concluding that the sea bottom is, for the most part, utterly dark, but that there are scattered areas, often of considerable, extent where animal life is aggregated in masses, and where the phosphorescent light is of sufficient quantity to render the colors, laid on as we have seen in broad patterns, visible to animals with functional eyes. These colors would then be of the same utility to their possessors as in the upper world, and act as protective, aggressive, directive, attractive and alluring agencies. We are further justified in maintaining that phosphorescence is in all cases of direct utility to its possessors, and that in the fixed eyeless forms it serves to attract food, and perhaps in some cases to warn enemies of the presence of the irritating nettling cells.

As a sort of compensation for the feebleness of the phosphorescent light, and for its absence over vast areas, many animals, especially fishes and crustaceans, are furnished with very large eyes, or with organs which serve as lanterns, or with enormous mouths and stomachs to make the most of a very occasional square meal, or with greatly elongated feelers or tactile organs. Others still are provided with a luminous bait to attract the prey.

The main thing that I would impress upon you this evening is the fact that we have a right to expect to find utility for every character, not rudimentary, possessed by animals, a utility not necessarily to the individual, but certainly to the species. And I would protest most vigorously against the vain and impotent conclusion that anything is useless simply because we have been too ignorant or too indolent to find its function. I have small patience with a statement such as the following taken from a recent writer on animal coloration : "The inevitable conclusion, therefore, from these facts appears to be that the brilliant and varied colorations of deep-sea animals are totally devoid of meaning ; they can not be of advantage for protective purposes or as warning colors, for the simple and sufficient reason that they are invisible."

This sort of thing is deeply injurious to science, because it is a helpless surrender of one of the most powerful of all incentives to research. If we can loll back in our easy chairs and declare that natural phenomena of widespread occurrence are meaningless, or, what amounts to the same thing, that Nature is guilty of a lot of vapid nonsense, we have indeed sold our scientific birthright for a mess of exceedingly thin pottage, and have stultified ourselves in the eyes of the thinking world.*

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REMINISCENT REMARKS ON THE TOP.

Some time ago, I wrote a short article in this journal,† in which among other things I endeavored to give an intelligible explanation of all that, from an elementary point of view, is interesting in the dynamics of the top. The treatment of this famous and ubiquitous apparatus in all text-books known to me is too sketchy and, didactically considered, useless. In my judgment this is a real gap and well worth filling. But my friends have so frequently and even quite recently taken me to task for my explanation, that I feel bound to reassert its correctness here.

Everybody will agree that up to the second order of approximation, and a vigorously spinning top or gyroscope, in which $\dot{\theta}$ is the polar velocity and φ and ψ the parameters of azimuth and altitude,

* Most of the facts and sometimes whole paragraphs concerning the coloration of deep-sea animals and phosphorescence, have been taken from the following papers by the author : 'The Color of Deep Sea Animals,' *Proc. Iowa Acad. of Sci.*, Vol. VI. ; 'The Utility of Phosphorescence in Deep Sea Animals,' *Am. Nat.*, Oct., 1899.

† Science, V., pp. 171–5.

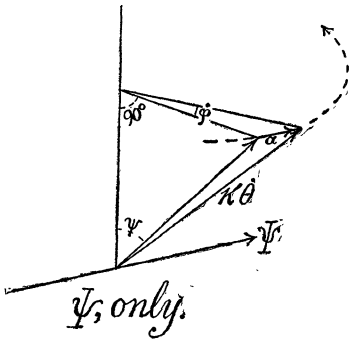
$$\Psi = K\dot{\theta} \sin \psi \cdot \dot{\varphi}, \qquad (1)$$

$$\Phi = -K\dot{\theta} \sin \psi \cdot \dot{\psi}, \qquad (2)$$

will express the motion. Here Ψ and Φ are the torques around the horizontal and vertical axes, respectively, and K the polar moment of inertia; friction is disregarded.* Hence $\dot{\theta}$, $\dot{\varphi}$, $\dot{\psi}$, are of the first order of small quantities, $\ddot{\varphi}$ and $\ddot{\psi}$ of the second order.

The point is now to show that these equations are reproduced in my geometrical constructions relative to the common theorem of the equivalence of torque, and the change of angular momentum per second around any particular direction.

I. *Torque* (Ψ) *around the horizontal axis only. Precession.*—Take any two positions of the top axis, a second of time in position apart. Lay off the angular momenta $K\dot{\theta}$,



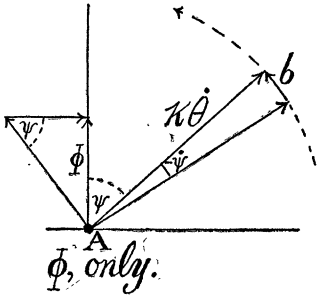
along these axes. They are equal by the premises. Hence the horizontal arc, a, is the rate of change of angular momentum due to the torque Ψ around the parallel axis shown. In the second of time stated, the angle of azimuth has changed by $\dot{\varphi}$, as shown in the figure. Therefore, the arc

$$a = K\dot{\theta} \sin \psi \cdot \dot{\varphi},$$

and hence

$$\Psi = K\dot{\theta} \sin \psi \cdot \dot{\varphi},$$

II. *Torque Φ around the vertical axis only.*—This requires gyroscopic mounting. Let the horizontal axis be seen end on at A. Take two positions of the top axis, a second of time in position apart. Lay off the (equal) angular momenta, $K\dot{\theta}$, along these



axes. Then the arc b is the rate of change of angular momentum, and the angle, $-\dot{\psi}$, subtended, the speed in altitude. Hence

$$b = -K\dot{\theta} \cdot \dot{\psi}.$$

The arc, b, cannot be resolved with advantage, for there is no way of accounting for both components. Φ, however, may be resolved; for if one component is made parallel to b, this is the equivalent of b; whereas if the other component tends to twist across A (in a plane at right angles to 'around A'), *i. e.*, in the plane of this axis, it can produce stress only, but no motion. Hence, as seen in the figure, the effective *component* is $\Phi/\sin \psi$, a result a little subtle, I grant, but none the less logically straightforward. Therefore

$$\Phi/\sin \Phi = b,$$

whence

$$\Phi = -K\dot{\theta} \sin \psi \cdot \dot{\psi}.$$

Of course my explanation was intended for the man interested in *spinning* tops. The other man, who prefers the top toppling through all stages of inebriety, may take such solace as comes from products of parameter speeds and accelerations. But for him I have no message other than my blessing.

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* Otherwise, if there is polar acceleration, $\Phi = -K(\dot{\theta}\,\dot{\psi} \sin \psi - \cos \psi \cdot \ddot{\theta})$ may be deduced without difficulty by the method of § II.