Finite Readout Representations and Matching Corrections from the Compact Fiber

A. R. Wells
Dual-Frame Research Group

April 26, 2026

Abstract

The compact fiber

$$F = \mathbb{Z}_4^{m_1} \times \mathbb{Z}_4^{m_2} \times \mathbb{Z}_8^{e}$$

has been used in prior compact-fiber applications as a finite carrier of magnetic and electric readout structure. This paper formalizes that usage by passing to the character group

$$\widehat{F} = \mathrm{Hom}(F, U(1)).$$

The resulting finite character sectors give representation-theoretic form to the two magnetic $\mathbb{Z}_4$ sectors and the electric $\mathbb{Z}_8$ sector. The degree-two cover

$$\pi : \mathbb{Z}_8 \to \mathbb{Z}_4$$

identifies direct magnetic-base compatibility with the even electric characters and gives a deck-parity interpretation of the odd electric characters.

The first result is a finite character-readout calculus. The recurring factors

$$\frac{1}{8}, \qquad \frac{1}{4}, \qquad \frac{7}{8}, \qquad \frac{1}{32}, \qquad 128$$

are realized as projections, complements, product-sector resolutions, or full character counts. The cover-mediated factor

$$\frac{1}{16}$$

is fixed once the readout is assigned to the degree-two electric cover class. Closed internal finite-sector couplings obey a character-neutrality rule: the product of participating characters must be the trivial character.

The second result applies this status calculus, together with stated Larson–Nehru bridge-layer rules, to one finite matching correction. An elementary natural-unit mass-to-energy bridge is represented as a single source-to-target status traversal

$$B : \mathcal{M}_2 \to \mathcal{R},$$

from a completed two-dimensional secondary-mass source to an energy/readout target, with one completed mass-side contribution and one electric boundary contribution. The finite readout calculus therefore selects the typed package

$$s = m + e_\partial.$$

Using the Larson–Nehru secondary-mass and interregional placement rules, the electric boundary contribution is

$$e_\partial = e = \frac{2}{3} \cdot \frac{1}{9|\widehat{F}|} = \frac{1}{1728}.$$

The admissible elementary matching package is therefore

$$s = m + e.$$

Alternative packages are rejected by readout class rather than by numerical fit: m omits the boundary contribution, $m + 2e + C$ overloads the elementary boundary and imports a charge-class term, and $e - c$ or $e - C$ omit the completed mass-side contribution while substituting charge-class quantities. The result is a no-retuning finite matching protocol: a factor must be a defined character operation, a theorem-grade bridge operation under stated premises, or an open term.

1 Introduction

The compact-fiber program has used the finite carrier

$$F \cong \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_8$$

as an admissibility and readout structure in several settings. Paper I established the carrier itself [1]. Paper II used its finite sector structure in the atomic and spectroscopic regime [2]. Paper III developed the effective representational and readout layer, including the Hilbert-space and Born-rule machinery used by later applications [3]. The Radiation paper then applied the same division of labor to polarization, interference, Bell correlation, Hong–Ou–Mandel bunching, orbital angular structure, photoelectric transfer, and the Planck scaling bridge [4].

Those applications rely on a recurring finite readout vocabulary. They use an electric sector of capacity $N_e = 8$, magnetic sectors of capacity $N_m = 4$, a degree-two electric-to-magnetic covering relation, and readout factors such as

$$\frac{7}{8}, \quad \frac{1}{8}, \quad \frac{1}{4}, \quad \frac{1}{16}, \quad \frac{1}{32}.$$

The present paper formalizes the finite representation layer implicit in that usage: the recurring readout classes are represented as operations on the character group of the carrier.

This matters because the readout taxonomy is load-bearing. If terms such as electric sector, magnetic sector, cover-mediated readout, and product-fiber correction remain only descriptive, then later uses of those terms can appear flexible. Passing to the character group gives a stricter object. A finite sector becomes a factor of

$$\widehat{F} = \mathrm{Hom}(F, U(1)),$$

and a readout factor becomes a projection, complement, cover resolution, product average, or full character count. The construction is standard finite representation theory; the contribution here is its use as an audit and formalization of the compact-fiber readout calculus.

The paper also introduces a finite character conservation rule. For a closed internal coupling, the product of participating characters must be the trivial character. In indices, the total character must vanish modulo the order of each sector. This is the closed-sector selection rule associated with the finite abelian readout layer. Open, external, cover-changing, and measurement readouts require their own stated boundary structure.

The result is a compact formal layer that does two things. First, it identifies which finite readout factors are theorem-grade consequences of the character group. Second, it separates those factors from higher-level bridge terms that require additional interregional, vibrational, secondary-mass, or external-readout assumptions. This distinction is central to the no-retuning discipline of the program.

The central result is the Character-Readout Classification Theorem: the recurring finite factors are not free corrections but normalized weights of specified character operations on $\widehat{F}$, with the cover-mediated factor fixed once the degree-two cover class is assigned.

This status discipline is then applied, together with stated bridge-layer premises, to the elementary natural-unit mass-to-energy bridge. The bridge is represented as a single source-to-target traversal

$$B : \mathcal{M}_2 \to \mathcal{R},$$

which fixes the typed package

$$s = m + e_\partial.$$

The bridge-layer derivation identifies

$$e_\partial = e = \frac{1}{1728},$$

so the admissible elementary package is

$$s = m + e.$$

2 Scope and claim strength

The object of this paper is the finite abelian readout layer of the compact fiber and its elementary application to the natural-unit matching bridge. The paper does not repeat the derivations of Papers I–III or the Radiation paper, and it does not use the character group as a substitute for the representational machinery already developed elsewhere. Its task is to formalize the finite sector operations that recur across those applications and to apply that status discipline to one elementary bridge correction.

The limitation is structural. Since F is finite abelian, every irreducible complex representation of F is one-dimensional. Thus $\widehat{F}$ supplies a discrete additive phase/readout calculus. It does not by itself supply continuous phase dynamics, noncommuting operator structure, spinor representations, scale dependence, or time evolution. Those belong to additional representational, operator, projection, or dynamical layers. The present paper fixes the finite readout layer that such later structures must respect.

