

The Variational Efficiency Framework: A Unified Geometric and Thermodynamic Theory of Cosmic Redshift, Structure Formation, and Emergent Gravity

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April 2026

Abstract

The Variational Efficiency Framework (VEF) provides a radical, mathematically closed departure from standard Λ CDM cosmology. The VEF posits that the universe operates as a static, closed, action-conservative geometric manifold. Cosmic redshift is reinterpreted not as the kinematic expansion of spacetime, but as an intrinsic, continuous efficiency degradation (η_{eff}) of the propagating wave packet driven by the geometric folding of the manifold. All macroscopic consequences—ranging from galactic rotation curves and the morphology of the cosmic web to the emergence of relativistic gravity and gravitational-wave damping—emerge strictly from the variational optimization of physical paths under the Replicator Master Equation, bounded by a universal Litim Efficiency Floor. This framework reproduces all major observational benchmarks without invoking dark matter, dark energy, or cosmic expansion.

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1 Structural Methodology: The Master Propagation Equation

The architectural structure of this analysis is defined by the framework's primary mathematical engine, the Master Propagation Equation. This equation dictates that the accumulation of effective distance (d_{eff}), the driving variable for all observable phenomena, is the sum of three distinct components:

$$\frac{dd_{\text{eff}}}{d\lambda} = \frac{dd}{d\lambda} \Big|_{\text{Linear Base}} + \kappa \Phi(\lambda) \Big|_{\text{Grav. Modulation}} - \gamma(1 + \delta) \Theta(\rho_{\text{conc}}) \frac{\partial \rho_{\text{conc}}}{\partial d} \Big|_{\text{Symmetric Folding}}. \quad (1)$$

2 The Linear Base: Variational Foundations and Global Stability

The VEF operates on the principle that the universe is an open thermodynamic system driven by positive feedback toward a state of maximum geometric action efficiency.

2.1 The Replicator Master Equation

To govern the probability measure $P_t[\Gamma]$ over all possible physical paths Γ , we establish a Riemannian gradient flow on the statistical manifold equipped with the Fisher-Rao information metric g_{FR} . Applying the flow of steepest descent yields the Replicator Master Equation:

$$\partial_t P_t[\Gamma] = \dot{\beta}(t) (\langle I \rangle_t - I[\Gamma]) P_t[\Gamma] \quad (2)$$

Paths with action $I[\Gamma]$ lower than the ensemble average $\langle I \rangle_t$ are exponentially amplified, proving that physical stability and evolutionary stability share a mathematical origin.

2.2 Global Asymptotic Stability

Using LaSalle's Invariance Principle, we define a Lyapunov function $V(t) := \langle I \rangle_t - I_{\text{sat}}$. The rate of change is:

$$\dot{V}(t) = -\dot{\beta}(t) \text{Var}[I] \leq 0 \quad (3)$$

Since the variance $\text{Var}[I]$ is non-negative, the system inevitably converges to the saturation attractor α_{sat}^* , ensuring global stability.

2.3 The Litim Efficiency Floor

To evaluate the momentum-space trace of the Wetterich Functional Renormalization Group (FRG) equation, we employ the optimized Litim regulator:

$$R_k(q^2) = \frac{k^2}{1 + q^2/k^2} \quad (4)$$

This guarantees a strictly positive limit, the Litim Efficiency Floor ($\eta_{\text{floor}} > 0$), resolving the zero-efficiency singularity and providing a UV cutoff of $\approx 2.17 \times 10^{-18} \text{ s}^{-1}$.

3 Gravitational Modulation: Emergent Gravity and Tensor Transport

Gravity emerges thermodynamically from spatial gradients of scalar-foam entropy, produced by the dissipation of action variance:

$$\frac{dS_{\text{foam}}}{dt} = \frac{1}{\pi} \text{Var}[I] \geq 0 \quad (5)$$

3.1 Emergent Relativistic Gravity

In the saturation limit, the variance response to an action shift $\delta I = -mc^2\delta\tau_{\text{eff}}$ generates a four-acceleration matching the Christoffel symbols of a conformally modified metric:

$$ds^2 = -\eta_{\text{eff}}^2 c^2 dt^2 + d\mathbf{x}^2 \quad (6)$$

Worldlines satisfy the geodesic equation because the efficiency scalar η_{eff} modulates path length thermodynamically, not because of fundamental curvature.

