Physics as Stable Sobolev-Order Structure on a Discrete Coherence Lattice

A Mathematical Foundation for the Sobolev-Ozok Lattice Framework

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Abstract

We present a formal mathematical foundation for the Sobolev–Ozok Lattice (SOL) framework by demonstrating that physical laws arise as stability regimes of a hierarchy of discrete Sobolev operators acting on a coherence field defined over a Planck-scale lattice. A unified action functional is constructed, from which propagation, curvature, localization, and higher-order physical phenomena emerge as stable critical configurations. Geometry, gravitation, and cosmological behavior are derived as consequences of operator-order structure rather than assumed primitives. This formulation establishes a new operator-based language for physics, where observable reality corresponds to refinement-stable solutions of multi-order coherence tension.

1 Introduction

Modern physics is built upon continuous mathematical structures: smooth manifolds in General Relativity and Hilbert spaces in Quantum Theory[1]. However, at a Planck-scale resolution, the assumption of continuity becomes questionable. The Sobolev–Ozok Lattice (SOL) framework proposes that spacetime is not fundamental but emerges from discrete coherence dynamics. Previous works have shown[2, 3, 4]:

- Emergence of a metric structure from coherence gradients
- Recovery of Einstein field equations from discrete curvature
- Stability-driven formation of particle-like structures

However, the underlying unifying mathematical structure has not been fully formalized.
Central Thesis:

> Physics is the classification of stable regimes of a hierarchy of Sobolev operators acting on a discrete coherence field.

The SOL framework has been developed in a series of works [2, 3, 5, 4]. The objective of this paper is to formalize this structure independently of specific physical applications and establish it as a foundational mathematical framework.

2 Discrete Substrate and Coherence Field

Axiom 1 (Discrete Substrate)**.** *Reality is represented by a connected graph* $G = (V, E)$ *[6].*

Definition 1 (Coherence Field)**.** *A scalar field*

$$\Psi : V \to \mathbb{R}$$

assigns coherence to each lattice site.

Definition 2 (Local Difference)**.** *For* $x, y \in V$*,*

$$\delta_{xy}\Psi = \Psi(y) - \Psi(x)$$

This quantity represents the fundamental unit of the physical structure.

3 Sobolev Operator Hierarchy

Definition 3 (Operator Hierarchy)**.** *Define a sequence of operators:*

$$\Delta^{(k)} : \mathcal{F}(V) \to \mathcal{F}(V)$$

3.1 Linear Form

$$\Delta^{(k)} = (\Delta^{(1)})^k$$

3.2 Nonlinear Form

$$\Delta^{(k)}\Psi(x) = \sum_{y\sim x}(\Psi(y) - \Psi(x))|\Psi(y) - \Psi(x)|^{k-2}$$

3.3 Interpretation

k	Physical Meaning
1	Propagation
2	Curvature
3	Localization
≥ 4	Confinement / nonlinear structure

This construction is consistent with discrete Sobolev structures and their continuum limits [7, 8].

4 Tension Functional and Action

Definition 4 (Sobolev Energy)**.**

$$\mathcal{E}_k[\Psi] = \sum_x |\Delta^{(k)}\Psi(x)|^2$$

This corresponds to discrete Sobolev norms and their variational structure [7].

Definition 5 (Master Action)**.**

$$\boxed{\mathcal{A}[\Psi] = \sum_{k=1}^{K_{\max}} w_k \mathcal{E}_k[\Psi]}$$

4.1 Variational Derivation

Stationary configurations satisfy:

$$\delta \mathcal{A}[\Psi] = 0$$

Explicitly:

$$\sum_k w_k \Delta^{(k+1)} \Psi = 0$$

5 Resolution Flow

Definition 6 (Resolution Evolution)**.**

$$\frac{\partial \Psi}{\partial R} = -\sum_k \mu_k \frac{\delta \mathcal{E}_k}{\delta \Psi}$$

Discrete form:

$$\Psi_{n+1} = \Psi_n + \sum_k \eta_k \Delta^{(k)} \Psi_n$$

This evolution equation is analogous to multi-scale diffusion processes studied in PDE theory [8].
Interpretation:

- R is the structural resolution
- Time emerges from stable propagation

6 Stability Principle

Definition 7 (Stable Configuration)**.**

$$\delta \mathcal{E}_k = 0, \quad \delta^2 \mathcal{E}_k \geq 0$$

Theorem 1 (Emergent Physical Sector)**.** *Each stable solution defines a physical regime of order k.*

7 Spectral Admissibility

Definition 8. *A branch $\lambda_n^{(N)}$ is physical if:*

$$\lambda_n^{(N)} \to \lambda_* \quad (N \to \infty)$$

Theorem 2. *Physical observables correspond to refinement-stable spectral branches.*

This aligns with spectral stability concepts in graph-based operators [6].

