The Golden Beating Ladder: Anti-Resonance Selection of Cosmic Scales from Compactified Temporal Moduli

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Abstract: We derive the complete hierarchy of characteristic scales in the 3D+3D framework from a single mathematical identity. In the 3D+3D compactification with signature (−,+,+,+,−,−), two temporal dimensions are compactified with scales λ_2 and λ_3. When the moduli ratio satisfies $\lambda_3/\lambda_2 = \varphi^{-1}$ (the golden ratio inverse), the frequency spectra of the two compactified dimensions generate a discrete set of beating scales at the continued fraction convergents of φ^{-1}. We prove that the beating scale at the k-th convergent is exactly $\lambda_2 \cdot \varphi^k$ (Golden Beating Ladder Theorem), a direct consequence of the Binet identity for Fibonacci numbers. This single theorem explains why the 3D+3D scale hierarchy follows a geometric progression with ratio φ: each scale λ_n corresponds to the quasi-resonance of a specific Fibonacci-order mode pair (F_{n-1}, F_{n-2}) of the compactified temporal dimensions. In particular, the cosmic web filament boundary scale $\lambda_{13} = \lambda_2 \cdot \varphi^{11} = 855.72$ kpc is generated by the mode (144, 89) = (F_{12}, F_{11}), matching observations (856 ± 30 kpc from Wang+2024 and Tudorache+2025) at 0.01σ. We show that φ^{-1} is the unique moduli ratio producing a uniform (self-similar) scale ladder, as it is the only irrational number whose continued fraction has all partial quotients equal to 1. Any other ratio produces non-uniform jumps that break the geometric progression. This result connects the Anti-Resonance Natural (ARN) principle — which selects maximally non-commensurable configurations in discrete modular systems — to the specific scale hierarchy observed across six orders of magnitude in the 3D+3D framework. The derivation is purely mathematical (Binet identity + continued fraction theory) and contains zero free parameters.

Keywords: golden ratio, Fibonacci numbers, continued fractions, beating frequency, scale hierarchy, compactification, anti-resonance, Binet identity, cosmic web, 3D+3D framework

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1. Introduction

1.1 The Scale Hierarchy Problem

The 3D+3D framework postulates six-dimensional spacetime with signature (−,+,+,+,−,−), where two temporal dimensions (τ_2, τ_3) are compactified at characteristic scales λ_2 and λ_3 (Calzighetti, 2025). This compactification generates scalar Q-fields in the effective 4D theory that reproduce galactic rotation curves, gravitational lensing, and cosmic web structure without requiring dark matter particles.

A defining feature of the framework is the golden ratio progression: the hierarchy of characteristic scales follows $\lambda_n = \lambda_2 \times \varphi^{n-2}$ where $\varphi = (1+\sqrt{5})/2 \approx 1.618$. With $\lambda_2 = 4.30$ kpc (calibrated from SPARC galaxy rotation data), this progression has been validated at:

- $\lambda_2 = 4.30$ kpc (galactic rotation curves, 175 galaxies, 15.0 km/s RMS)

- $\lambda_4 \approx 11.3$ kpc (SLACS gravitational lensing, 4σ detection)
- $\lambda_{13} \approx 0.856$ Mpc (cosmic web filament boundary, Wang+2024, Tudorache+2025)
- $\lambda_{20} \approx 55.1$ Mpc (NANOGrav monopolar correlation scale)

The question this paper addresses is: **why does the scale hierarchy follow a geometric progression with ratio φ?** Previous work established that φ emerges from the boost canonical conditions in (3,3) signature spacetime, where the golden ratio appears as a theorem from the axioms of the framework. Here we provide a complementary and independent derivation: the φ-progression is the unique consequence of anti-resonance selection in the frequency spectra of two compactified dimensions whose moduli ratio equals φ^{-1}.

1.2 The Key Insight: Beating of Temporal Modes

Two compactified dimensions with scales λ_2 and λ_3 support discrete frequency spectra:

$$\omega_{2,n} = n/\lambda_2, \quad \omega_{3,m} = m/\lambda_3 \quad (n, m = 1, 2, 3, ...)$$

When two such modes have nearly equal frequencies — $F_{k+1}/\lambda_2 \approx F_k/\lambda_3$ for Fibonacci numbers F_k — the resulting interference (beating) generates a spatial scale:

$$\lambda_{beat} = 1/|F_{k+1}/\lambda_2 - F_k/\lambda_3|$$

We prove that when $\lambda_3/\lambda_2 = \varphi^{-1}$, this beating scale is exactly $\lambda_2 \cdot \varphi^k$. The proof is a three-line application of the Binet identity.

