Many-fermion quantum entanglement

in

strange metals

and

in black holes

Rutgers University

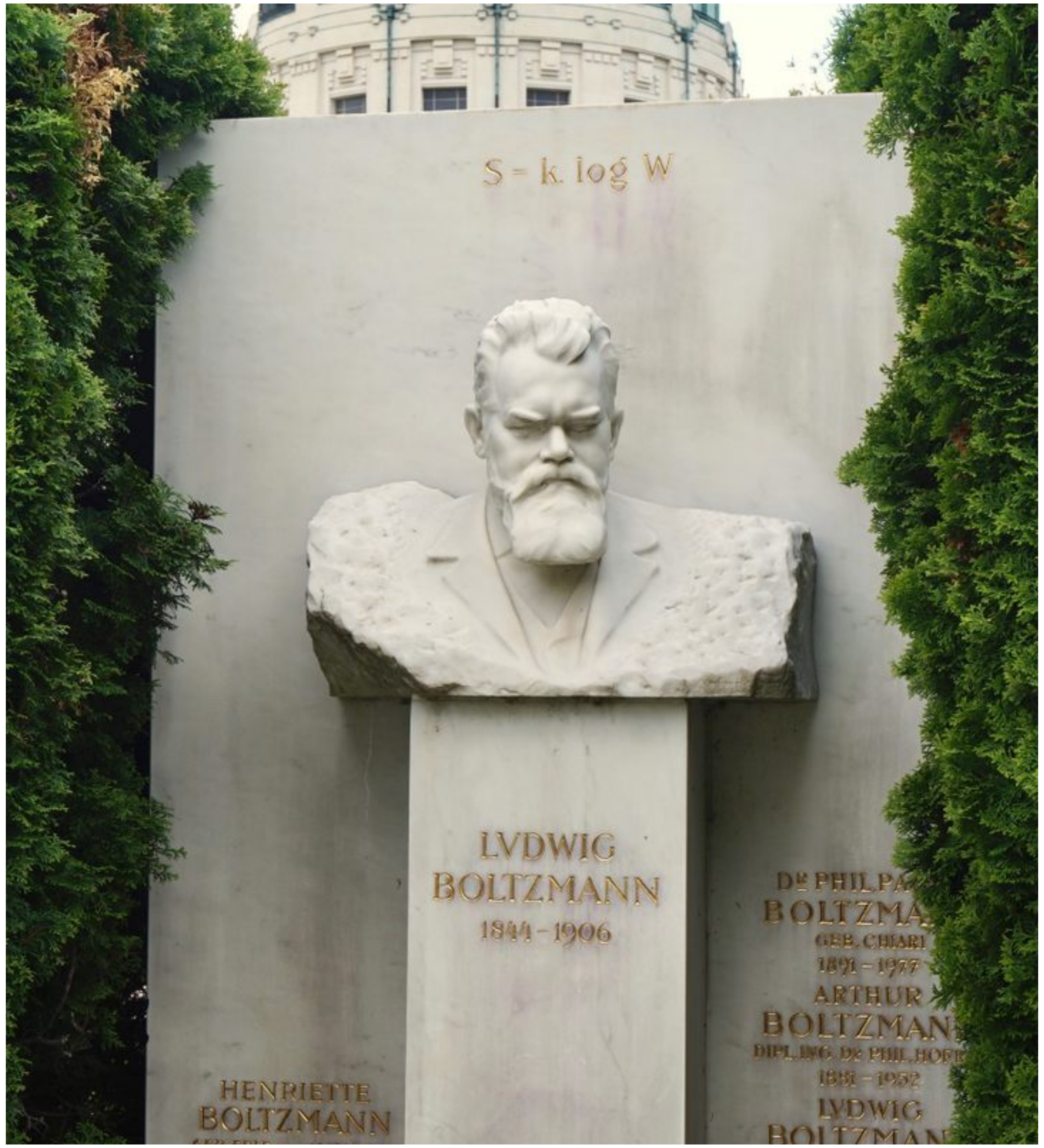
April 22, 2026

Subir Sachdev

PHYSICS
HARVARD

Boltzmann-Landau theory of ordinary metals: Cu, Ag

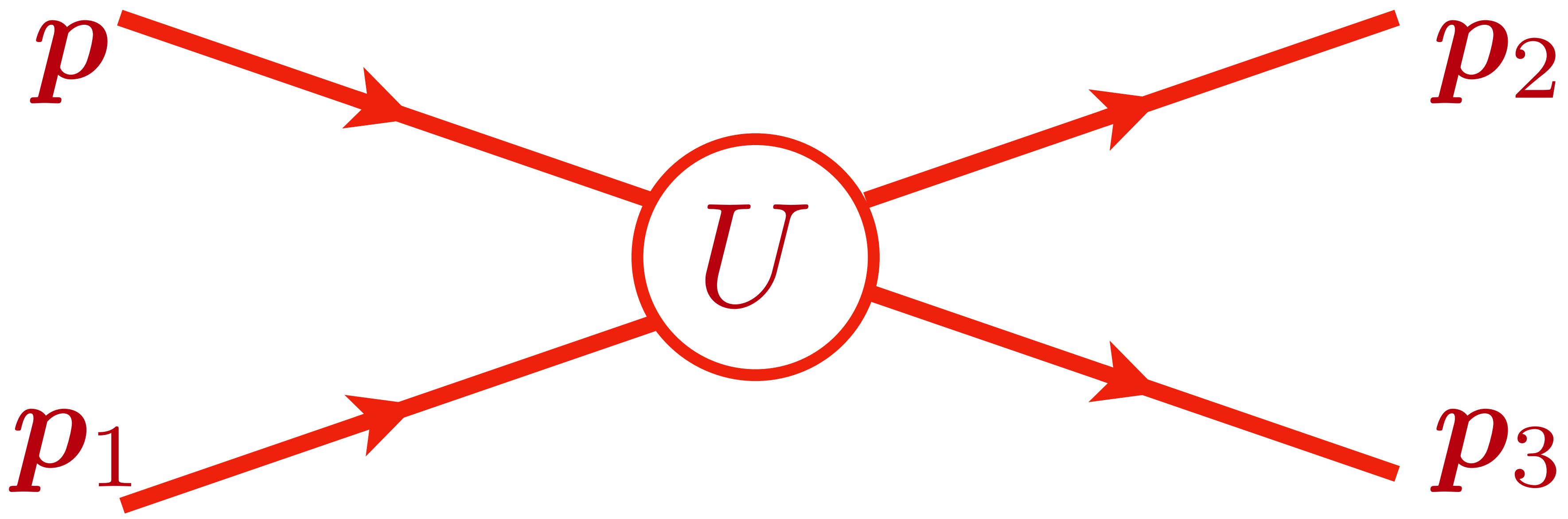
Quantum Boltzmann equation (Landau) Dense gas of electrons



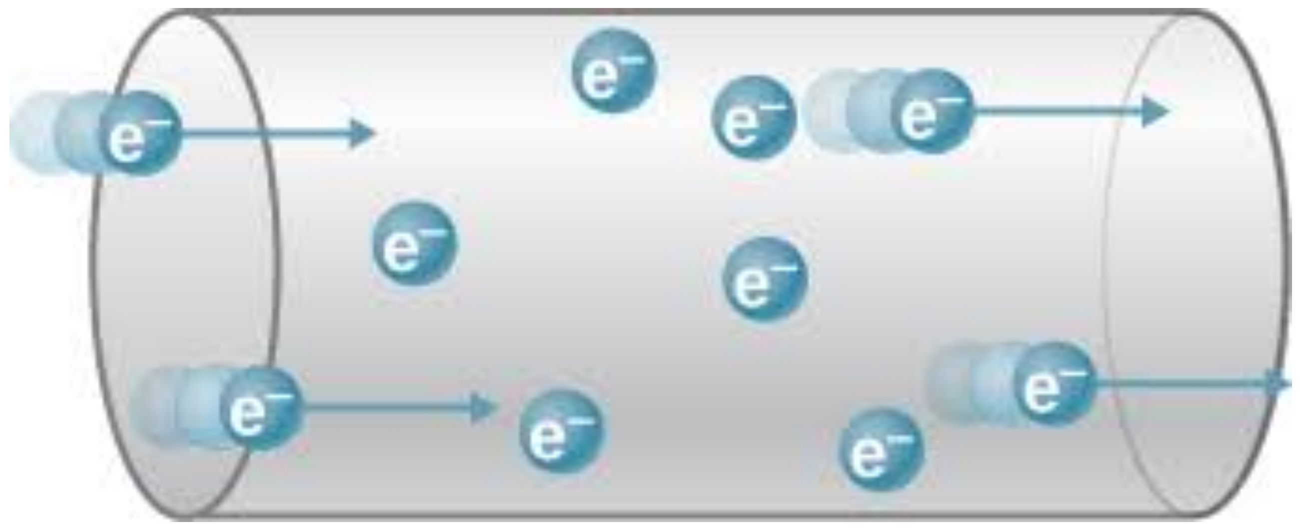
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Collisions are also rare in a dense quantum gas at low temperatures because of the Pauli exclusion principle.

Neglect quantum interference (entanglement) between successive collisions



Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

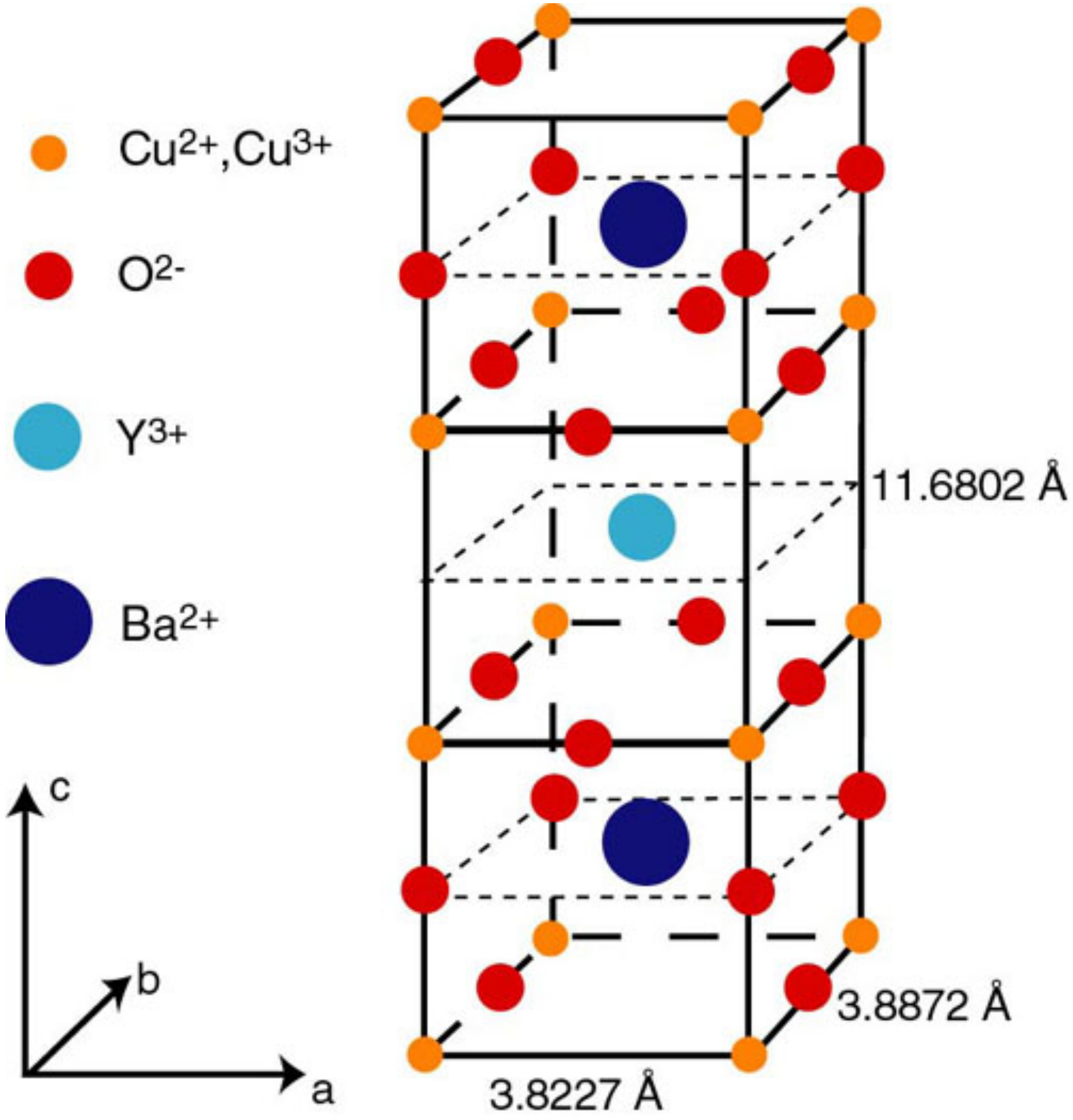
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

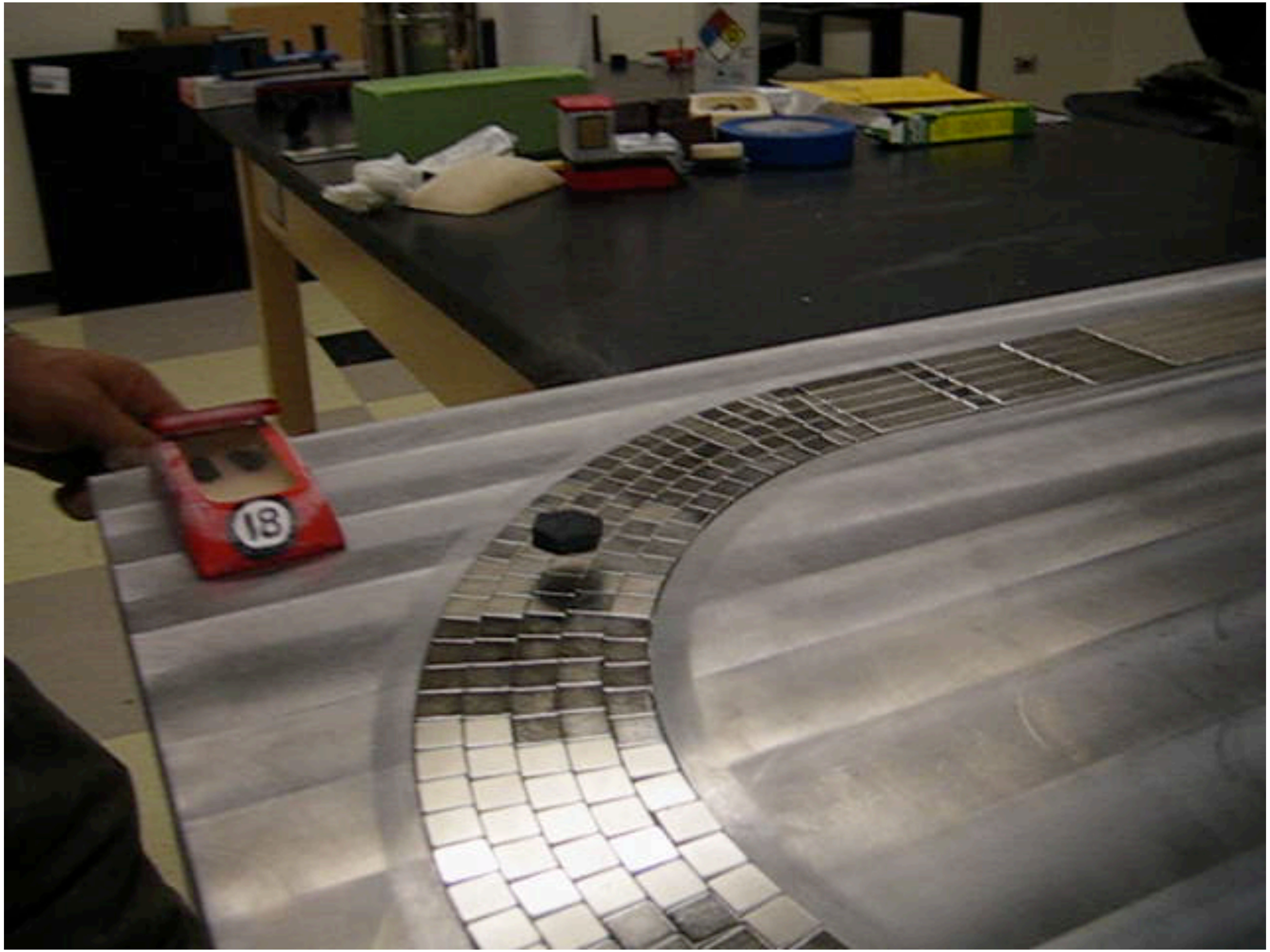
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

Strange metals: the cuprates

Cuprate high temperature superconductors



$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

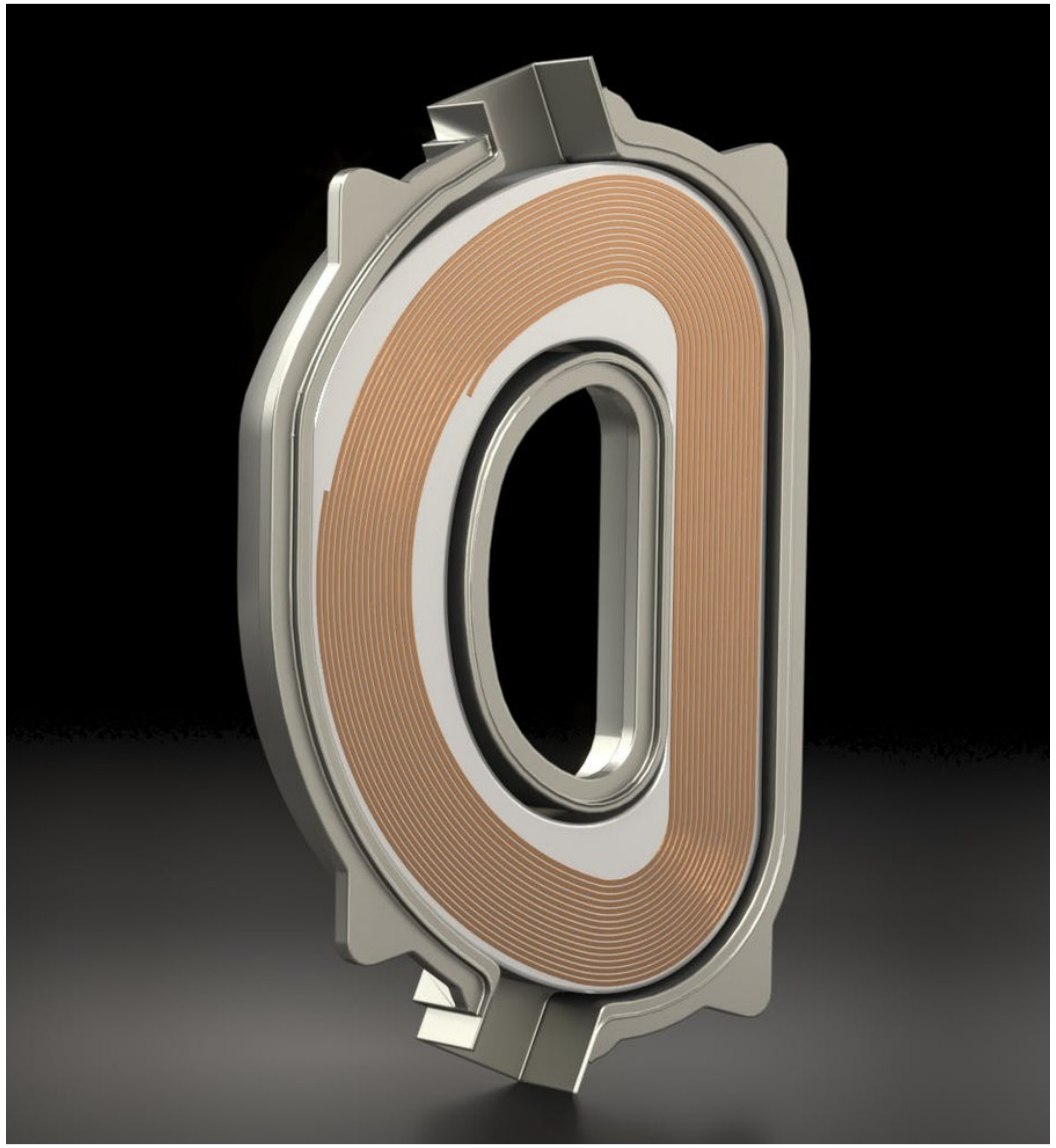
Julian Hetel and Nandini Trivedi, Ohio State University

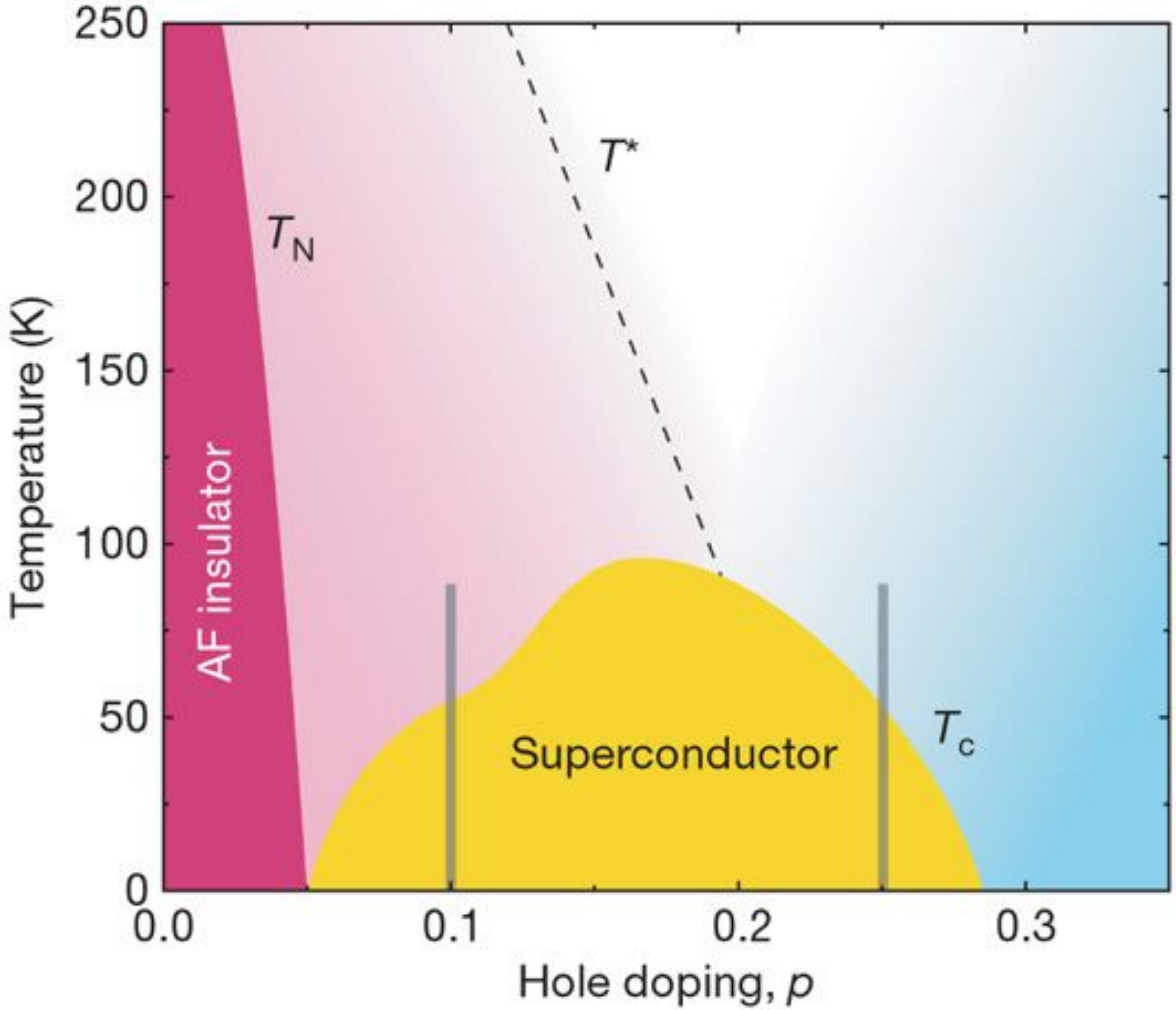
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
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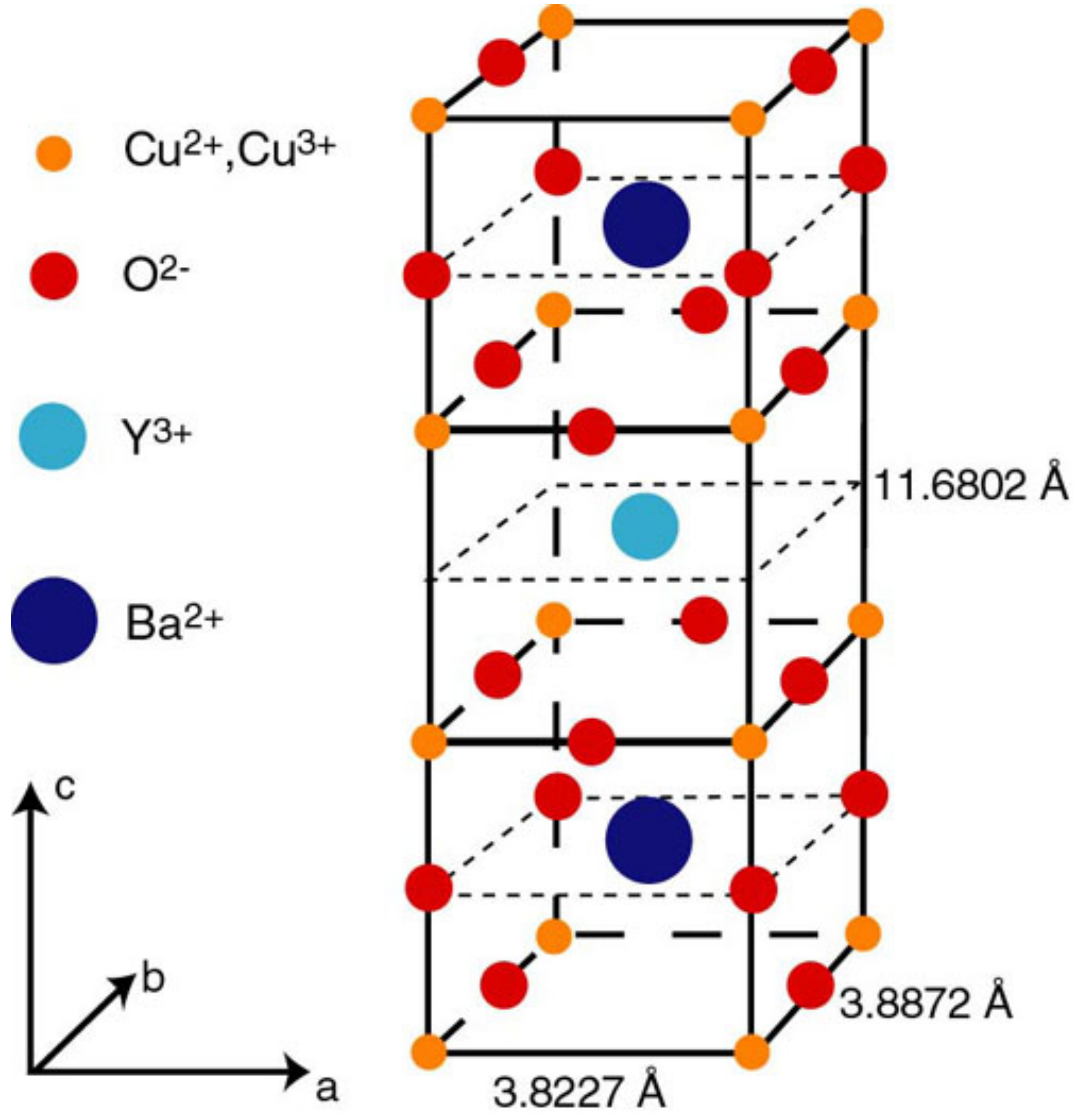
Hole doping, p
T*
TN
AF insulator
Superconductor
Tc



Cu2+,Cu3+
O2-
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c
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a
11.6802 Å
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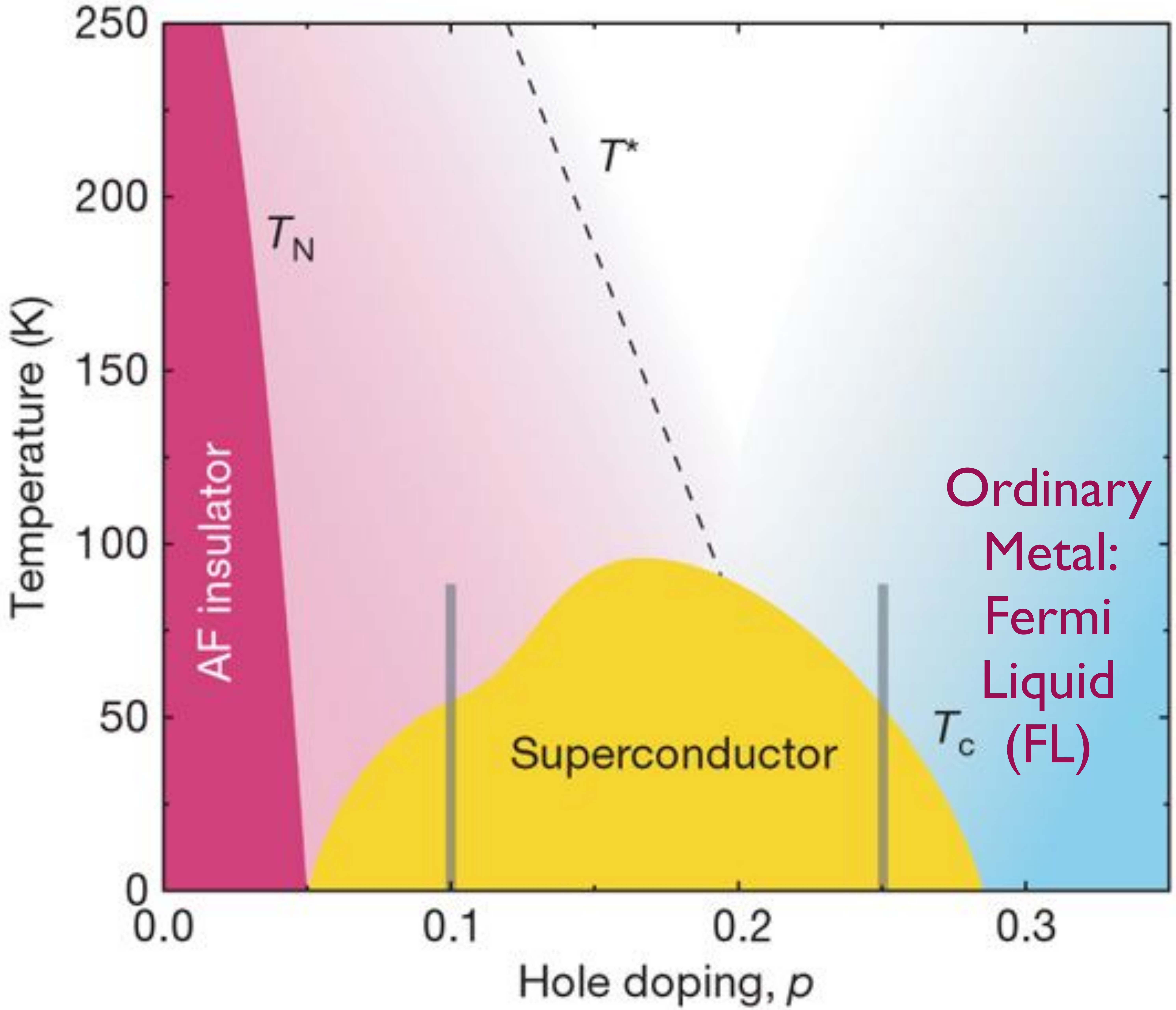
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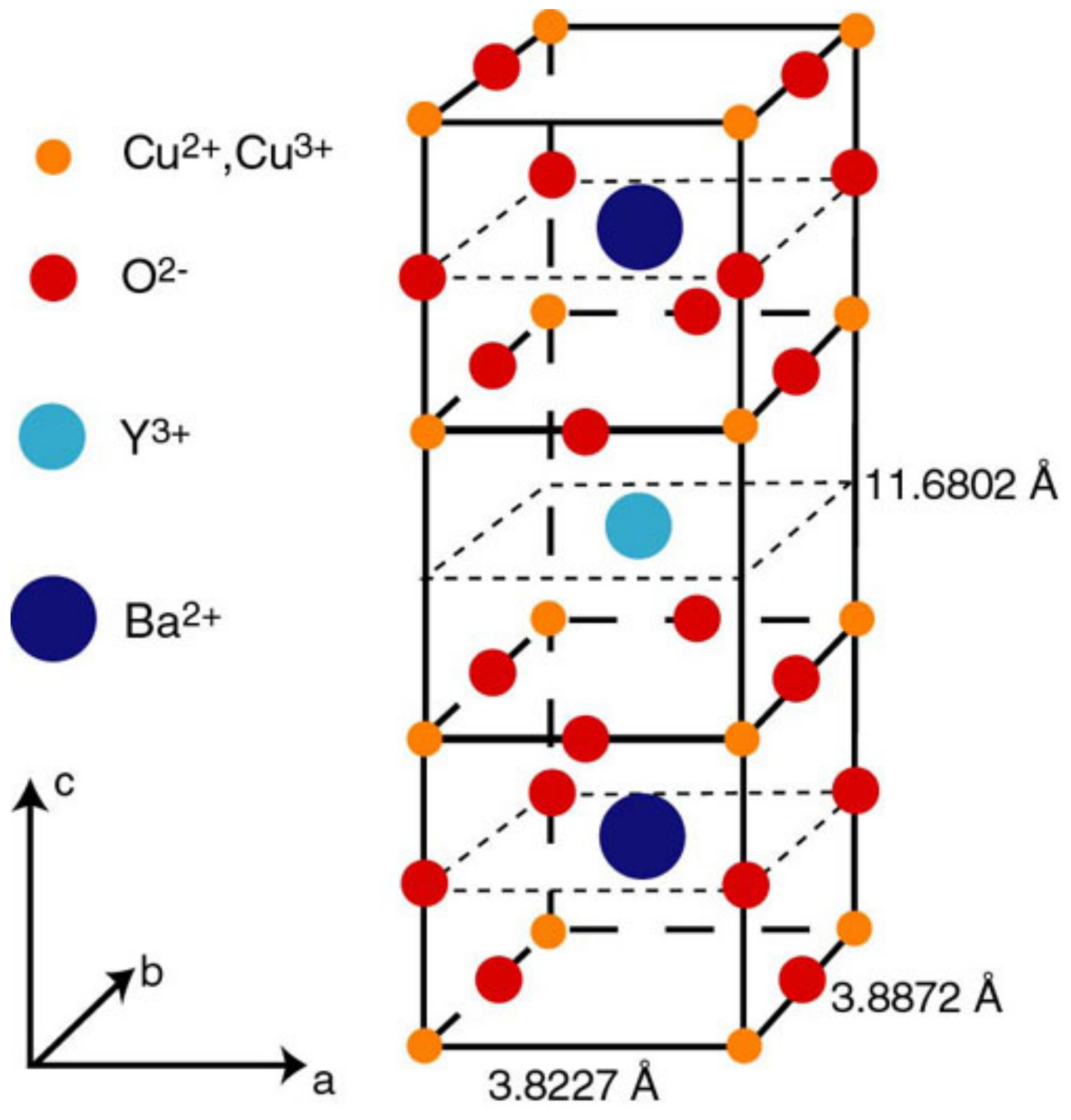
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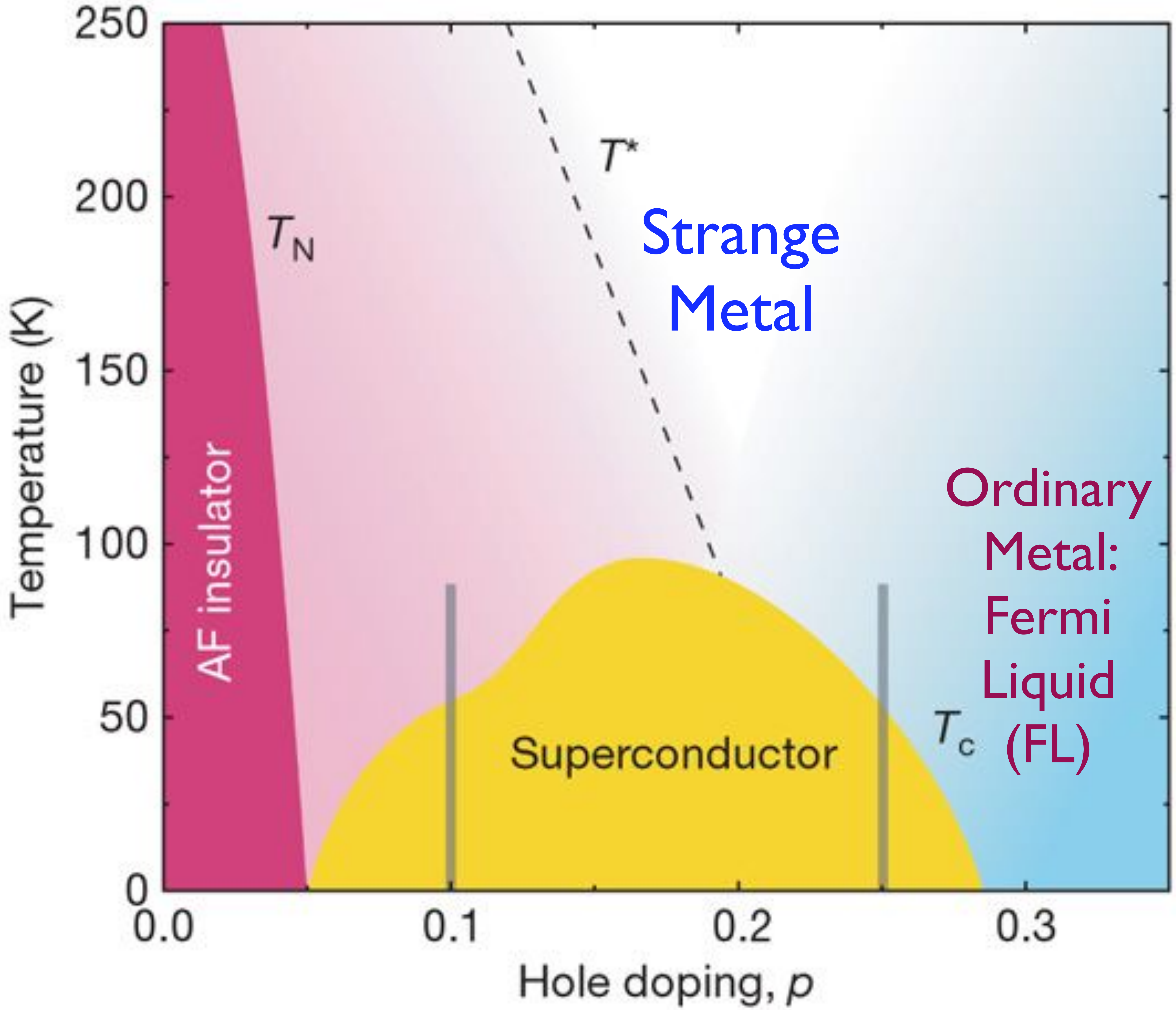