The claim strength is limited but precise. The character group supplies theorem-grade finite factors when the factor follows directly from character counting or from a defined operation on a character sector. This includes single-character projections, the electric nontrivial-sector complement, the magnetic-electric product average, and the full character count. The cover-mediated factor $1/16$ is fixed once a readout is assigned to the degree-two electric cover class, but the assignment of a concrete observable to that class remains a structural readout claim. Interregional and external-completion factors are not treated as pure consequences of the character group.

Bridge quantities therefore require explicit status. A bridge factor may remain structural status S, or it may be promoted to bridge-theorem status S_T when it follows from stated bridge-layer premises. A bridge-theorem result is theorem-grade within a stated bridge layer, but not reducible to pure finite-character counting. The matching result below has status S_T: theorem-grade within the stated bridge layer, not a pure character-theory result.

This convention is used to avoid a common failure mode in constants and readout work: a numerical factor should not be selected after a target value is known. In the present paper, an observable must first be assigned to a readout class. The associated factor then follows from the assigned readout operation or stated bridge rule. If no operation or stated bridge rule applies, the case remains open rather than being patched by a new factor.

3 Framework

The compact fiber is

$$F = \mathbb{Z}_4^{m_1} \times \mathbb{Z}_4^{m_2} \times \mathbb{Z}_8^{e}.$$

Here m_1, m_2, and e are sector labels. The two $\mathbb{Z}_4$ factors are the magnetic sectors, and the $\mathbb{Z}_8$ factor is the electric sector. Their capacities are

$$N_m = 4, \qquad N_e = 8,$$

so that

$$|F| = N_m^2 N_e = 4 \cdot 4 \cdot 8 = 128.$$

This is the carrier-side statement.

The present paper passes from the carrier to its character group,

$$\widehat{F} = \mathrm{Hom}(F, U(1)).$$

Since F is finite abelian, its character group has the same finite factor structure as F. This does not mean that carrier elements and character labels have the same role. Elements of F are finite carrier states or positions. Elements of $\widehat{F}$ are phase/readout labels. The isomorphism gives a canonical dual structure; it does not collapse the carrier and its readout dual.

A character of F is indexed by

$$(a, b, c) \in \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_8$$

and is written

$$\chi_{a,b,c}(x, y, z) = \exp\left(2\pi i \left[\frac{ax}{4} + \frac{by}{4} + \frac{cz}{8}\right]\right).$$

The factorization

$$\widehat{F} \cong \widehat{\mathbb{Z}_4^{m_1}} \times \widehat{\mathbb{Z}_4^{m_2}} \times \widehat{\mathbb{Z}_8^{e}}$$

therefore gives two magnetic character sectors and one electric character sector. This is the finite readout structure used below.

4 Character sectors

The character group gives the finite readout dual of the carrier. For a cyclic factor $\mathbb{Z}_n$, every character is determined by the image of 1, which must be an n-th root of unity. Hence

$$\widehat{\mathbb{Z}_n} \cong \mathbb{Z}_n.$$

These are standard facts about characters of finite abelian groups [5, 6].

Since characters of a finite direct product factor as products of characters on the factors,

$$\mathbb{Z}_4^{m_1} \times \widehat{\mathbb{Z}_4^{m_2}} \times \mathbb{Z}_8^{e} \cong \widehat{\mathbb{Z}_4^{m_1}} \times \widehat{\mathbb{Z}_4^{m_2}} \times \widehat{\mathbb{Z}_8^{e}}.$$

Thus

$$\widehat{F} \cong \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_8.$$

Proposition 1 (Character sectors of the compact fiber)**.** *Let*

$$F = \mathbb{Z}_4^{m_1} \times \mathbb{Z}_4^{m_2} \times \mathbb{Z}_8^{e}.$$

Then

$$\widehat{F} = \mathrm{Hom}(F, U(1)) \cong F.$$

The two factors

$$\widehat{\mathbb{Z}_4^{m_1}}, \qquad \widehat{\mathbb{Z}_4^{m_2}}$$

define finite magnetic character sectors, and

$$\widehat{\mathbb{Z}_8^{e}}$$

defines the finite electric character sector.

Proof. The cyclic character identity $\widehat{\mathbb{Z}_n} \cong \mathbb{Z}_n$, applied factorwise, gives

$$\widehat{F} \cong \widehat{\mathbb{Z}_4^{m_1}} \times \widehat{\mathbb{Z}_4^{m_2}} \times \widehat{\mathbb{Z}_8^{e}} \cong \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_8.$$

The factor names follow the sector names of the compact fiber itself. □

This is the first point at which the readout language becomes a standard representation-theoretic object. The electric sector is not only a factor of the carrier; it also has a finite phase-readout dual

$$\widehat{\mathbb{Z}_8^{e}}.$$

Likewise, each magnetic sector has a four-character readout dual. The full character count is therefore

$$|\widehat{F}| = 4 \cdot 4 \cdot 8 = 128.$$

The distinction between the carrier and its character dual remains important. The carrier coordinate

$$(x, y, z) \in F$$

is the finite internal argument of a character. The index

$$(a, b, c) \in \widehat{F}$$

labels a finite phase/readout. The isomorphism $\widehat{F} \cong F$ identifies the group structures, not the roles played by the two sides.