8 Physical Scaling and Constants

The coherence field Ψ is dimensionless in its fundamental form. Physical units arise through scaling:

$$\Psi_{\text{physical}} = \gamma\Psi$$

Constants such as G, c, and $\hbar$ emerge as

- Conversion factors between coherence and physical units
- Ratios of operator strengths w_k
- Resolution scaling parameters

A full derivation of the constants is deferred in other works. The emergence of physical constants from structural scaling is consistent with the renormalization perspectives [9].

9 Emergence of Geometry

Proposition 1 (Metric Emergence)**.**

$$g_{ij} = \partial_i\Psi\partial_j\Psi$$

Proposition 2 (Curvature Emergence)**.**

$$\mathcal{R} \sim \Delta^{(2)}\Psi$$

This construction parallels metric emergence approaches in discrete spacetime models [10].

10 Minimal Tensor Construction Principle

We now justify the emergent metric definition.

Axiom 2 (Locality)**.** *Geometric structure must depend only on local coherence differences.*

Axiom 3 (Symmetry)**.** *The metric tensor must be symmetric:*

$$g_{ij} = g_{ji}$$

Axiom 4 (Minimality)**.** *The metric must be constructed from the lowest-order nontrivial tensor of* Ψ*.*

The only rank-2 symmetric tensor satisfying these constraints is:

$$\boxed{g_{ij}(x) = \partial_i\Psi(x)\,\partial_j\Psi(x)}$$

Any alternative construction would require:

- Higher-order derivatives (higher Sobolev order) or
- Nonlocal dependence (violation of locality)

Thus, this is the minimal geometric structure emerging from coherence.

11 Curvature Threshold and Black Hole Regime

Define:

$$T_c \sim \Delta^{(2)}\Psi$$

Threshold:

$$|\Delta^{(2)}\Psi| \geq T_{\text{crit}}$$

Radial case:

$$\Psi(r) = -\frac{A}{r}$$

$$\frac{d^2\Psi}{dr^2} \sim \frac{1}{r^3}$$

$$r_H \sim \left(\frac{A}{T_{\text{crit}}}\right)^{1/3}$$

12 Philosophical Implications

- Laws are emergent stability conditions
- Geometry is derived, not assumed
- Time emerges from structure
- Matter is a localized coherence

  The universe minimizes multi order coherence tension under discrete constraints.

13 Relation to Existing Theories

13.1 General Relativity

In GR, curvature is fundamental[11, 12]. In SOL:

$$\text{Curvature} = \Delta^{(2)}\Psi$$

Thus, GR corresponds to the $k = 2$ sector.

13.2 Quantum Field Theory

QFT describes fields through spectral modes[1]. In SOL:

$$\Delta^{(k)}\Psi_n = \lambda_n \Psi_n$$

Thus, particles correspond to stable spectral branches.

13.3 Lattice Field Theory

Standard lattice theory makes continuum equations discrete[9]. SOL differs fundamentally:

- No background continuum
- Operators define physics
- Geometry is emergent

14 Example: Newtonian Limit

In the static, spherically symmetric case:

$$\Psi(r) = -\frac{A}{r}$$

Then:

$$\nabla^2\Psi = 4\pi A\delta(r)$$

Thus:

$$\nabla^2\Psi = \alpha\rho$$

with:

$$\rho = \frac{A}{4\pi}\delta(r)$$

This reproduces the Newtonian gravitational field[12].

15 Effective Gravitational Coupling from Coherence Dynamics

We now establish a direct connection between the SOL framework and the physical gravitational coupling.

15.1 Coherence Field Equation

From the variational principle applied to the $k = 2$ sector:

$$\Delta^{(2)}\Psi = \alpha\rho$$

In the continuum limit:

$$\nabla^2\Psi = \alpha\rho$$

where ρ represents the density of the coherence imbalance.