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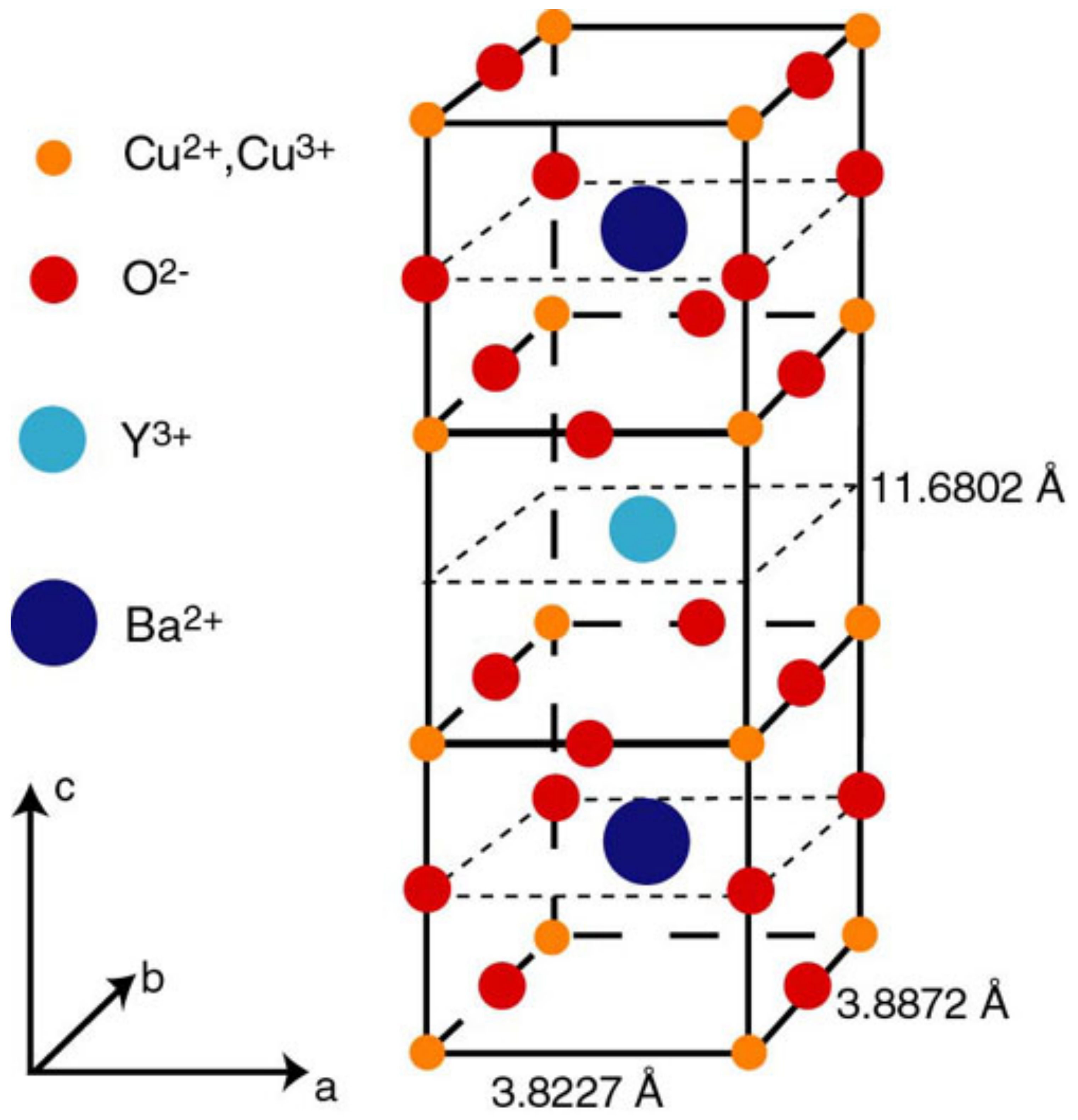

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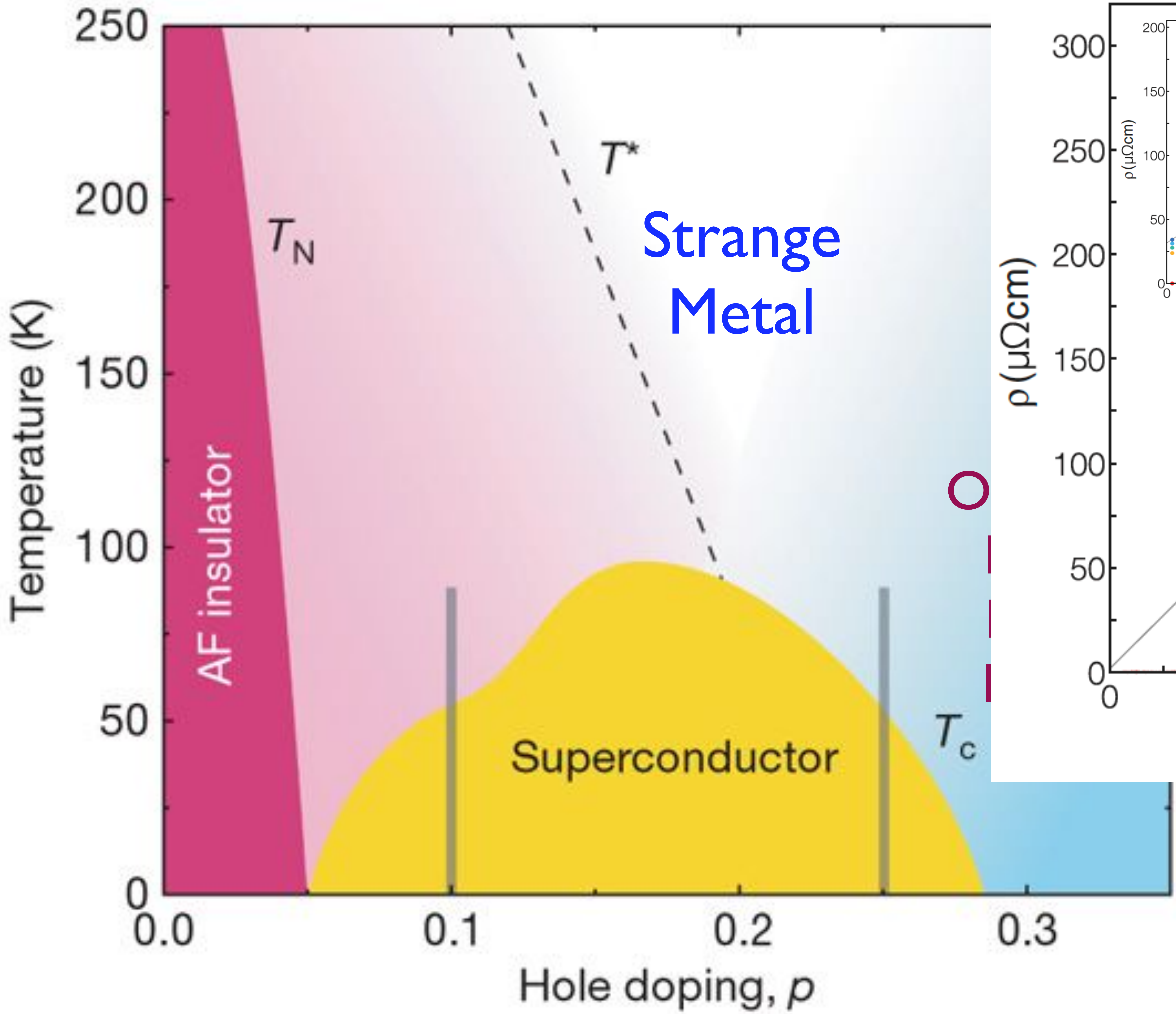
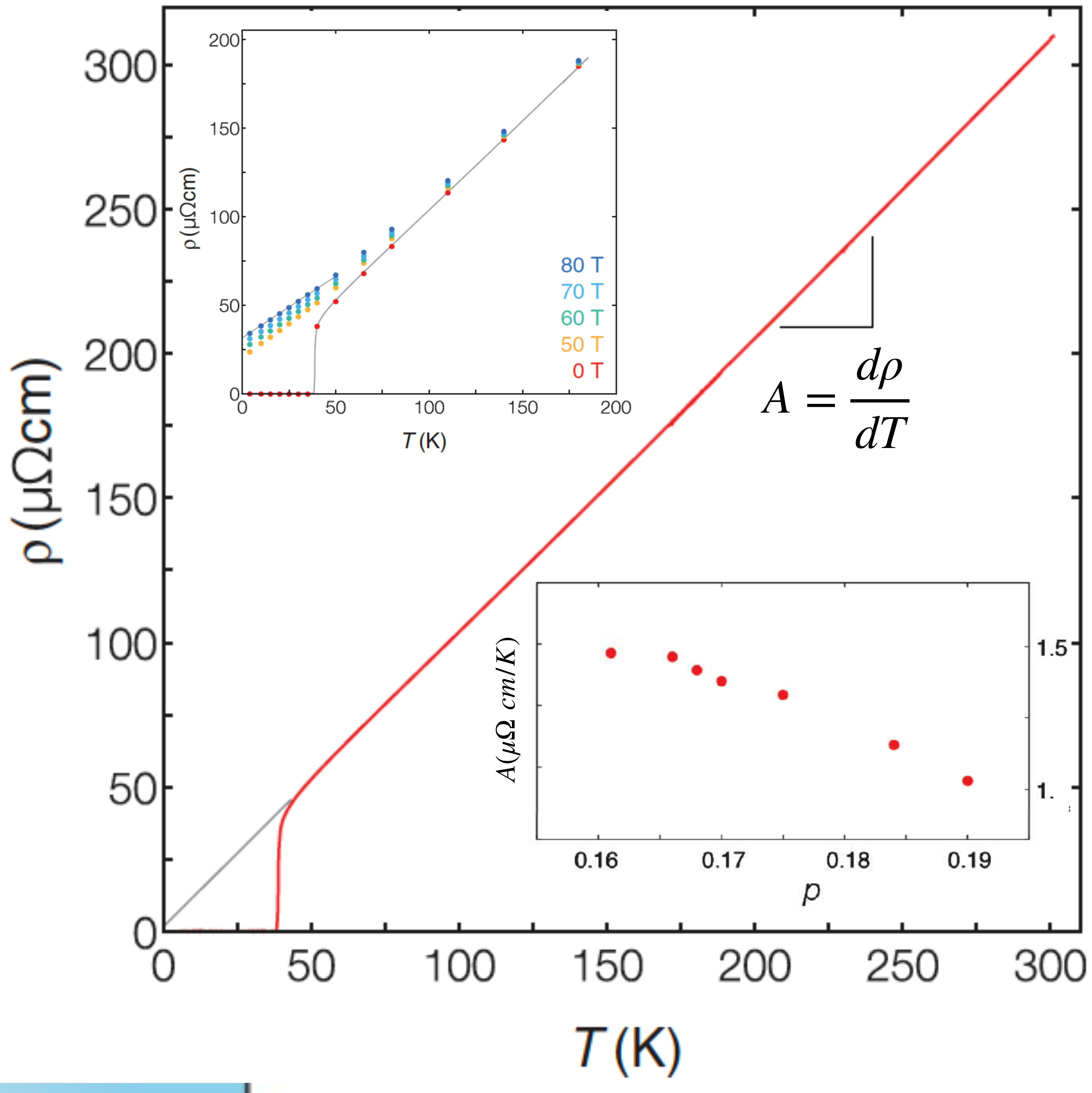




LSCO: Giraldo-Gallo et al. 2018

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

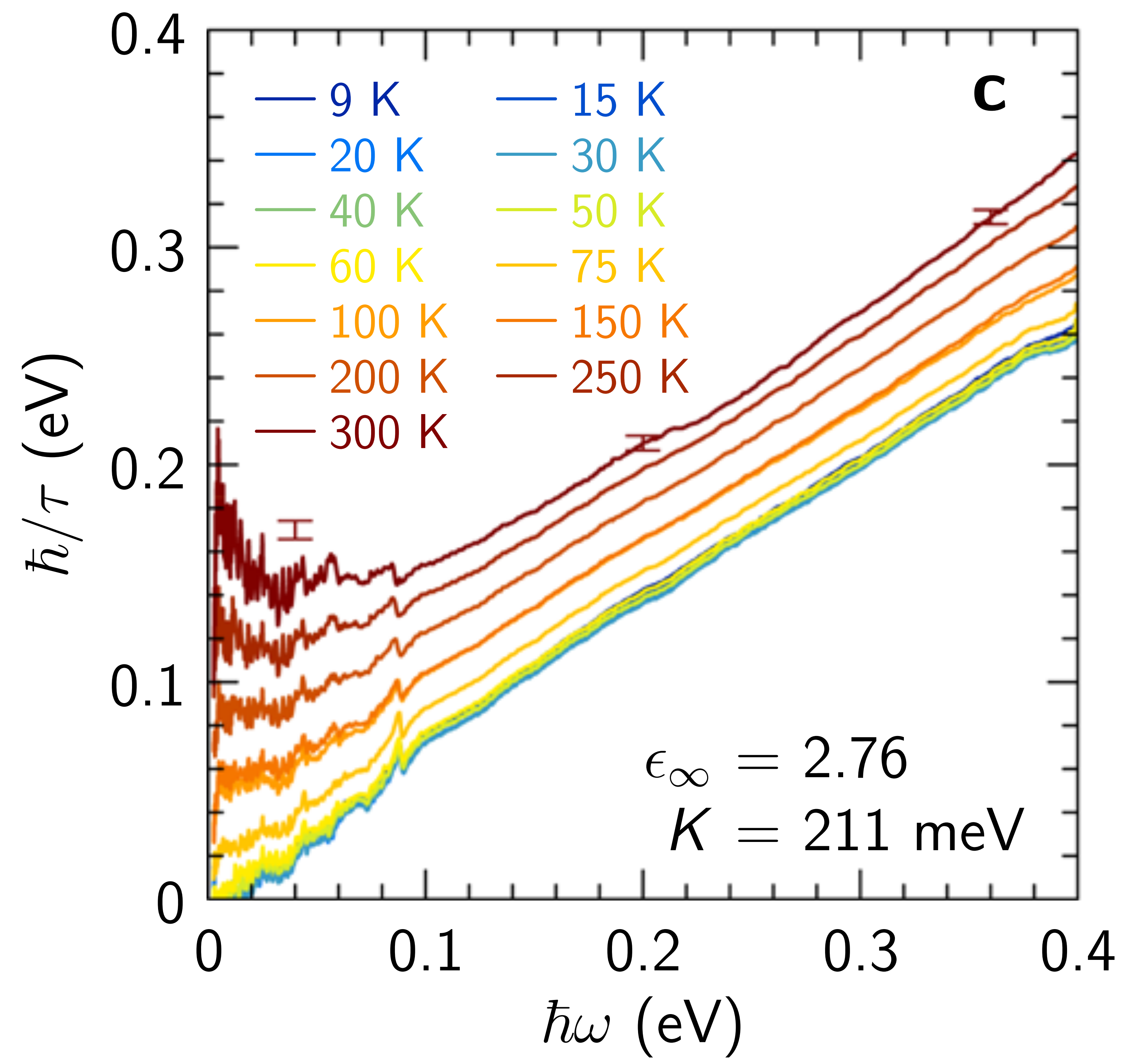
Nature Communications **14**, Article number: 3033 (2023)

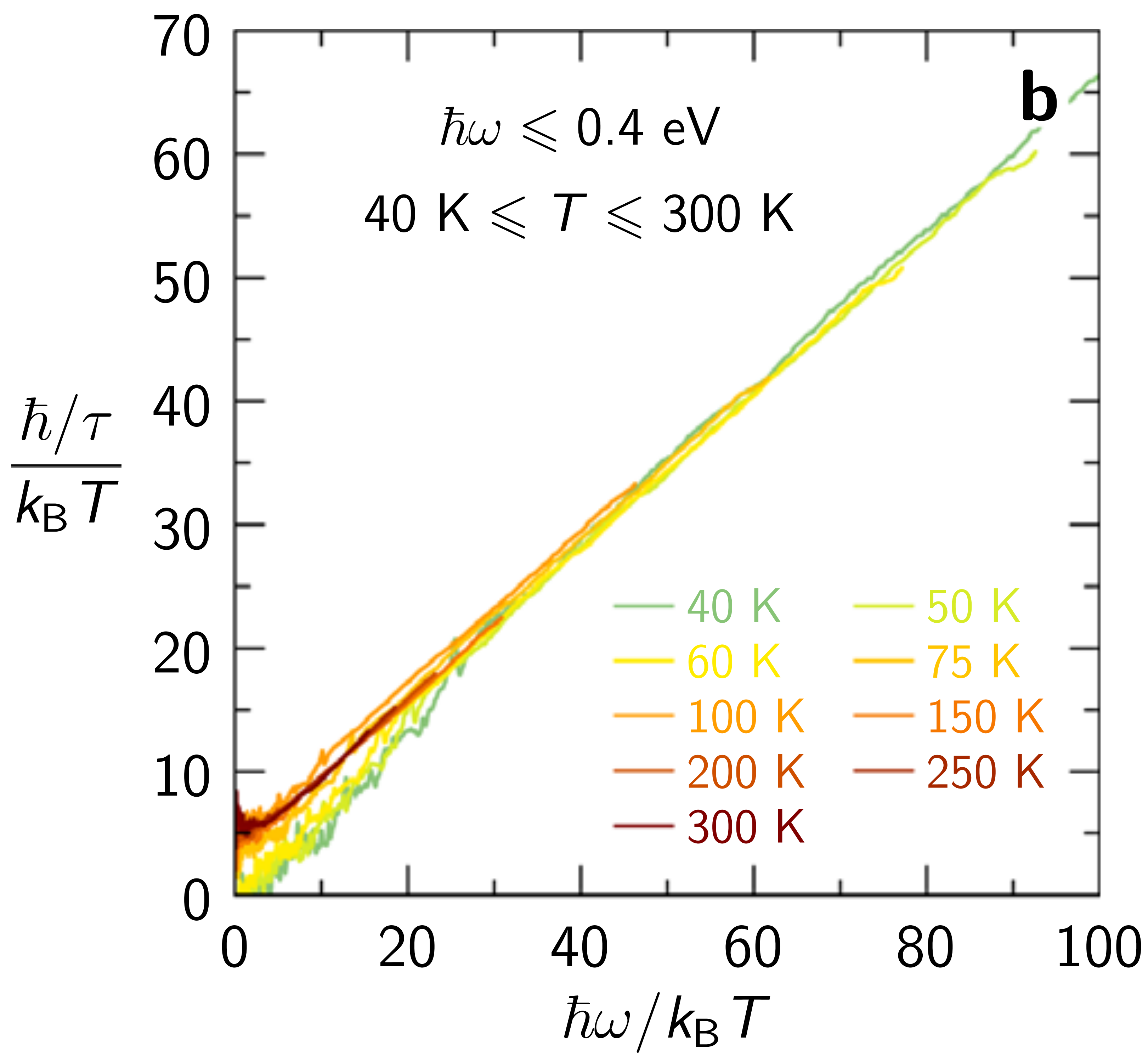
$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_BT}F\left(\frac{\hbar\omega}{k_BT}\right)$$

The time τ appears to be *independent* of interaction strength, contrary to Boltzmann.





Central questions:

What is the origin of the strange metal and why is it ubiquitous in correlated electron quantum materials?

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Much progress has been made in addressing the first question in last 3 decades.

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What is special about the strange metal in the hole-doped cuprates that leads to the highest temperature superconductivity?

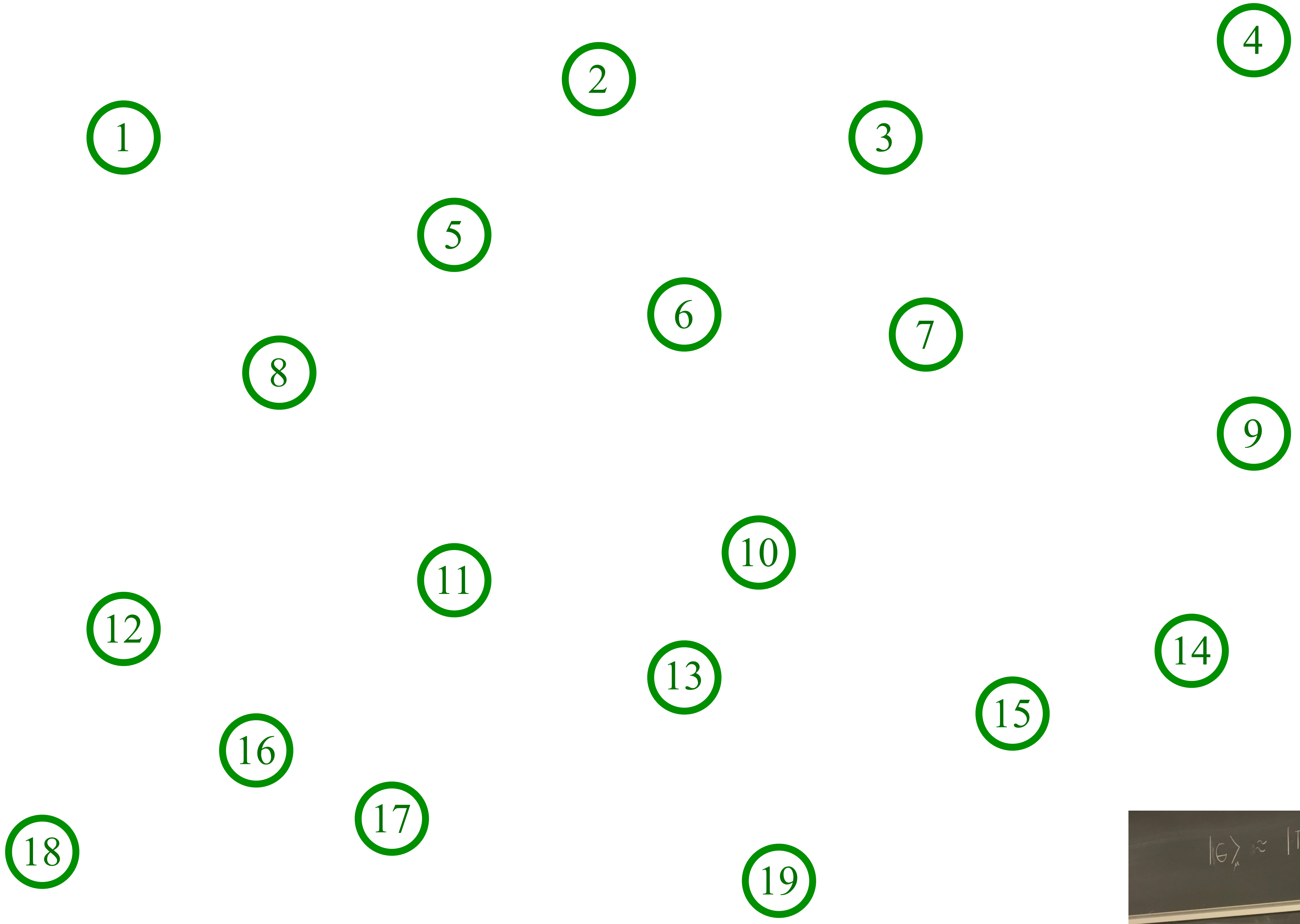
Much progress has been made in addressing the first question in last 3 decades.

But an unexpected bi-product has been a much deeper understanding of the quantum theory of many particles, which has impacted numerous other fields of physics, including the quantum theory of black holes, and quantum error correction

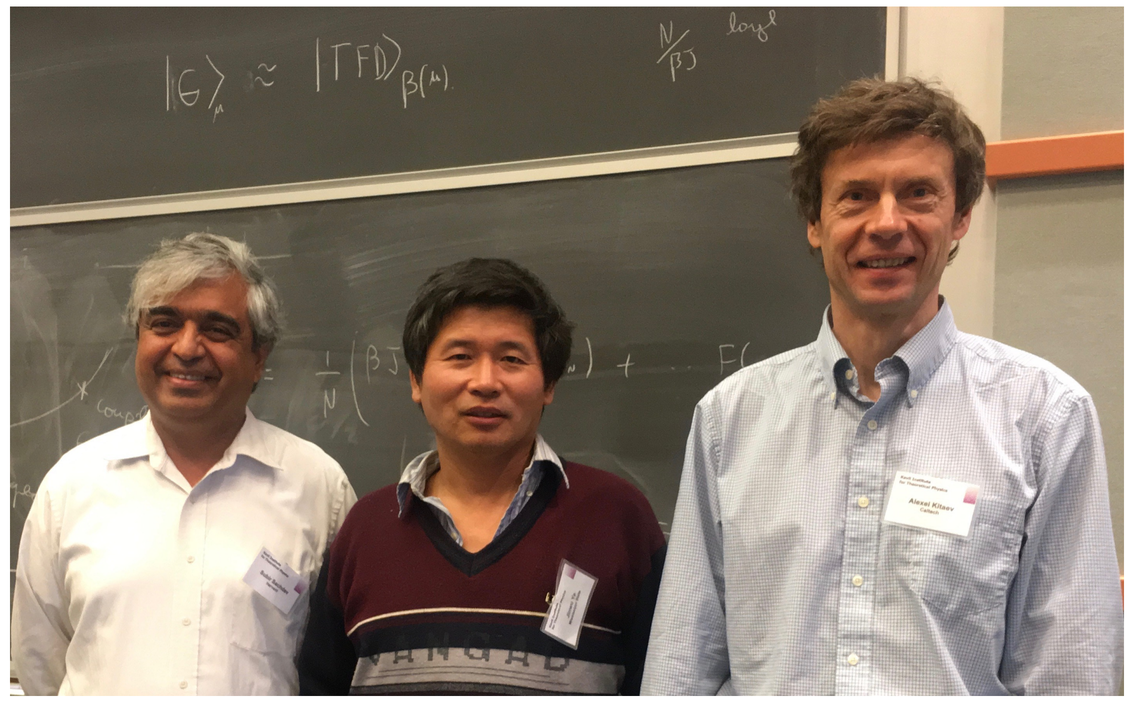
The Sachdev-Ye-Kitaev model: solvable Planckian dynamics

The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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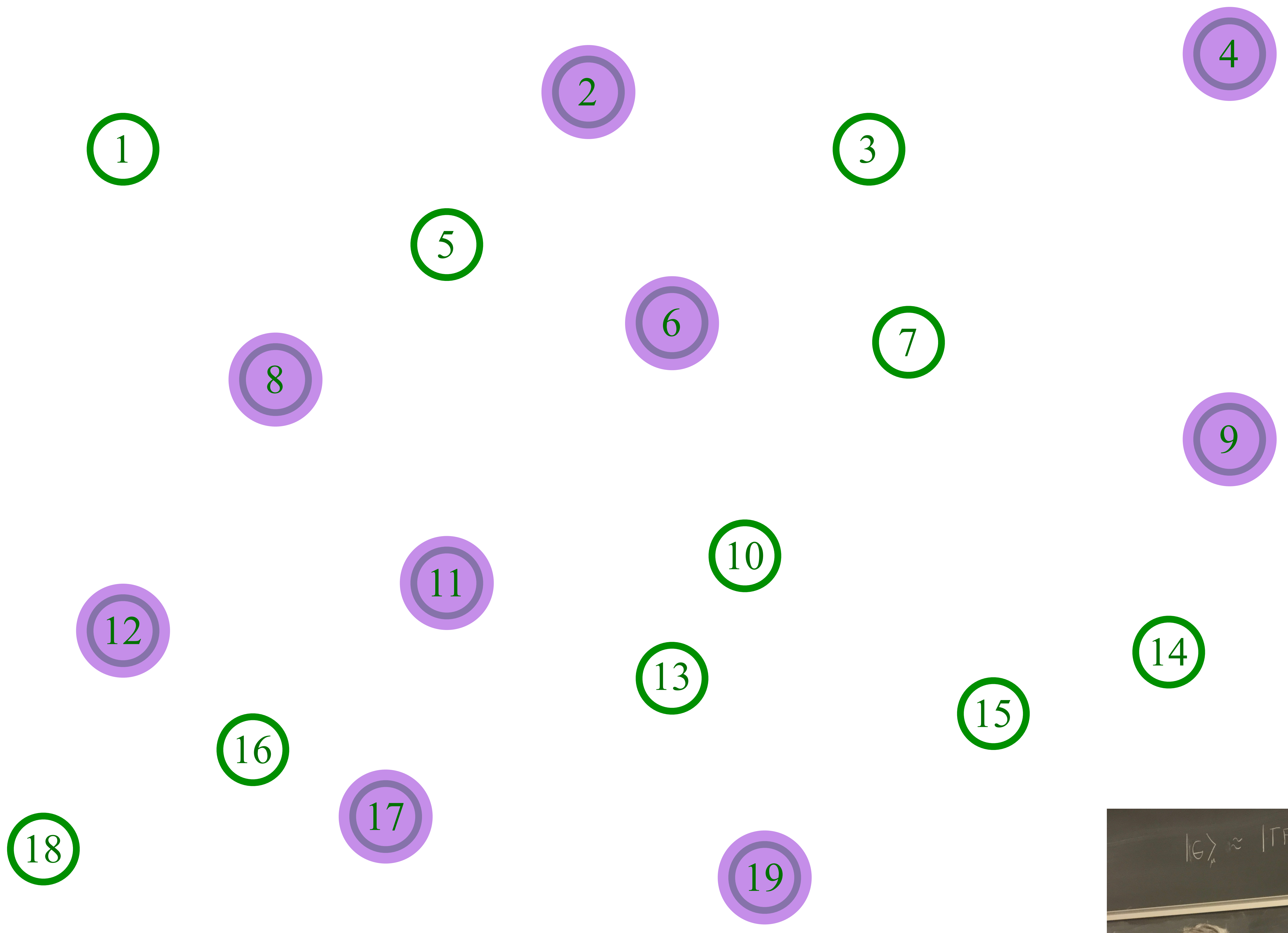
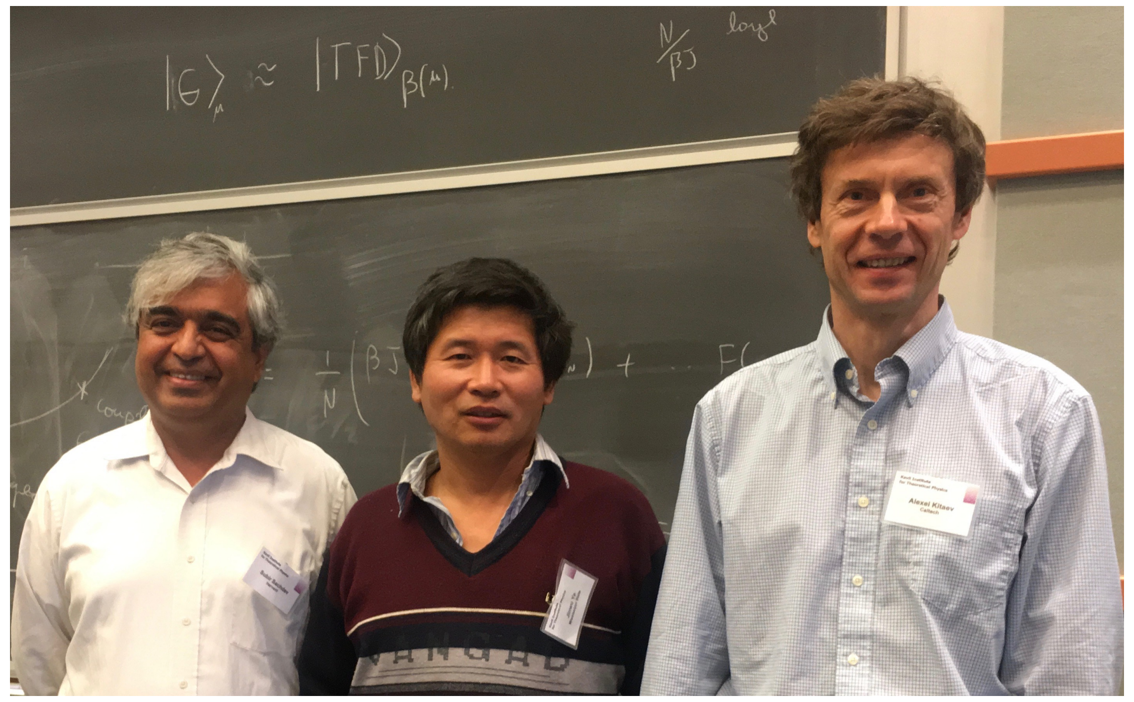


Place electrons randomly on some sites



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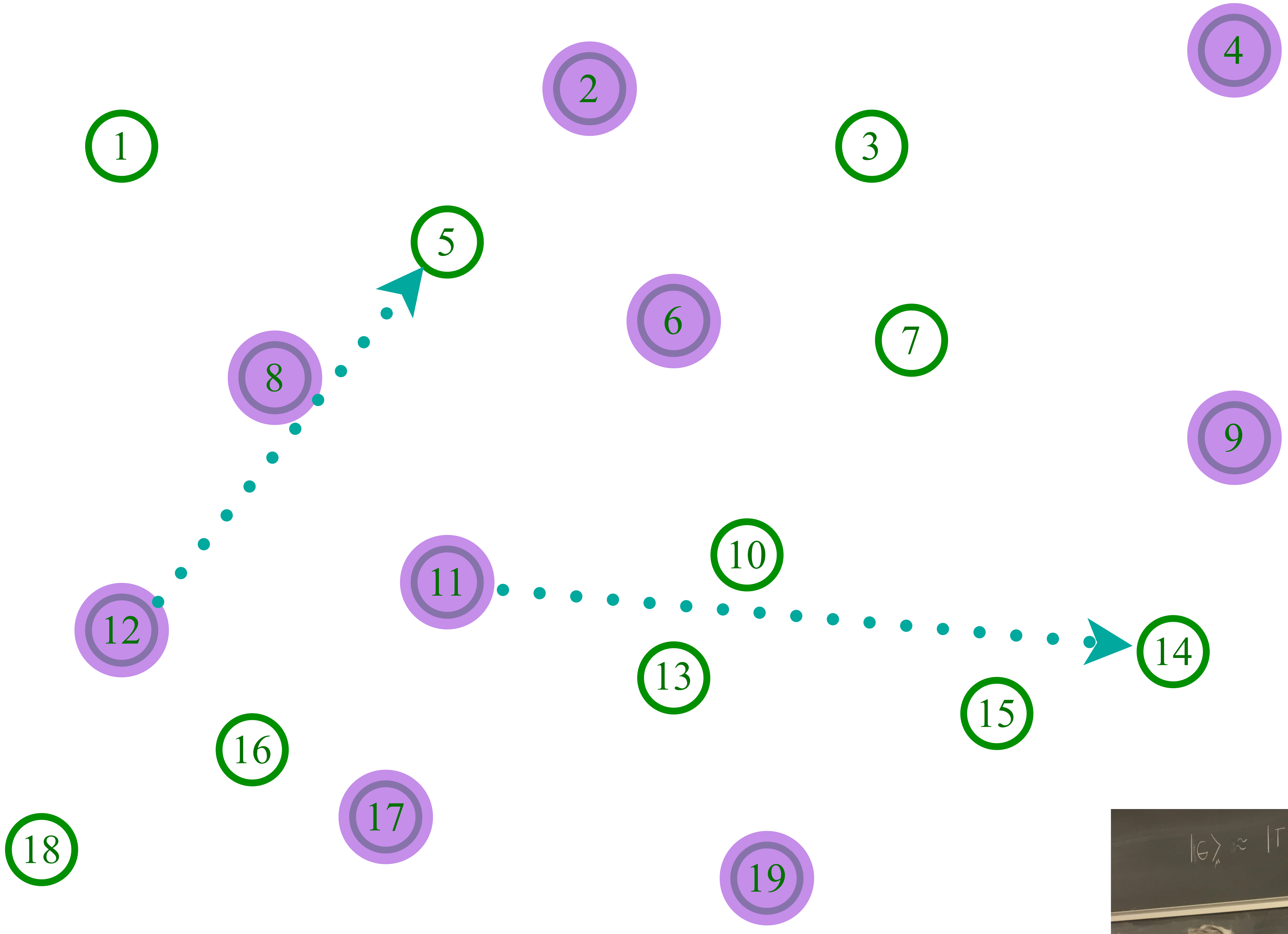
$$U_{11,12;5,14}$$