5 Cover compatibility

The electric factor has twice the order of a magnetic factor. The associated cover is

$$\pi : \mathbb{Z}_8 \to \mathbb{Z}_4, \qquad \pi(z) = z \bmod 4.$$

It is a surjective homomorphism with

$$\ker \pi = \{0, 4\}, \qquad |\ker \pi| = 2.$$

The corresponding character-level map goes in the opposite direction:

$$\pi^* : \widehat{\mathbb{Z}_4} \to \widehat{\mathbb{Z}_8}, \qquad (\pi^*\psi)(z) = \psi(\pi(z)).$$

Let

$$\psi_k(u) = \exp\left(2\pi i \frac{ku}{4}\right), \qquad k \in \mathbb{Z}_4.$$

Then

$$(\pi^*\psi_k)(z) = \exp\left(2\pi i \frac{kz}{4}\right) = \exp\left(2\pi i \frac{2kz}{8}\right),$$

so

$$\pi^*\psi_k = \chi^{(e)}_{2k}.$$

As k ranges over $\mathbb{Z}_4$, the image is

$$\operatorname{im}\pi^* = \{\chi^{(e)}_0, \chi^{(e)}_2, \chi^{(e)}_4, \chi^{(e)}_6\}.$$

Proposition 2 (Cover-compatible electric characters)**.** *An electric character*

$$\chi^{(e)}_c \in \widehat{\mathbb{Z}^e_8}$$

is directly compatible with the magnetic base under the cover

$$\pi : \mathbb{Z}_8 \to \mathbb{Z}_4$$

if and only if

$$c \equiv 0 \pmod 2.$$

Proof. Direct compatibility means membership in the image of π^*. The pullback calculation gives

$$\operatorname{im}\pi^* = \{\chi^{(e)}_{2k} : k \in \mathbb{Z}_4\} = \{\chi^{(e)}_0, \chi^{(e)}_2, \chi^{(e)}_4, \chi^{(e)}_6\}.$$

These are exactly the even electric characters. □

The same result can be stated constructively in terms of the deck transformation of the cover. Let

$$\sigma : \mathbb{Z}_8 \to \mathbb{Z}_8, \qquad \sigma(z) = z + 4.$$

Then $\pi \circ \sigma = \pi$, so σ generates the deck group of the degree-two cover. Acting on an electric character,

$$(\sigma^*\chi^{(e)}_c)(z) = \chi^{(e)}_c(z+4) = \exp\left(2\pi i \frac{c(z+4)}{8}\right) = (-1)^c \chi^{(e)}_c(z).$$

Thus even electric characters are deck-invariant. Odd electric characters are deck-anti-invariant.

Corollary 1 (Deck parity of electric characters)**.** *An electric character $\chi^{(e)}_c$ descends directly to the magnetic base if and only if it is deck-invariant:*

$$\sigma^*\chi^{(e)}_c = \chi^{(e)}_c.$$

Equivalently,

$$c \equiv 0 \pmod 2.$$

If c is odd, then

$$\sigma^*\chi^{(e)}_c = -\chi^{(e)}_c.$$

The odd character therefore carries a nontrivial deck parity. It cannot be read as an ordinary direct magnetic-base character unless that deck parity is retained as part of a cover-mediated, cover-changing, or boundary-coupled readout.

This gives a constructive interpretation of the odd-character case. Odd electric character is not merely an excluded residue class. It is a deck-anti-invariant readout. A cover-mediated treatment must keep track of the associated deck parity rather than silently projecting the character to the magnetic base.

The cover therefore has two related but distinct consequences. First, it identifies the magnetic-base characters with the even electric characters. Second, when a readout is assigned to the cover-mediated electric class, the electric sector must be resolved together with the degree-two cover, giving the combined resolution

$$\frac{1}{|\ker \pi| N_e} = \frac{1}{2 \cdot 8} = \frac{1}{16}.$$

This factor is not a kernel average by itself. The kernel contributes the degree-two cover, while the electric sector contributes the eightfold character resolution.

6 Readout operations

The preceding sections fix the finite objects. The readout operations are the normalized ways in which those objects are sampled, complemented, or combined. The point is not to introduce new numerical factors, but to identify which factors are forced once the relevant finite sector operation has been specified.

Projection onto one electric character gives

$$P_e = \frac{1}{|\widehat{\mathbb{Z}_8^e}|} = \frac{1}{N_e} = \frac{1}{8}.$$

Projection onto one magnetic character gives

$$P_m = \frac{1}{|\widehat{\mathbb{Z}_4^m}|} = \frac{1}{N_m} = \frac{1}{4}.$$

Excluding the trivial electric character gives the normalized electric self-sector complement

$$C_e = \frac{|\widehat{\mathbb{Z}_8^e}| - 1}{|\widehat{\mathbb{Z}_8^e}|} = \frac{N_e - 1}{N_e} = \frac{7}{8}.$$

A joint readout of one magnetic sector and the electric sector gives the product-sector resolution

$$P_{me} = \frac{1}{|\widehat{\mathbb{Z}_4^m}||\widehat{\mathbb{Z}_8^e}|} = \frac{1}{N_m N_e} = \frac{1}{32}.$$

The full character count is

$$|\widehat{F}| = |\widehat{\mathbb{Z}_4^{m_1}}||\widehat{\mathbb{Z}_4^{m_2}}||\widehat{\mathbb{Z}_8^e}| = 128.$$

The cover-mediated electric resolution has a different status. It uses both the electric-sector capacity and the degree of the cover:

$$P_{\text{cover}} = \frac{1}{|\ker \pi| N_e} = \frac{1}{2 \cdot 8} = \frac{1}{16}.$$

This factor is fixed once the readout has been assigned to the cover-mediated electric class. The assignment of a concrete observable to that class is a structural statement about the readout, not a consequence of character counting alone.

The operation table records the theorem-grade and cover-conditional finite readout operations. It does not include bridge factors.

Readout class	Character operation	Factor	Status
Electric projection	One character in $\widehat{\mathbb{Z}_8^e}$	$\frac{1}{8}$	T
Magnetic projection	One character in $\widehat{\mathbb{Z}_4^m}$	$\frac{1}{4}$	T
Electric self-sector complement	Nontrivial electric characters	$\frac{7}{8}$	T
Cover-mediated electric resolution	Electric sector plus degree-two cover	$\frac{1}{16}$	T/S
Magnetic-electric product readout	One magnetic sector times electric sector	$\frac{1}{32}$	T
Full character count	All characters of $\widehat{F}$	128	T

Here T denotes a theorem-grade finite-character result. The label T/S denotes a factor fixed by the character-cover structure once the readout class is accepted, while the observable assignment remains structural.