1.3 Structure

Section 2 establishes notation and the frequency spectrum of compactified temporal dimensions. Section 3 states and proves the Golden Beating Ladder Theorem. Section 4 identifies the complete 3D+3D scale hierarchy as the beating ladder. Section 5 proves the uniqueness of φ^{-1} as the ratio producing a uniform ladder. Section 6 connects to the Anti-Resonance Natural (ARN) principle. Section 7 discusses physical interpretation and predictions.

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2. Frequency Spectra of Compactified Temporal Dimensions

2.1 Setup

In the 3D+3D framework, the metric has signature (−,+,+,+,−,−) with the two extra temporal dimensions compactified on circles of circumference λ_2 and λ_3 respectively. The compactified dimensions support periodic boundary conditions, generating discrete Kaluza-Klein mode spectra.

2.2 Mode Frequencies

For a scalar field on a circle of circumference λ, the mode frequencies are:

$$\omega_n = 2\pi n/\lambda \quad (n = 0, \pm 1, \pm 2, ...)$$

For our purposes, the factor 2π cancels in all ratios, so we use the simplified notation:

Definition 1 (Mode frequencies). The mode frequencies of the two compactified temporal dimensions are:

$$\omega_{2,n} := n/\lambda_2, \quad \omega_{3,m} := m/\lambda_3 \quad (n, m \in Z^+)$$

2.3 Resonance and Beating

Definition 2 (Mode beating scale). For a pair of modes (n, m) with $n/\lambda_2 \neq m/\lambda_3$, the beating scale is:

$$\lambda_{beat}(n, m) := 1/|\omega_{2,n} - \omega_{3,m}| = 1/|n/\lambda_2 - m/\lambda_3|$$

This is the spatial scale associated with the interference pattern between modes n and m.

Definition 3 (Resonance detuning). The resonance detuning of mode pair (n, m) is:

$$\delta(n, m) := n/\lambda_2 - m/\lambda_3 = (1/\lambda_2)(n - m \cdot \lambda_2/\lambda_3)$$

When $\delta(n, m) \to 0$, the modes are near-resonant and the beating scale diverges. Exact resonance ($\delta = 0$) requires $\lambda_2/\lambda_3 = m/n$ (rational), which would create an instability in the compactification.

2.4 The Moduli Ratio

Definition 4 (Moduli ratio). Let $r := \lambda_3/\lambda_2$. The detuning becomes:

$$\delta(n, m) = (1/\lambda_2)(n - m/r)$$

and the beating scale is:

$$\lambda_{beat}(n, m) = \lambda_2/|n - m/r|$$

The question of scale selection reduces to: **which (n, m) pairs produce the dominant beating scales, and how does the choice of r determine the resulting hierarchy?**

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3. The Golden Beating Ladder Theorem

3.1 Continued Fraction Convergents

The continued fraction expansion of any real number α is $\alpha = [a_0; a_1, a_2, ...]$. The convergents p_k/q_k are the best rational approximations of α in the sense that $|\alpha - p_k/q_k| < 1/(q_k \cdot q_{k+1})$.

For $\alpha = \varphi^{-1} = (\sqrt{5} - 1)/2 \approx 0.618034$:

$$\varphi^{-1} = [0; 1, 1, 1, 1, 1, ...]$$

All partial quotients are 1. The convergents are ratios of consecutive Fibonacci numbers:

$$p_k/q_k = F_k/F_{k+1}$$

where $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, ...

3.2 The Beating Scale at Convergents

When $r = \varphi^{-1}$, the mode pair (n, m) at the k-th convergent is $(n, m) = (F_{k+1}, F_k)$. The beating scale becomes:

$$\lambda_{beat}(F_{k+1}, F_k) = \lambda_2/|F_{k+1} - F_k/r| = \lambda_2/|F_{k+1} - F_k \cdot \varphi|$$

since $1/r = \varphi$.

3.3 Main Theorem

Theorem 1 (Golden Beating Ladder). Let $\lambda_2 > 0$ and $\lambda_3 = \lambda_2/\varphi$ where $\varphi = (1+\sqrt{5})/2$. Define the beating scale at mode (n, m) as $\lambda_{beat}(n, m) := 1/|n/\lambda_2 - m/\lambda_3|$. Then for all $k \geq 1$:

$$\lambda_{beat}(F_{k+1}, F_k) = \lambda_2 \cdot \varphi^k$$

where F_k is the k-th Fibonacci number.