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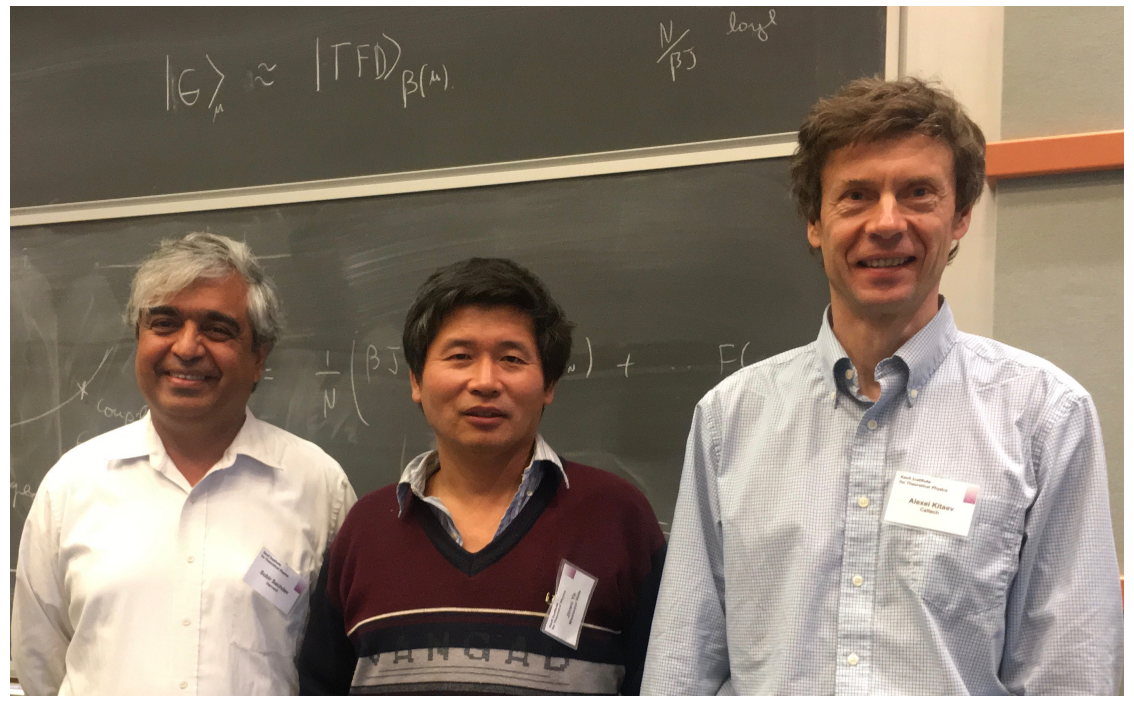

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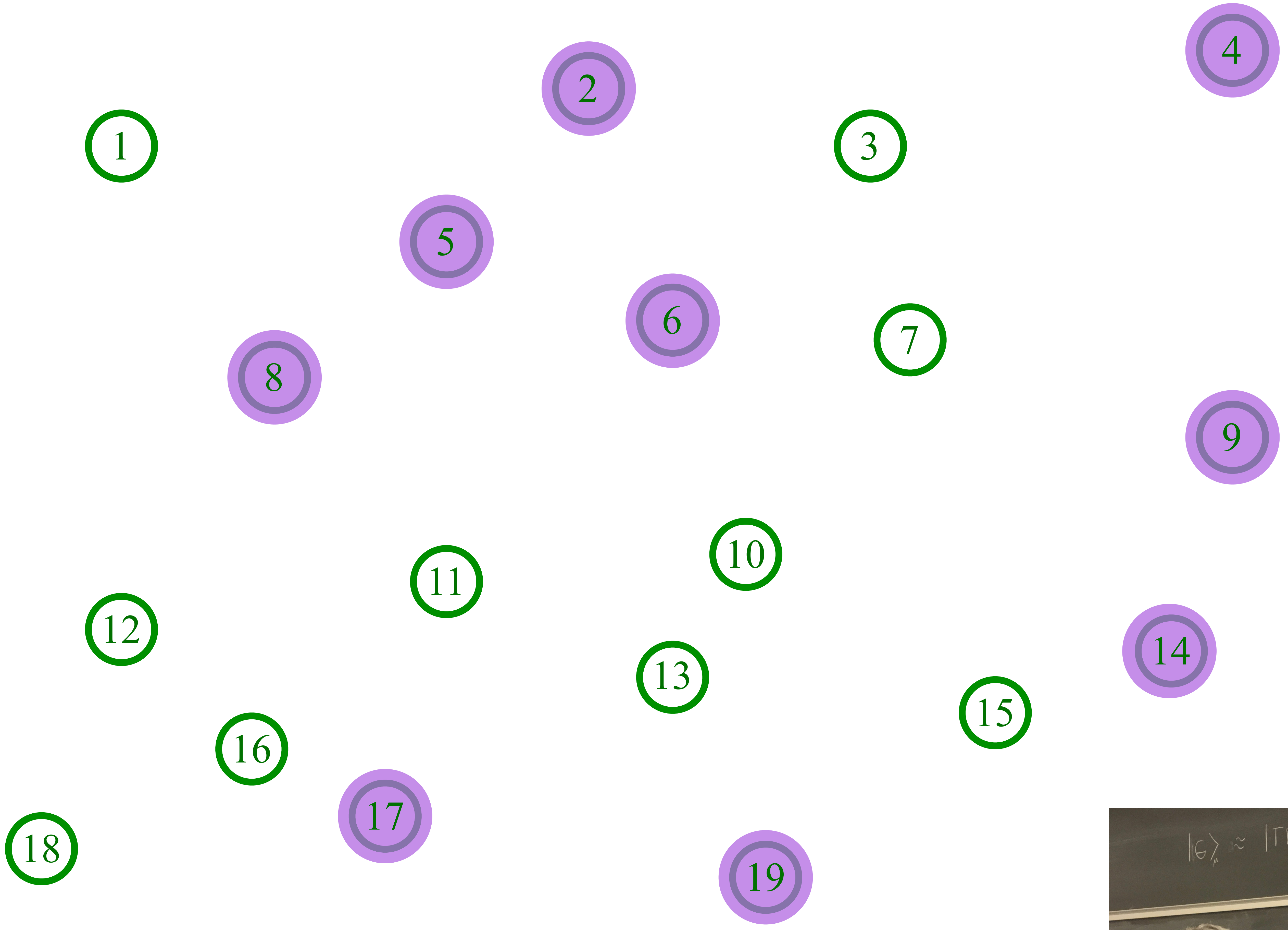
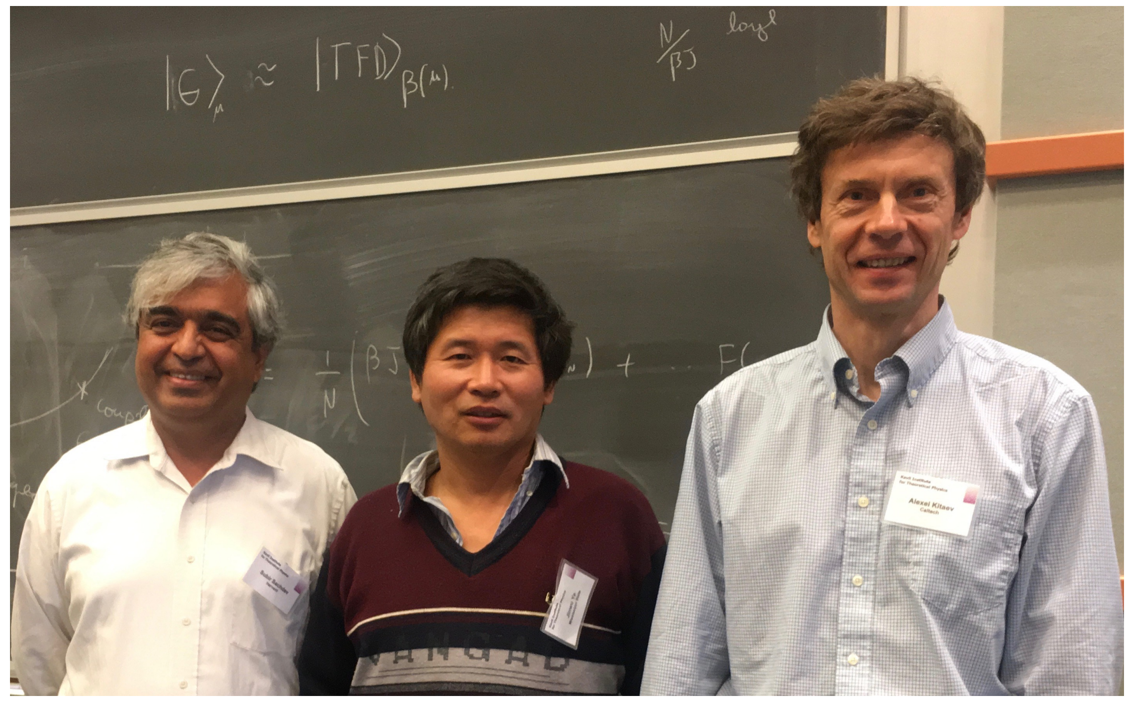


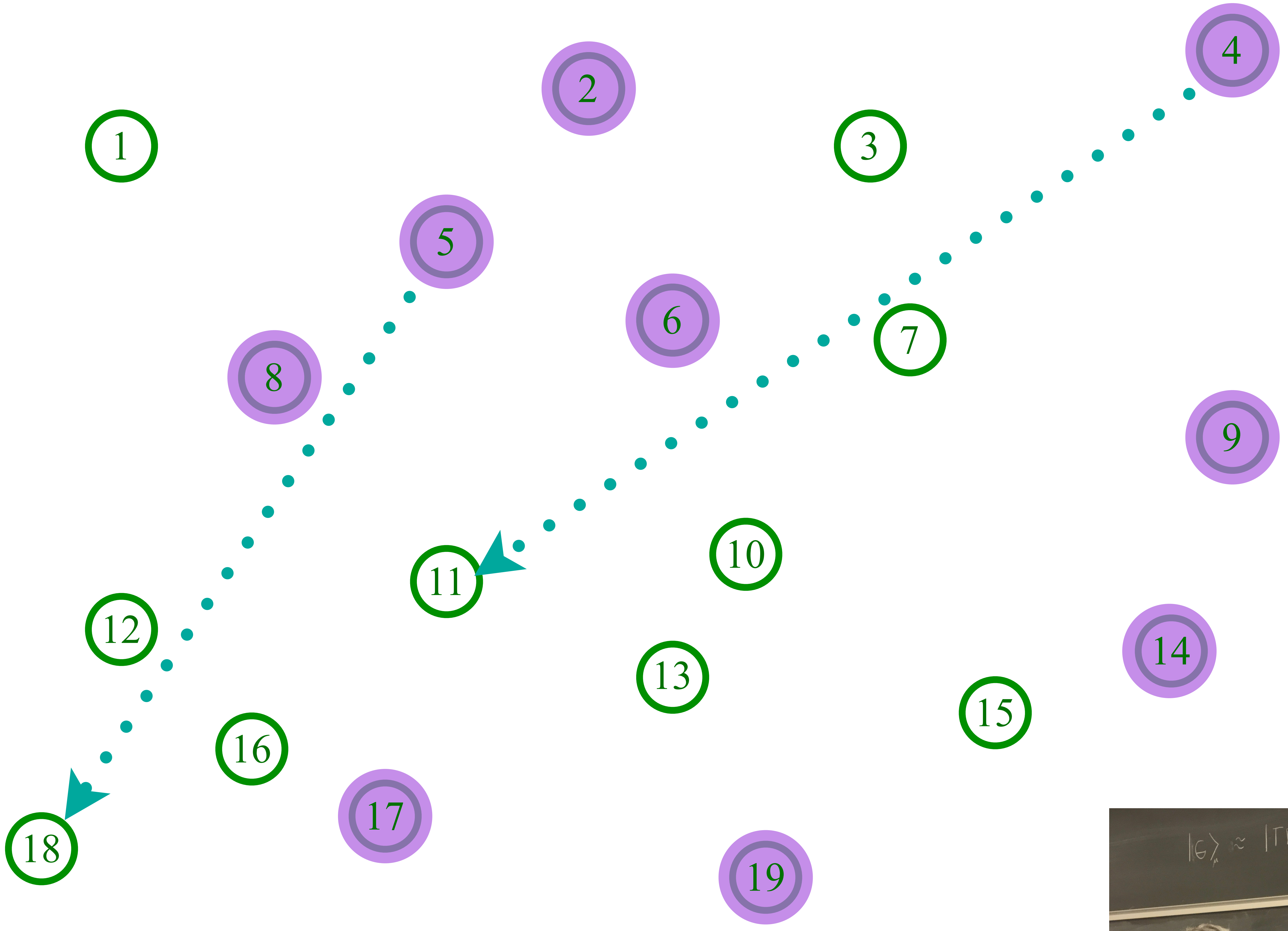
Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



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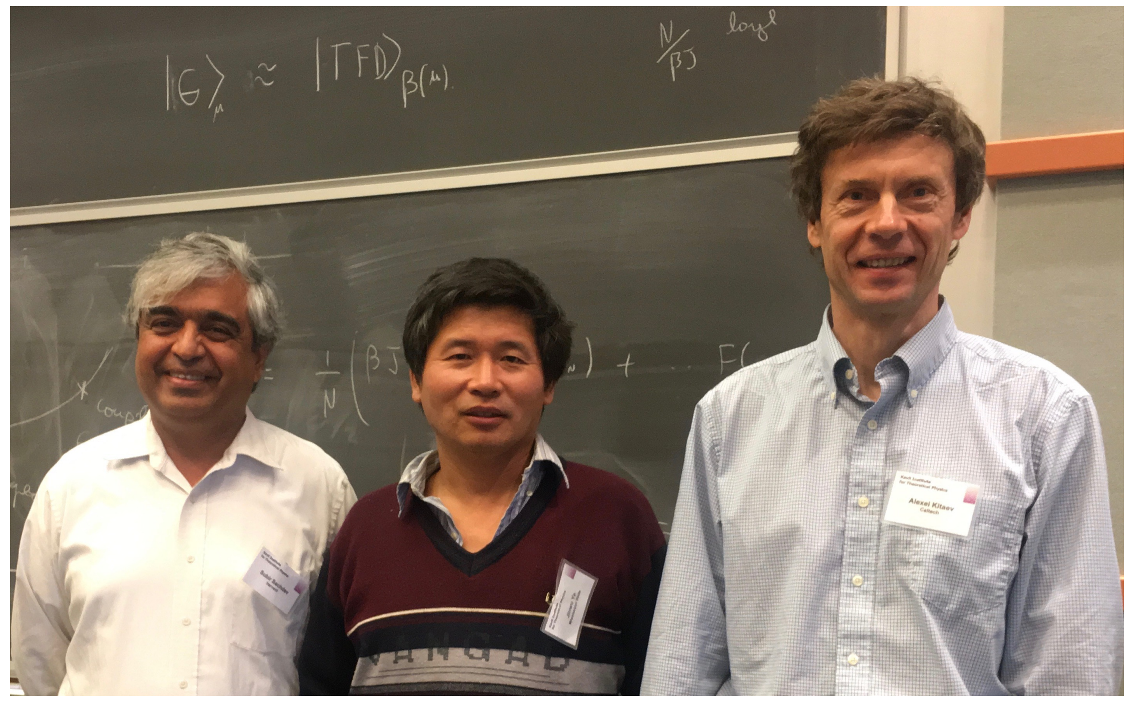

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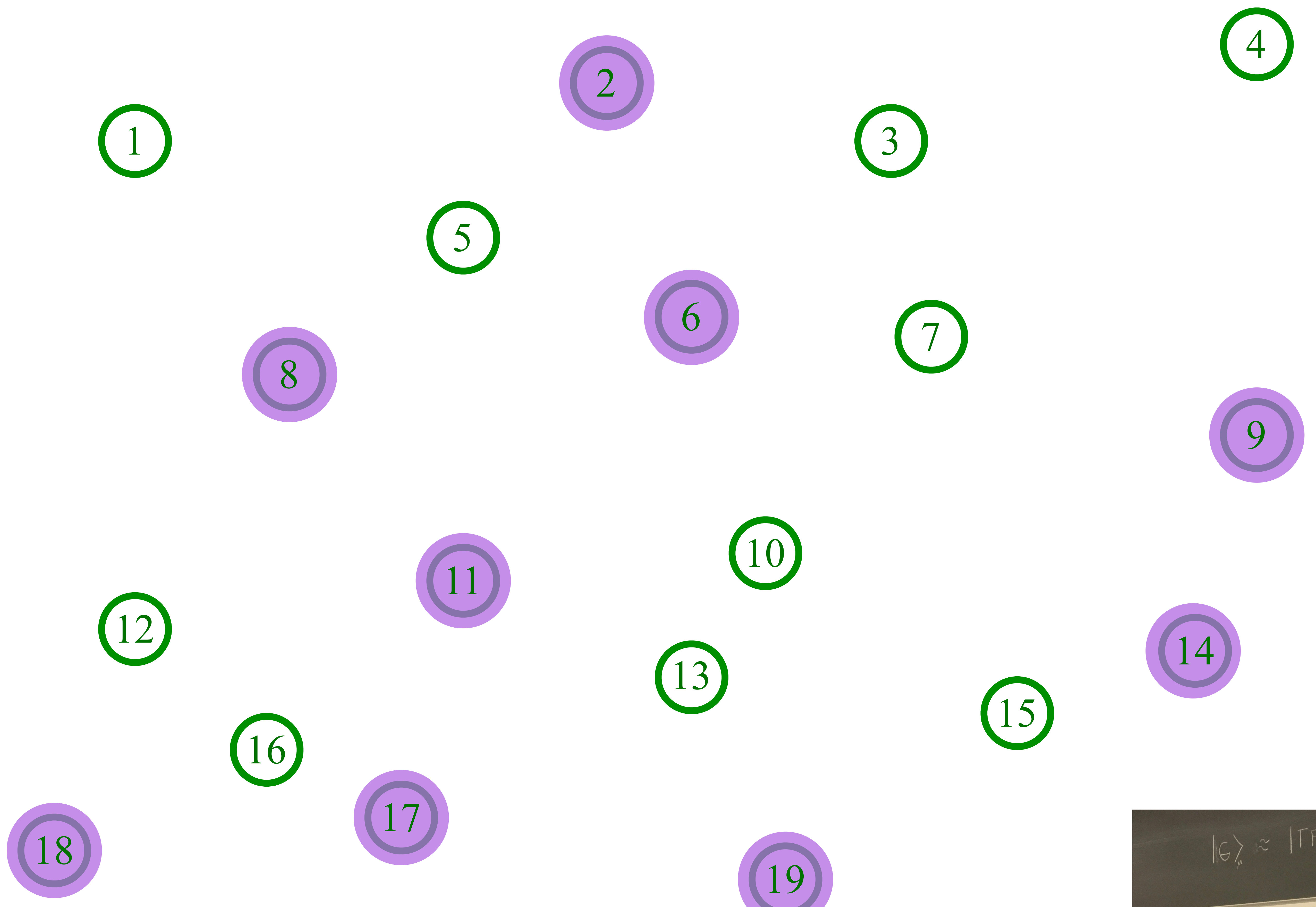

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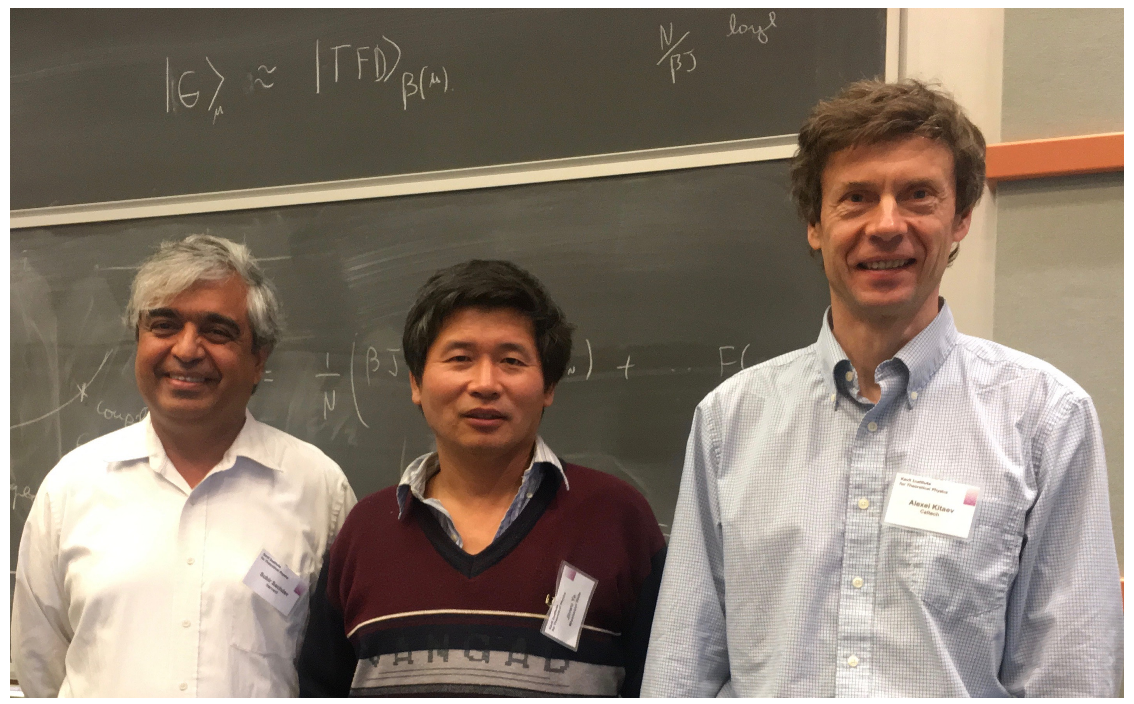

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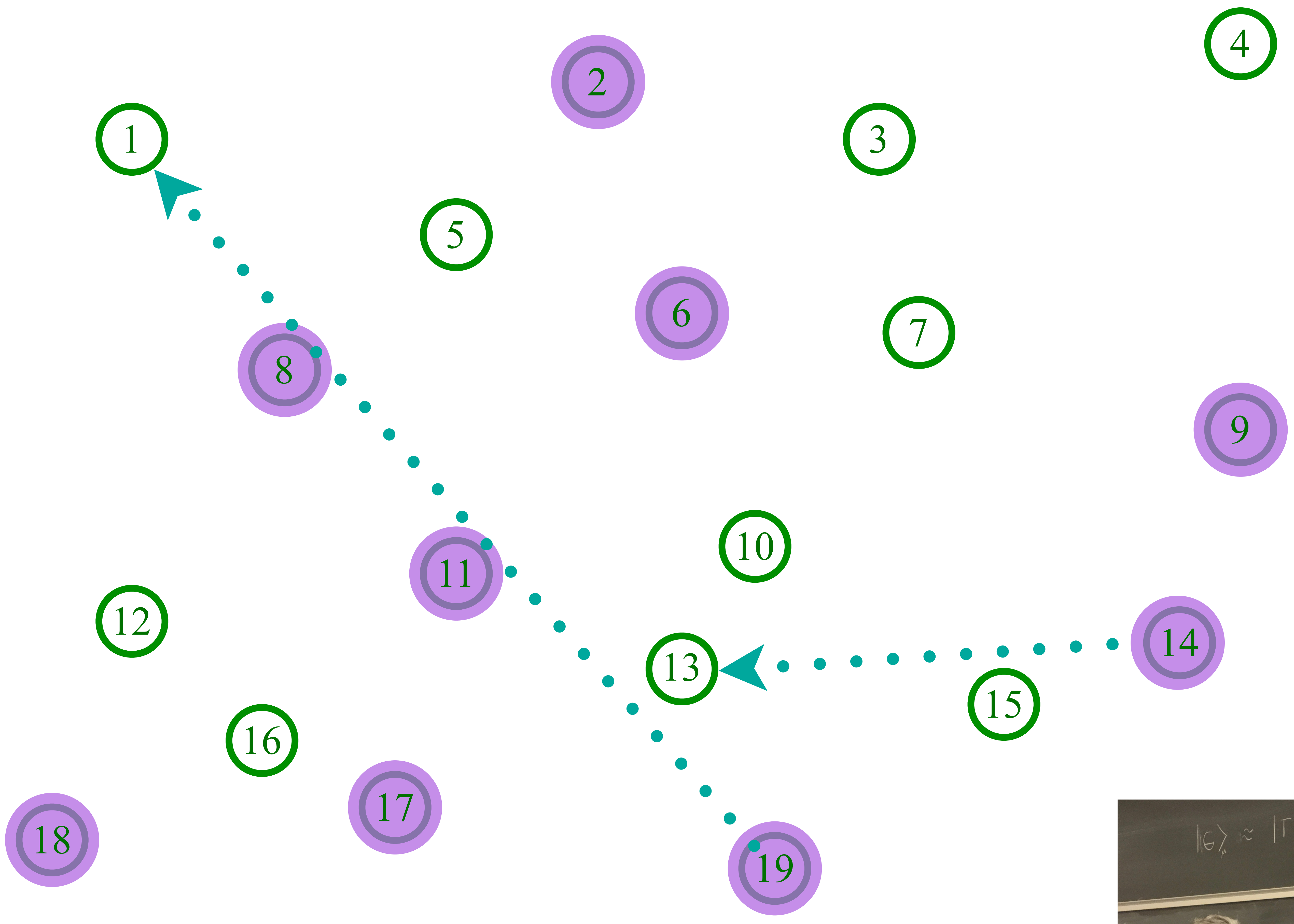

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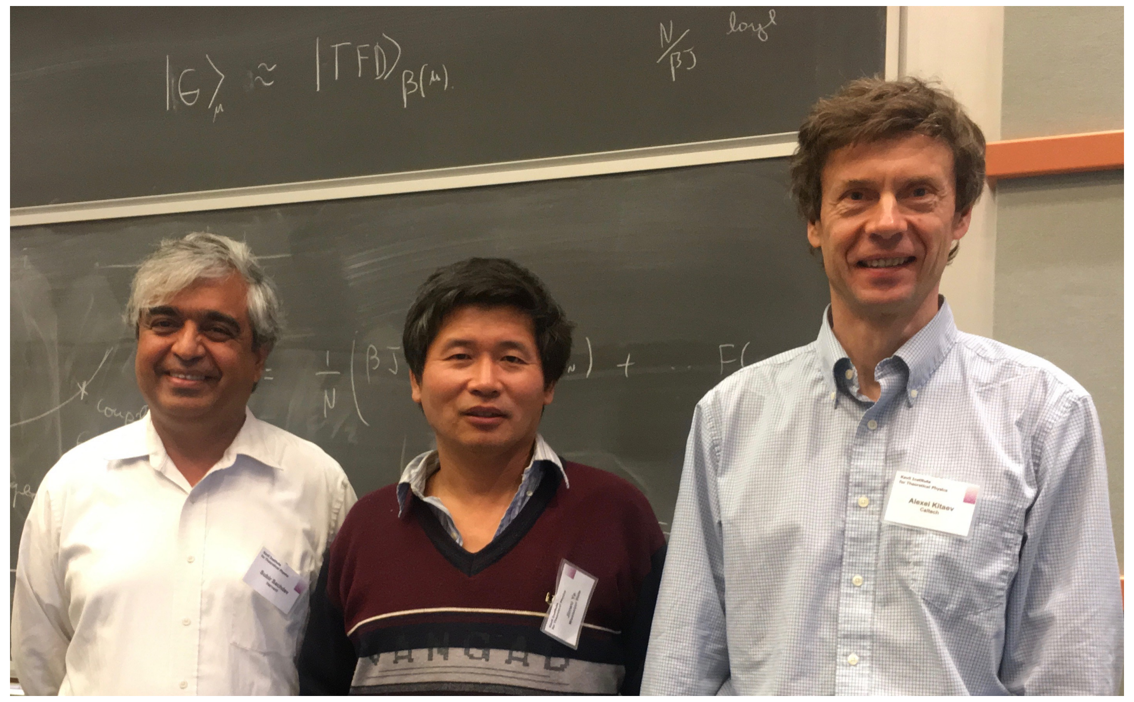

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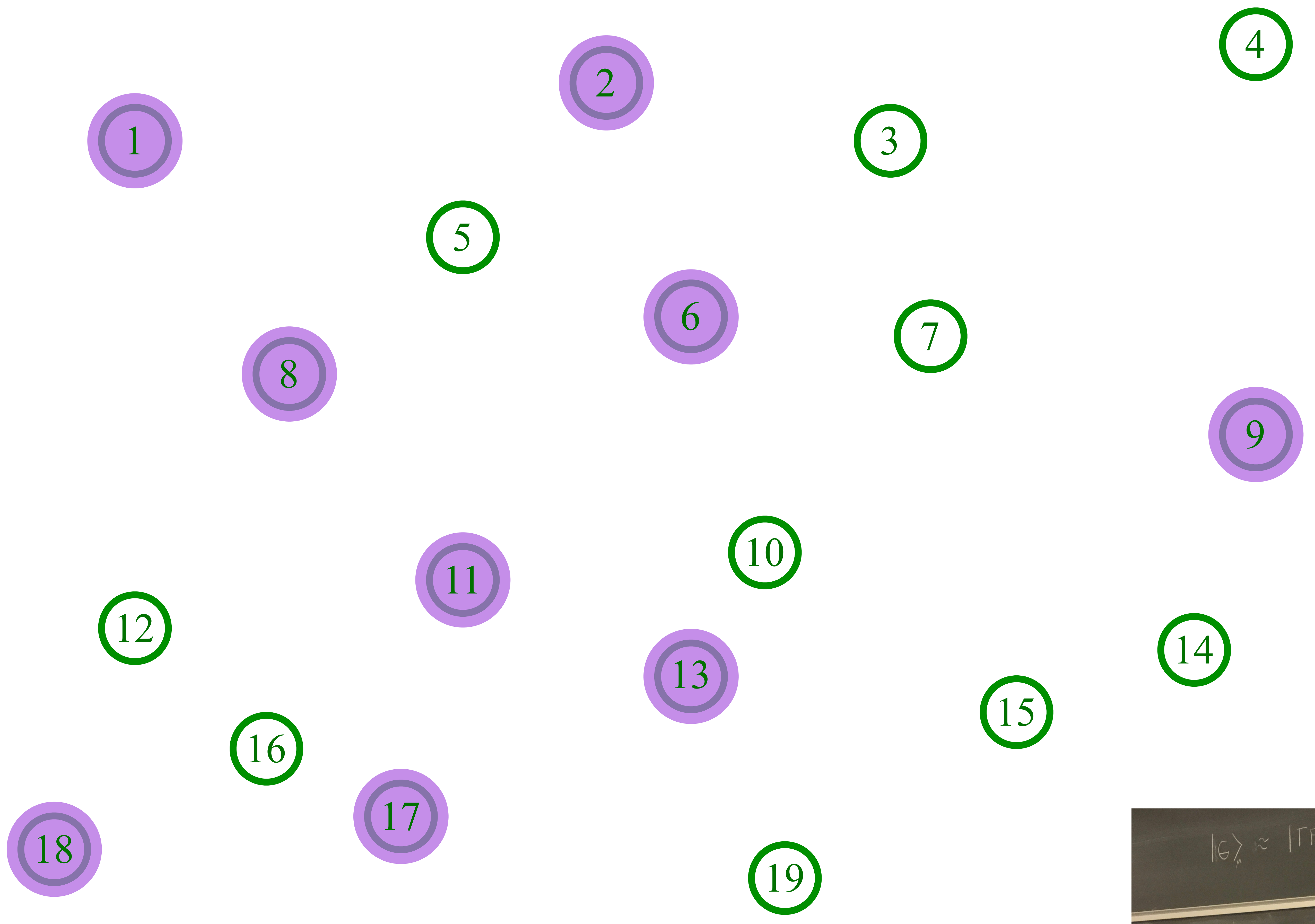

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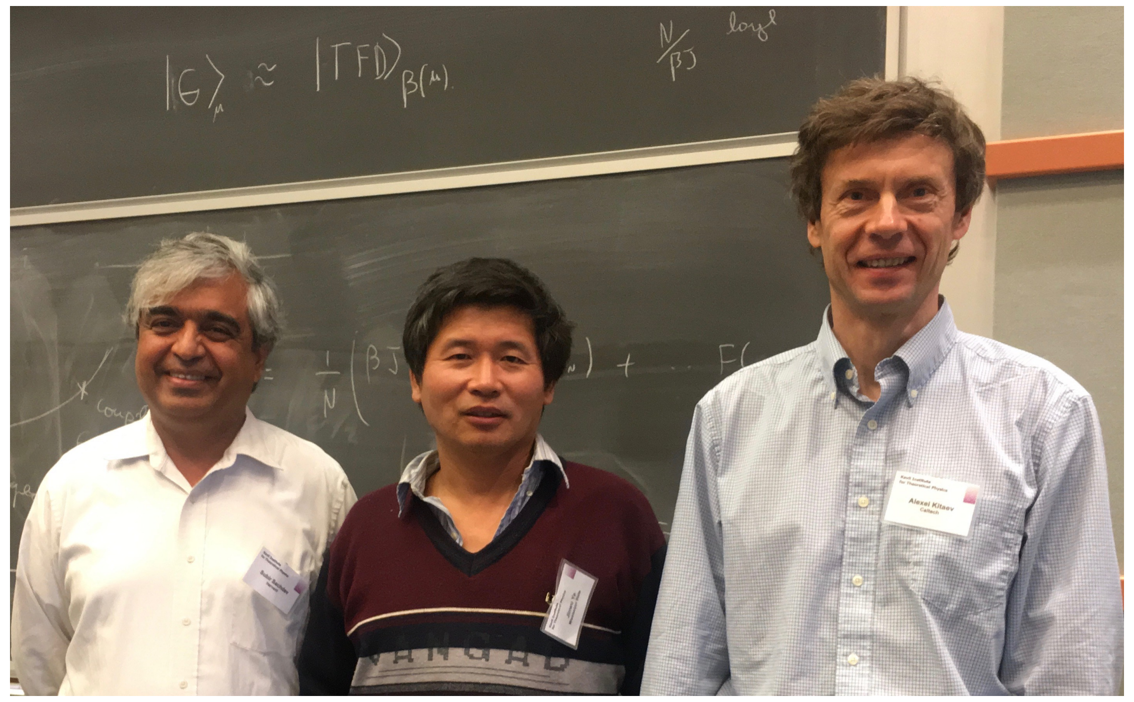

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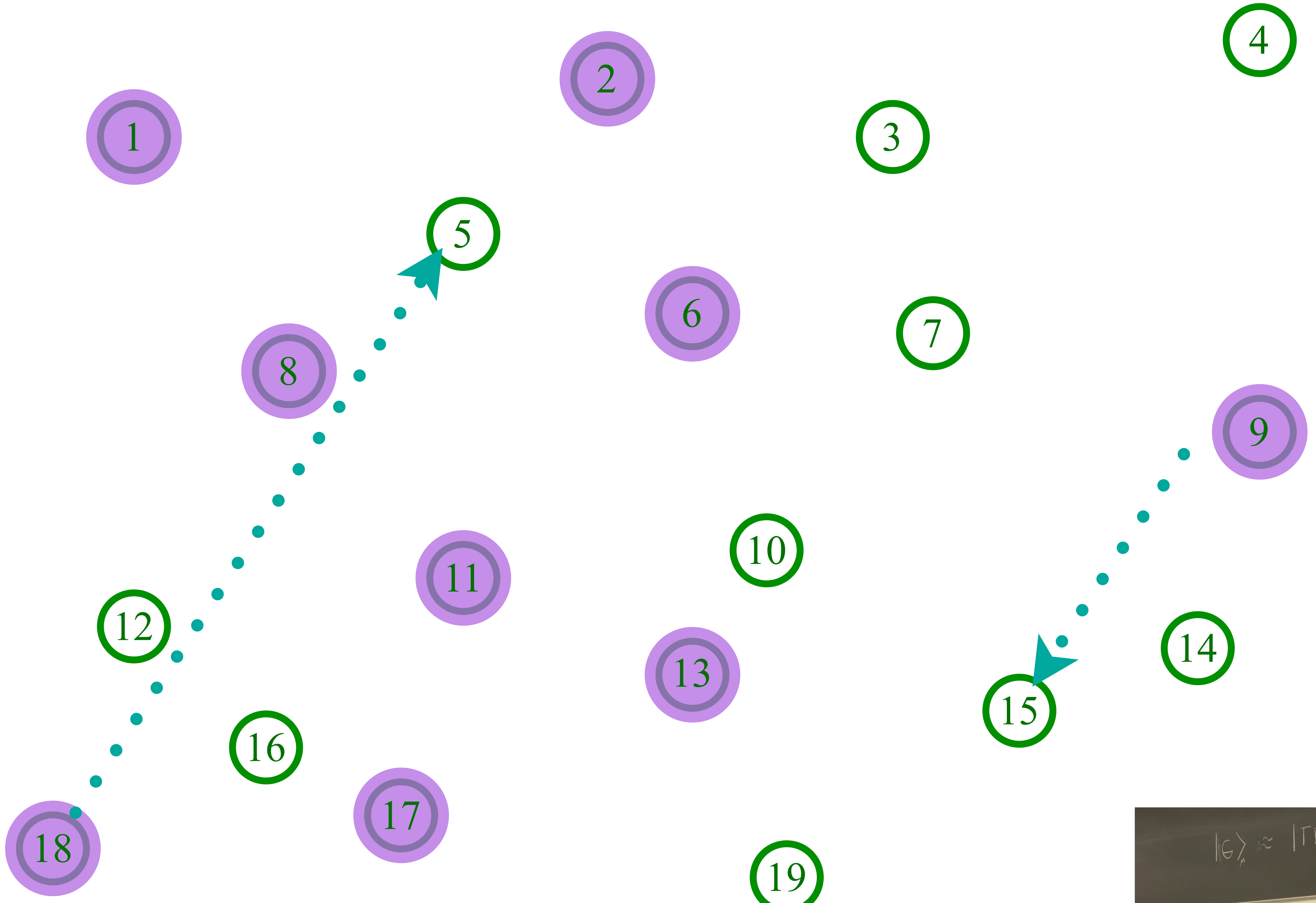
$$U_{9,18;5,15}$$