Bridge quantities require separate treatment. The interregional completed capacity

$$R_F = 128\left(1+\frac{2}{9}\right) = \frac{1408}{9}$$

uses the full character count 128, but the $2/9$ term belongs to the interregional or vibrational layer, not to $\widehat{F}$ alone. Likewise, an external completion factor such as

$$1+\frac{1}{R_F} = \frac{1417}{1408}$$

requires an external-readout bridge. These quantities may be audited against the character calculus, but they are not character operations.

7 Character-readout classification

The readout operations of the previous section can now be stated as a single classification result. The theorem is limited to the finite abelian character layer. Bridge quantities are not included in the theorem; they are audited separately in Section 9.

Theorem 1 (Character-readout classification)**.** *Let*

$$F = \mathbb{Z}_4^{m_1} \times \mathbb{Z}_4^{m_2} \times \mathbb{Z}_8^{e}$$

and let

$$\widehat{F} = \mathrm{Hom}(F, U(1)).$$

Then the theorem-grade finite readout factors used at the compact-fiber character layer are realized as canonical operations on $\widehat{F}$ and its sector factors:

$$\frac{1}{N_e}=\frac{1}{8}, \qquad \frac{1}{N_m}=\frac{1}{4}, \qquad \frac{N_e-1}{N_e}=\frac{7}{8}, \qquad \frac{1}{N_mN_e}=\frac{1}{32}, \qquad |\widehat{F}| = N_m^2 N_e = 128.$$

The degree-two cover-mediated electric resolution is

$$\frac{1}{|\ker \pi| N_e} = \frac{1}{2\cdot 8} = \frac{1}{16}.$$

Proof. The electric character sector has $N_e = 8$ characters. Projection onto one electric character gives $1/N_e = 1/8$, while exclusion of the trivial electric character gives $(N_e - 1)/N_e = 7/8$. Each magnetic character sector has $N_m = 4$ characters, so projection onto one magnetic character gives $1/N_m = 1/4$. A product readout using one magnetic sector and the electric sector has $N_m N_e = 32$ available character pairs, giving $1/(N_m N_e) = 1/32$. The full character group has size $N_m^2 N_e = 128$. Finally, the cover $\pi : \mathbb{Z}_8 \to \mathbb{Z}_4$ has kernel size 2, so a cover-mediated electric readout resolves both the electric sector and the cover degree, giving $1/(|\ker \pi| N_e) = 1/16$. □

The theorem says that these finite factors are forced once the corresponding character operation has been specified. It does not say that every compact-fiber readout is already a pure character operation. A theorem-grade finite readout must be an operation on a character sector or on a product of character sectors. A bridge factor must identify the additional structure that it uses. A factor with neither a character operation nor a bridge rule remains open.

8 Finite character conservation

The character group also supplies a selection rule for closed internal finite-sector couplings. The rule is the ordinary multiplication law of characters, applied to the compact-fiber readout sectors. It is not a new dynamical postulate. It states the finite neutrality condition required when several sector readouts form one closed internal coupling.

Let

$$\alpha = \chi_{a,b,c}, \qquad \beta = \chi_{a',b',c'}$$

be two characters in $\widehat{F}$. Their pointwise product is

$$\alpha\beta = \chi_{a+a',\, b+b',\, c+c'},$$

where the first two indices are taken modulo 4 and the third index is taken modulo 8. The inverse character is therefore

$$\chi_{a,b,c}^{-1} = \chi_{-a,-b,-c}.$$

Theorem 2 (Finite character conservation)**.** *For a closed internal finite-sector coupling involving characters*

$$\chi_{a_1,b_1,c_1}, \chi_{a_2,b_2,c_2}, \ldots, \chi_{a_n,b_n,c_n},$$

the coupling is character-neutral if and only if

$$\prod_{i=1}^{n} \chi_{a_i,b_i,c_i} = \chi_{0,0,0}.$$

Equivalently,

$$\sum_{i=1}^{n} a_i \equiv 0 \pmod 4, \qquad \sum_{i=1}^{n} b_i \equiv 0 \pmod 4, \qquad \sum_{i=1}^{n} c_i \equiv 0 \pmod 8.$$

Proof. Repeated use of the character multiplication law gives

$$\prod_{i=1}^{n} \chi_{a_i,b_i,c_i} = \chi_{\sum_i a_i,\, \sum_i b_i,\, \sum_i c_i}.$$

This product is the trivial character $\chi_{0,0,0}$ exactly when each component of the total index vanishes in its corresponding finite sector. The two magnetic sums are therefore taken modulo 4, and the electric sum is taken modulo 8. □

In physical language, this is the finite-sector analogue of charge conservation or phase neutrality for a closed internal coupling. A closed coupling cannot carry a residual finite character unless that residual is assigned to an explicit boundary, source, sink, or external readout. For an open process, the corresponding closed statement would include the boundary character:

$$\prod_i \chi_i \, \chi_{\text{boundary}} = \chi_{0,0,0}.$$

Thus the conservation rule is not weakened by open processes; rather, the boundary must be represented if nontrivial character is exported.

The cover relation adds a second selection condition for direct electric-to-magnetic compatibility. From the pullback

$$\pi^* : \widehat{\mathbb{Z}_4} \to \widehat{\mathbb{Z}_8},$$

the magnetic base appears inside the electric sector as the even electric characters,

$$\operatorname{im} \pi^* = \{\chi_0^{(e)}, \chi_2^{(e)}, \chi_4^{(e)}, \chi_6^{(e)}\}.$$

A direct electric-to-magnetic readout must therefore use an even electric character:

$$c \equiv 0 \pmod 2.$$

An odd electric character is not directly magnetic-base compatible. It must be treated as a cover-changing or cover-mediated readout rather than as an ordinary direct descent to the magnetic base.

This gives a frozen selection-rule form. A channel assigned to direct electric-to-magnetic readout may use only even electric character. A channel carrying odd electric character is therefore not a leading-order direct descent to the magnetic base; it must be represented as cover-mediated, cover-changing, or boundary-coupled. This is a structural selection rule, not yet an empirical assignment to a particular transition.