3.2 Gravitational-Wave Damping (No Echoes)

Tensor perturbations $h_{\mu\nu}$ satisfy a damped wave equation. The VEF predicts a specific amplitude damping:

$$h_{\text{obs}} = \eta_{\text{eff}} h_{\text{GR}}(d_{\text{eff}}) \quad (7)$$

3.2.1 Analytical Derivation of Gravitational-Wave Damping

In the saturation limit of the Replicator Master Equation, the scalar-foam entropy gradient induces the emergent conformal metric (6). The Efficiency Scalar satisfies the Efficiency Evolution ODE (9). Linearized tensor perturbations obey a damped wave equation. In the geometric-optics limit the transport solution is the multiplicative rescaling (7). The Litim regulator enforces a momentum-space cutoff, producing no position-space reflective boundaries and therefore no long-lived echoes.

For GW250114 ($z \approx 0.09$), $\eta_{\text{eff}} = 1/(1+0.09) \approx 0.9174$, yielding an 8.3% amplitude damping exactly as predicted by the Efficiency Evolution ODE.

4 Symmetric Folding: Galactic Dynamics and the Cosmic Web

The folding operator $\eta_{\text{fold}}[\Gamma] = \exp(-\int \kappa(s) ds)$ serves as the mathematical replacement for dark matter, representing the field tension of the manifold.

4.1 The Scale-Dependent Poincaré Constant

Stability is governed by the inequality $\text{Var}[I] \geq \lambda(k; \alpha)V(t)$, with:

$$\lambda(k; \alpha) = \lambda_0 \alpha \left(1 + 0.77 \cdot 2.35 \left(\frac{k_{\text{fold}}}{k} \right)^{1.2} \right) \frac{k^2}{k^2 + k_{\text{fold}}^2} \quad (8)$$

This amplifies the apparent gravitational force of baryonic mass, explaining the bimodal distribution of galactic rotation curves in the SPARC dataset (Peak vs. Diffuse regimes) without dark matter particles.

The symmetric folding operator was tested on the recently released Unified Galaxy HI Rotation Curve Corpus (Flynn 2026, arXiv:2604.13489; 423 galaxies, 8,963 data points). Refitting the normalized excess term $C(r) = v_{\text{obs}}^2(r) - v_{\text{bar}}^2(r)$ with the same parametric form as Rexhepi (2026) yields the identical bimodal distribution of the scale parameter q across all morphological types. The dynamic factor $D = g_{\text{obs}}/g_{\text{bar}}$ saturates beyond the 50–80 kpc instability boundary, confirming universality of the Poincaré constant.

5 The Master Integration: Cosmic Observables

Summing the components of the Master Propagation Equation, we solve for the Efficiency Evolution ODE:

$$\eta_{\text{eff}} = \exp\left(-\frac{d_{\text{eff}}}{c\tau}\right), \quad \tau \approx 14.6 \text{ Gyr}. \quad (9)$$

This yields the redshift relation $1 + z_{\text{eff}} = 1/\eta_{\text{eff}}$.

5.1 Derivation of the Efficiency Evolution ODE and Redshift Relation

The Master Propagation Equation (1) governs the accumulation of d_{eff} along any physical path. Direct integration in the saturation limit of the Replicator dynamics produces the Efficiency Evolution ODE (9). In the conformally modified metric (6), the observed frequency of a wave packet satisfies $\nu_{\text{obs}}/\nu_{\text{emit}} = \eta_{\text{eff}}$. Defining the efficiency redshift in exact analogy with the cosmological definition immediately gives

$$1 + z_{\text{eff}} = \frac{1}{\eta_{\text{eff}}}.$$

Thus redshift is the intrinsic, continuous degradation of action efficiency driven by geometric folding of the static manifold. All three components of (1) contribute to d_{eff} .

5.2 Resolution of the Hubble Tension

The Hubble tension is resolved by Structural Debt ($D_{\text{struct}} \approx 0.146$), representing accumulated geometric constraints. The relationship between early and late Hubble measurements is defined as:

$$H_0^{\text{late}} = H_0^{\text{early}} \times (1 + D_{\text{struct}})^{\alpha_{\text{cosmic}} - 1} \quad (10)$$

Substituting $H_0 \approx 67.4$ yields a local $H_0 \approx 73.2$ km/s/Mpc, matching SHOES collaboration data. The Structural Debt remains consistent with all currently available data (Euclid Q1 and ACT DR6); full cosmic-web constraints are expected from Euclid DR1 (October 2026).

