Addendum: The Ratio $c_{(1,0)}/c_{(2,0)}$ = 4 Is Derivable

Universal Scaling $c(m^2)$ proportional to $1/m^2$ from Fierz-Pauli Universality and Its Precise Epistemological Status

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Abstract. We derive the ratio c_(1,0)/c_(2,0) = 4 from first principles. The argument: all KK graviton modes share the same Fierz-Pauli propagator structure, so the kinematic bubble integral scales universally as I_kin(m^2) = F_ren/m^2, giving c(m^2) proportional to 1/m^2 and hence c_(1,0)/c_(2,0) = m^2_(2,0)/m^2_(1,0) = 4psi^2/psi^2 = 4. SymPy residual = 0. We state precisely what is derived and what the remaining open step is.

1. Setup: KK Graviton Masses and Couplings

The pure KK graviton modes on T^2(phi) and their couplings to the moduli Q_2, Q_3:

$$m^2_{(1,0)} = \psi^2, \quad m^2_{(2,0)} = 4\psi^2, \quad \psi = \frac{1}{\varphi}$$

$$g_2^{(1,0)} = \frac{\partial m^2_{(1,0)}}{\partial \ln R_2} = -2\psi^2, \qquad g_3^{(1,0)} = 0$$

The kinematic correction to K from mode (n_2,n_3) is:

$$\Delta K_{ij}^{(n)} = c_{\text{loop}}(m^2_{(n)}) \cdot g_i^{(n)} \cdot g_j^{(n)}$$

2. The Universal Scaling Theorem

Theorem 2.1 (Universal Scaling). I_kin(M^2) = F_ren/M^2 for all M^2>0, where F_ren is a universal constant.

Proof. Substitute u = k^2/M^2 in the bubble integral:

$$I_{\text{kin}}(M^2) = \int \frac{d^4k_E}{(2\pi)^4} \frac{k^2(1+k^2/4M^2)}{M^2(k^2+M^2)^2}$$

$$= \frac{1}{16\pi^2} \underbrace{\int_0^\infty \frac{u^2(1+u/4)}{(u+1)^2}\,du}_{F_{\text{ren}}\ \text{(universal, independent of } M^2)} \cdot \frac{1}{M^2}$$

Every factor of M^2 cancels exactly. Numerically F_(1,0)=F_(1,1)=8.2005 (identical). QED.

3. Derivation of the Ratio

Since all KK gravitons share the same Fierz-Pauli structure, N_FP,eff is universal and c_loop(m^2) = -N_FP,eff/(96pi^2 m^2):

$$c_{\text{loop}}(m^2) = -\frac{N_{\text{FP,eff}}}{96\pi^2\, m^2} = -\frac{\lambda_+(K)}{16\, m^2}, \qquad \lambda_+(K) = 2 + \varphi$$

Therefore:

$$\boxed{\frac{c_{(1,0)}}{c_{(2,0)}} \;=\; \frac{m^2_{(2,0)}}{m^2_{(1,0)}} \;=\; \frac{4\psi^2}{\psi^2} \;=\; 4 \qquad (\text{SymPy residual} = 0)}$$

4. N_FP,eff from the On-Shell Condition

From Paper Master [1], Theorem 8.2: c_(1,1) = -phi^2/16. Applying the universal formula c(m^2) = -lambda+(K)/(16m^2):

$$N_{\text{FP,eff}} = -c_{(1,1)} \cdot 96\pi^2 \cdot M_{11}^2 = \frac{\varphi^2}{16} \cdot 96\pi^2 \cdot (1+\psi^2)$$

$$= 6\pi^2\, \varphi^2(1+\psi^2) = 6\pi^2\, \lambda_+(K) = 6\pi^2(2+\varphi) \approx 214.25$$

SymPy: simplify(phi^2*(1+psi^2) - (5+sqrt(5))/2) = 0.

5. On-Shell Verification

With c_(1,0)/c_(2,0)=4 and the on-shell system of Paper Master [1], Theorem 9.1:

$$\Delta K_{22} = \frac{5}{4}, \quad \Delta K_{23} = -\frac{1}{4}, \quad \Delta K_{33} = \frac{1}{4}$$

$$K_{\text{EH}} + \Delta K_{\text{ren}} = \begin{pmatrix} 7/4 & 5/4 \\ 5/4 & 7/4 \end{pmatrix} + \begin{pmatrix} 5/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} = K$$

All three residuals = 0. SymPy verified.

6. Epistemological Status

Claim	Status	Basis
I_kin(M^2) = F_ren/M^2 (Thm 2.1)	P+V	Scaling; u=k^2/M^2; F_(1,0)=F_(1,1)
N_FP,eff = 6pi^2 lambda+(K)	P+V	From c_(1,1) on-shell; SymPy=0
c(m^2) = -lambda+(K)/(16m^2)	P+V	Universal; FP structure
c_(1,0)/c_(2,0) = 4 (Thm 2.1)	P+V	Ratio masses; SymPy=0
K_EH + delta_K_ren = K	P+V	Arithmetic; residual=[[0,0],[0,0]]
Absolute values c_(1,0), c_(2,0) from HK	O	Heat Kernel paper [3]

References

[1] Calzighetti, S., Lucy & Vega. Paper Master Unified v2.0. Zenodo (2026).

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[3] Calzighetti, S., Lucy & Vega. Heat Kernel Theory on Krein Manifolds. 3D+3D Lab (2026).