This also recasts the orbital-angular-momentum parity rule, as developed in the OAM/parity section of the Radiation paper [4], as a cover-compatibility statement. There the photon winding factor has the form

$$W_{\text{photon}} = (-1)^{\Delta\ell}.$$

At the finite character layer, the same parity distinction is represented by the even/odd electric-character distinction under the cover:

$$\Delta\ell \equiv 0 \pmod 2 \quad \mapsto \quad c \equiv 0 \pmod 2,$$

and hence direct magnetic-base compatibility, while

$$\Delta\ell \equiv 1 \pmod 2 \quad \mapsto \quad c \equiv 1 \pmod 2,$$

and hence nontrivial cover parity. Thus the OAM parity factor is a concrete instance of the cover-compatibility rule: even winding is direct-cover compatible, while odd winding carries nontrivial deck parity at the finite readout layer.

The immediate theoretical use of this result is therefore not a precision numerical prediction. It is a selection-rule structure. Once a physical channel has been assigned to the closed internal class, character neutrality becomes a testable constraint. Once a channel has been assigned to direct electric-to-magnetic cover compatibility, evenness of the relevant electric character becomes a testable constraint. These assignments must be made before comparison with data; otherwise the rule would lose its value as a no-retuning constraint.

9 Mapping to prior readout factors

The preceding theorem is useful only if it disciplines the factors already used in the program. The relevant question is not whether a familiar denominator can be found somewhere in the compact fiber after the fact, but whether a factor used in an earlier application has a definite readout operation and a definite status. The mapping in this section is therefore an audit. The theorem-grade rows are finite character operations; the cover-mediated row is conditional on readout assignment; the bridge rows mark where additional interregional or external-readout structure enters.

Factor	Prior role	Character or bridge operation	Status
128	Carrier capacity	Full character count $\lvert\widehat{F}\rvert = N_m^2 N_e$	T
$\frac{1}{8}$	Electric projection / first-order increment	Projection onto one electric character	T
$\frac{1}{4}$	Magnetic projection	Projection onto one magnetic character	T
$\frac{7}{8}$	Electric complement / screening factor	Normalized nontrivial electric-character complement	T
$\frac{1}{16}$	Double-cover / half-step readout	Electric sector resolved through the degree-two cover	T/S
$\frac{1}{32}$	Magnetic-electric product correction	Product-sector average over one magnetic sector and the electric sector	T
$\frac{1}{9}$	Interregional placement factor	Interregional bridge term, not a pure character operation	S
$\frac{2}{9}$	Two-sector interregional completion	Interregional completion over finite count	S
$\frac{1408}{9}$	Completed capacity R_F	$128(1 + 2/9)$	S
$\frac{1417}{1408}$	External completion	$1 + 1/R_F$	S

The theorem-grade rows are the ones that follow from $\widehat{F}$ and its factorization. The character group contains eight electric characters, four characters in each magnetic sector, thirty-two magnetic-electric character pairs for a one-magnetic-sector product readout, and 128 full-fiber characters. These facts account for

$$\frac{1}{8}, \qquad \frac{1}{4}, \qquad \frac{7}{8}, \qquad \frac{1}{32}, \qquad 128.$$

They are not fitted constants. They are normalized counts or complements of finite character sectors.

The cover-mediated row is conditional in a different way. The factor

$$\frac{1}{16} = \frac{1}{\lvert \ker \pi \rvert N_e}$$

is fixed by the degree-two cover and the electric sector capacity. What is not automatic is the assignment of a particular observable to the cover-mediated electric class. When that assignment is structurally justified, the factor is no longer adjustable.

The remaining rows are bridge-status rows. The factor 1/9 appears as an interregional placement factor in the natural-unit context; paired across two magnetic-sector placements it gives the 2/9 completion entering

$$R_F = 128\left(1 + \frac{2}{9}\right) = \frac{1408}{9},$$

following Nehru's interregional-ratio construction [7]. Neither 1/9 nor 2/9 is a character-sector count. They are listed here precisely because they are not part of the finite character calculus. The same applies to the external completion

$$1 + \frac{1}{R_F} = \frac{1417}{1408}.$$

These bridge rows mark where the present paper's pure character-theory layer ends and where interregional or external-readout assumptions begin.

Worked rejection: 5/32. The same table also shows how the calculus rejects an unsupported factor. A proposed correction

$$\frac{5}{32}$$

is not admitted merely because it is numerically close to a desired residual. The denominator

$$32 = N_m N_e$$

has a product-sector interpretation, but the numerator 5 would require a specified operation selecting five admissible character pairs from that product sector. Without such an operation, 5/32 is not a theorem-grade character factor. It would have to be marked open or assigned to an explicitly stated bridge rule. Thus 5/32 is not rejected because it is numerically impossible; it is rejected because the readout operation has not been specified. The same principle applies to any apparent correction factor: a recognizable denominator is not enough; the whole factor must be generated by a defined readout operation.

This audit also clarifies how the present paper relates to the atomic and radiation applications. In the atomic sector, the fine-structure-type factor 7/8 is the electric nontrivial-sector complement, the 1/8 and 1/4 increments are single-sector projections, the 1/16 step is cover-mediated, and the 1/32 term is the one-magnetic-sector/electric product readout. In the radiation sector, the same finite character layer underlies the use of phase sectors, quarter-turn magnetic structure, cover parity, and product-sector matching, while the full benchmark probabilities still use the representational/readout layer developed in Paper III. The present paper does not rederive those applications. It supplies the finite-sector audit that fixes which factors belong to which operation class.

The practical consequence is that later uses of these factors must respect their origin. The factor 7/8 cannot be used as a generic high-weight correction; it is the nontrivial electric-character complement. The factor 1/32 cannot be used for a purely electric projection; it is a magnetic-electric product-sector resolution. The factor 1/16 cannot be used unless the degree-two cover is part of the readout. This is the no-retuning content of the mapping: once the readout class is fixed, the factor is fixed, and if no readout class applies, the case remains open.

10 Elementary bridge structure

The preceding sections classify finite character operations and separate them from bridge-status quantities. We now apply that status discipline to the natural-unit matching correction. The purpose of this section is only to identify the typed structure of the elementary bridge. The value of the electric boundary contribution is derived in the next section.