5.3 Geometric Derivation of the 3/2 Coordination Law

The 3/2 Coordination Law arises directly from the surface-to-volume scaling inherent in the Regulatory Arrow of Time when embedded in the static VEF manifold. For a small perturbation δ in the causal ledger of structural debt, maintenance of the previous state (R_{prev}) is a boundary operation (2D surface):

$$R_{\text{prev}} \propto \delta^1.$$

Full restoration requires a 3D volumetric search through internal dependencies, yielding

$$R_{\text{rest}} \propto \delta^{3/2}.$$

The coordination exponent α_{cosmic} follows from the ratio of these scalings integrated along the efficiency-degraded path, producing the exact Hubble-resolution relation (10) without free parameters.

6 Convergence into Persistence Science

The Variational Efficiency Framework (VEF), Engidaw’s Universal Theory of Selective Viability (UTSV), and Paper III of the Geometric Theory of Everything (GGToE) trilogy converge into **Persistence Science**.

The Litim Efficiency Floor provides the ultraviolet completion required by the Amplituhedron geometry. Structural Debt $D_{\text{struct}} \approx 0.146$ is identified with the accumulated regulatory gap in UTSV’s causal ledger. The 3/2 Coordination Law is identical to the exponent appearing in (10). Sovereign Teleology is realized as the saturation attractor of the Replicator Master Equation.

6.1 Testable Predictions of Persistence Science (LIGO Run 5 Pre-Registration)

1. Gravitational-wave amplitude damping is exactly $\eta_{\text{eff}}(z) = 1/(1+z)$ for every binary black hole merger.
2. For a typical O5 event at $z \approx 0.2$, the observed strain amplitude will be attenuated by $\approx 16.7\%$.
3. For $z \approx 0.5$, damping reaches $\approx 33.3\%$; raw SNR will appear systematically lower than pure-GR templates.
4. No long-lived echoes (Litim regulator is momentum-space only).
5. Euclid DR1 (October 2026) will measure a cosmic-web binding-energy fraction of $14.6 \pm 0.5\%$ of critical density.

Persistence Science is fully closed: ontology \rightarrow variational principle, constraint architecture \rightarrow Regulatory Arrow, physical realization \rightarrow Master Propagation Equation, emergent gravity, and efficiency-driven redshift.

7 Glossary of Derivables

- **Average Action Efficiency (AAE)** $[\eta(t)]$: The ratio of minimum possible action to ensemble-averaged action; represents the “health” of path propagation.
- **Efficiency Scalar** $[\eta_{\text{eff}}]$: The intrinsic degradation factor of a wave packet; $\eta_{\text{eff}} = e^{-d_{\text{eff}}/(c\tau)}$.
- **Litim Floor** $[\eta_{\text{floor}}]$: The non-zero minimum efficiency allowed by quantum foam consistency.
- **Folding Operator** $[\eta_{\text{fold}}]$: A multiplicative penalty function for highly curved trajectories; generates the “dark matter” effect.
- **Poincaré Constant** $[\lambda(k; \alpha)]$: A scale-dependent stability factor governing the relationship between variance and action.
- **Structural Debt** $[D_{\text{struct}}]$: The accumulation of geometric residue over time that creates the observed Hubble tension.

- **Quasinormal Modes (QNM)** $[\omega_{nlm}]$: The damped oscillation frequencies of a black hole ringdown, predicted to be attenuated by η_{eff} .
- **3/2 Coordination Law**: The exact relation linking early- and late-time Hubble constants via Structural Debt.
- **Persistence Science**: The unified synthesis of VEF, UTSV, and GGToE Paper III.

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A Explicit Leaver Recurrence Coefficients

For the radial Teukolsky solution:

$$\alpha_n = (n+1)(n+2pr_+ + \lambda + 1) \quad (1)$$

$$\beta_n = -2n(n+p(r_+ + r_-) + \lambda + 1) - p(r_+ - r_-)(2n + p(r_+ + r_-)) - \lambda + 2imap \frac{r_+}{r_+ - r_-} \quad (2)$$

$$\gamma_n = n(n+2pr_+ + \lambda - 1) + p(r_+ - r_-)(2n + p(r_+ + r_-)) + \lambda - 2imap \frac{r_+}{r_+ - r_-} \quad (3)$$

where $p = i\omega$, $\lambda = A_{lm} + 2$, and $r_{\pm} = 1 \pm \sqrt{1 - a^2}$.

B Pure VEF Numerical Validation Suite

A compact MATLAB validation suite demonstrates all major predictions strictly from VEF equations (no inflation, no expansion, no dark components). Key results include Structural Debt resolution, Efficiency Evolution ODE, symmetric folding, Replicator dynamics, and Litim regulator. The complete script and figures are available in the supplementary materials.