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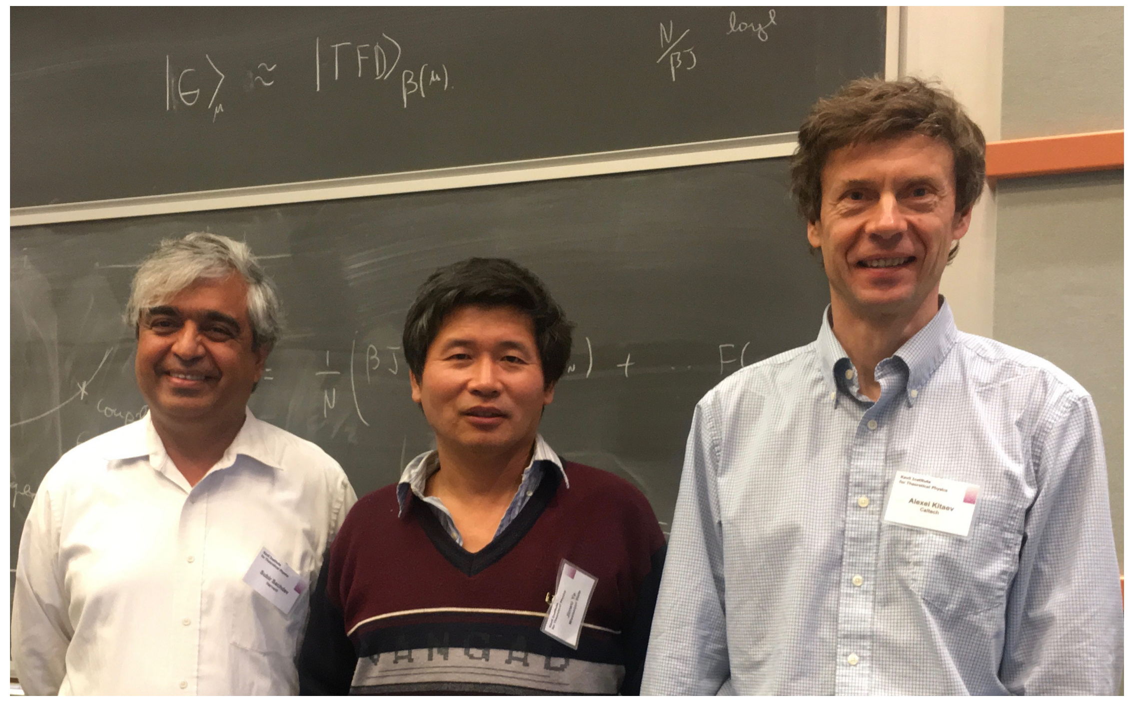

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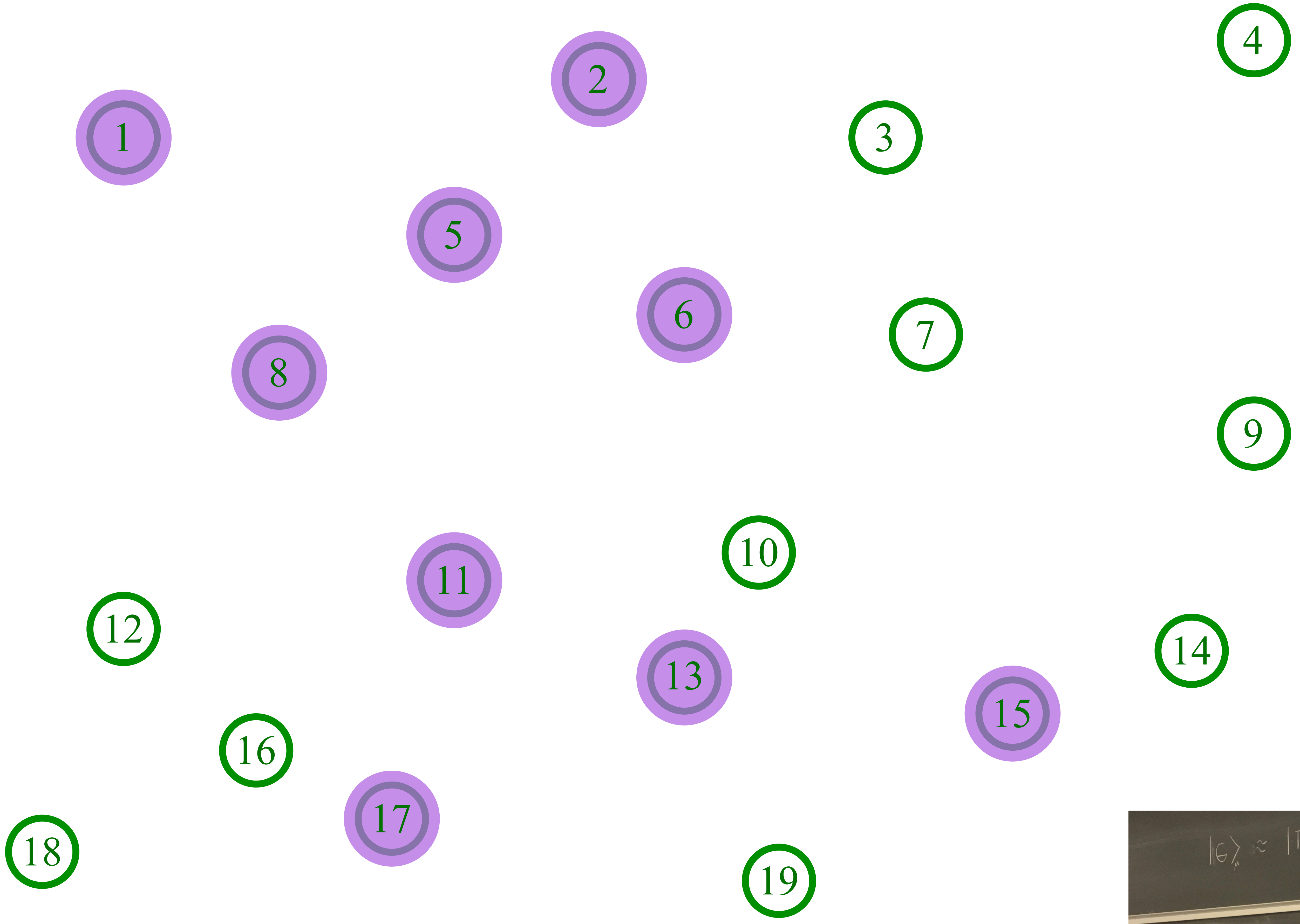

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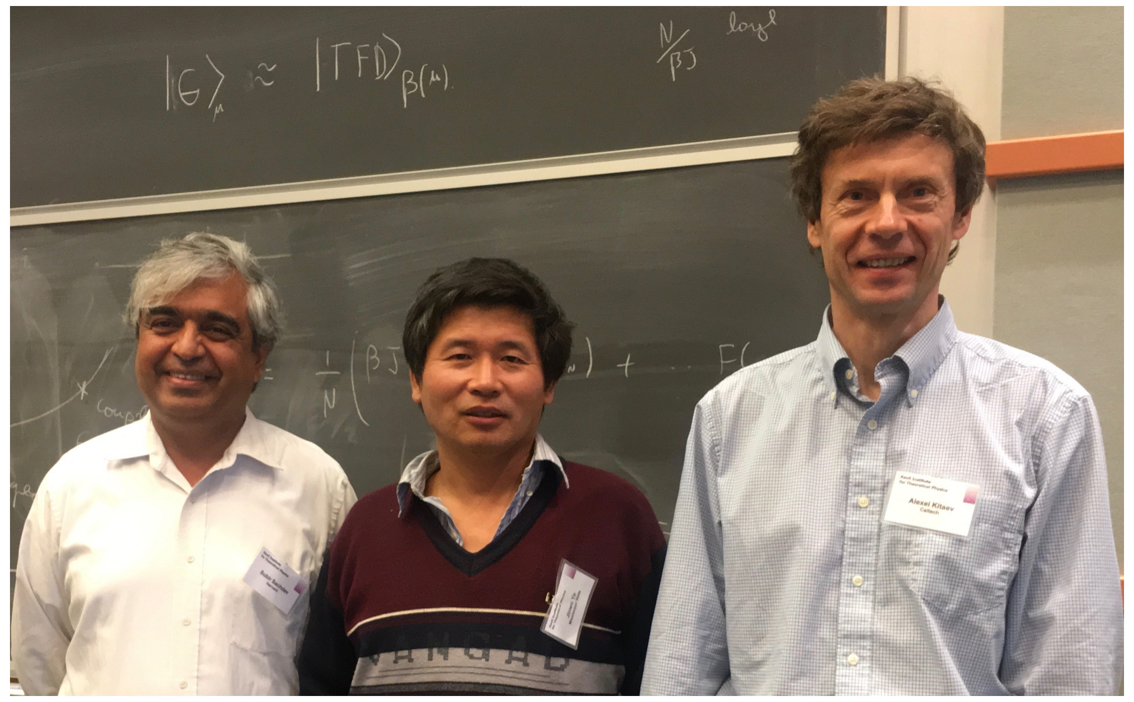
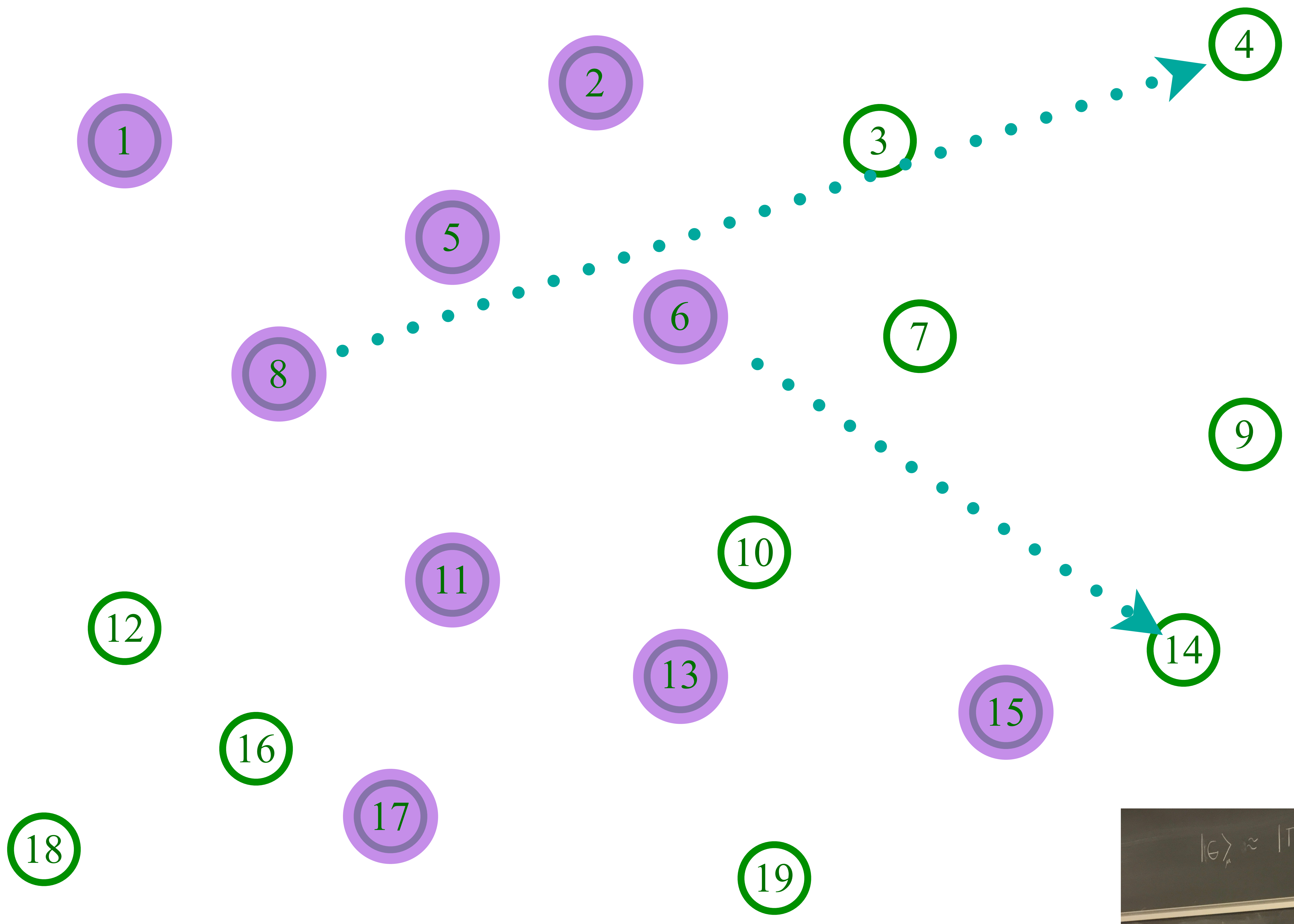
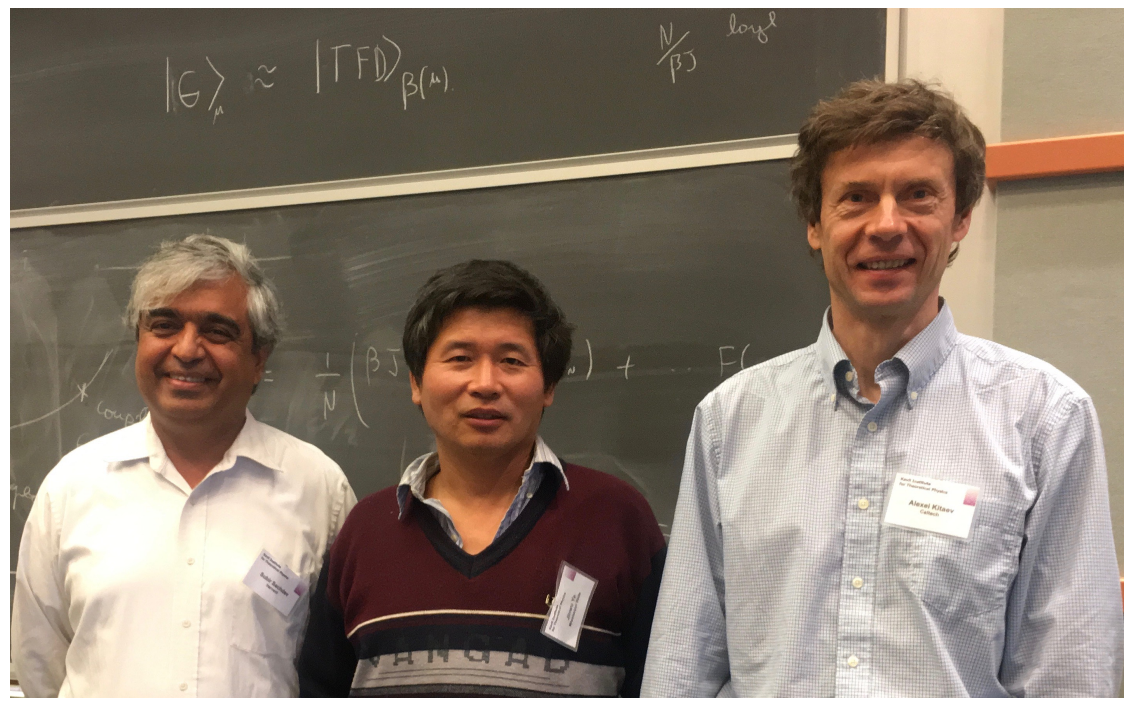


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$





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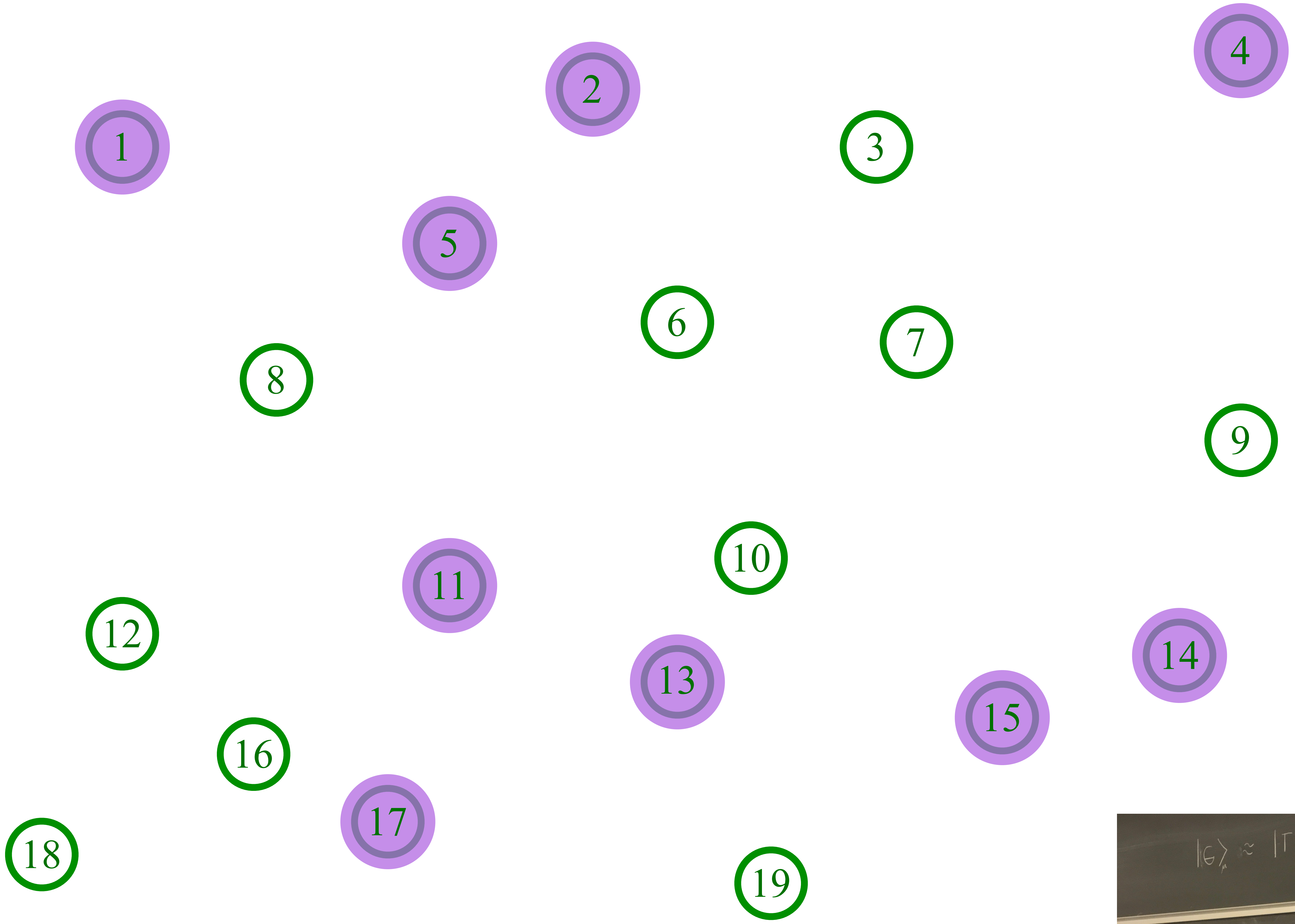

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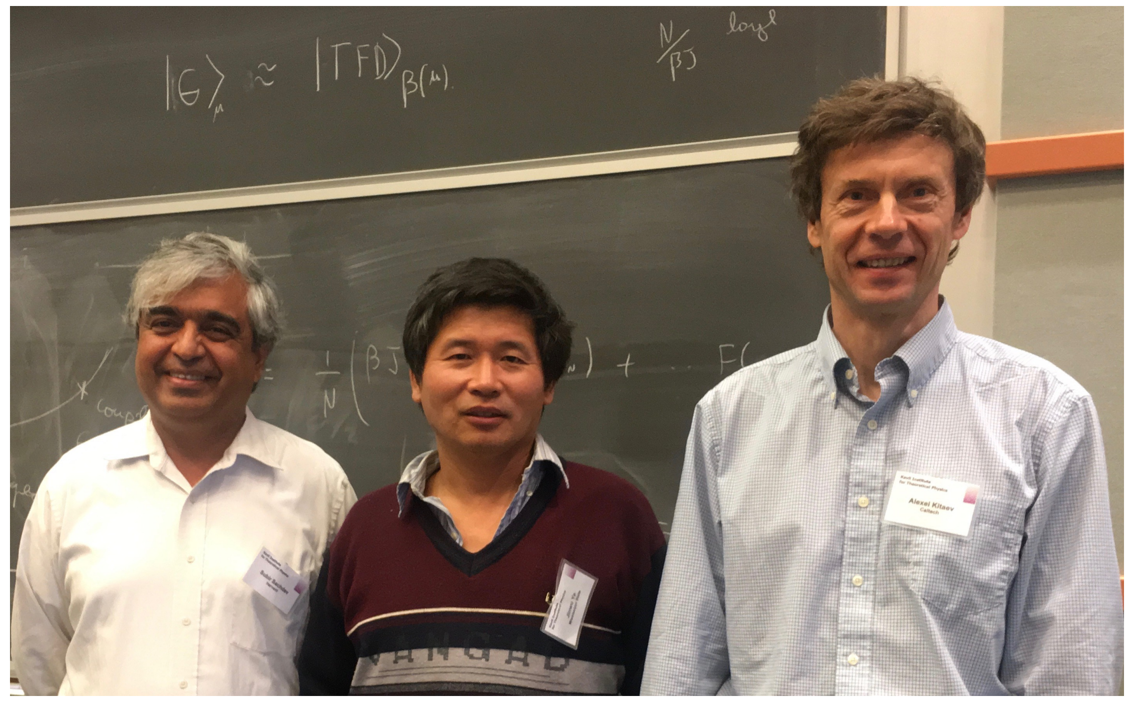


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

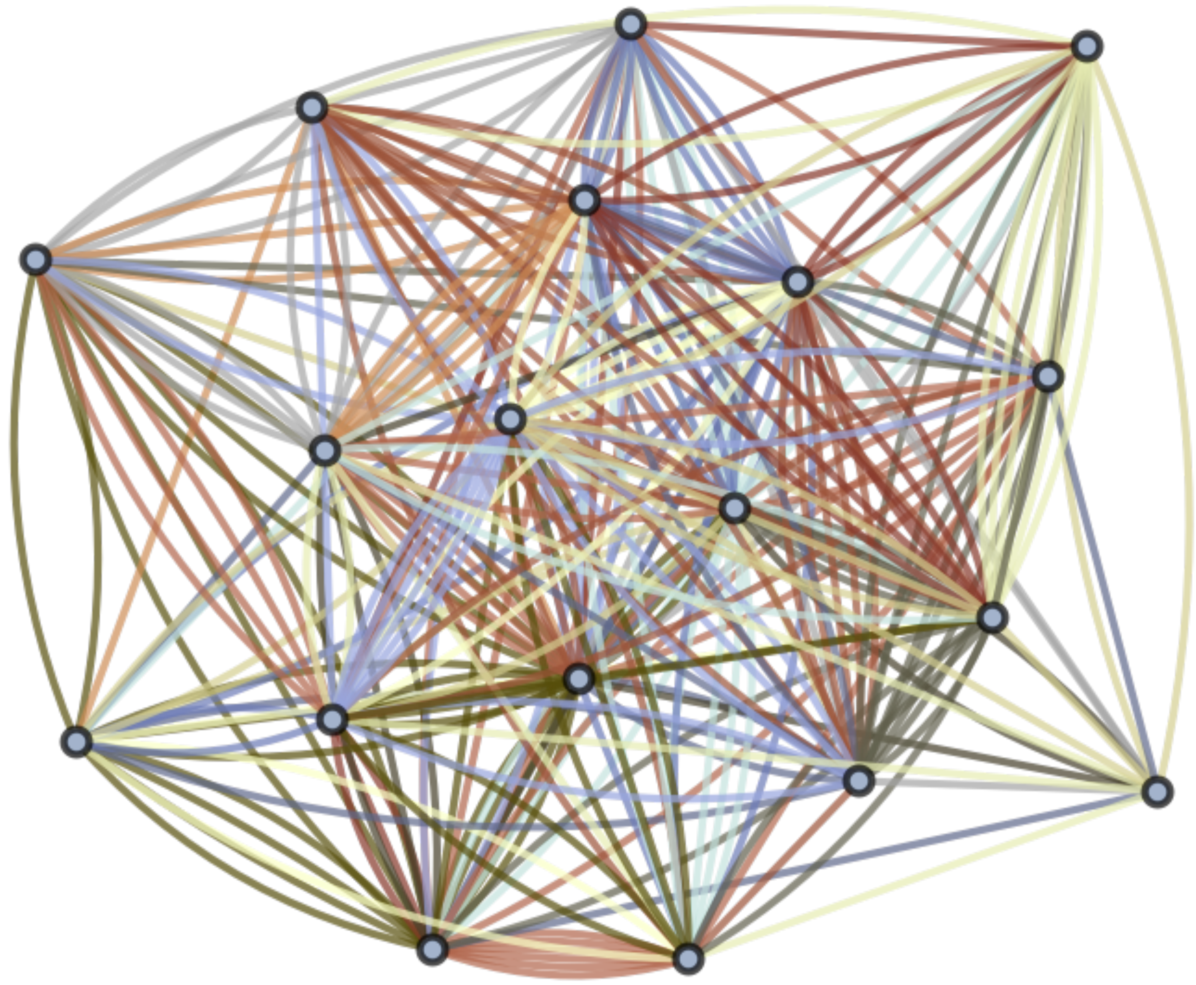
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

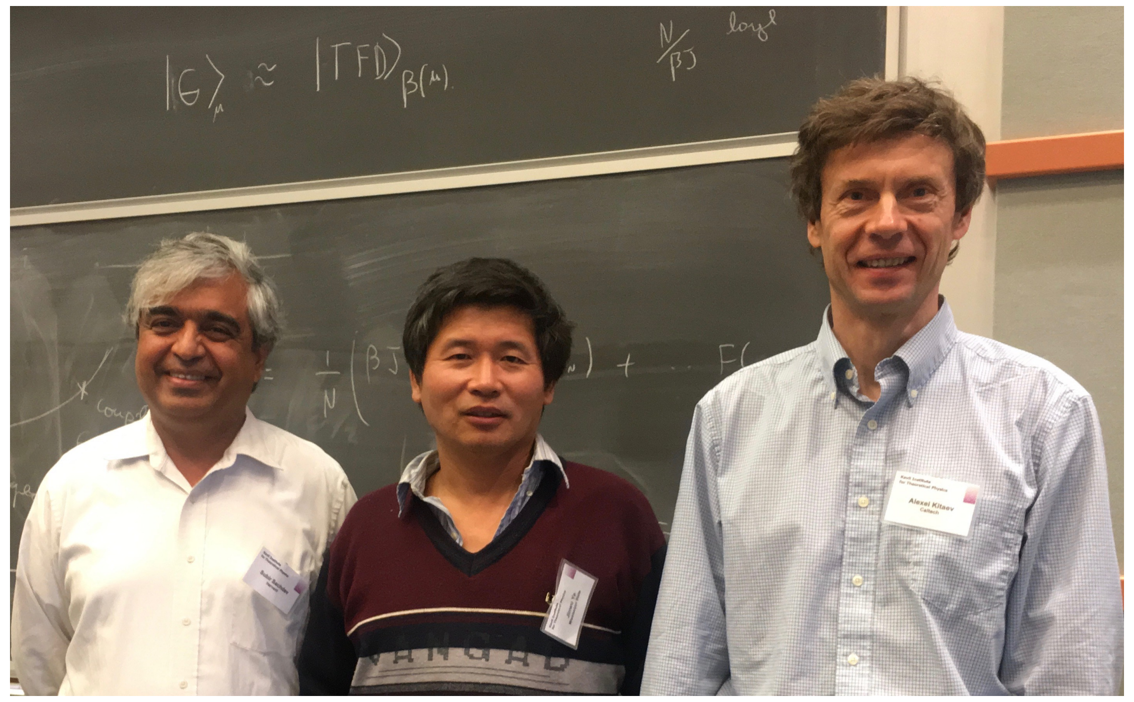
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \,; \quad [\mathcal{H}, \mathcal{Q}] = 0\,; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



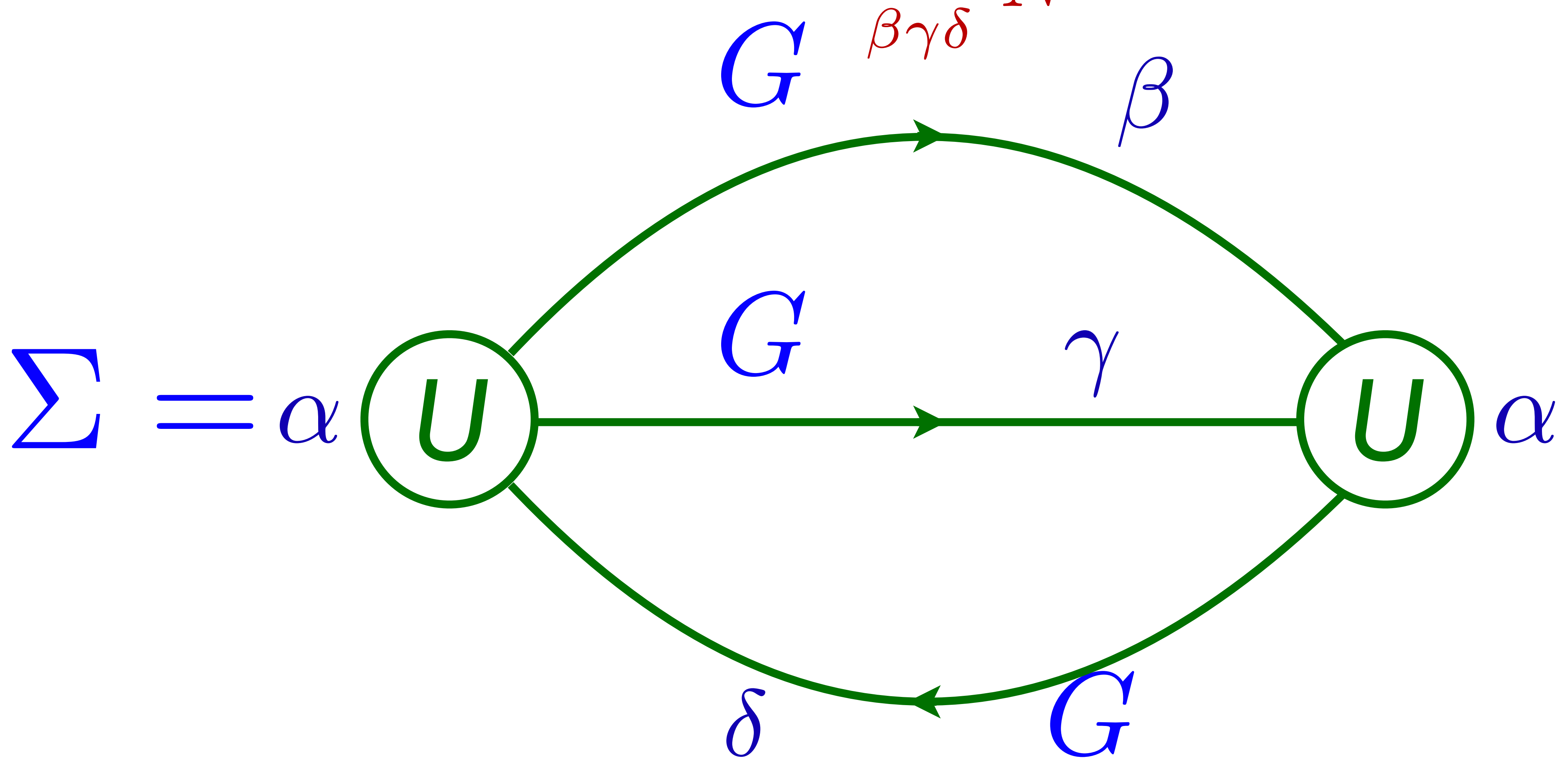


The Sachdev-Ye-Kitaev (SYK) model

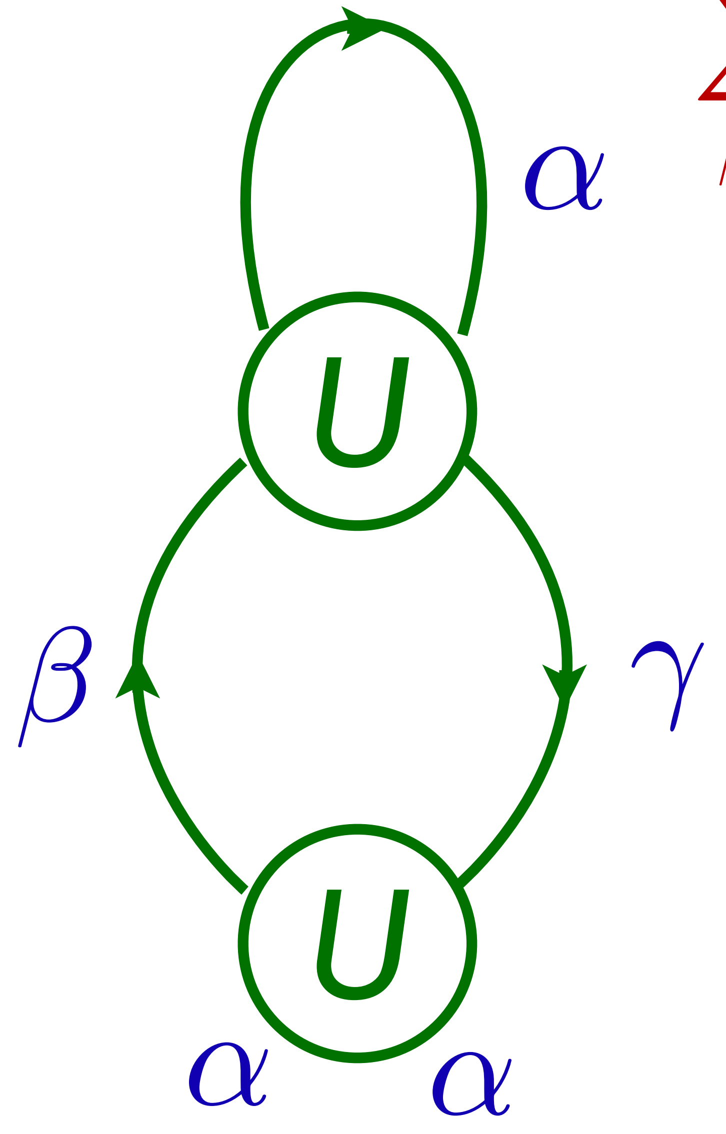
Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$$\sum_{\beta\gamma\delta} \frac{U^2}{N^3} = U^2$$



$$\sum_{\beta\gamma} \frac{U^2}{N^3} = \frac{U^2}{N}$$



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

Current is carried by an "entangled quantum soup"

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

A key consequence of the absence of the particle-like excitations is *Universal Planckian Dissipation*.
The relaxation time, τ, when perturbed at a frequency ω is given by

$$\tau = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

where $\hbar$ is Planck's constant, T is temperature, and the function F is independent of the strength of interaction between the particles.