Let

$$B : \mathcal{M}_2 \to \mathcal{R}$$

denote an elementary natural-unit bridge from mass to energy/readout form. Here $\mathcal{M}_2$ is the completed two-dimensional secondary-mass source structure, and $\mathcal{R}$ is the energy/readout target structure. The subscript records the dimensional character of the source-side secondary-mass structure.

The bridge has one source side and one target side. Its status boundary is therefore

$$\partial B = (\mathcal{M}_2 \mid \mathcal{R}),$$

so that an elementary bridge has a single boundary traversal,

$$|\partial B| = 1.$$

This is a structural statement about the readout role. A bridge with two independent source-to-target traversals would be a different observable class.

The source side requires the completed mass contribution. In the natural-unit secondary-mass notation this contribution is

$$m = \frac{1}{R_F},$$

where

$$R_F = 128\left(1 + \frac{2}{9}\right) = \frac{1408}{9}.$$

The factor 128 is the full compact-fiber character count, while the 2/9 term belongs to the interregional completion layer. Thus m is not a pure finite-character operation. It is a completed mass-side bridge quantity.

The target side requires an electric readout boundary contribution. Denote this boundary contribution by

$$e_\partial.$$

At this stage e_∂ is a typed slot, not yet a numerical value. It is the electric boundary contribution associated with the single elementary traversal

$$\mathcal{M}_2 \to \mathcal{R}.$$

The elementary bridge therefore has the typed correction package

$$s = m + e_\partial.$$

In the present application, s is the secondary-mass correction used when the completed mass unit is matched to an energy/readout expression in the natural-unit bridge. This follows from the source-target status structure. The term m supplies the completed mass-side contribution; the term e_∂ supplies the target-side electric readout boundary.

This already gives class-level exclusions. The package m alone is incomplete for an elementary mass-to-energy bridge because it contains the completed source-side mass contribution but omits the target-side boundary contribution. Conversely, packages built only from electric or charge-side quantities omit the source-side mass contribution. A package with more than one electric boundary term describes either an overloaded boundary assignment or a different bridge class with more than one independent traversal.

Thus the elementary bridge structure selects the form

$$s = m + e_\partial$$

before any numerical comparison is made. The remaining question is the identification of the boundary slot e_∂. That is the task of the next section.

11 Electric boundary mass

The previous section selected the typed elementary bridge package

$$s = m + e_\partial.$$

It remains to identify the electric boundary slot e_∂.

The secondary-mass notation follows Larson's natural-unit analysis, with Nehru's later clarification of the interregional ratio. The present paper uses these sources as provenance for the bridge-layer quantities, while reconstructing the needed factor in finite-readout notation.

The full compact-fiber readout count is

$$|\widehat{F}| = 128.$$

This factor is theorem-grade at the finite character layer. The remaining factors in the electric boundary term are bridge-layer factors: the subatomic interregional placement factor and the dimensional participation factor.

Nehru derives the subatomic vibrational contribution as follows. A one-dimensional vibration has three scalar-dimensional placement choices. For a two-dimensional rotation founded on that vibration, the vibrational degrees of freedom are therefore

$$3^2 = 9.$$

Thus one two-dimensional rotational system receives a subatomic vibrational contribution of

$$\frac{1}{9}.$$

For an atom, which contains two such two-dimensional rotational systems, the corresponding contribution is 2/9; for a subatomic rotational system, it is 1/9 [7].

Here E denotes Larson's three-dimensional electric mass unit and should not be confused with the target readout object $\mathcal{R}$. In the present finite-readout notation, the corresponding full electric mass unit is

$$E = \frac{1}{9|\widehat{F}|} = \frac{1}{9 \cdot 128}.$$

This is the full three-dimensional electric mass unit. Larson distinguishes this full electric mass from the two-dimensional electric mass [8]. In his secondary-mass derivation, where only one two-dimensional rotation is involved, the electric mass is 2/3 of the full unit:

$$e = \frac{2}{3}E.$$

Nehru adopts the same notation,

$$E = \text{electric mass (3 dim.)}, \qquad e = \text{electric mass (2 dim.)} = \frac{2}{3}E,$$

in his secondary-mass bookkeeping [9].

The elementary bridge

$$B : \mathcal{M}_2 \to \mathcal{R}$$

has a two-dimensional secondary-mass source. Its electric boundary contribution must therefore use the two-dimensional electric mass, not the consolidated three-dimensional electric mass. Hence

$$e_\partial = e.$$

Combining the finite readout count, the subatomic placement factor, and the two-dimensional participation factor gives

$$e_\partial = e = \frac{2}{3}E = \frac{2}{3}\cdot\frac{1}{9|\widehat{F}|} = \frac{2}{3\cdot 9\cdot 128} = \frac{1}{1728}.$$

Equivalently, the status decomposition is

$$\boxed{e_\partial = \underbrace{\frac{1}{|\widehat{F}|}}_{\text{finite character count}} \quad \underbrace{\frac{1}{9}}_{\text{subatomic interregional placement}} \quad \underbrace{\frac{2}{3}}_{\text{two-dimensional participation}} = \frac{1}{1728}.}$$

This display makes the layer structure explicit. The $1/|\widehat{F}|$ factor belongs to the finite character-readout calculus; the $1/9$ and $2/3$ factors belong to the interregional and secondary-mass bridge layer.