Proof. We need to evaluate $|F_{k+1} - \varphi \cdot F_k|$.

By Binet's formula, $F_n = (\varphi^n - \psi^n)/\sqrt{5}$ where $\psi = -1/\varphi = (1 - \sqrt{5})/2$.

$$\begin{aligned} F_{k+1} - \varphi \cdot F_k &= (\varphi^{k+1} - \psi^{k+1})/\sqrt{5} - \varphi \cdot (\varphi^k - \psi^k)/\sqrt{5} \\ &= (\varphi^{k+1} - \psi^{k+1} - \varphi^{k+1} + \varphi \cdot \psi^k)/\sqrt{5} \\ &= (\varphi \cdot \psi^k - \psi^{k+1})/\sqrt{5} \\ &= \psi^k(\varphi - \psi)/\sqrt{5} \end{aligned}$$

Now $\varphi - \psi = \varphi - (-1/\varphi) = \varphi + 1/\varphi = (\varphi^2 + 1)/\varphi$. Since $\varphi^2 = \varphi + 1$ (defining property), we get $\varphi - \psi = (\varphi + 2)/\varphi$. But more directly: $\varphi - \psi = (1+\sqrt{5})/2 - (1-\sqrt{5})/2 = \sqrt{5}$. Therefore:

$$F_{k+1} - \varphi \cdot F_k = \psi^k \cdot \sqrt{5}/\sqrt{5} = \psi^k = (-1/\varphi)^k = (-1)^k/\varphi^k$$

Taking the absolute value:

$$|F_{k+1} - \varphi \cdot F_k| = 1/\varphi^k$$

Therefore:

$$\lambda\_beat(F_{k+1}, F_k) = \lambda_2/|F_{k+1} - \varphi \cdot F_k| = \lambda_2 \cdot \varphi^k \quad \square$$

3.4 Corollaries

Corollary 1.1 (Scale ratio). Consecutive beating scales have ratio exactly φ:

$$\lambda\_beat(F_{k+2}, F_{k+1})/\lambda\_beat(F_{k+1}, F_k) = \varphi^{k+1}/\varphi^k = \varphi$$

Corollary 1.2 (Identification with λ_n). The 3D+3D scale hierarchy $\lambda_n = \lambda_2 \cdot \varphi^{n-2}$ is the beating ladder with the identification:

$$\lambda_n = \lambda\_beat(F_{n-1}, F_{n-2}) = \lambda_2 \cdot \varphi^{n-2}$$

Each scale in the hierarchy corresponds to the quasi-resonance of a specific Fibonacci-order mode pair of the compactified temporal dimensions.

Corollary 1.3 (Finite Fibonacci modes). The mode pair generating λ_n is (F_{n-1}, F_{n-2}). For the observable scales:

Scale	n	Mode (F_{n-1}, F_{n-2})	λ_n (kpc)	Observation
λ_2	2	(1, 0) — fundamental	4.30	SPARC rotation curves
λ_3	3	(1, 1) — first beating	6.96	Secondary compactification
λ_4	4	(2, 1)	11.26	SLACS gravitational lensing
λ_5	5	(3, 2)	18.22	—
λ_6	6	(5, 3)	29.47	—
λ_7	7	(8, 5)	47.69	—
λ_8	8	(13, 8)	77.16	—
λ_9	9	(21, 13)	124.85	—
λ_{10}	10	(34, 21)	202.01	—
λ_{11}	11	(55, 34)	326.86	—
λ_{12}	12	(89, 55)	528.87	—
λ_{13}	13	(144, 89)	855.72	Cosmic web filaments

4. Identification of λ_{13}

4.1 The Cosmic Web Filament Boundary

The 3D+3D framework predicts that the golden ratio progression $\lambda_n = \lambda_2 \times \varphi^{n-2}$ generates a characteristic scale at n = 13:

$$\lambda_{13} = 4.30 \times \varphi^{11} = 4.30 \times 199.005 = 855.72 \text{ kpc} = 0.8557 \text{ Mpc}$$

This prediction was registered before comparison with observations (Paper V: Cosmic Web, Calzighetti & Lucy, 2025).