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$\boxed{\begin{array}{c} G(i\omega_n) = \dfrac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \dfrac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{array}}$$

Make the low frequency ansatz

$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK model

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

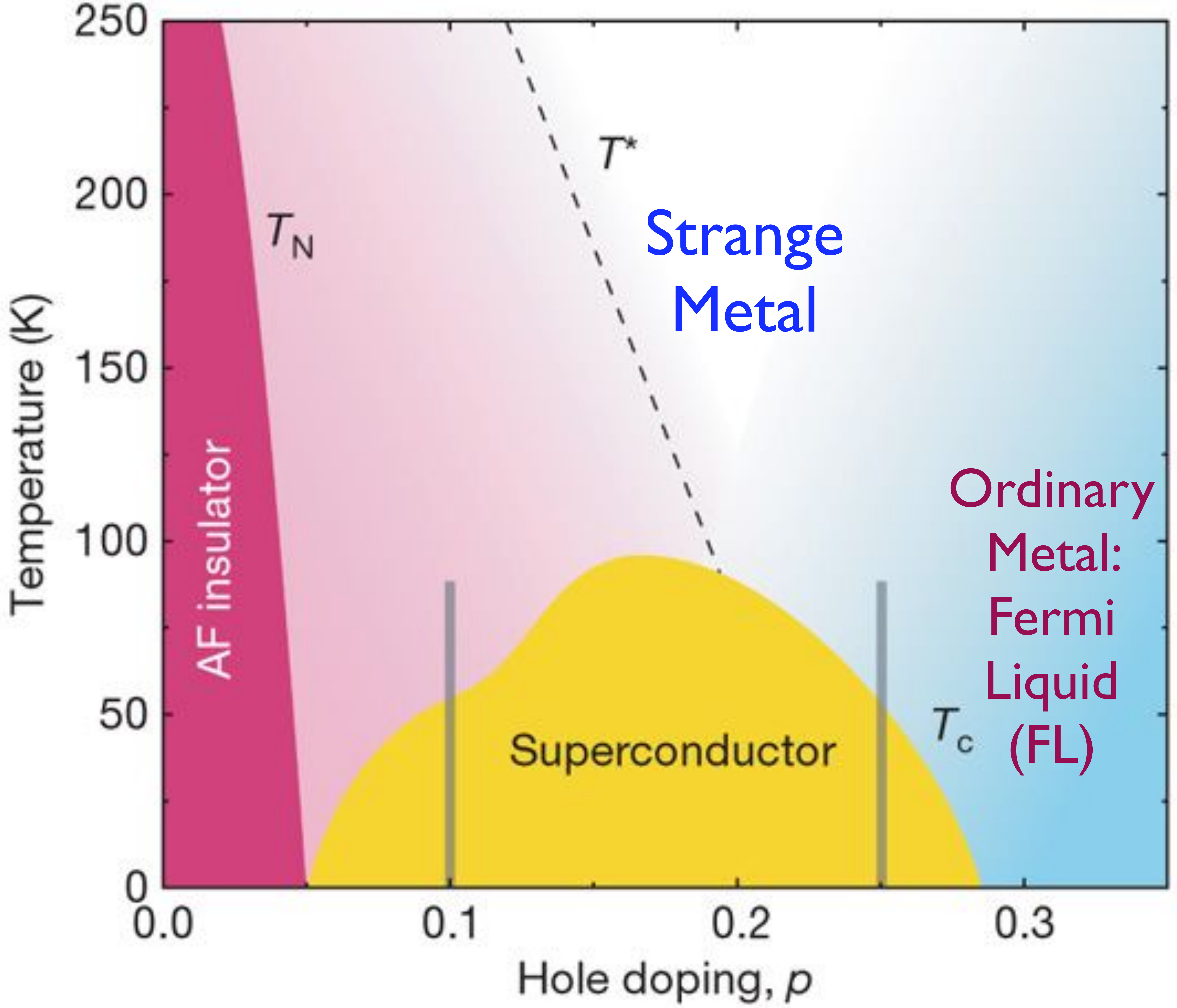
A key consequence of the absence of the particle-like excitations is *Universal Planckian Dissipation*.
The relaxation time, τ, when perturbed at a frequency ω is given by

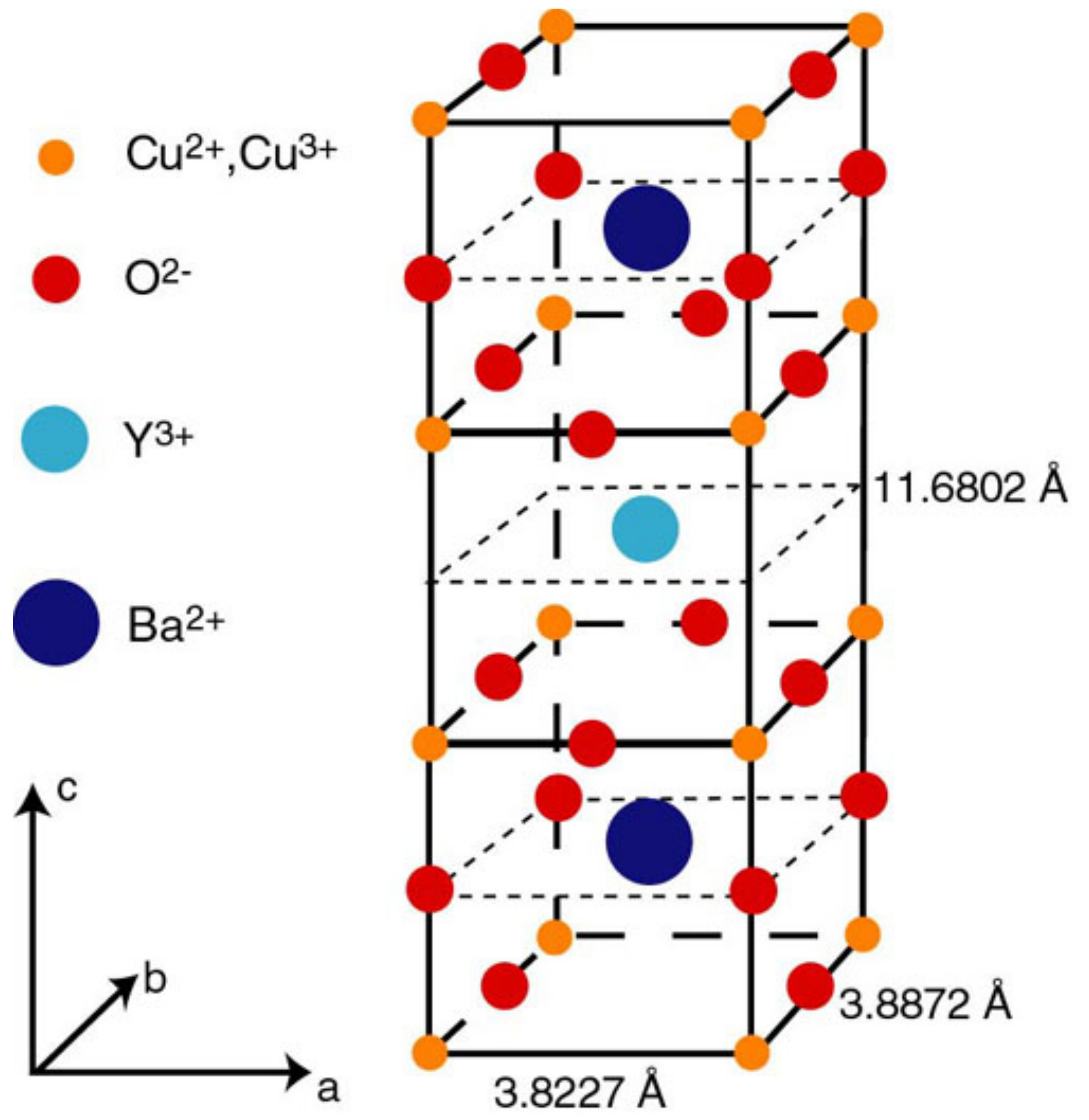
$$\tau = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

where $\hbar$ is Planck's constant, T is temperature, and the function F is independent of the strength of interaction between the particles.

Universal theory of strange metals:

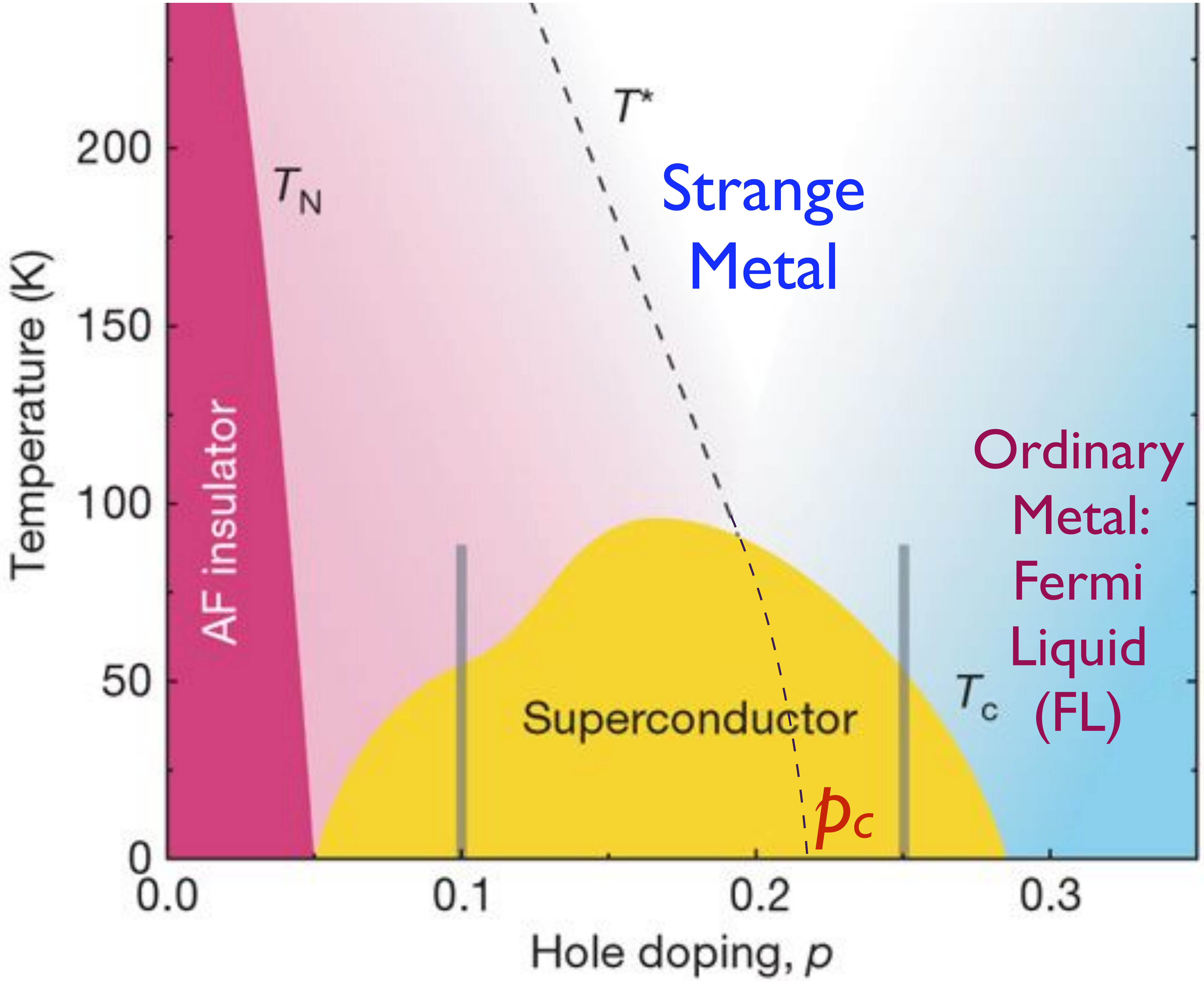
Quantum phase transitions
in inhomogeneous metals
described by the
two-dimensional Yukawa-SYK model

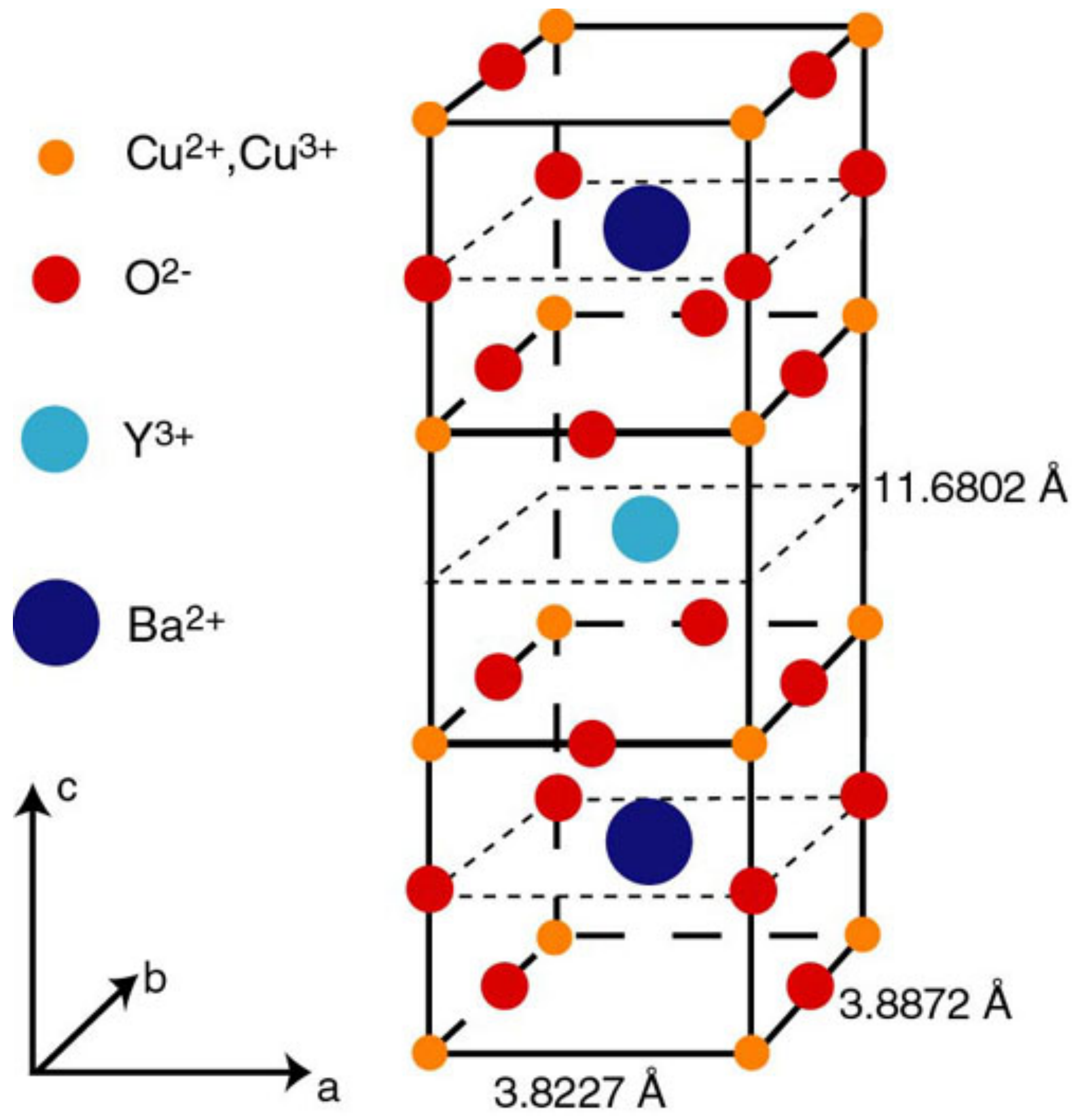

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal: Fermi Liquid (FL)
Superconductor
T_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

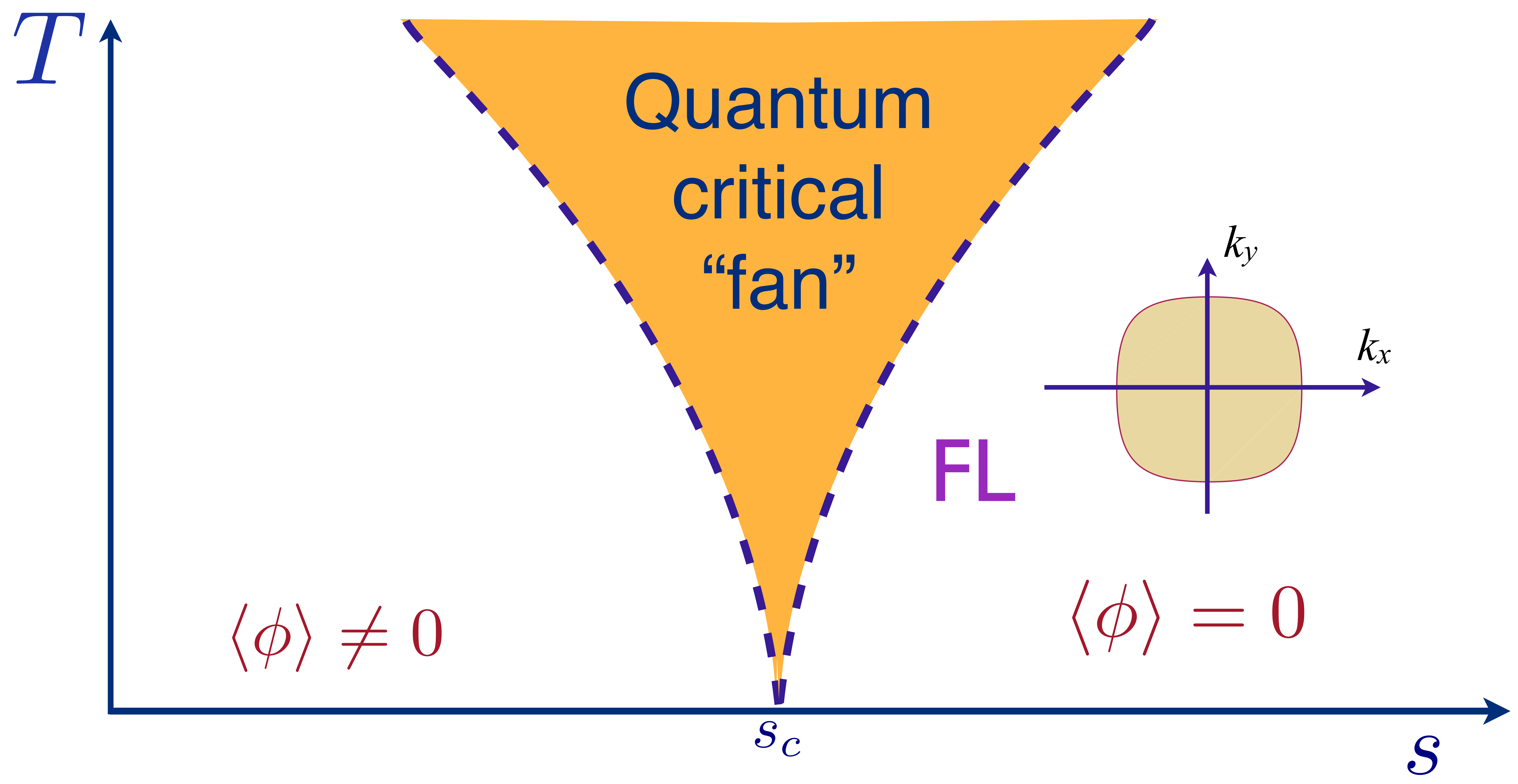
Quantum phase transitions in two-dimensional metals



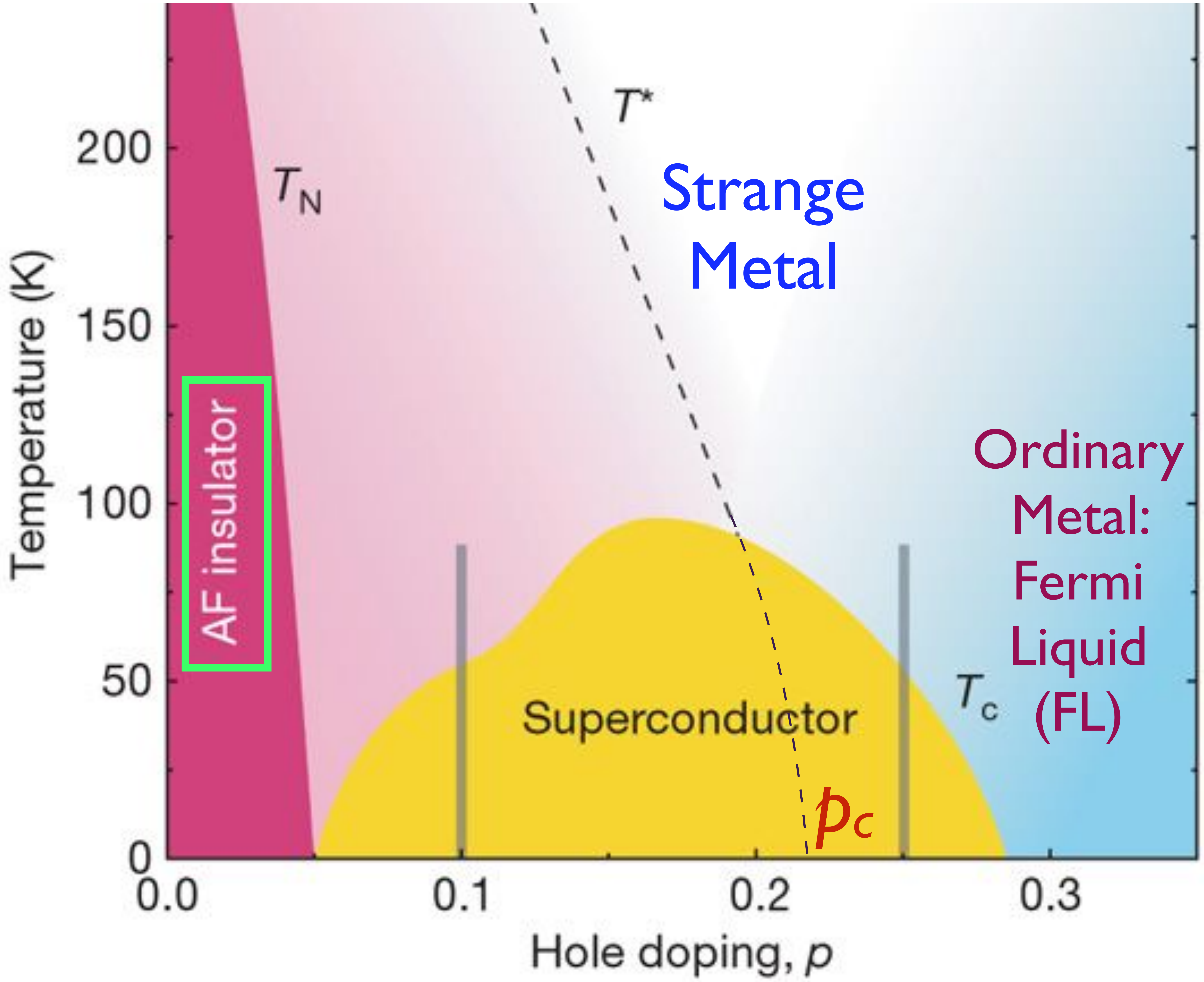


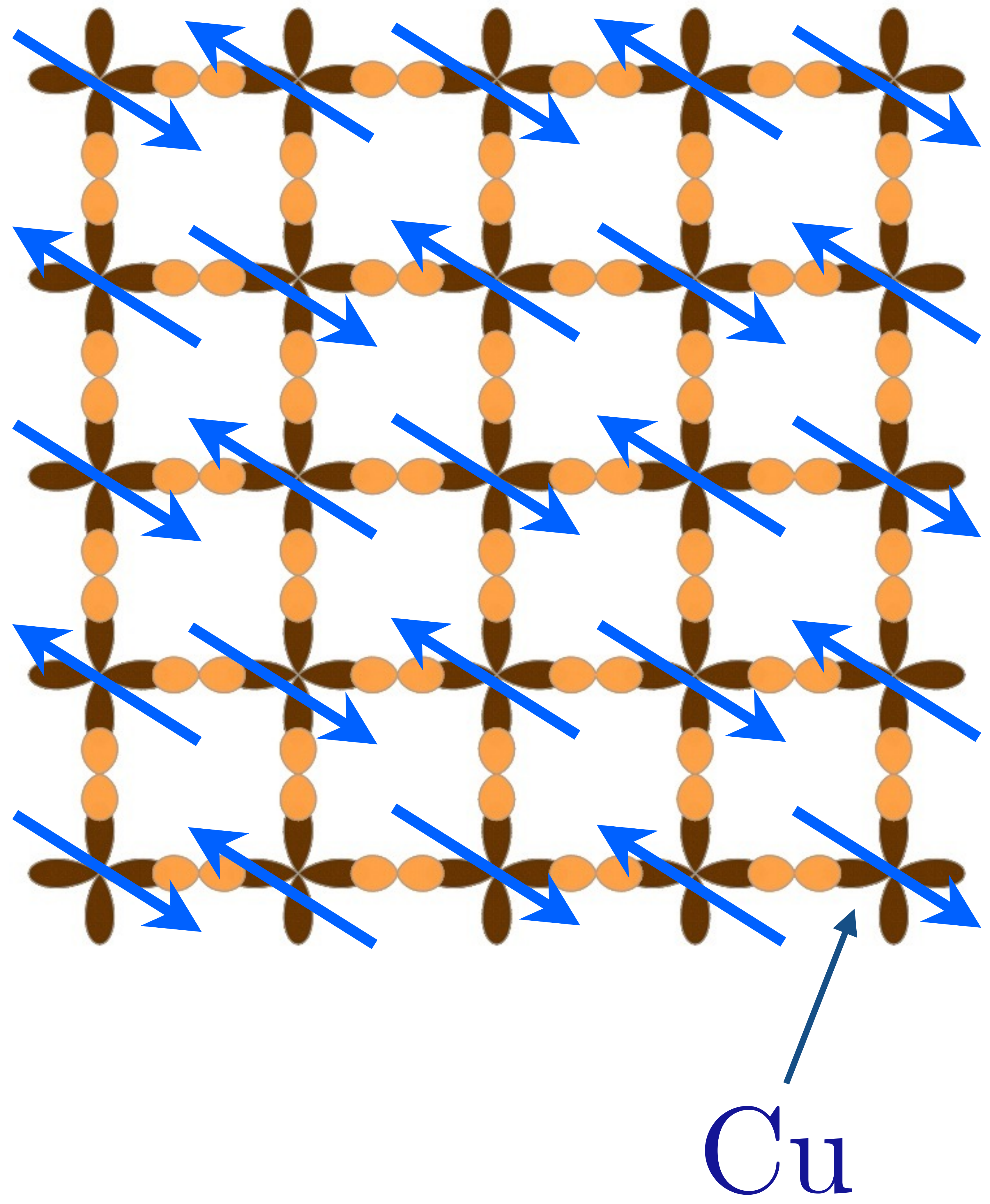
$YBa_2Cu_3O_{6+x}$

Quantum phase transitions in two-dimensional metals



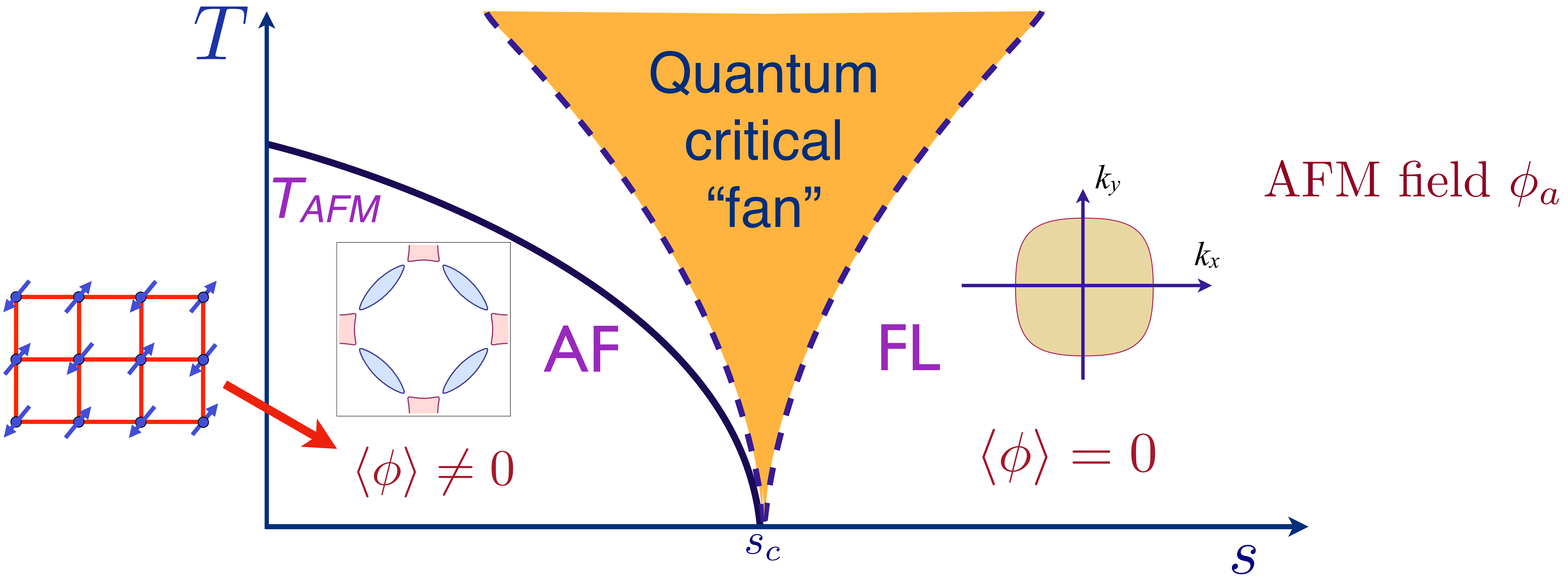
Quantum phase transitions in two-dimensional metals





Quantum phase transitions in two-dimensional metals

With symmetry breaking



Quantum phase transitions in two-dimensional metals

Without symmetry breaking

FL*: fractionalized Fermi liquid

Applies to hole-doped cuprates

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004); Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Yukawa-SYK model

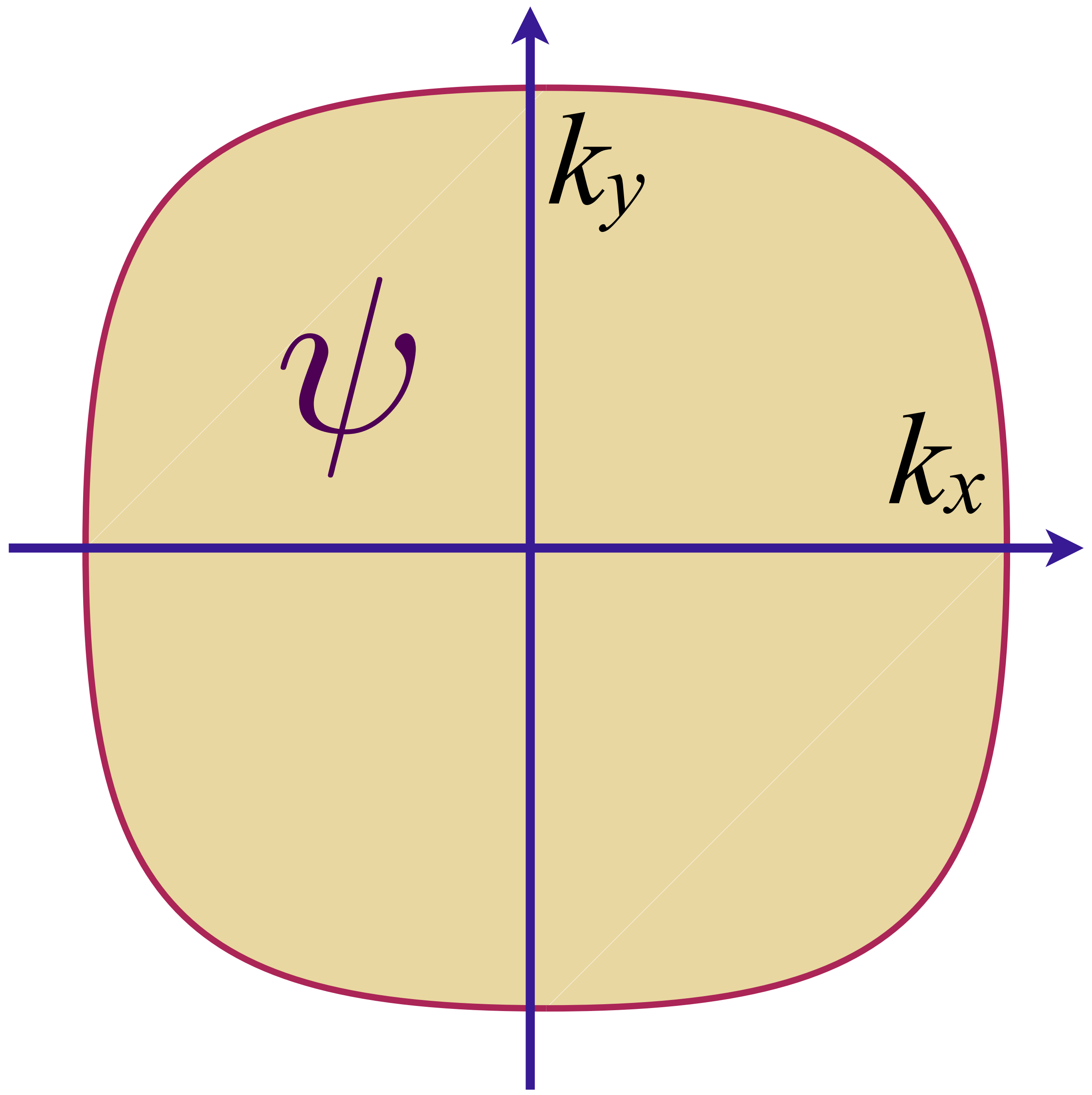
$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

2d-YSYK model: Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



$$+s\,[\phi(\boldsymbol{r})]^2 + [g \qquad\qquad]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Hertz 1976

With or without symmetry breaking, but without impurities

Quantum phase transitions in two-dimensional metals

With or without symmetry breaking, but without impurities

- In the absence of impurities, there is strong *drag* between the fermions ψ and the bosons ϕ, leading to an emergent conserved total momentum at low energies.

S.A. Hartnoll, P.K. Kovtun, M. Muller and S. S., PRB **76** (2007) 144502; S.A. Hartnoll, R. Mahajan, M. Punk and S. S., PRB **89** (2014) 155130; A.A. Patel and S. S., PRB **90** (2014) 165146

Quantum phase transitions in two-dimensional metals

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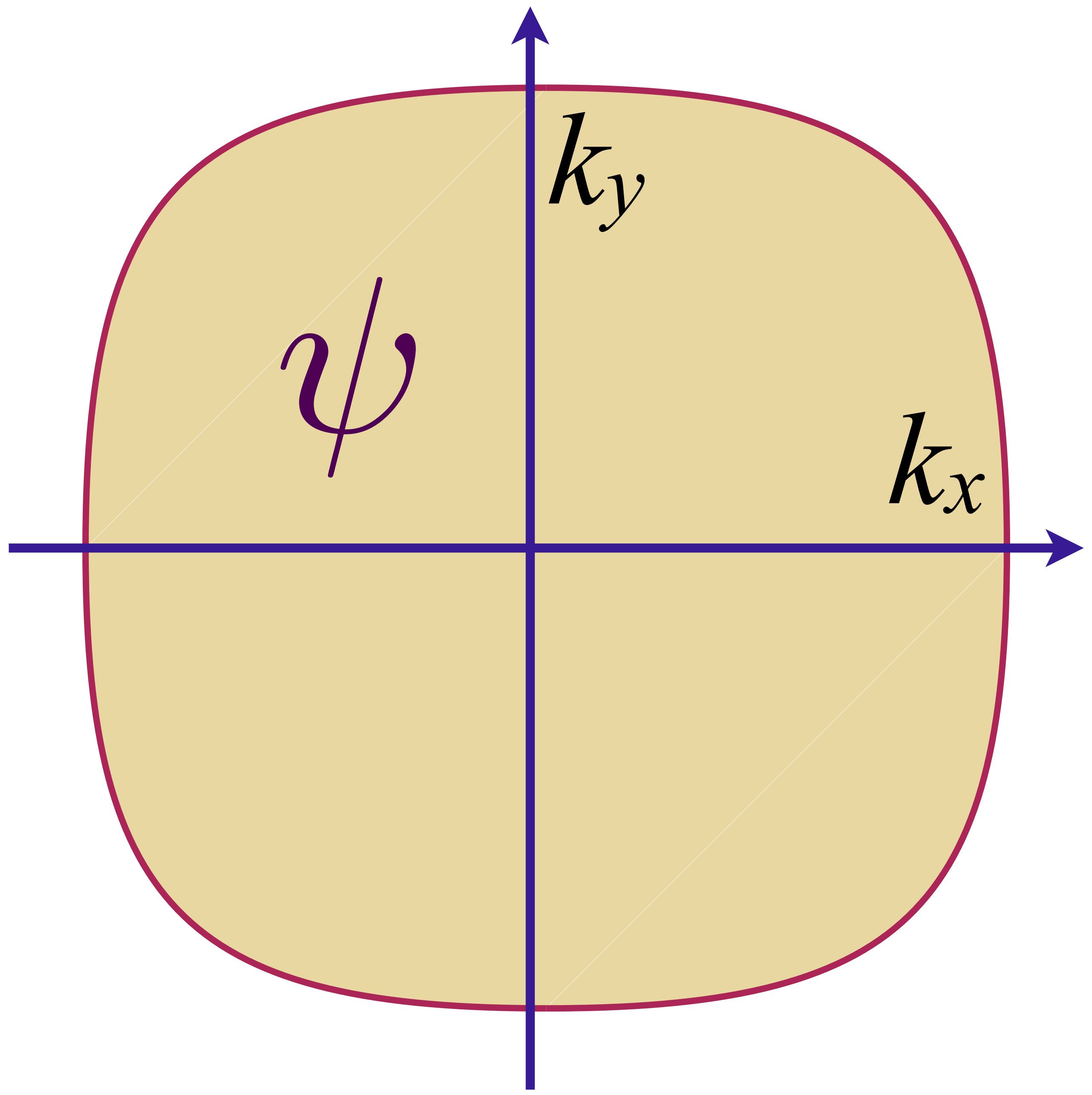
- The most important effect of impurities is Harris disorder: spatial disorder in the tuning parameter across the quantum phase transition.

S.A. Hartnoll, P.K. Kovtun, M. Muller and S. S., PRB **76** (2007) 144502; S.A. Hartnoll, R. Mahajan, M. Punk and S. S., PRB **89** (2014) 155130; A.A. Patel and S. S., PRB **90** (2014) 165146

2d-YSYK model: Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



$$+s\,[\phi(\boldsymbol{r})]^2 + [g \qquad]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

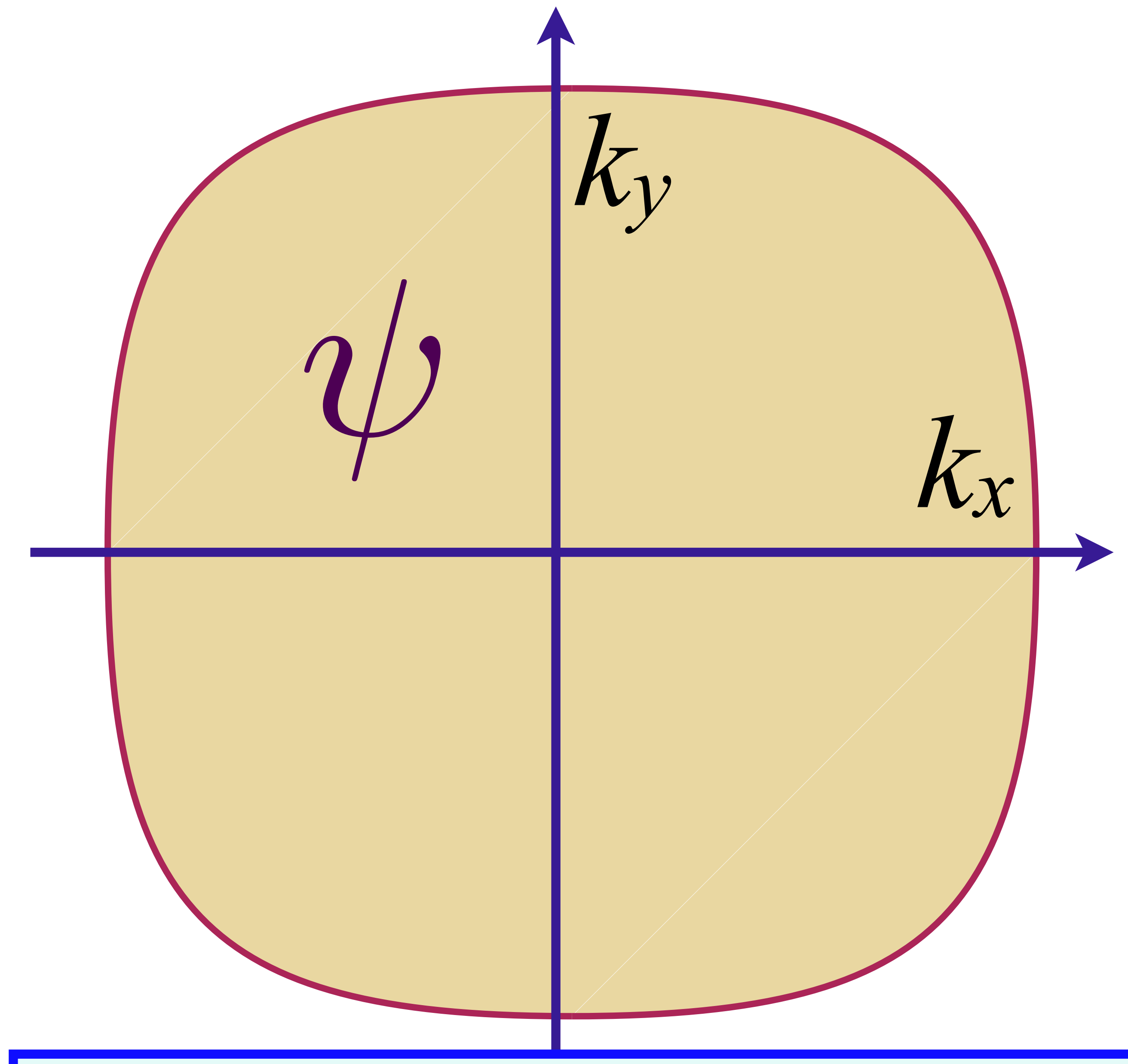
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Hertz 1976

With or without symmetry breaking, but without impurities

2d-YSYK model: Fermi surface + critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

$g'(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder): the two-dimensional Yukawa-Sachdev-Ye-Kitaev model.