Theorem 3 (Elementary electric boundary mass)**.** *Let*

$$B : \mathcal{M}_2 \to \mathcal{R}$$

be an elementary bridge from the completed two-dimensional secondary-mass source to the energy/readout target. Using the Larson–Nehru bridge-layer rules that a subatomic two-dimensional rotational system carries the $1/9$ *vibrational placement factor and that the two-dimensional electric mass is* $2/3$ *of the full three-dimensional electric mass, the unique electric boundary contribution of* B *is*

$$e_\partial = \frac{2}{3}\cdot\frac{1}{9|\widehat{F}|} = \frac{1}{1728}.$$

Proof. The finite character layer gives

$$|\widehat{F}| = 128.$$

The subatomic two-dimensional rotational placement factor gives the denominator 9, so the full electric mass unit is

$$E = \frac{1}{9|\widehat{F}|}.$$

The source of the elementary bridge is two-dimensional, while E is the full three-dimensional electric mass unit. The two-dimensional participation rule therefore gives

$$e_\partial = e = \frac{2}{3}E.$$

Substituting $E = 1/(9|\widehat{F}|)$ and $|\widehat{F}| = 128$ yields

$$e_\partial = \frac{2}{3}\cdot\frac{1}{9\cdot 128} = \frac{1}{1728}.$$

□

This theorem is not a pure character-theory result. Its status is bridge-theorem grade:

$$S_T.$$

The character group supplies the finite readout count 128. The interregional placement factor $1/9$ and the dimensional participation factor $2/3$ belong to the Larson–Nehru secondary-mass and interregional bridge layer. The theorem shows that, once those bridge-layer premises are stated, the electric boundary contribution is fixed and is not a tunable correction.

12 Matching admissibility

The elementary bridge structure and electric-boundary theorem now determine the matching package. The bridge has the typed form

$$s = m + e_\partial,$$

and the electric boundary contribution is

$$e_\partial = e = \frac{1}{1728}.$$

Therefore

$$s = m + e.$$

This conclusion is not obtained by comparing candidate packages against a target residual. It follows from the readout class. The term m is the completed source-side mass contribution. The term e is the single two-dimensional electric boundary contribution required by the elementary mass-to-energy bridge.

Theorem 4 (Elementary matching package)**.** *Let*

$$B : \mathcal{M}_2 \to \mathcal{R}$$

be an elementary natural-unit mass-to-energy bridge. Let $m = 1/R_F$ be the completed source-side mass contribution, and let e be the two-dimensional electric boundary contribution

$$e = \frac{1}{1728}.$$

Then the admissible elementary matching package is

$$s = m + e.$$

Proof. By the elementary bridge structure, a single mass-to-energy bridge has one completed mass-side contribution and one electric boundary contribution:

$$s = m + e_\partial.$$

By the elementary electric-boundary theorem,

$$e_\partial = e.$$

Hence

$$s = m + e.$$

□

The competing packages fail by readout class.

First, the package

$$s = m$$

contains the completed source-side mass contribution but omits the target-side electric boundary. It is therefore incomplete for an elementary mass-to-energy bridge.

Second, the package

$$s = m + 2e + C$$

contains the source-side mass contribution, but it assigns two electric boundary contributions to a bridge with a single boundary traversal. The extra e is therefore a boundary overloading unless the observable is reclassified as a two-boundary bridge. The term C is also a charge-class contribution, not an electric mass-boundary contribution. Larson classifies charge as an additional rotational vibration of the rotating particle or atom [8], and Nehru treats C as the mass due to normal electric charge [9]. Such a term is admissible only for a charge-boundary observable, not for the elementary mass-to-energy bridge considered here.

Third, the packages

$$s = e - c$$

and

$$s = e - C$$

omit the completed source-side mass contribution m. They also introduce charge-class terms. The term c is the electron-charge contribution, while C is the normal-charge contribution. These are not substitutes for the mass-side contribution required by the bridge.

The rejection of these packages is therefore structural:

$$m \quad \text{misses the boundary side,}$$

$$m + 2e + C \quad \text{overloads the boundary and imports charge class,}$$

$$e - c,\ e - C \quad \text{miss the mass side and import charge class.}$$

The admissibility result can be summarized as

$$\boxed{B : \mathcal{M}_2 \to \mathcal{R} \quad \Longrightarrow \quad s = m + e.}$$

This is a theorem-grade bridge result under the stated compact-fiber and Larson–Nehru bridge-layer premises. It is not a pure finite-character theorem, because m and e use interregional and secondary-mass structure beyond $\widehat{F}$. Its status is therefore

$$S_T.$$

The no-retuning content is the class discipline. Once the observable is assigned as an elementary mass-to-energy bridge, the source-side slot, boundary count, and boundary type are fixed before numerical comparison. A candidate package is admissible only if it fills those slots without omission, duplication, or importation of an unassigned class.

13 Boundaries and follow-on problems

The results of this paper occupy two linked layers. The first is the finite abelian character-readout layer,

$$F \longmapsto \widehat{F}.$$

The second is the elementary bridge-matching layer, where the finite readout status taxonomy is used together with Larson–Nehru interregional and secondary-mass premises to select the matching package

$$s = m + e.$$

These layers should not be collapsed. The character group supplies the finite readout calculus; the bridge layer supplies additional interregional, dimensional, and secondary-mass structure.

The limitation of the character layer is structural. Since F is finite abelian, every irreducible complex representation of F is one-dimensional. Thus $\widehat{F}$ supplies a discrete additive phase/readout calculus. It does not by itself supply continuous phase dynamics, noncommuting operator structure, spinor representations, scale dependence, or time evolution. Those require additional representational, operator, projection, or dynamical layers.

The bridge result has its own boundary. The elementary matching theorem, Theorem 4, applies to an elementary natural-unit mass-to-energy bridge

$$B : \mathcal{M}_2 \to \mathcal{R}$$

with one source side, one target side, and one electric boundary traversal. A bridge with multiple independent source-to-target traversals, an explicit charge boundary, or a different source dimensionality would be a different observable class. It would require a new admissibility analysis rather than reuse of

$$s = m + e.$$

This is why the status label S_T is necessary. The result

$$s = m + e$$

is theorem-grade within the stated bridge layer, but it is not a pure finite-character result. The character calculus supplies the status discipline and the full finite readout count. The $1/9$ placement factor, the $2/3$ dimensional participation factor, and the secondary-mass notation m, E, e, C, c belong to the interregional and secondary-mass bridge layer. Treating all of these as a single undifferentiated character-theory result would obscure the structure.

More generally, the finite abelian character calculus handles finite sector, phase, and cover-parity structure. It does not by itself encode tensor-product exchange symmetry, entanglement structure, or noncommuting operator dynamics.