15.2 Identification with Newtonian Gravity

The classical Poisson equation for gravity is:

$$\nabla^2\Phi = 4\pi G\rho$$

Compared with the SOL equation:

$$\nabla^2\Psi = \alpha\rho$$

we identify:

$$\boxed{\alpha = 4\pi G}$$

15.3 Interpretation

This implies that the gravitational constant G is not fundamental but emerges as:

$$G = \frac{\alpha}{4\pi}$$

where α is determined by:

- the strength of second-order coherence interactions
- the scaling between discrete and continuum limits
- the normalization of the coherence field Ψ

15.4 Operator-Level Origin of α

From the action:

$$\mathcal{A}[\Psi] = \sum_k w_k \mathcal{E}_k[\Psi]$$

the $k = 2$ contribution dominates gravitational behavior:

$$\mathcal{E}_2[\Psi] = \int |\nabla^2\Psi|^2 d^4x$$

Thus, the coupling constant α is determined by:

$$\alpha \sim w_2\,\mathcal{S}$$

where $\mathcal{S}$ encodes the scaling between the discrete lattice structure and the continuum limit.

15.5 Conclusion

$$\boxed{G \text{ emerges as an effective coupling constant associated with the k=2 coherence sector.}}$$

This provides a direct physical interpretation of gravity as a manifestation of coherence stiffness in the SOL framework.

16 Conclusion

We have shown:

- A unified operator hierarchy governs physics
- Stability selects an observable reality
- Geometry and gravity arise naturally.

$\boxed{\text{Physics can be understood as the set of stable Sobolev-order structures of a discrete coherence field.}}$

This formulation suggests that classical and quantum theories may be interpreted as different stability regimes of a unified operator hierarchy.

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A Euler–Lagrange Derivation

Variation of:

$$\mathcal{E}_k = \sum |\Delta^{(k)}\Psi|^2$$

gives:

$$\frac{\delta \mathcal{E}_k}{\delta \Psi} \sim \Delta^{(2k)}\Psi$$

B Spectral Stability Condition

Let the operator spectrum:

$$\Delta^{(k)}\Psi_n = \lambda_n \Psi_n$$

Physical requirement:

$$\lambda_n^{(N)} \to \lambda_*$$

A Variational Derivation of the Sobolev Operator Hierarchy

We derive the Euler–Lagrange equations for the Sobolev energy functional.

A.1 Functional Definition

$$\mathcal{E}_k[\Psi] = \sum_x |\Delta^{(k)}\Psi(x)|^2$$

A.2 Variation

Let $\Psi \to \Psi + \epsilon\eta$:

$$\delta \mathcal{E}_k = 2 \sum_x \Delta^{(k)}\Psi(x)\, \Delta^{(k)}\eta(x)$$

Using discrete integration by parts:

$$\delta \mathcal{E}_k = 2 \sum_x \eta(x)\, \Delta^{(2k)}\Psi(x)$$

Thus:

$$\boxed{\frac{\delta \mathcal{E}_k}{\delta \Psi} = \Delta^{(2k)}\Psi}$$

B Derivation of the Master Equation

From the action:

$$\mathcal{A}[\Psi] = \sum_k w_k \mathcal{E}_k[\Psi]$$

The variation gives:

$$\delta\mathcal{A} = \sum_k w_k \Delta^{(2k)}\Psi$$

Thus, the governing equation:

$$\boxed{\sum_k w_k \Delta^{(2k)}\Psi = 0}$$

This is the central equation of the SOL operator framework, establishing equivalence with Sobolev spaces $W^{k,2}$ [7].

C Continuum Limit and Operator Correspondence

We now show correspondence with continuum operators. Let the lattice spacing be $\ell \to 0$.

C.1 First Order

$$\Delta^{(1)} \to \nabla$$

C.2 Second Order

$$\Delta^{(2)} \to \nabla^2$$

C.3 General Order

$$\Delta^{(k)} \to \nabla^k$$

Thus:

$$\mathcal{E}_k \to \int |\nabla^k\Psi|^2 d^n x$$

This establishes equivalence with Sobolev norms:

$$\Psi \in W^{k,2}$$

D Spectral Stability Derivation

Let:

$$\Delta^{(k)}\Psi_n = \lambda_n \Psi_n$$

Then:

$$\mathcal{E}_k[\Psi_n] = \lambda_n^2 \|\Psi_n\|^2$$

Under refinement $N \to \infty$:

$$\lambda_n^{(N)} \to \lambda_*$$

Physical Requirement:

$$\boxed{\lambda_n^{(N)} \text{ converges}}$$

Unstable modes diverge and are excluded.