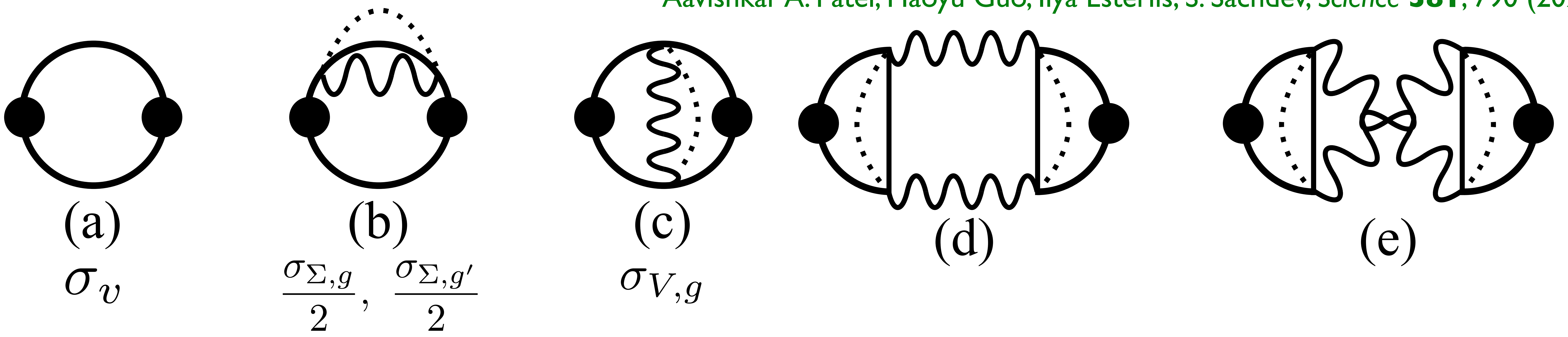
2d-YSYK model: Fermi surface + critical boson with disorder

SYK-type self-consistent equations

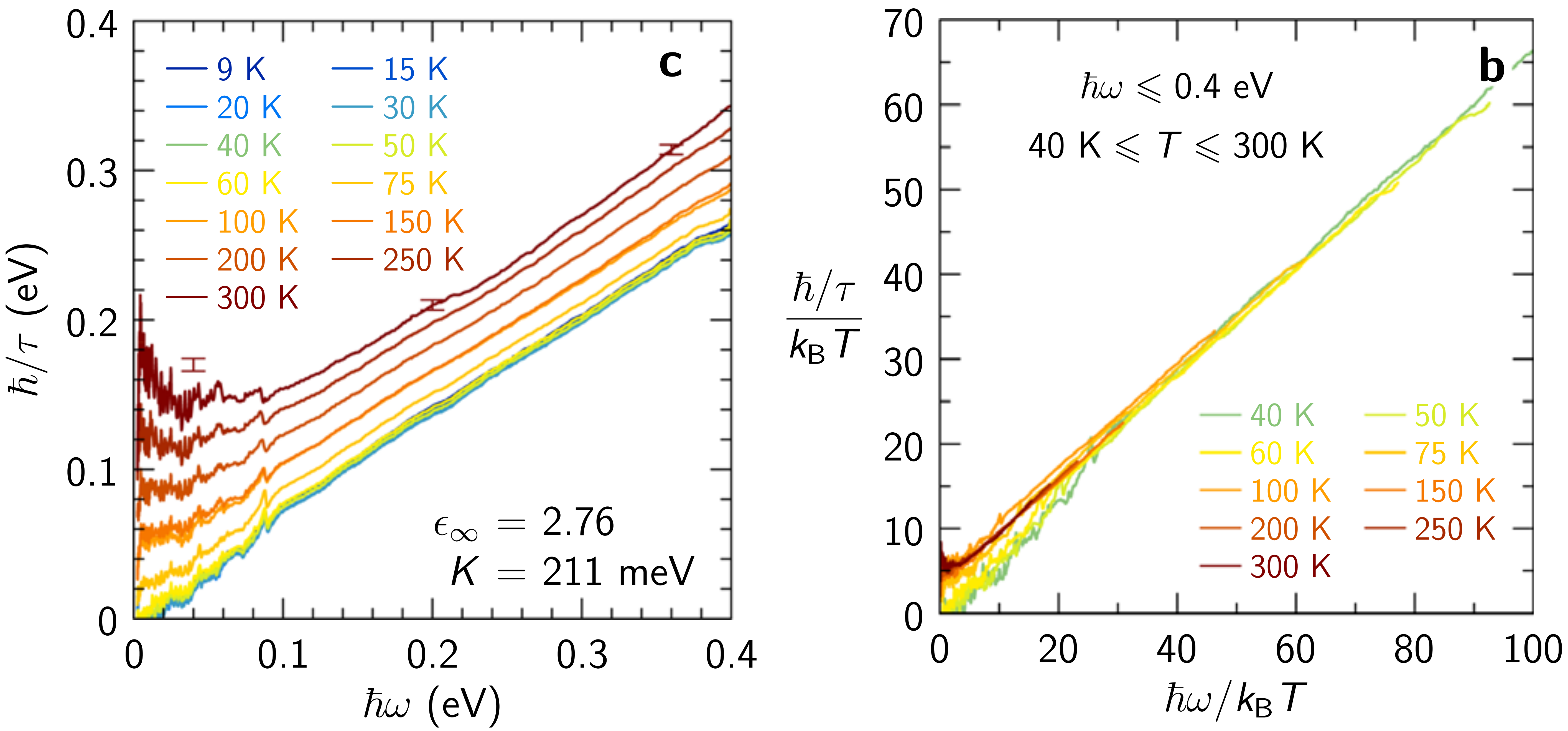
$$\Sigma(\tau,\mathbf{r}) = g^2 D(\tau,\mathbf{r}) G(\tau,\mathbf{r}) + v^2 G(\tau,\mathbf{r})\delta^2(\mathbf{r}) + g'^2 G(\tau,\mathbf{r}) D(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$
$$\Pi(\tau,\mathbf{r}) = -g^2 G(-\tau,-\mathbf{r}) G(\tau,\mathbf{r}) - g'^2 G(-\tau,\mathbf{r}) G(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$
$$G(i\omega,\mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega,\mathbf{k})},$$
$$D(i\Omega,\mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega,\mathbf{q})}.$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Conductivity:

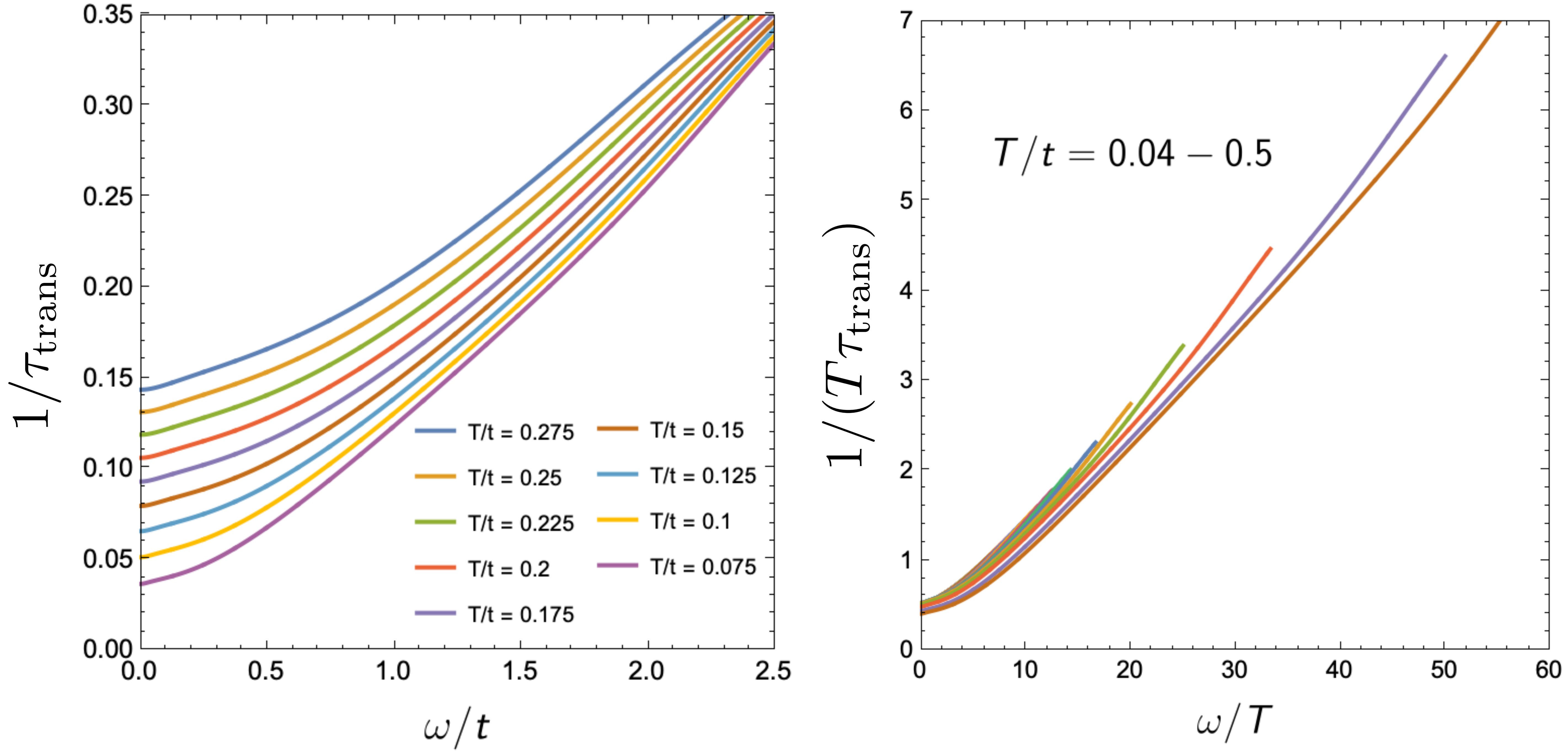


+ all ladders and bubbles…..



$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$

From optical conductivity data of Michon et al. (2023)



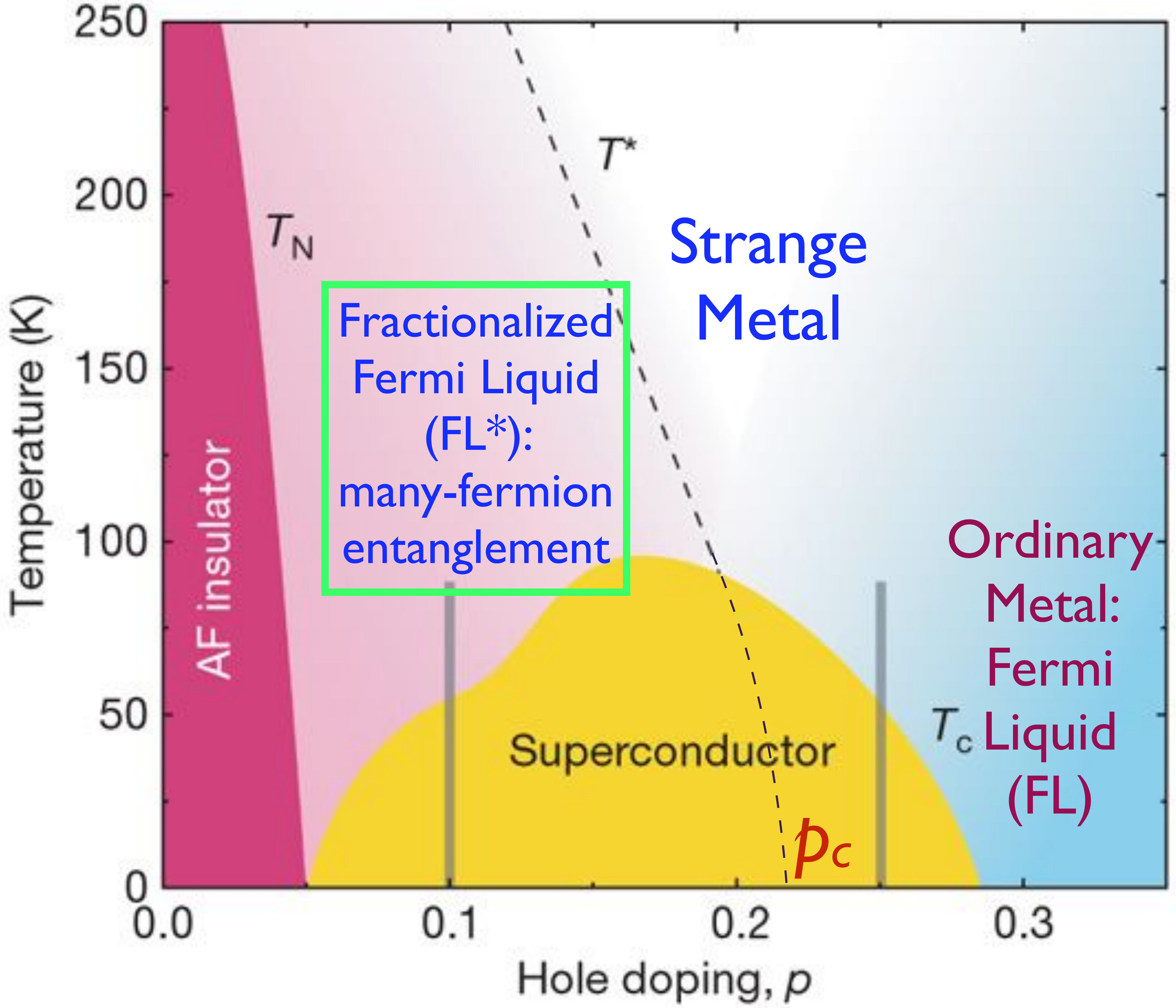
$$\frac{\hbar}{\tau(\omega)} = k_B T\,\Phi_\tau\left(\frac{\hbar\omega}{k_B T}\right)$$

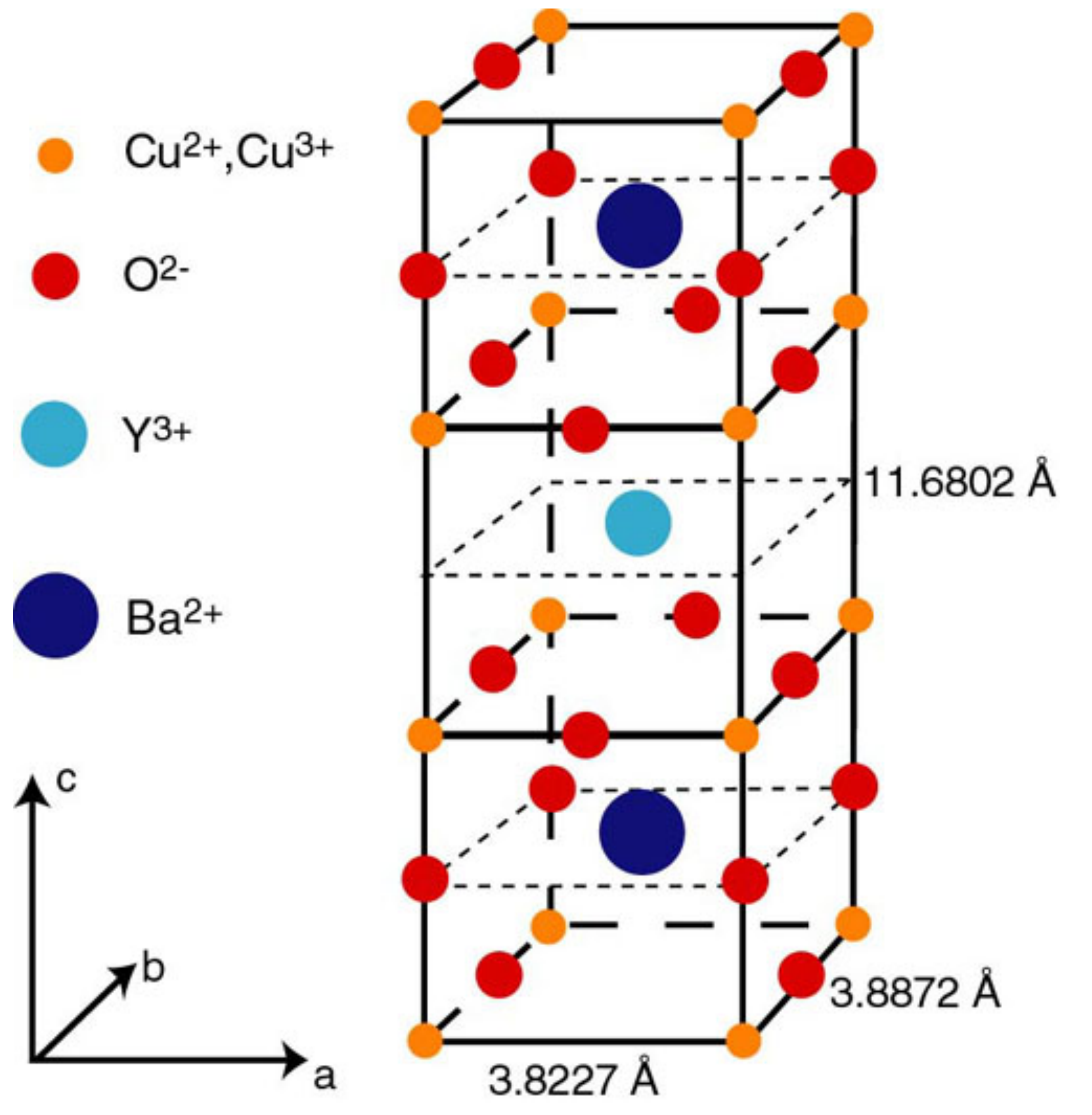
2d-YSYK theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

Fractionalized
Fermi liquid (FL*)
in the hole-doped cuprates

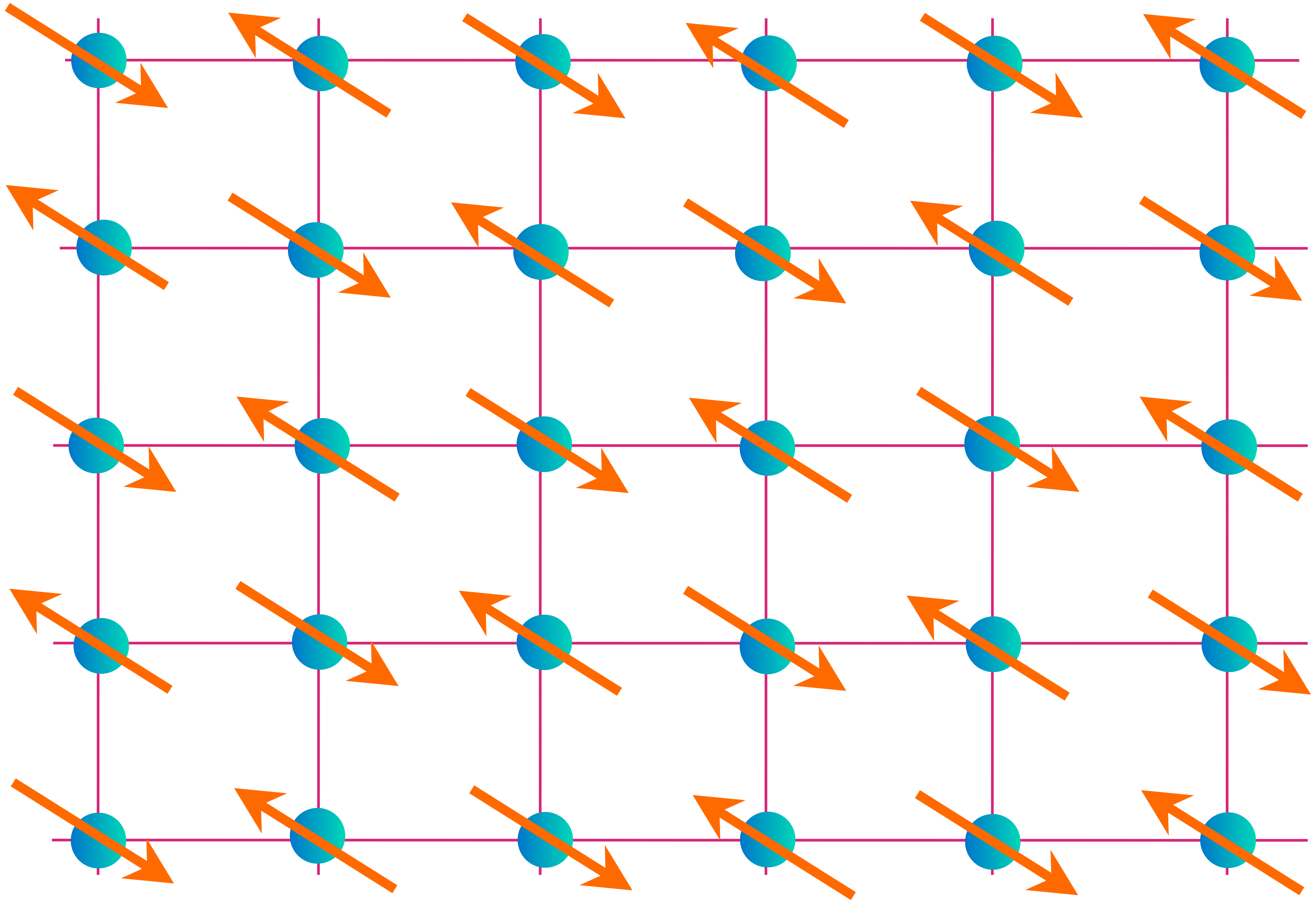




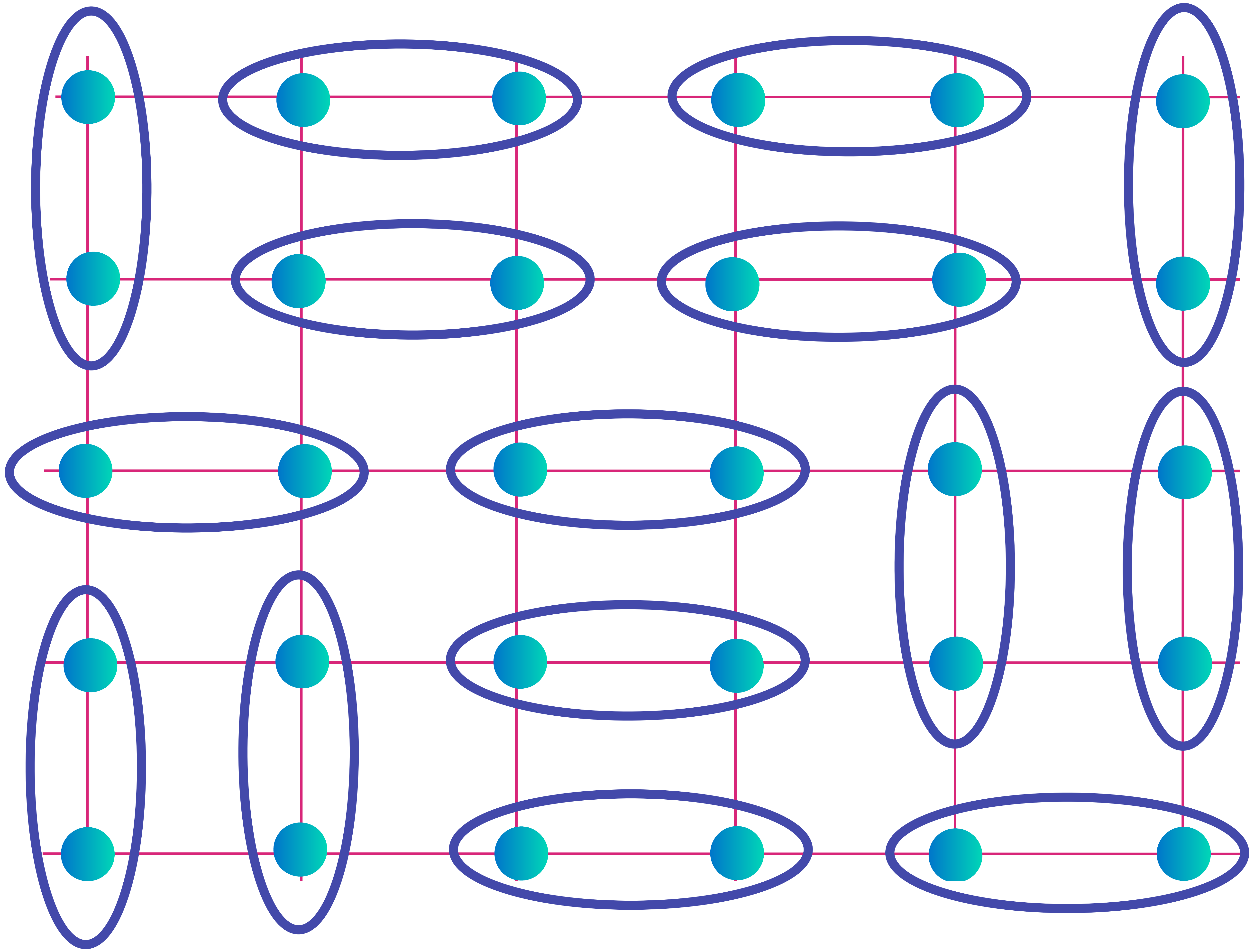
$YBa_2Cu_3O_{6+x}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

<u>Antiferromagnet</u>

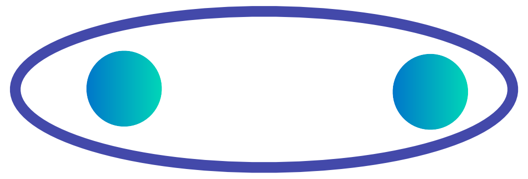


Anderson's Resonating Valence Bond (1972,1987)

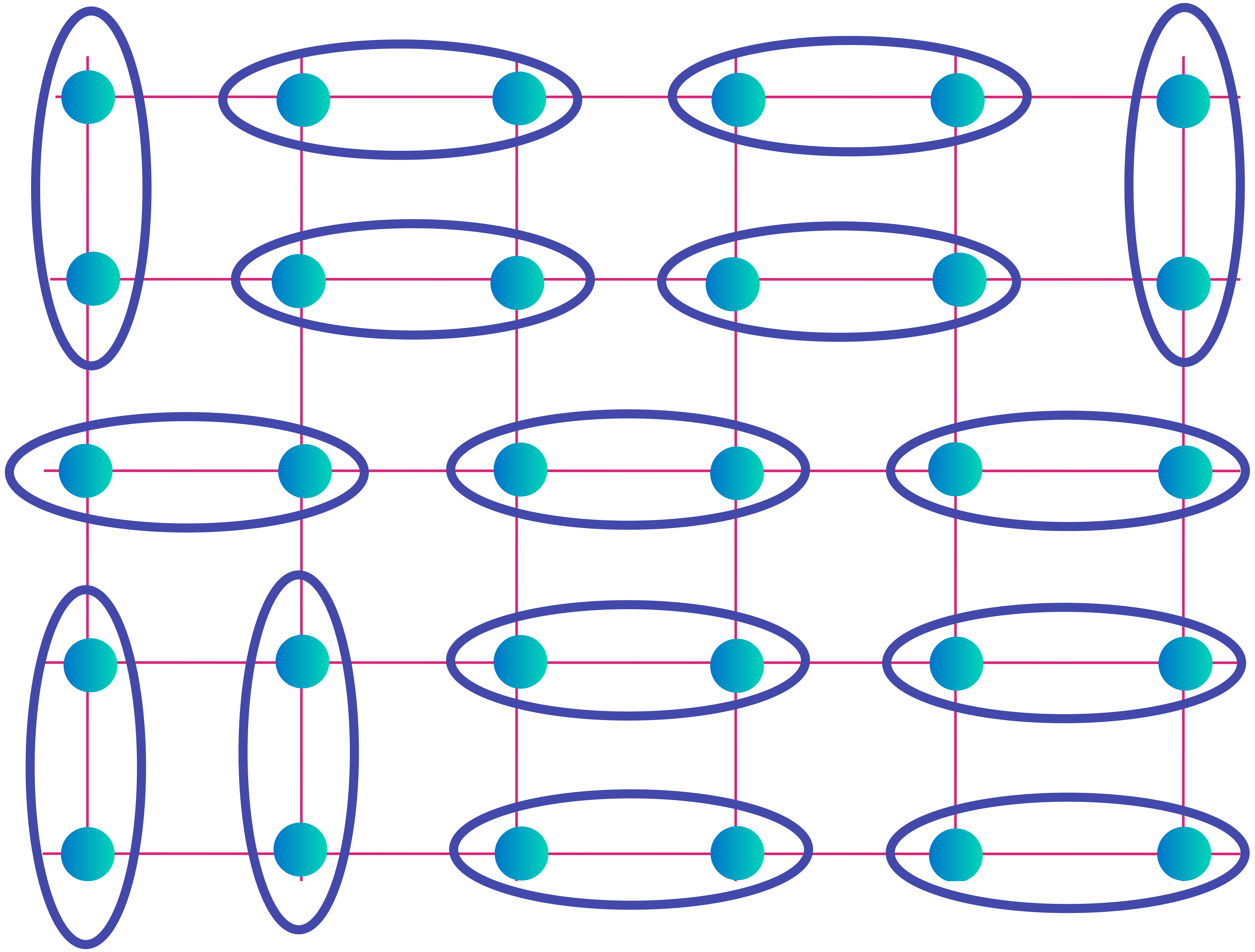


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)



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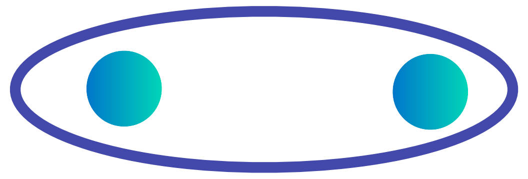 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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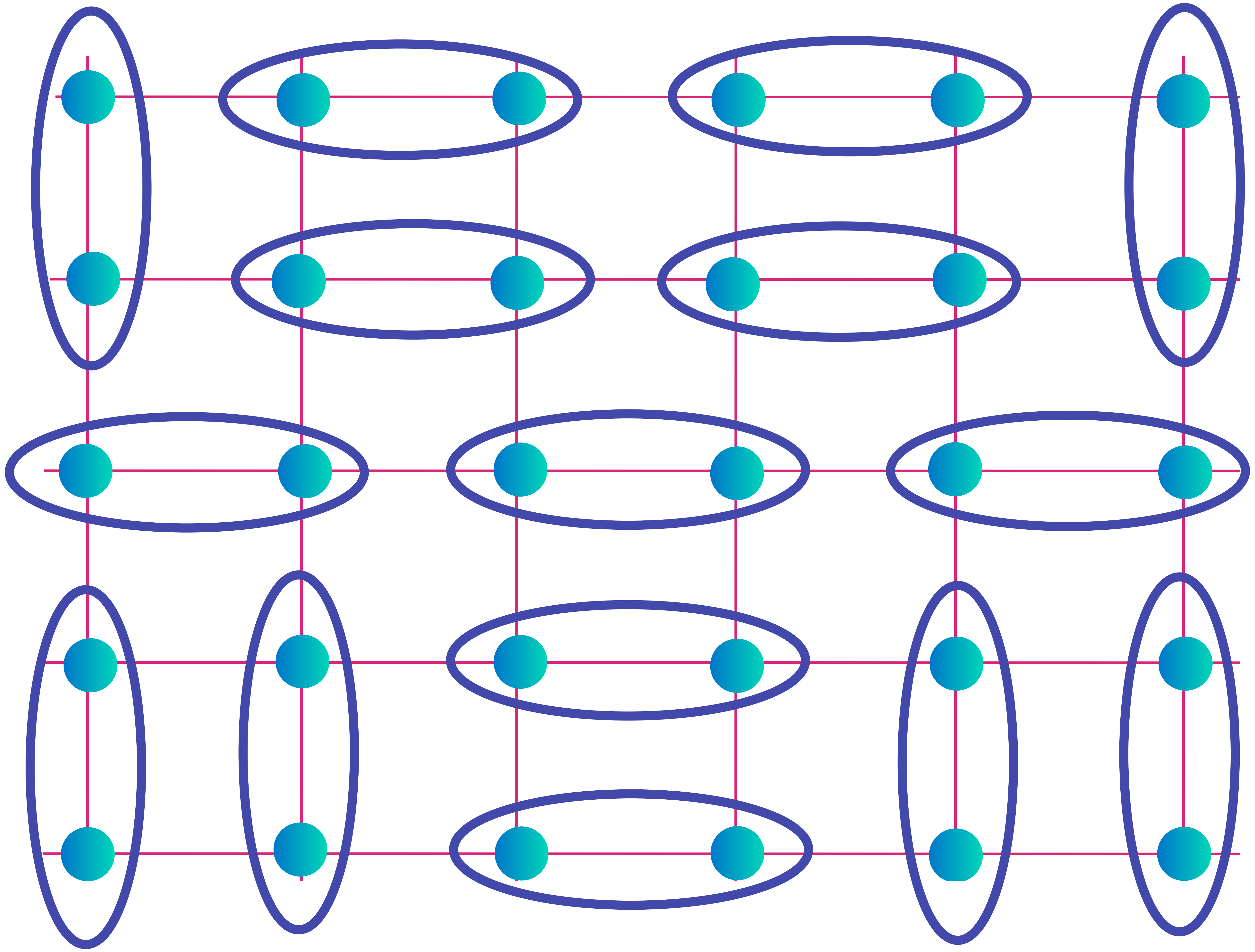

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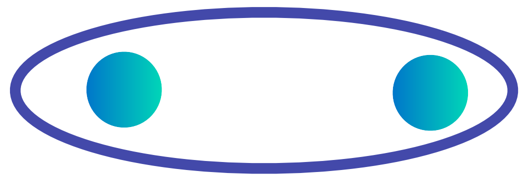 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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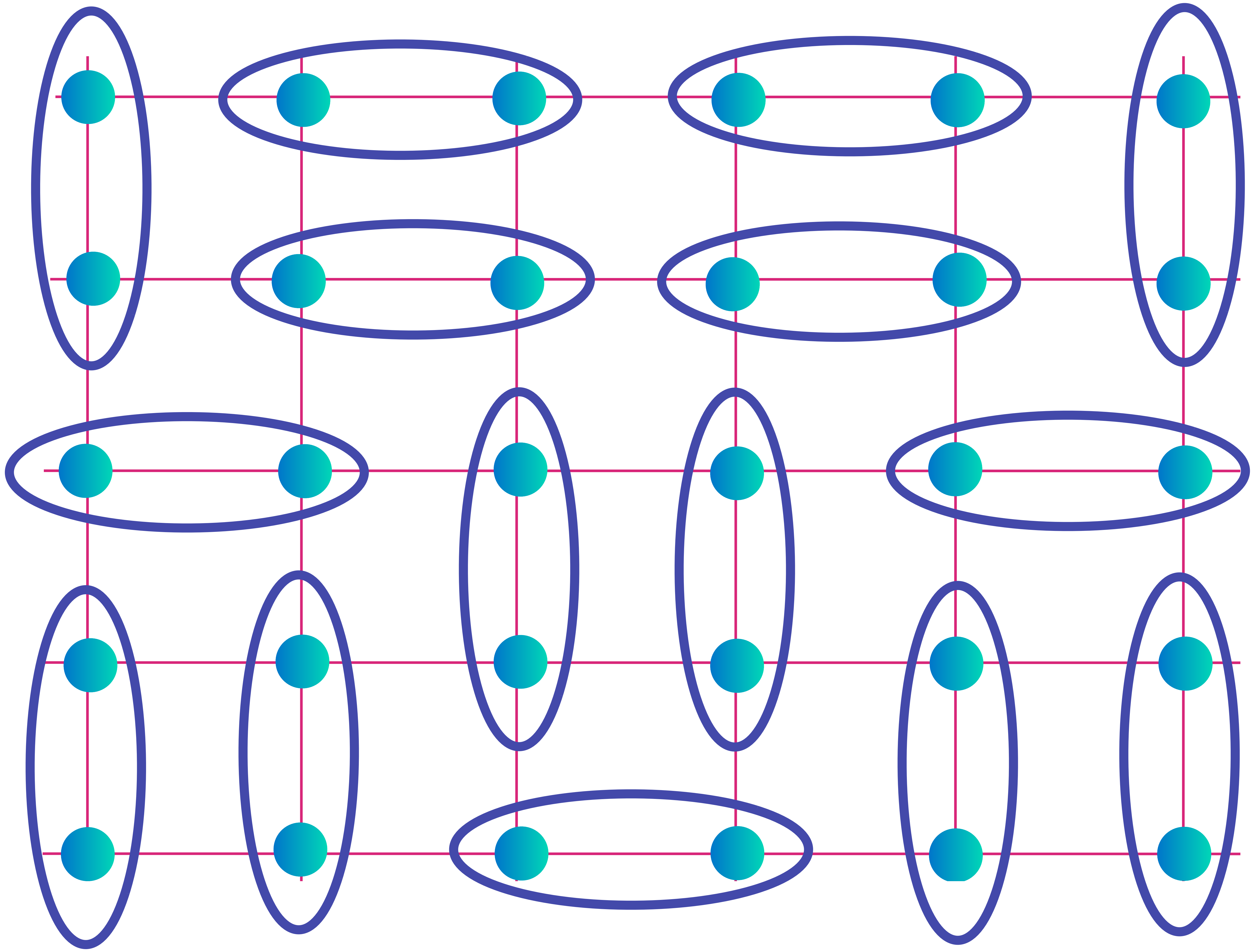