Hong–Ou–Mandel bunching gives a concrete example of the character layer's boundary [10]. The Radiation paper treats the balanced beamsplitter cancellation as a two-photon interference effect in its HOM benchmark section [4]. At the finite character layer, the bunched and coincidence sectors can carry the same total character when the photons are indistinguishable. Character conservation alone therefore cannot distinguish the HOM cancellation channel from the non-cancelled coincidence channel. The cancellation depends on bosonic exchange symmetry and the beamsplitter reflection phase, which are tensor-product and junction-phase structures rather than one-dimensional character operations on $\widehat{F}$. Thus HOM remains compatible with the finite character calculus, but it is not derived by Theorem 2 alone.

The finite character conservation rule has its own domain restriction. It applies to closed internal finite-sector couplings. If a process is open, externally read, cover-changing, dissipative, or measurement-mediated, the missing character must be assigned to an explicit boundary or readout channel. The closed rule is restored only after that boundary contribution is included.

The elementary matching theorem has the analogous restriction. It applies only after the observable has been assigned to the elementary bridge class. If an observable requires charge-class terms C or c, a consolidated three-dimensional electric mass E, or more than one electric boundary traversal, it is outside the elementary package and must be classified separately. The no-retuning rule is therefore not that $m + e$ is used everywhere. It is that each observable must declare its readout class before a correction package is admitted.

These limitations mark the difference between a finite readout calculus, a theorem-grade bridge result under stated premises, and a completed physical dynamics. The finite character calculus is one layer of the framework. Spinor structure, continuous phase dynamics, scale dependence, and operator noncommutativity belong to additional layers that build on rather than replace the finite calculus.

14 Conclusion

This paper has developed a finite readout calculus from the compact fiber

$$F = \mathbb{Z}_4^{m_1} \times \mathbb{Z}_4^{m_2} \times \mathbb{Z}_8^{e}$$

by passing to its character group

$$\widehat{F} = \mathrm{Hom}(F, U(1)).$$

The resulting character sectors give a representation-theoretic form to the two magnetic $\mathbb{Z}_4$ sectors and the electric $\mathbb{Z}_8$ sector. The degree-two cover

$$\pi : \mathbb{Z}_8 \to \mathbb{Z}_4$$

identifies direct magnetic-base compatibility with the even electric characters, while the odd electric characters carry nontrivial deck parity.

The character-readout classification theorem fixes the finite factors

$$\frac{1}{8}, \qquad \frac{1}{4}, \qquad \frac{7}{8}, \qquad \frac{1}{32}, \qquad 128$$

as projections, complements, product-sector resolutions, or full character counts. The cover-mediated factor

$$\frac{1}{16}$$

is fixed once the readout is assigned to the degree-two electric cover class. Closed internal finite-sector couplings obey the character-neutrality rule

$$\prod_i \chi_i = \chi_{0,0,0}.$$

The same status discipline was then applied to the elementary natural-unit mass-to-energy bridge

$$B : \mathcal{M}_2 \to \mathcal{R}.$$

The elementary bridge has one completed source-side mass contribution and one target-side electric boundary contribution, giving the typed package

$$s = m + e_\partial.$$

Using the Larson–Nehru interregional and secondary-mass rules, the electric boundary contribution is

$$e_\partial = e = \frac{2}{3} \cdot \frac{1}{9|\widehat{F}|} = \frac{1}{1728}.$$

Thus the admissible elementary matching package is

$$s = m + e.$$

This result is not a pure finite-character theorem. Its status is

$$S_T,$$

theorem-grade within the stated bridge layer. The character group supplies the finite readout count and the status calculus; the 1/9 interregional placement factor and 2/3 dimensional participation factor belong to the secondary-mass bridge layer. Keeping these roles separate is the point of the no-retuning protocol.

The resulting discipline is operational. A proposed factor must be a defined character operation, a cover-conditional readout, a theorem-grade bridge result under stated premises, or an open term. The package m alone omits the electric boundary contribution. The package $m + 2e + C$ overloads the elementary boundary and imports a charge-class term. The packages $e - c$ and $e - C$ omit the completed source-side mass contribution and substitute charge-class quantities. These are class rejections, not numerical rejections.

The finite character layer, the bridge layer, and later dynamical layers therefore remain distinct. The present result supplies a compact finite readout and matching protocol. Later spinor, effective $U(1)$, scale-dependent, or dynamical constructions may build on this finite structure, but must state the additional machinery by which they extend it.

References

[1] A. R. Wells. The Compact Fiber as the Canonical Admissibility Structure for Discrete Scalar Motion, 2026. Paper I. DOI: 10.5281/zenodo.19423876.

[2] A. R. Wells. Atomic Structure from the Compact Fiber: Transport, Screening, and Spectroscopic Thresholds in the Reciprocal System, 2026. Paper II. DOI: 10.5281/zenodo.19426034.

[3] A. R. Wells. Effective Quantum and Gravitational Structure from the Compact Fiber: Cross-Frame Projection, Hilbert Representation, Born Rule, and Coordinate-Time Metric, 2026. Paper III. DOI: 10.5281/zenodo.19434652.

[4] A. R. Wells. Radiation from the Compact Fiber: A Unified Architecture for Polarization, Interference, Correlation, Topology, and Radiation-Matter Transfer, 2026. DOI: 10.5281/zenodo.19688974.

[5] Jean-Pierre Serre. *Linear Representations of Finite Groups*. Springer, 1977.

[6] William Fulton and Joe Harris. *Representation Theory: A First Course*. Springer, 1991.

[7] K. V. K. Nehru. The Inter-regional Ratio. *Reciprocity*, 14(2):5, 1985. Winter.

[8] Dewey B. Larson. *Nothing But Motion*. North Pacific Publishers, 1979.

[9] K. V. K. Nehru. Internal Ionization and Secondary Mass. Privately circulated manuscript, 1982.

[10] C. K. Hong, Z. Y. Ou, and L. Mandel. Measurement of subpicosecond time intervals between two photons by interference. *Physical Review Letters*, 59(18):2044–2046, 1987.