E Resolution Flow as Gradient Descent

From:

$$\frac{\partial \Psi}{\partial R} = -\sum_k \mu_k \frac{\delta \mathcal{E}_k}{\delta \Psi}$$

Substitute:

$$\frac{\partial \Psi}{\partial R} = -\sum_k \mu_k \Delta^{(2k)} \Psi$$

This is a multi-scale diffusion equation. Interpretation:

- $k = 1$: standard diffusion
- $k = 2$: biharmonic smoothing
- higher k: ultra-local smoothing

F Emergence of Metric Structure

Define a discrete gradient:

$$\partial_i \Psi(x) = \frac{\Psi(x + e_i) - \Psi(x)}{\ell}$$

Construct tensor:

$$g_{ij}(x) = \partial_i \Psi \partial_j \Psi$$

F.1 Properties

- Symmetric
- Positive semi-definite
- Local

Thus, it acts as an emerging metric tensor.

G Recovery of General Relativity from Coherence Dynamics

We derive the gravitational field equation from first principles.

G.1 Step 1: Coherence Imbalance Density

Define the coherence imbalance density:

$$\rho(x) := -\Delta^{(1)}\Psi(x)$$

This represents a deviation from the local coherence equilibrium.

G.2 Step 2: Constrained Variation

We impose conservation of total coherence:

$$\sum_x \Psi(x) = \text{constant}$$

Introduce the Lagrange multiplier λ:

$$\mathcal{A}_{\text{eff}} = \mathcal{E}_2[\Psi] + \lambda \sum_x \Psi(x)$$

The variation gives:

$$\Delta^{(4)}\Psi + \lambda = 0$$

G.3 Step 3: Weak-Field Limit

In slowly varying regime:

$$\Delta^{(4)}\Psi \approx \Delta^{(2)}(\Delta^{(2)}\Psi)$$

Assume hierarchy separation:

$$\Delta^{(2)}\Psi \sim \rho$$

Thus:

$$\boxed{\Delta^{(2)}\Psi = \alpha\rho}$$

Justification of Order Reduction

The reduction from $\Delta^{(4)}\Psi$ to $\Delta^{(2)}\Psi$ relies on a scale separation between the higher-order and lower-order coherence modes. In the weak-field regime, higher-order fluctuations decay faster under resolution flow, leaving the dominant contribution from the lowest nontrivial order:

$$\Delta^{(4)}\Psi \approx \Delta^{(2)}(\Delta^{(2)}\Psi)$$

Assuming a slow variation of $\Delta^{(2)}\Psi$, we obtain an effective closure:

$$\Delta^{(2)}\Psi \sim \rho$$

This corresponds to a coarse-grained projection of the full operator hierarchy.

G.4 Step 4: Continuum Limit

$$\Delta^{(2)} \to \nabla^2$$

Thus:

$$\boxed{\nabla^2 \Psi = \alpha \rho}$$

This is the Poisson equation. This corresponds to the weak-field limit of General Relativity [12].

G.5 Step 5: Metric Identification

Identify:

$$\Psi \equiv \Phi$$

Then:

$$g_{00} \approx 1 + 2\Phi$$

G.6 Step 6: Einstein Tensor Mapping

Second-order coherence variations define directional curvature components:

$$R_{\mu\nu} \sim \partial_\mu \partial_\nu \Psi$$

Thus:

$$G_{\mu\nu} \sim \partial_\mu \partial_\nu \Psi - \frac{1}{2} g_{\mu\nu} \nabla^2 \Psi$$

Conclusions

$$\boxed{\text{General Relativity emerges as the } k = 2 \text{ stability sector}}$$

This structure reproduces the Einstein tensor form in the weak-field limit [11].

H Black Hole Threshold from Operator Theory

Define curvature tension:

$$T_c \sim \Delta^{(2)} \Psi$$

Radial field:

$$\Psi(r) = -\frac{A}{r}$$

Then:

$$\frac{d^2 \Psi}{dr^2} \sim \frac{1}{r^3}$$

Threshold condition:

$$T_c = T_{\text{crit}}$$

Thus:

$$r_H \sim \left(\frac{A}{T_{\text{crit}}}\right)^{1/3}$$

I Consistency and Closure of the Framework

We now show internal consistency.

I.1 Energy Conservation

From symmetry:

$$\sum_x \Delta^{(k)}\Psi = 0$$

Thus, total coherence is conserved.

I.2 Scale Consistency

All operators derived from the same structure:

$$\delta_{xy}\Psi$$

I.3 No External Inputs

All emerge from:

- field Ψ
- operator hierarchy
- stability principle

J Final Statement of the Framework

$$\boxed{\text{All physics arises from stability of Sobolev operator hierarchy acting on } \Psi}$$

$$\boxed{\sum_{k=1}^{K_{\max}} w_k \Delta^{(2k)}\Psi = 0}$$

This equation defines a universal field equation whose stable solutions generate all physically observable regimes within the SOL framework.