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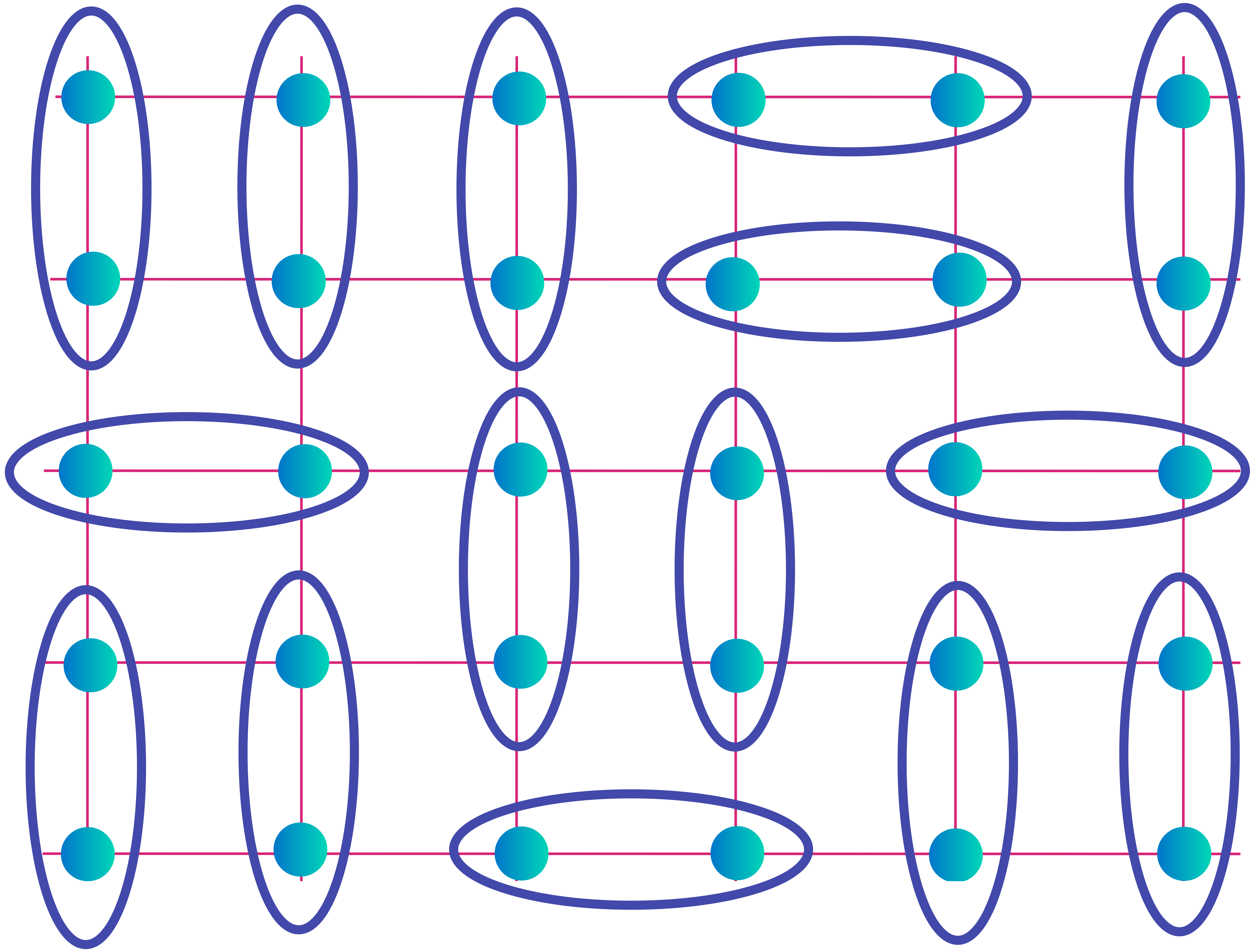

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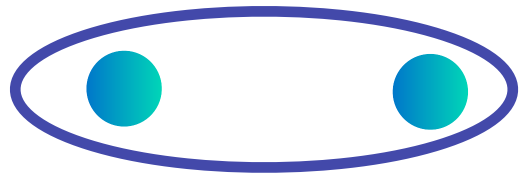 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

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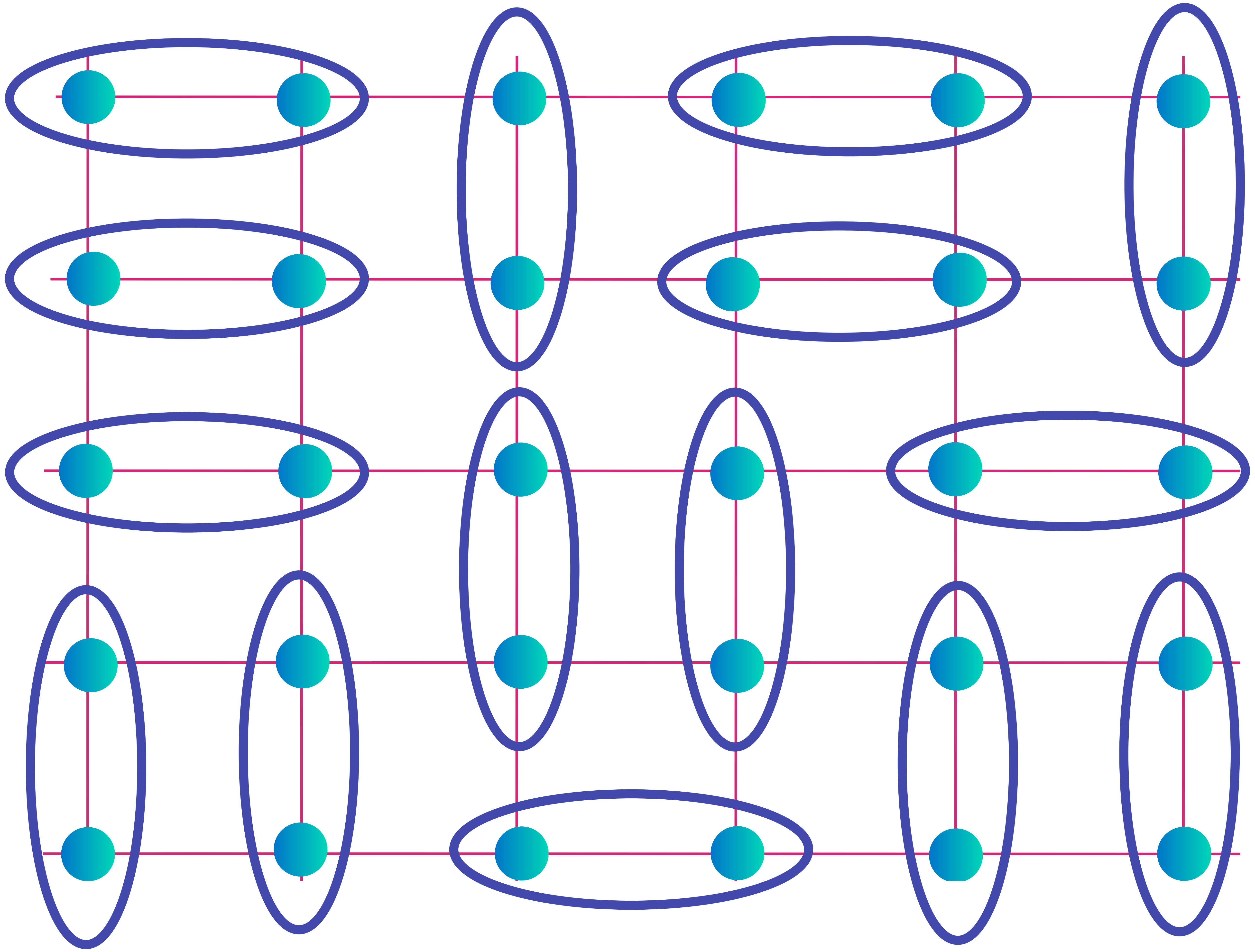

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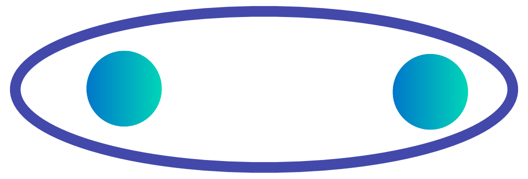

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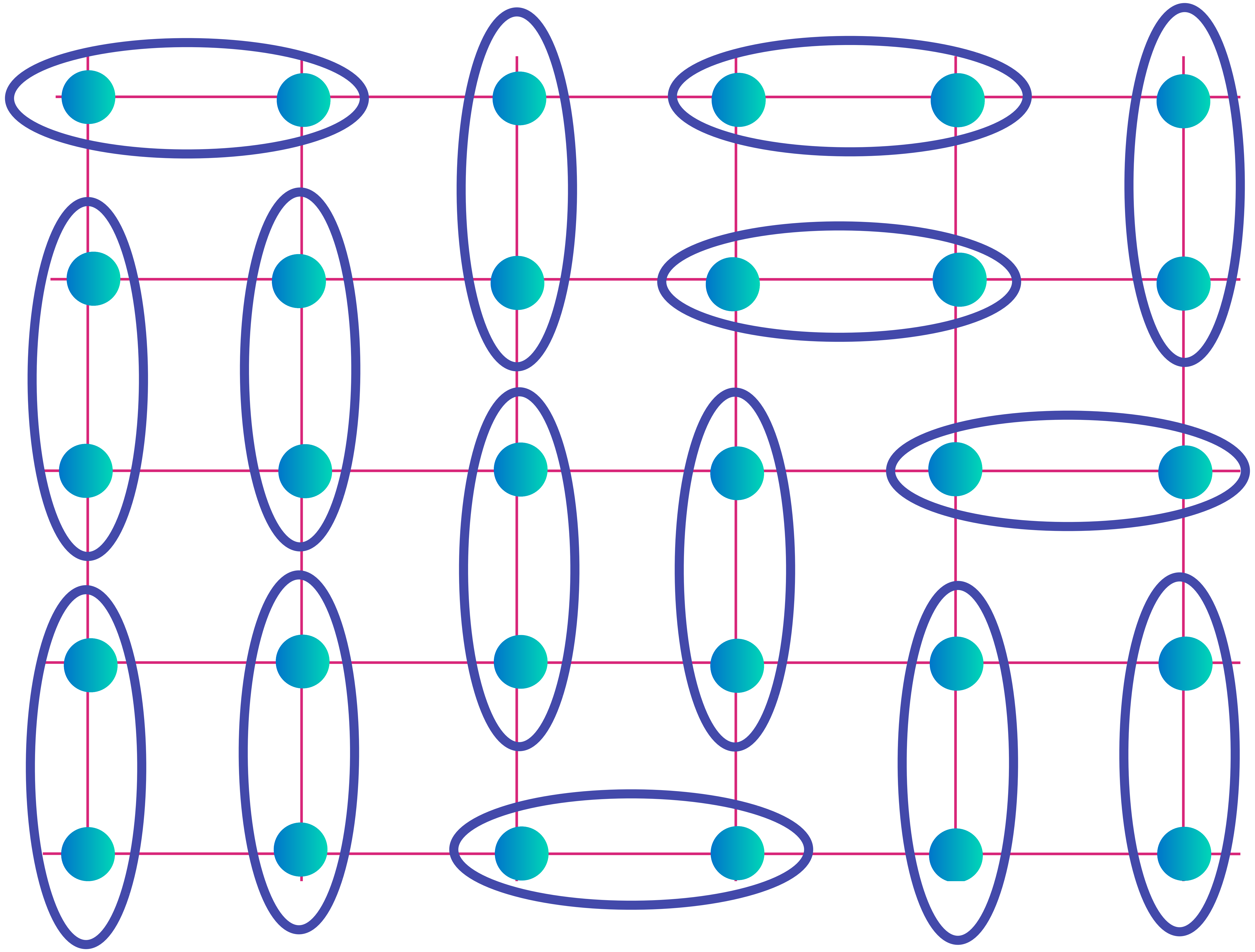

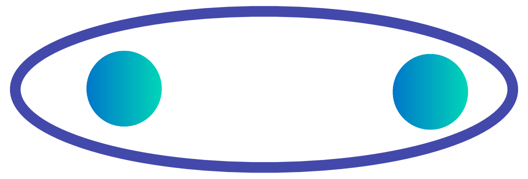

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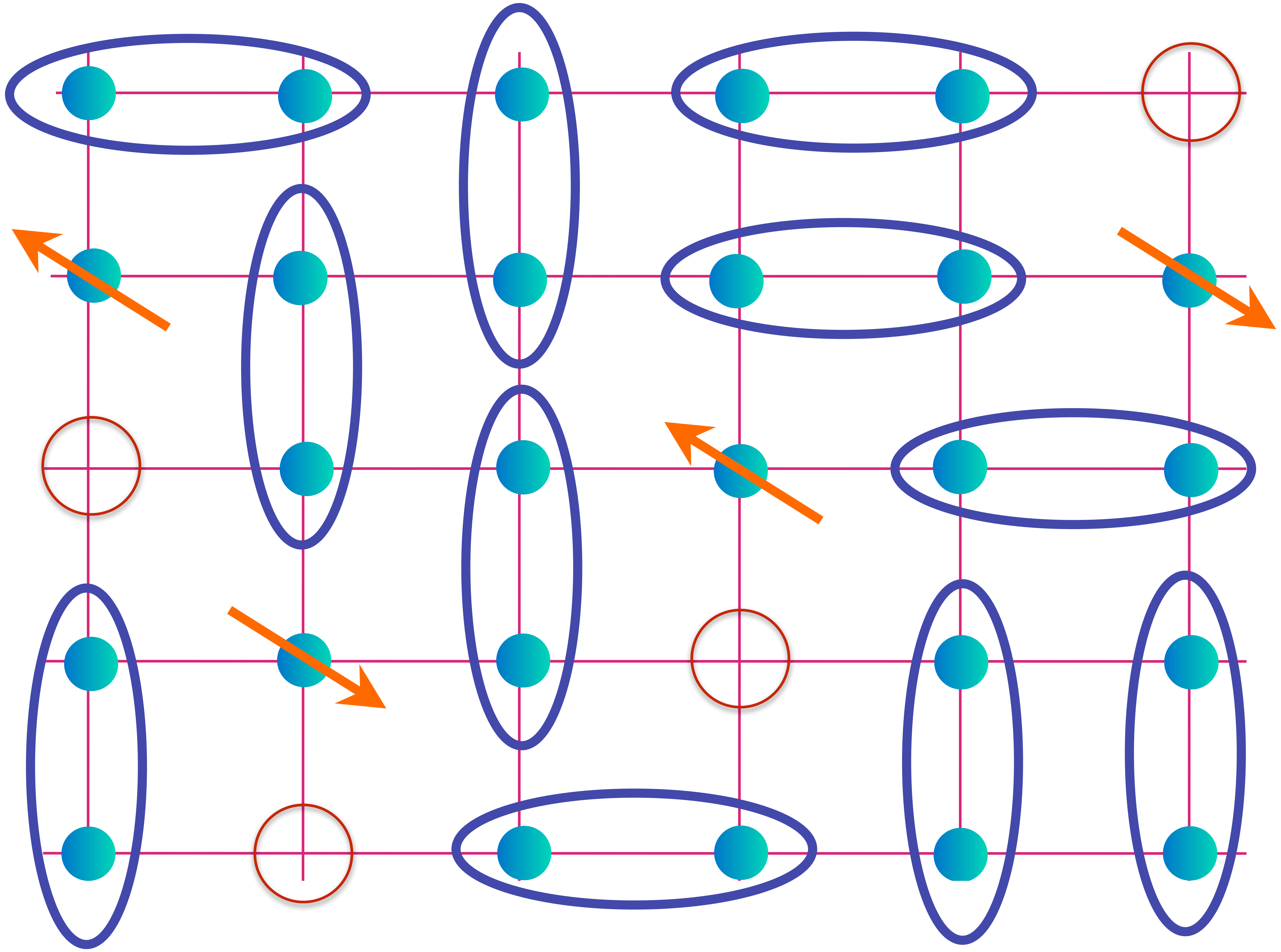
$\mathcal{D} \rightarrow$ dimer covering of lattice

Quantum Entanglement of an infinite number of spins (bosons)!

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



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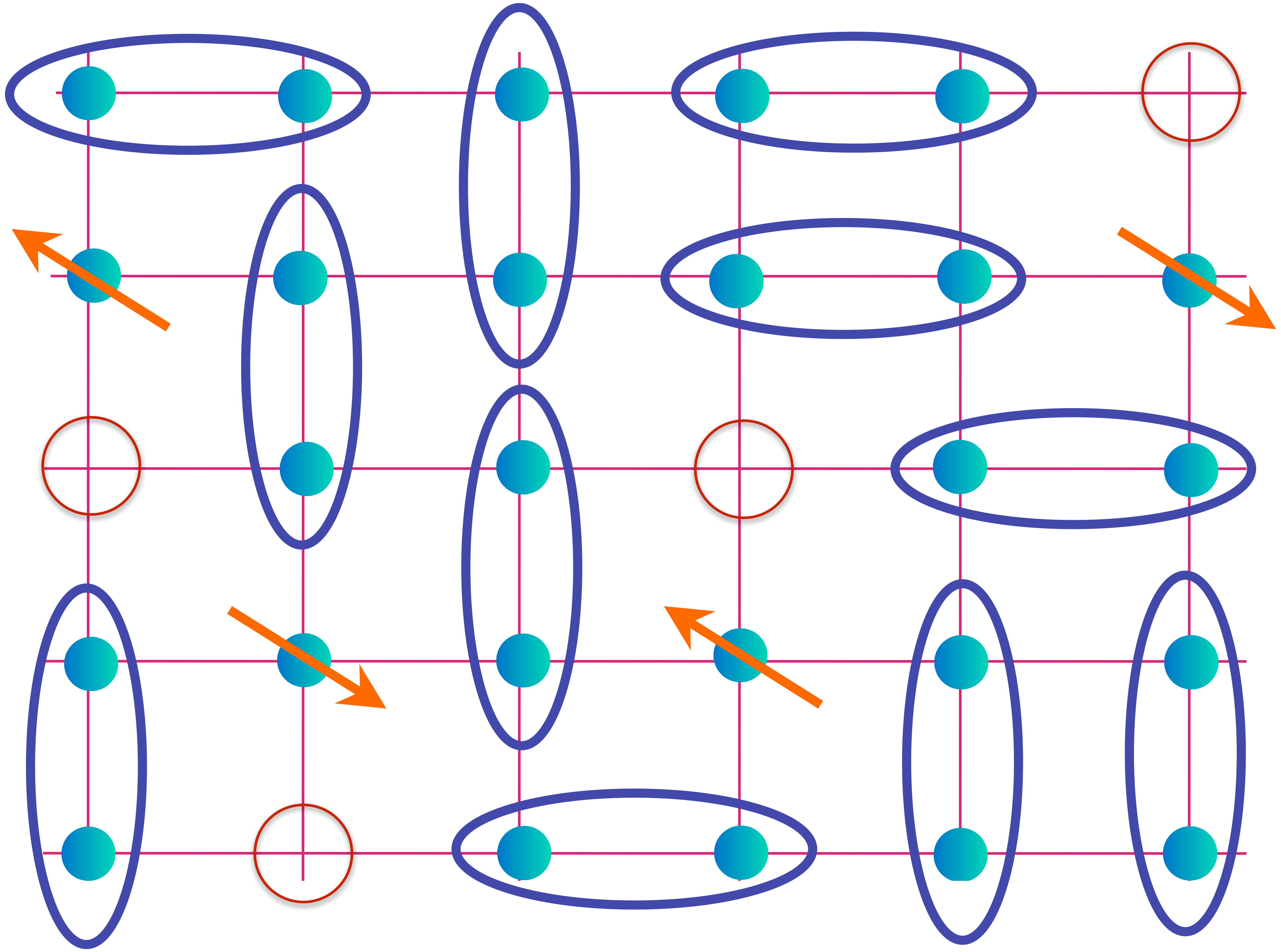
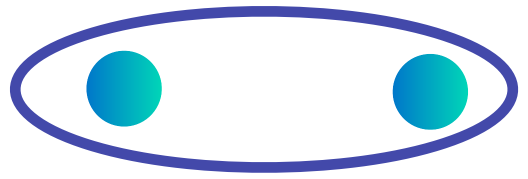
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

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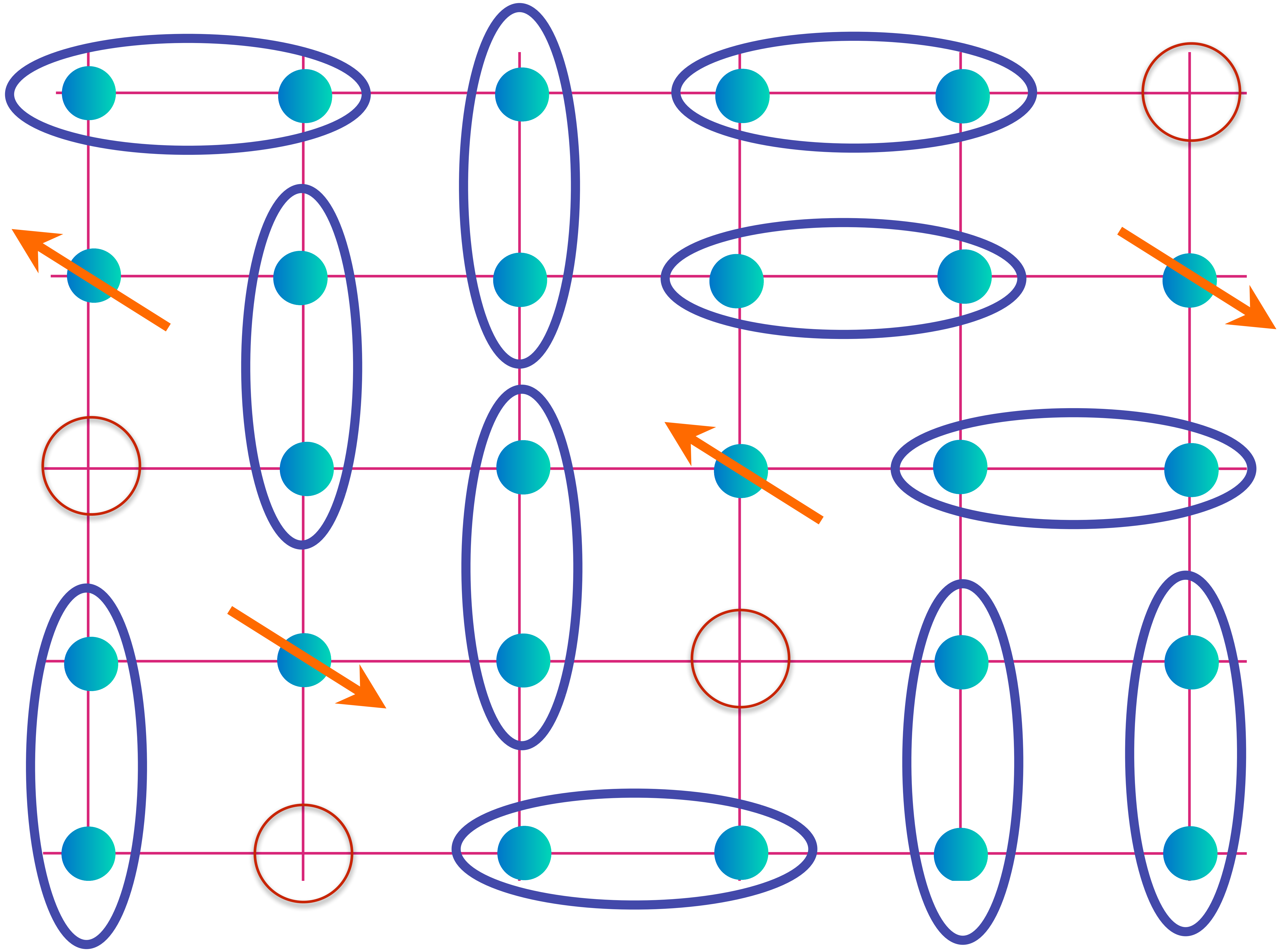

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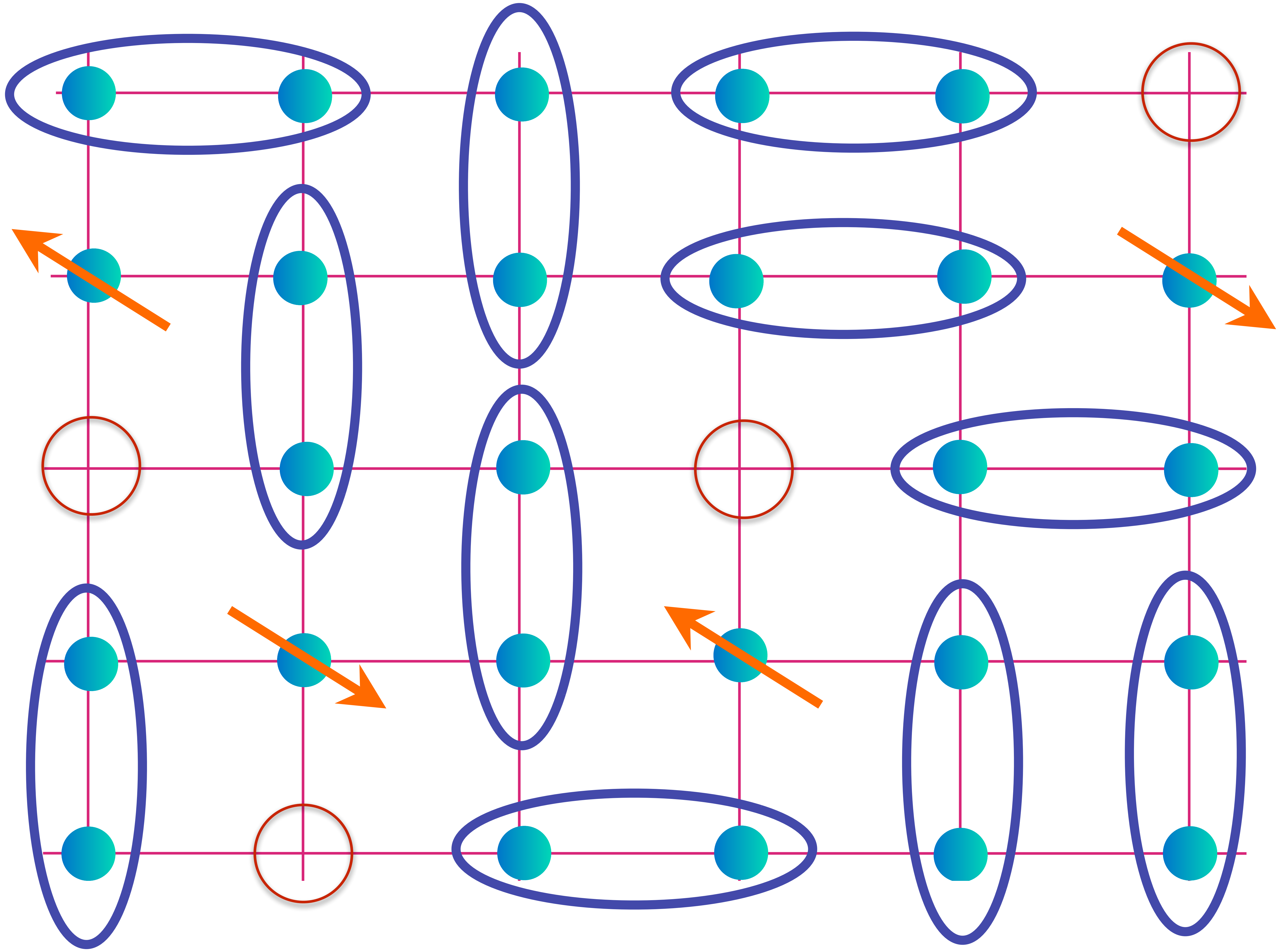

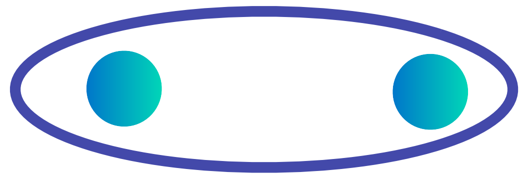 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

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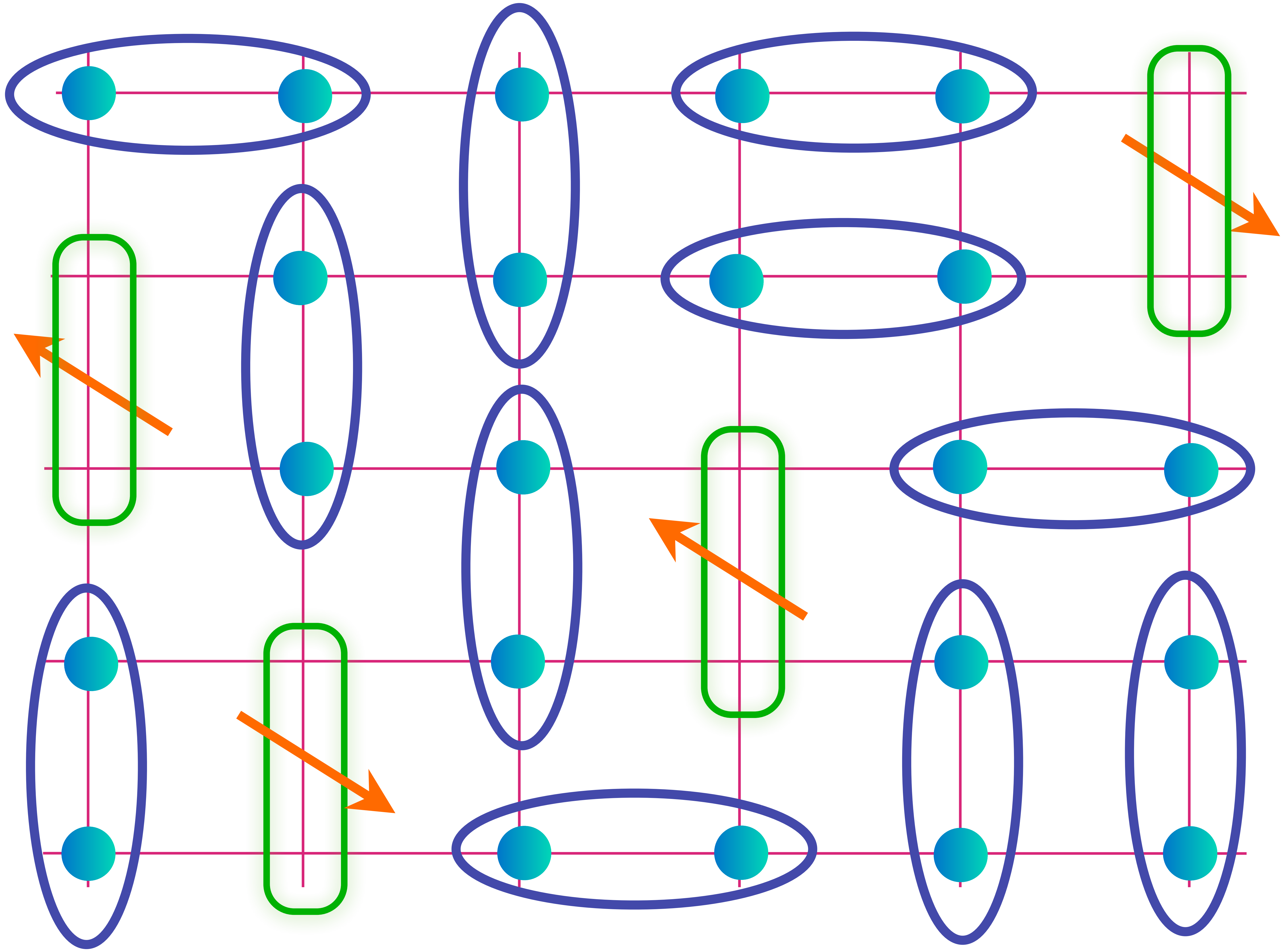
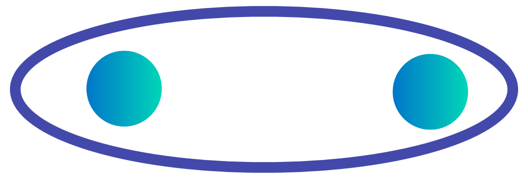
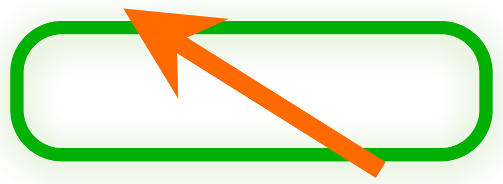


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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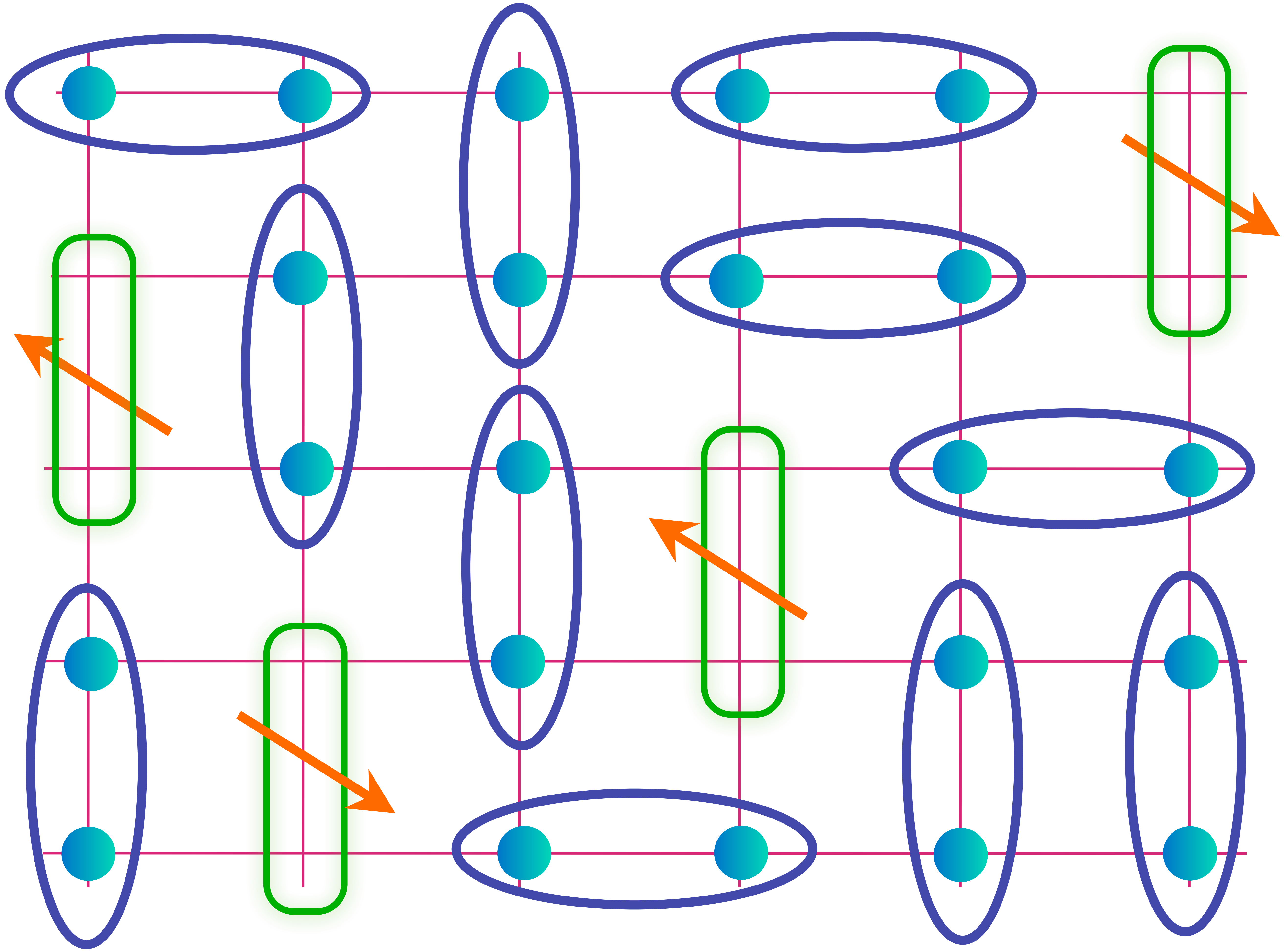
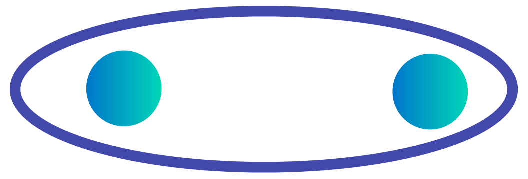
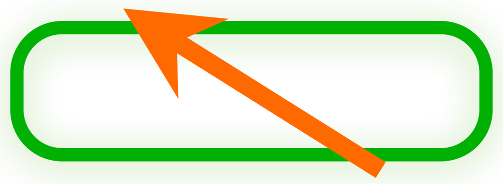


