / R_c          # = (sqrt(5)-2)/7 = 3.37%
assert abs(gap - (np.sqrt(5)-2)/7) < 1e-14

# Angle between u and v_+
cos_theta = abs(u @ v_p)
angle_deg = np.degrees(np.arccos(cos_theta)) # = 13.28 degrees

# Power decomposition
power_vp = (u @ v_p)**2          # = 0.947 = 94.7%
v_m = np.array([1, -phi]); v_m /= np.linalg.norm(v_m)
power_vm = (u @ v_m)**2          # = 0.053 = 5.3%
assert abs(power_vp + power_vm - 1) < 1e-14

print(f'R_c    = {R_c:.6f} (exact: 7/2 = 3.500000)')
print(f'R_max  = {R_max:.6f} (exact: 2+phi = 3.618034)')
print(f'gap    = {gap*100:.4f}% (exact: (sqrt(5)-2)/7 = 3.3724%)')
print(f'angle   = {angle_deg:.4f} deg (exact: arccos(0.9732))')
print(f'power v_+ = {power_vp*100:.1f}%')
print(f'power v_- = {power_vm*100:.1f}%')
print("ALL ASSERTIONS PASS")

```

Expected output:

R_c = 3.500000 (exact: $7/2 = 3.500000$)
R_max = 3.618034 (exact: $2+\phi = 3.618034$)
gap = 3.3724% (exact: $(\sqrt{5}-2)/7 = 3.3724\%$)
angle = 13.2822 deg (exact: $\arccos(0.9732)$)
power v_+ = 94.7%
power v_- = 5.3%
ALL ASSERTIONS PASS

All residuals < 1e-14. Verified March 10, 2026.

References

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- [2] Calzighetti, S. & Lucy. (2026). "The Modular-Fibonacci Decomposition $K = I + A^2$." Paper_Fibonacci_Decomposition_Lemma_v1_1. 3D+3D Laboratory.
- [3] Calzighetti, S. & Lucy. (2026). "Gravitational Bohr Radius and Connection Lemma." Paper_GravBohr_Q_Field_v1_0. 3D+3D Laboratory.
- [4] Calzighetti, S. & Lucy. (2025). "Moduli Stabilization." Paper VIII. 3D+3D Laboratory.
- [5] Lelli, F., McGaugh, S.S., Schombert, J.M. (2016). SPARC database. AJ 152, 157.
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"Non facciamo le cose a meta." — Simone Calzighetti

Paper_Spectral_Lemma_v1_0 3D+3D Laboratory, Abbiategrosso, Italy simone.calzighetti@3dplus3d.it — www.3dplus3d.it March 10, 2026

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