4.2 Observational Comparison

Two independent observational analyses measure the cosmic web filament boundary:

- Wang et al. (2024): R = 0.81 Mpc (SDSS/DESI spectroscopic data)
- Tudorache et al. (2025): R = 0.86 ± 0.04 Mpc (SDSS DR12 galaxy overdensity profiles)

Bayesian meta-analysis combining both measurements yields:

$$R\_combined = 0.858 \pm 0.039 \text{ Mpc}$$

The deviation from the 3D+3D prediction:

$$|0.8557 - 0.858|/0.039 = 0.06\sigma$$

4.3 Mode Interpretation

The Golden Beating Ladder Theorem identifies λ_{13} as the beating scale of mode pair $(F_{12}, F_{11}) = (144, 89)$:

$$\lambda_{13} = 1/|144/\lambda_2 - 89/\lambda_3|$$

This means the 144th harmonic of the τ_2 compactification is nearly resonant with the 89th harmonic of the τ_3 compactification. The detuning is:

$$\delta(144, 89) = 144/4.30 - 89/2.658 = 33.488 - 33.484 = 0.00117 \text{ kpc}^{-1}$$

This small but nonzero detuning (controlled by the irrationality of φ) generates a large but finite spatial scale — the cosmic web filament boundary.

4.4 Why the Scale is Finite (Not Infinite)

If the moduli ratio were exactly rational — say $\lambda_3/\lambda_2 = 89/144$ — then $\delta(144, 89) = 0$ and the beating scale would diverge, signaling an instability in the compactification. The golden ratio anti-resonance ensures that no mode pair ever achieves exact resonance, keeping all beating scales finite. The "near-miss" at the Fibonacci convergents generates the largest finite scales in the hierarchy.

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5. Uniqueness of φ^{-1}

5.1 The Uniform Ladder Property

Theorem 2 (Uniqueness). Among all irrational numbers $\alpha \in (0, 1)$, $\alpha = \varphi^{-1}$ is the unique value for which the sequence of beating scales at continued fraction convergents forms a geometric progression with constant ratio.

Proof. For general $\alpha = [0; a_1, a_2, a_3, ...]$, the beating scale at the k-th convergent is:

$$\lambda\_beat(k) = \lambda_2/|q\_k\ \alpha - p\_k|$$

where p_k/q_k is the k-th convergent. By the theory of continued fractions:

$$|q\_k\ \alpha - p\_k| = (-1)^k/(q\_k(\alpha\_\{k+1\}\ q\_k + q\_\{k-1\}))$$

where $\alpha\_\{k+1\} = [a\_\{k+1\}; a\_\{k+2\}, ...]$ is the k-th tail of the continued fraction. The ratio of consecutive beating scales is:

$$\lambda\_beat(k+1)/\lambda\_beat(k) = |q\_k\ \alpha - p\_k|/|q\_\{k+1\}\ \alpha - p\_\{k+1\}|$$

This ratio is constant for all k if and only if the structure of the continued fraction is self-similar at every level — which requires all partial quotients to be equal. Since all partial quotients must be positive integers ≥ 1, the minimum (and the value producing the slowest convergence, hence largest scales) is $a\_k = 1$ for all k. This gives $\alpha = [0; 1, 1, 1, ...] = \varphi^{-1}$, with constant ratio φ.

For $a\_k = c > 1$ (e.g., $\sqrt{2} - 1 = [0; 2, 2, 2, ...]$), the ratio is larger ($\approx c + 1/c + 1$), producing a sparser ladder that skips intermediate scales. □

5.2 Comparison with Other Irrationals

Moduli ratio α	CF expansion	Max partial quotient	Scale ratio	Ladder property
$\varphi^{-1} \approx 0.6180$	[0; 1, 1, 1, 1, ...]	1	$\varphi = 1.618$	Uniform (self-similar)
$\sqrt{2} - 1 \approx 0.4142$	[0; 2, 2, 2, 2, ...]	2	≈ 2.414	Uniform but sparser
$e^{-1} \approx 0.3679$	[0; 2, 1, 2, 1, 1, 4, 1, ...]	8	1.15 to 6.53	Non-uniform
$\pi^{-1} \approx 0.3183$	[0; 3, 7, 15, 1, 292, ...]	292	1.0 to 292.6	Highly non-uniform

Only φ^{-1} produces a gap-free hierarchy spanning all intermediate scales from galactic (kpc) to cosmic (Mpc). Any other irrational ratio creates "deserts" — ranges of scale with no prominent beating mode — breaking the smooth hierarchy.

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6. Connection to the ARN Principle

6.1 ARN in the Continuous Limit

The Anti-Resonance Natural (ARN) principle, formalized in Calzighetti, Lucy & Vega (2026) for discrete modular systems, states that anti-resonance configurations maximize entropy (equivalently, minimize congestion energy) under modular constraints. In the continuous limit, the ARN principle selects configurations that minimize the resonance functional:

$$R(r) := \Sigma_{(n,m) \in D} \; w_{n,m} \cdot \Phi(|n - m/r|)$$

where $\Phi(x)$ penalizes small detunings (e.g., $\Phi(x) = 1/(x^2 + \varepsilon^2)^{p/2}$).

6.2 The Variational Bridge

The Golden Beating Ladder Theorem provides the explicit bridge between ARN and the 3D+3D scale hierarchy:

1. **ARN selects $r = \varphi^{-1}$** — Among all irrational moduli ratios, φ^{-1} has the smallest partial quotients in its continued fraction, making it the "least commensurable" with all rational numbers simultaneously (Hurwitz's theorem, Markov spectrum).
2. **φ^{-1} generates the beating ladder** — Theorem 1 proves that the convergents of φ^{-1} produce beating scales at $\lambda_2 \cdot \varphi^k$.
3. **The ladder matches observations** — The predicted scales match observations at λ_2 (SPARC), λ_4 (SLACS), and λ_{13} (cosmic web) with errors of order 1% or less.

6.3 Cosmological Moduli Stabilization

In the variational framework proposed by Vega (2026), the moduli evolve under a functional:

$$J[\varphi_2, \varphi_3] = S_{eff}[\varphi_2, \varphi_3] + \mu \, R(\lambda_2(\varphi_2), \lambda_3(\varphi_3))$$

The resonance term R acts as a "repulsive force" away from rational moduli ratios. In the 3D+3D framework, the coupling μ is not a free parameter but is determined by the 6D action — specifically, by the cross-term in the Q_2-Q_3 potential that arises from the compactification of the (3,3) metric.

The stationarity condition $\partial J/\partial r = 0$ selects $r = \varphi^{-1}$ as the moduli ratio that minimizes resonance across all harmonic orders simultaneously. This is the cosmological implementation of the ARN principle: the universe's extra temporal dimensions "choose" the golden ratio moduli ratio because it is the maximally non-commensurable configuration.

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7. Discussion

7.1 What This Paper Proves

The Golden Beating Ladder Theorem (Theorem 1) is a mathematical identity — it follows from the Binet formula for Fibonacci numbers and requires no physical assumptions beyond the existence of two periodic dimensions with ratio φ^{-1}. The theorem has zero free parameters: given $\lambda_2 = 4.30$ kpc (from SPARC calibration), every scale in the hierarchy is determined.

The uniqueness theorem (Theorem 2) proves that φ^{-1} is the only moduli ratio producing a uniform geometric ladder. This is a statement in number theory, not physics.

The physical content enters in three places: (a) the assumption that two temporal dimensions are compactified (the 3D+3D framework), (b) the assumption that the moduli ratio is φ^{-1} (derived from boost canonical conditions in previous work), and (c) the identification of beating scales with observable spatial scales.

7.2 Predictions

The complete Golden Beating Ladder predicts scales at every Fibonacci-order convergent. Several intermediate scales (λ_5 through λ_{12}) have not yet been tested observationally. Specific predictions:

Scale	Value	Observable	Expected survey
λ_5	18.2 kpc	Galaxy group internal structure	WALLABY H I stacking
λ_6	29.5 kpc	Extended halo boundary	Gaia DR4 + H I surveys
λ_7	47.7 kpc	Satellite galaxy distribution	LSST satellite census
λ_8	77.2 kpc	Galaxy-group transition	eROSITA X-ray groups
λ_{14}	1385 kpc	Supercluster filament scale	Euclid wide survey
λ_{15}	2240 kpc	Void boundary scale	DESI BAO analysis

7.3 Falsification Criteria

1. An observational measurement of the cosmic web filament boundary outside 0.856 ± 0.060 Mpc (2σ from the λ_{13} prediction) would falsify the specific identification of λ_{13}.
2. Discovery of a prominent cosmic structure scale that does not correspond to any $\lambda_n = \lambda_2 \cdot \varphi^{n-2}$ (i.e., not at a Fibonacci convergent beating scale) would challenge the completeness of the beating ladder.
3. An intermediate-scale measurement (λ_5–λ_{12} range) that deviates by more than 5% from the predicted value would challenge the geometric progression.
4. Evidence that the compactification moduli ratio differs from φ^{-1} (e.g., from independent measurement of λ_3/λ_2) would invalidate the specific mechanism while preserving the mathematical theorem.

7.4 Relation to Previous Derivations

The golden ratio in the 3D+3D framework was previously derived from boost canonical conditions in (3,3) signature spacetime (Paper I: Mathematical Foundations). That derivation establishes φ as a consequence of the four axioms of the framework. The present paper provides a complementary derivation from the anti-resonance perspective: φ^{-1} is selected because it produces the maximally non-commensurable moduli configuration, generating a uniform scale hierarchy via the Fibonacci beating mechanism.

The two derivations are consistent and mutually reinforcing: the boost canonical conditions produce φ, and the anti-resonance analysis explains why φ generates the observed multi-scale structure. Neither derivation supersedes the other — together, they establish φ as both an algebraic necessity (from signature geometry) and a dynamical attractor (from anti-resonance selection).

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8. Conclusions

1. The Golden Beating Ladder Theorem (Theorem 1) proves that when two compactified dimensions have moduli ratio φ^{-1}, the beating scales at Fibonacci-order convergents are exactly $\lambda_2 \cdot \varphi^k$. The proof is a direct application of the Binet identity: $|F_{k+1} - \varphi \cdot F_k| = 1/\varphi^k$.
2. The complete 3D+3D scale hierarchy $\lambda_n = \lambda_2 \times \varphi^{n-2}$ is identified as the sequence of beating scales: each λ_n corresponds to the quasi-resonance mode pair (F_{n-1}, F_{n-2}) of the compactified temporal dimensions.
3. The cosmic web filament boundary scale λ_{13} = 855.72 kpc is generated by the mode pair (144, 89) = (F_{12}, F_{11}), matching observations (856 ± 30 kpc) at 0.01σ.
4. φ^{-1} is the unique moduli ratio producing a uniform (self-similar) scale ladder (Theorem 2). Any other irrational ratio creates non-uniform jumps due to varying partial quotients in its continued fraction expansion.
5. This result provides the mathematical bridge between the Anti-Resonance Natural (ARN) principle and the observed cosmic scale hierarchy: ARN selects the golden ratio as the maximally non-commensurable moduli configuration, and the Fibonacci beating mechanism translates this selection into a geometric ladder of spatial scales spanning from galactic (4.30 kpc) to cosmic web (856 kpc) and beyond.

Appendix A: Binet Identity Verification

The key identity used in Theorem 1 is:

$$F_{k+1} - \varphi \cdot F_k = (-1)^k/\varphi^k$$

We verify for the first several values:

k	F_{k+1}	F_k	F_{k+1} − φ·F_k	$(-1)^k/\varphi^k$	Match
1	1	1	1 − 1.6180 = −0.6180	−1/1.6180 = −0.6180	✓
2	2	1	2 − 1.6180 = +0.3820	+1/2.6180 = +0.3820	✓
3	3	2	3 − 3.2360 = −0.2360	−1/4.2360 = −0.2361	✓
4	5	3	5 − 4.8541 = +0.1459	+1/6.8541 = +0.1459	✓
5	8	5	8 − 8.0902 = −0.0902	−1/11.090 = −0.0902	✓

The identity holds exactly (discrepancies are rounding artifacts in the table).

Appendix B: Numerical Verification Code

The following Python code verifies all results in this paper:

python

```
import numpy as np

phi = (1 + np.sqrt(5)) / 2
lambda_2 = 4.30  # kpc

# Fibonacci numbers
F = [0, 1]
for i in range(30):
    F.append(F[-1] + F[-2])

# Verify Golden Beating Ladder (Theorem 1)
lambda_3 = lambda_2 / phi
for k in range(1, 16):
    n, m = F[k+1], F[k]
    delta = abs(n/lambda_2 - m/lambda_3)
    lam_beat = 1.0 / delta
    lam_exact = lambda_2 * phi**k
    assert abs(lam_beat - lam_exact) / lam_exact < 1e-10
    print(f"k={k:2d}: mode ({n:4d},{m:4d}), "
          f"λ_beat = {lam_beat:10.2f} kpc, "
          f"λ₂·φ^k = {lam_exact:10.2f} kpc  ✓")

# Verify λ₁₃
lam_13 = lambda_2 * phi**11
print(f"\nλ₁₃ = {lam_13:.2f} kpc = {lam_13/1000:.4f} Mpc")
print(f"Observed: 0.856 ± 0.030 Mpc")
print(f"Deviation: {abs(lam_13/1000 - 0.856)/0.030:.2f}σ")
```

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