FL*:
Fermi gas of holon-spinon bound states

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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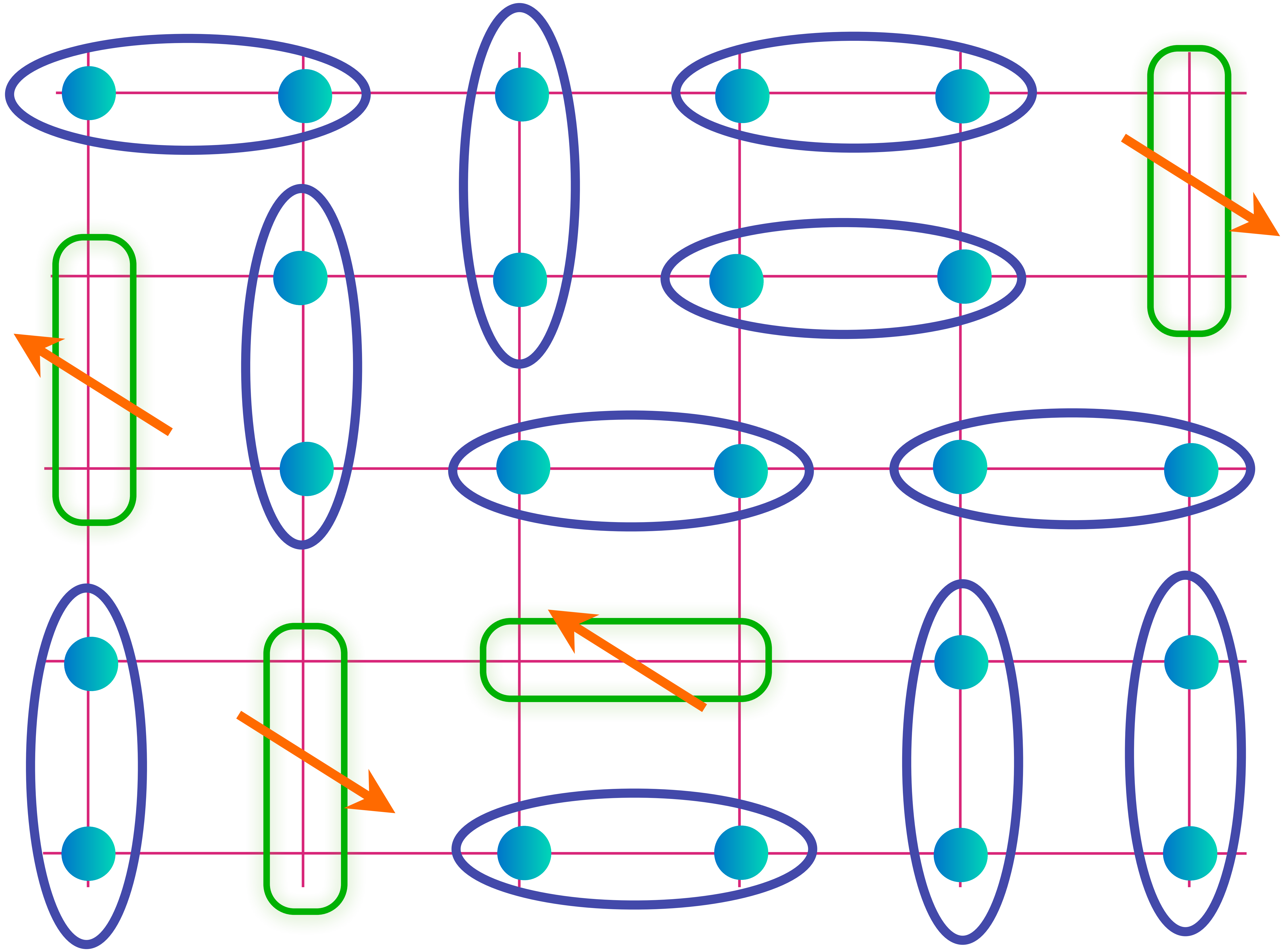

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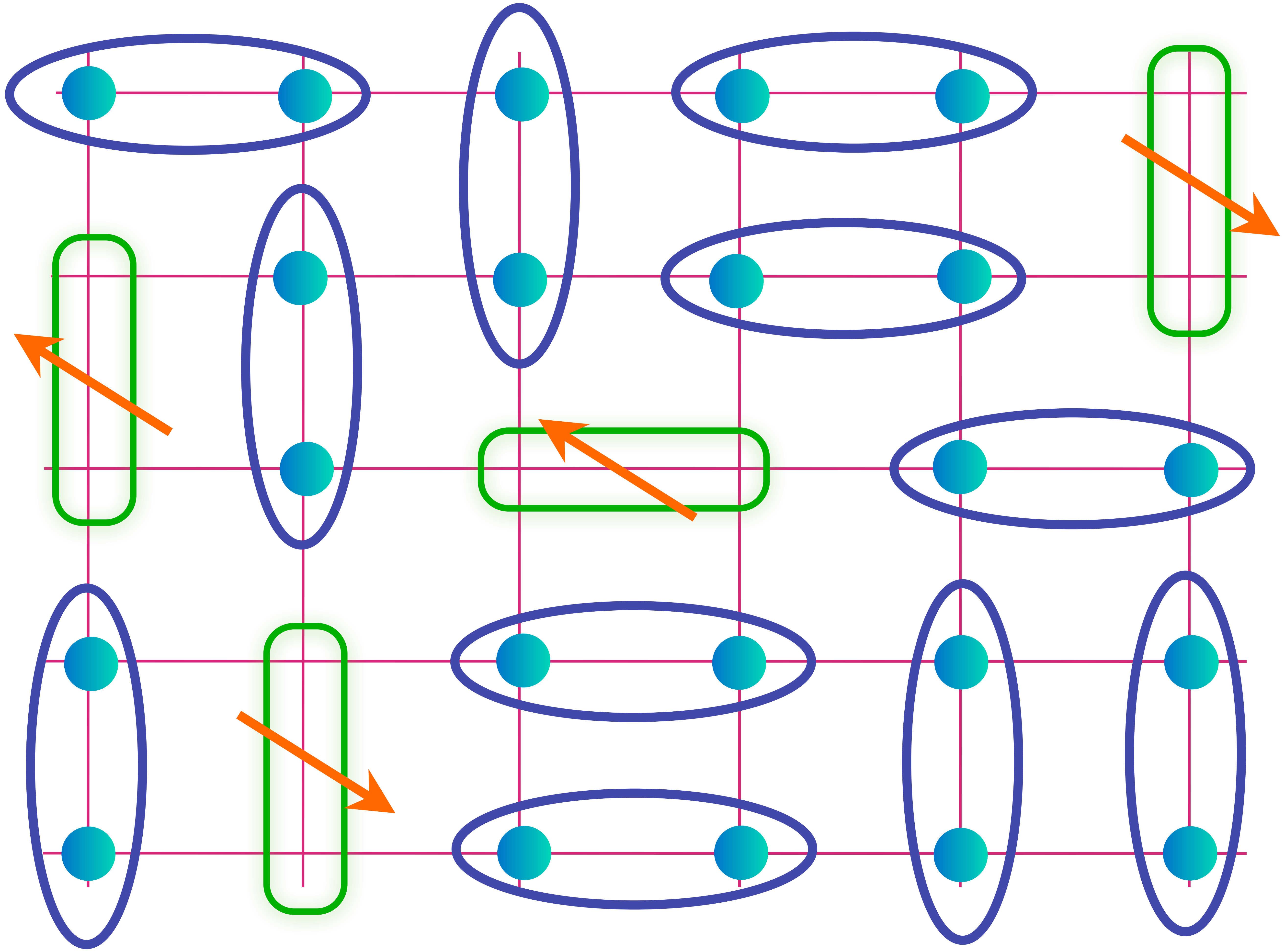

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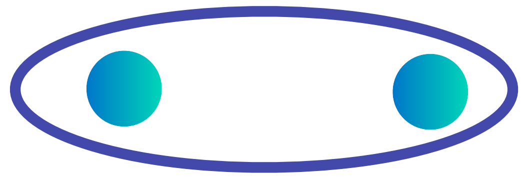
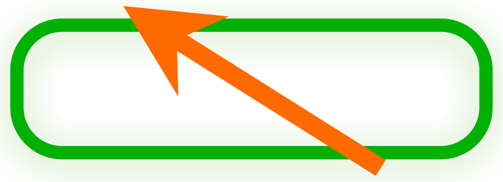
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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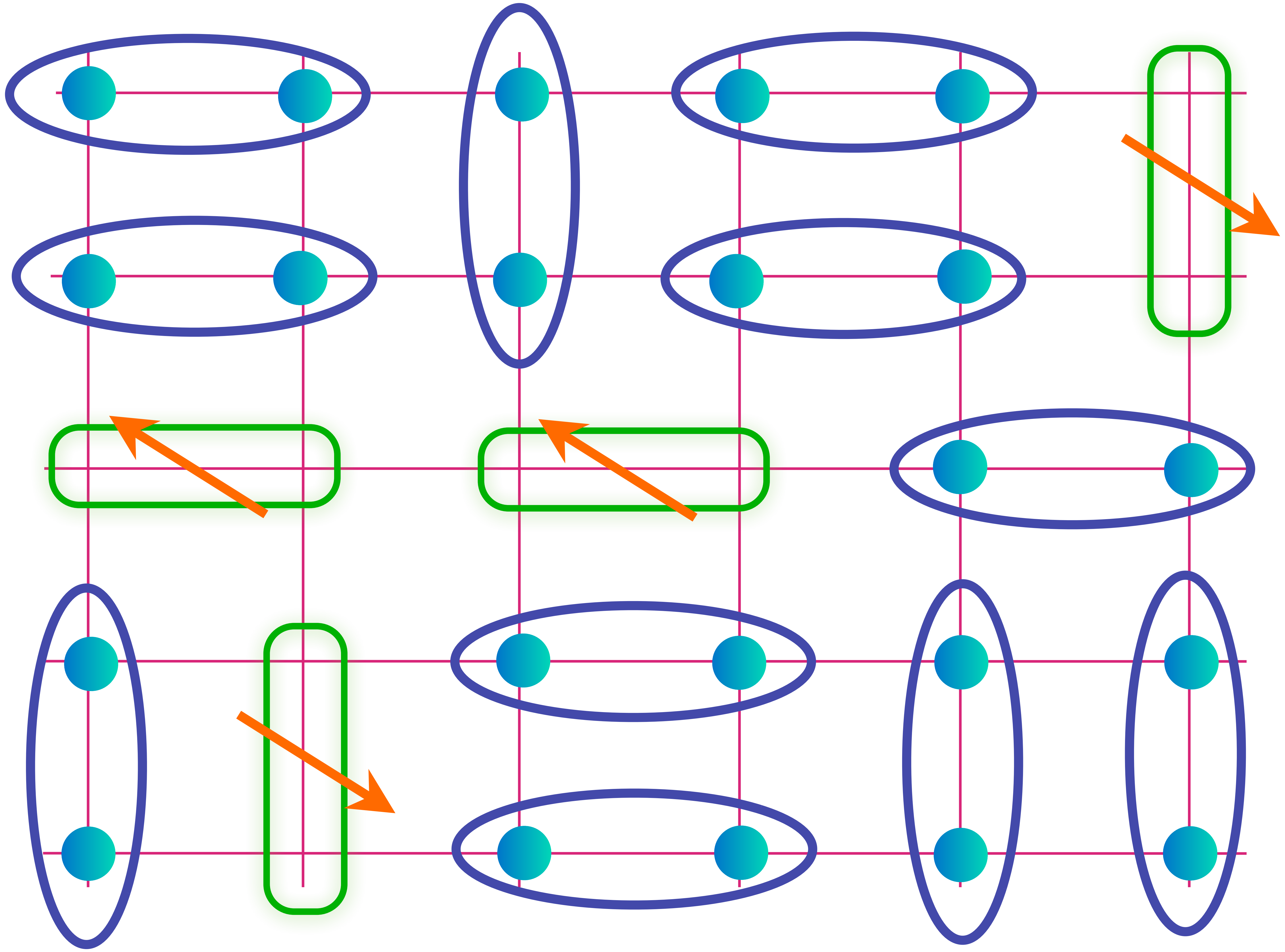

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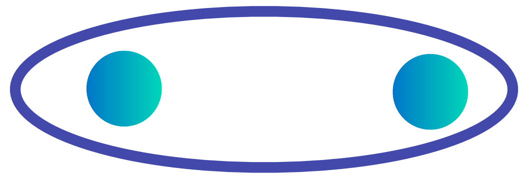
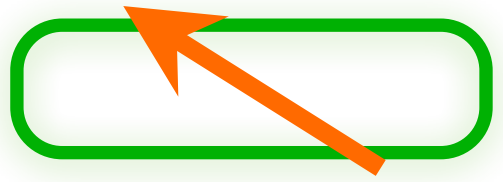
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$  $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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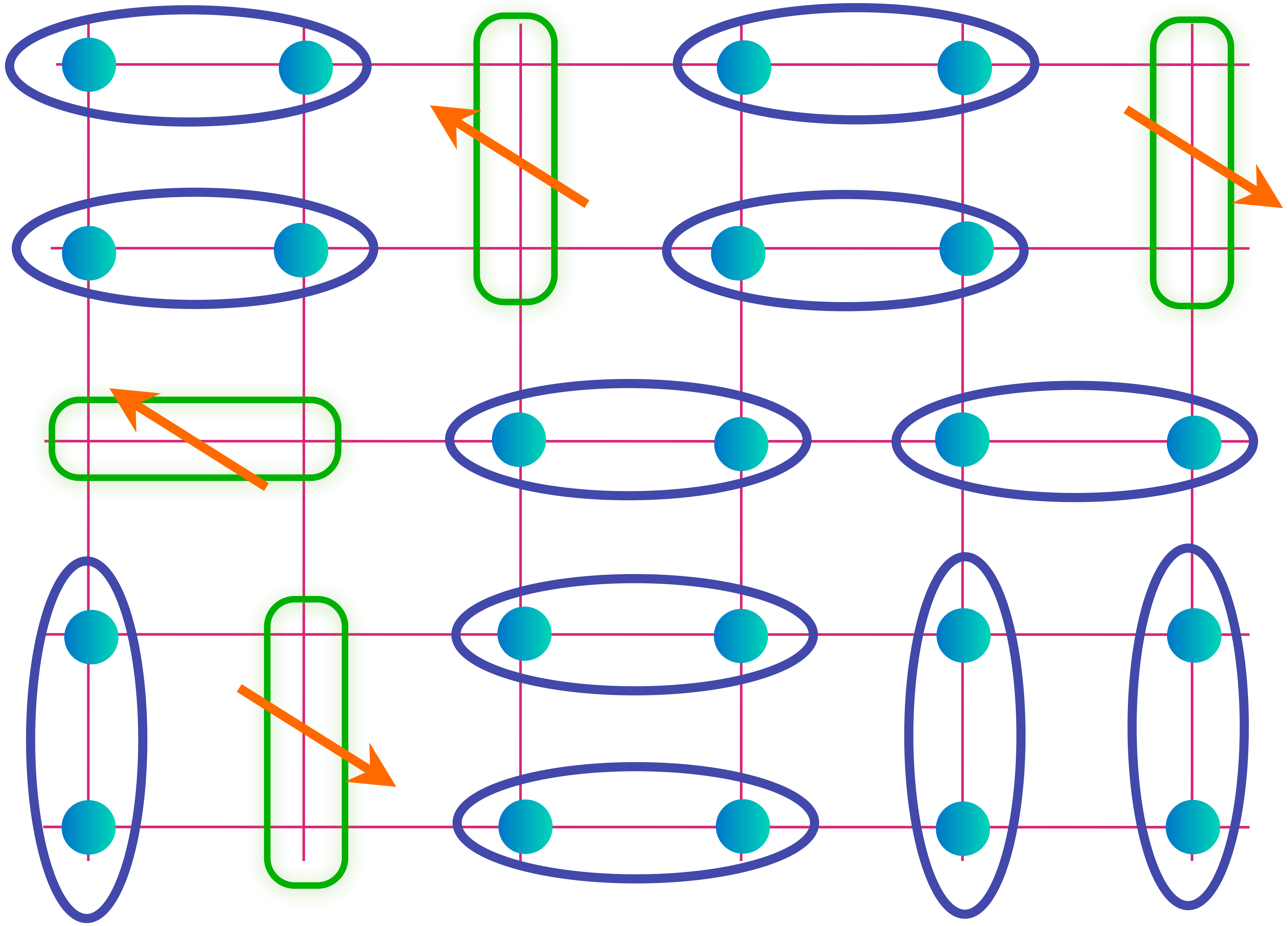

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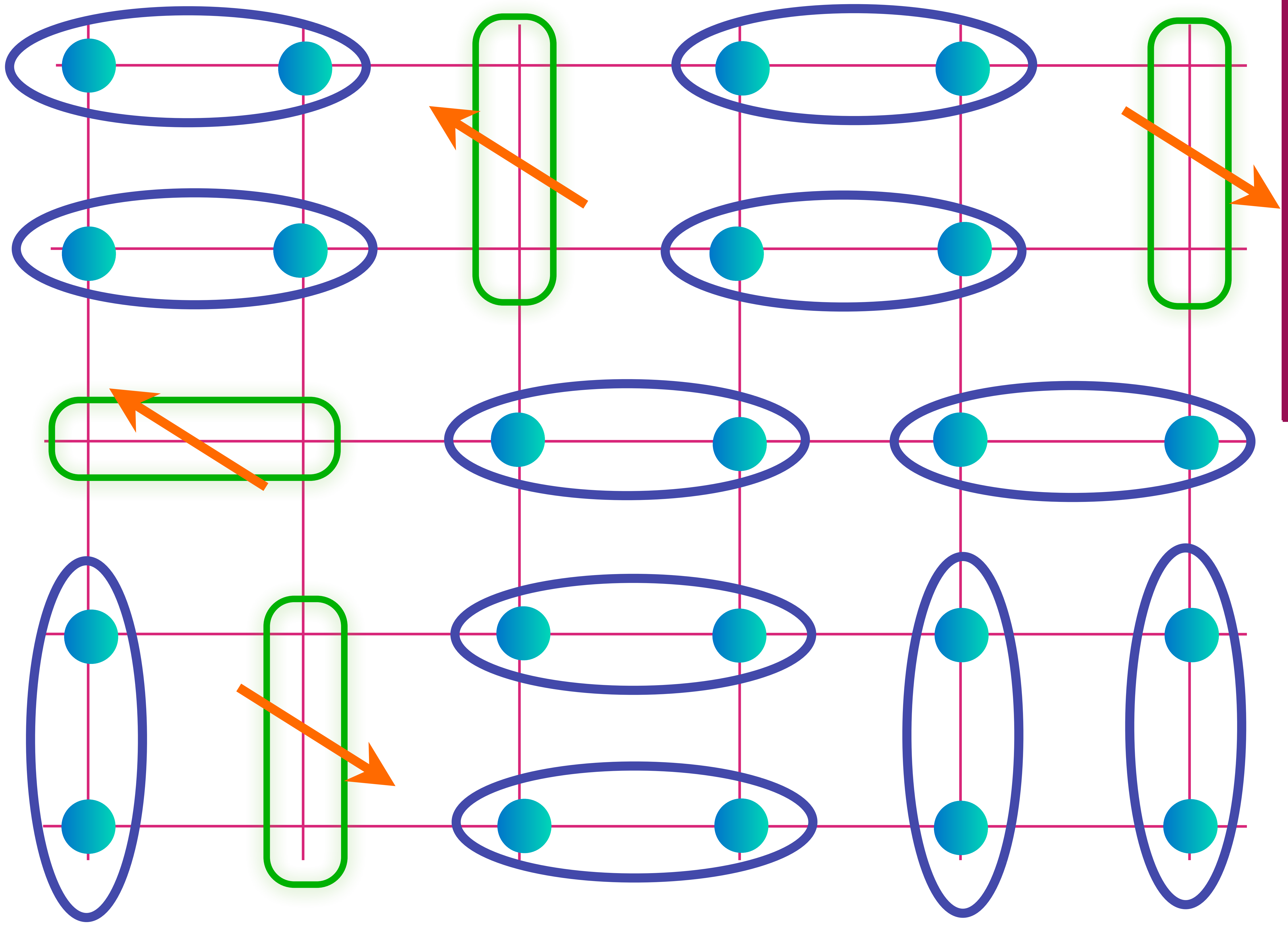
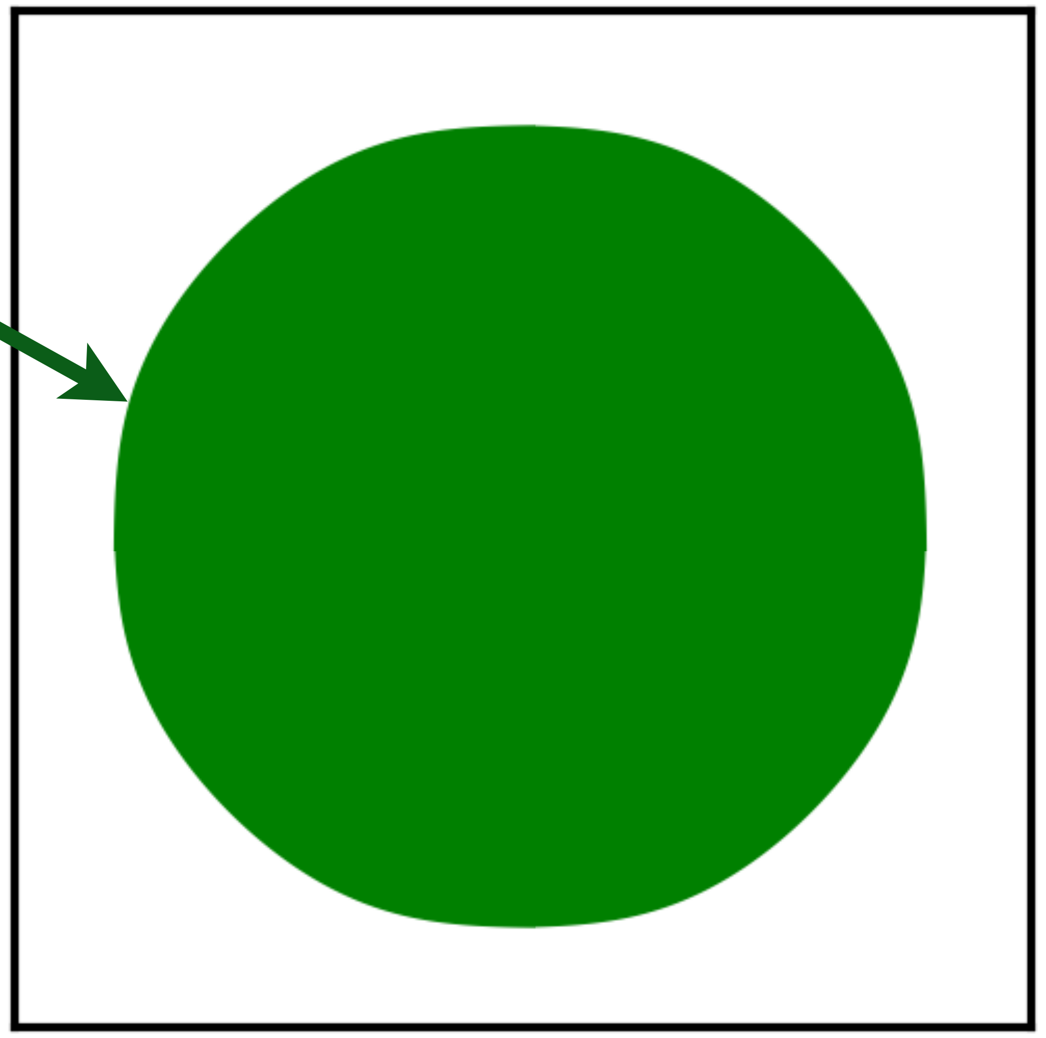


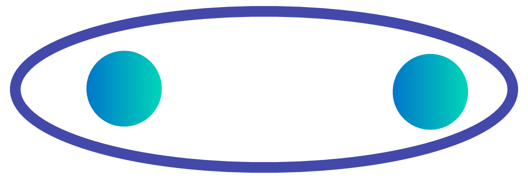
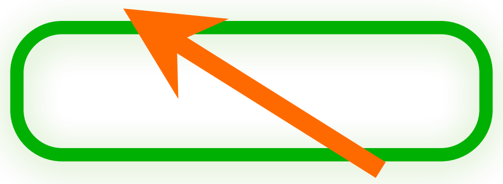
Luttinger area $(1+p)/2$



Count *all* electrons $= 1 - p$. Holes in a filled band $= 1 + p$.

FL

FL*: Fermi gas of holon-spinon bound states

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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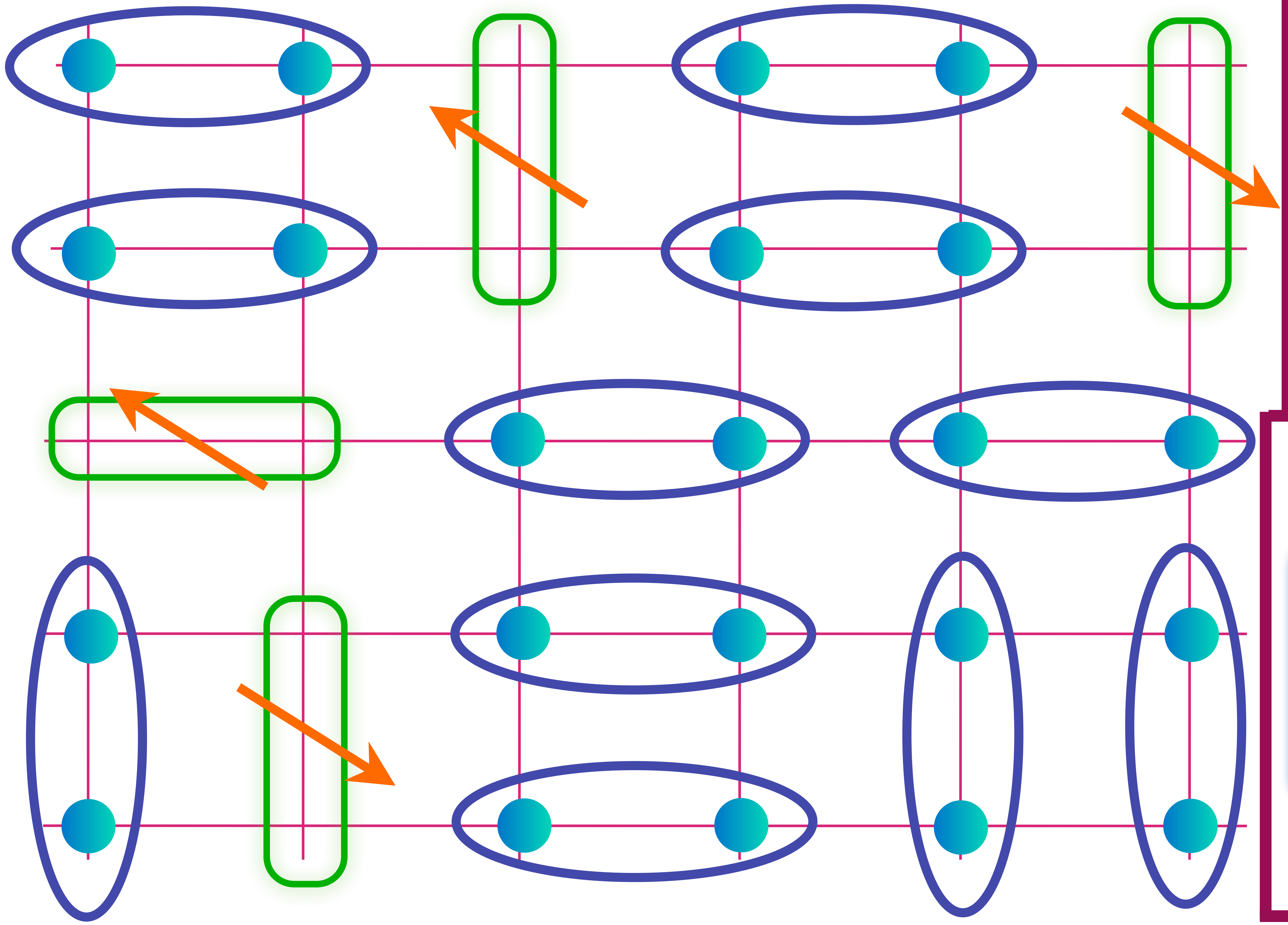

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

Luttinger area $(1+p)/2$

Count *all* electrons $= 1 - p$.

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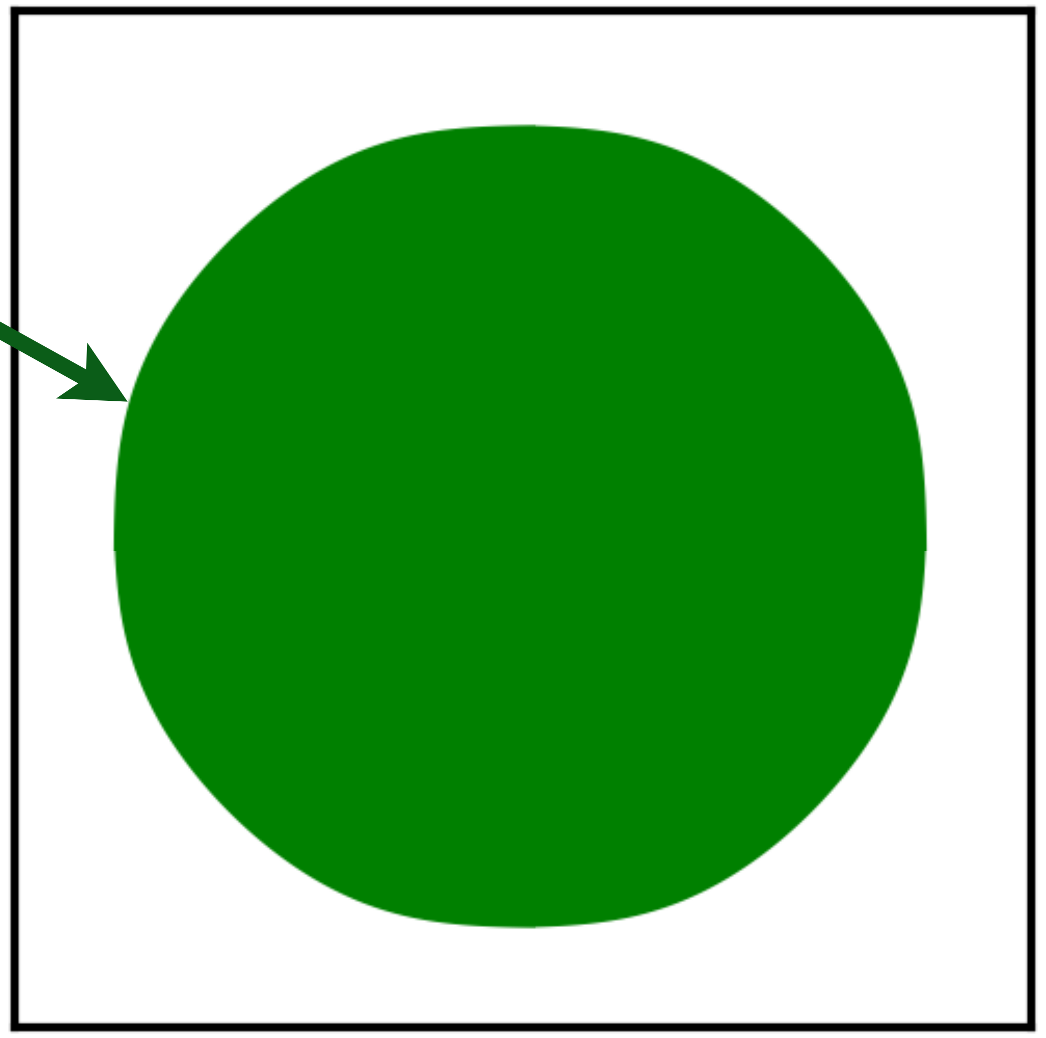
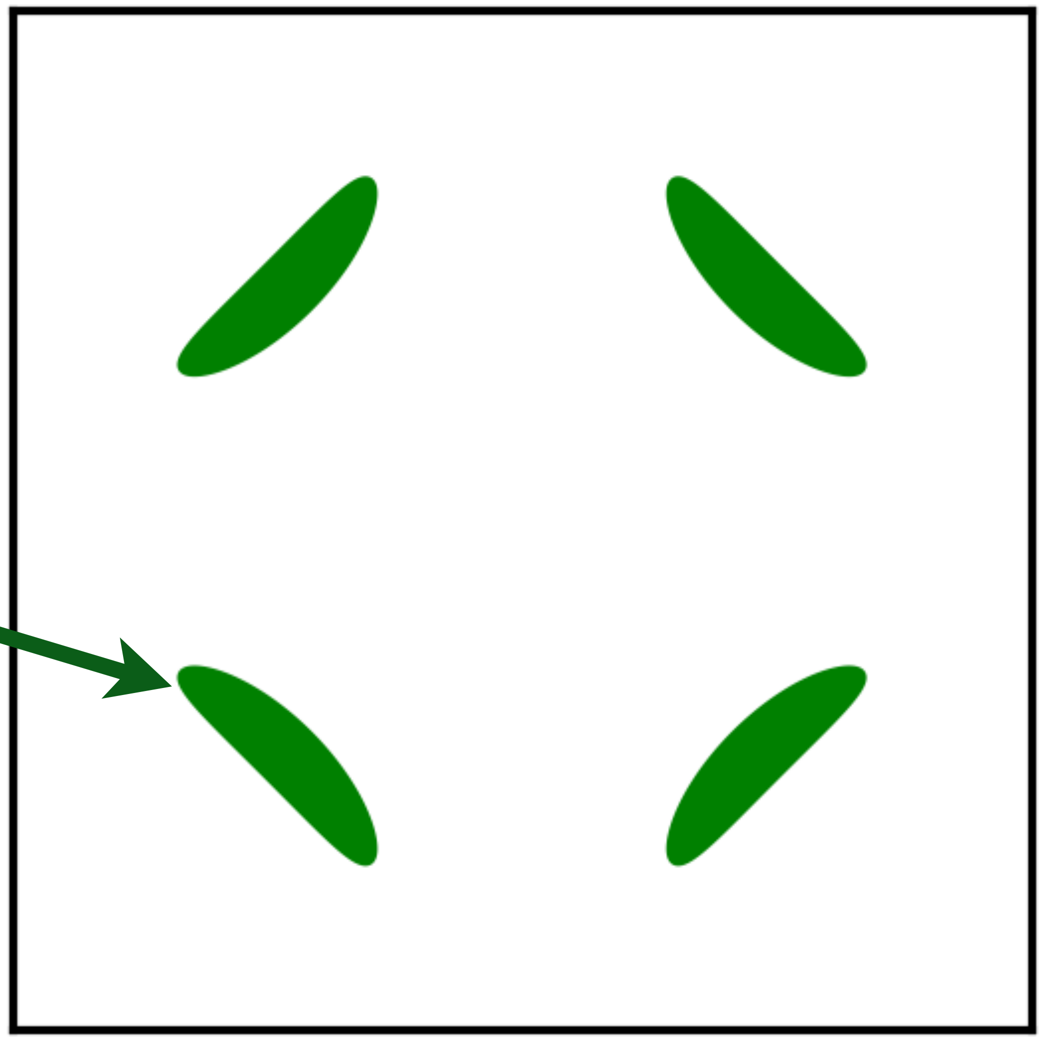


FL

Non-Luttinger area $p/8$



FL*

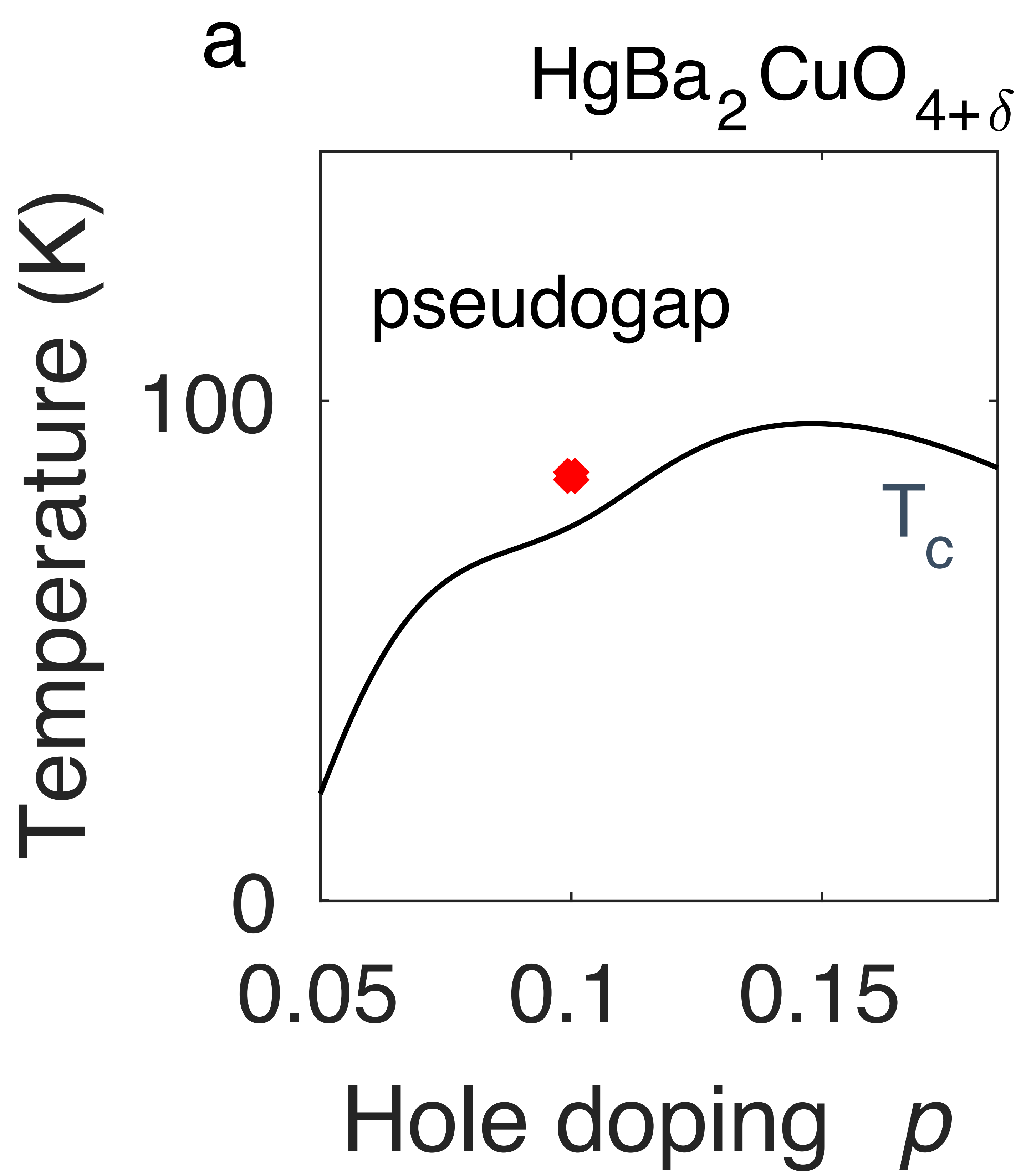
Count only green dimers

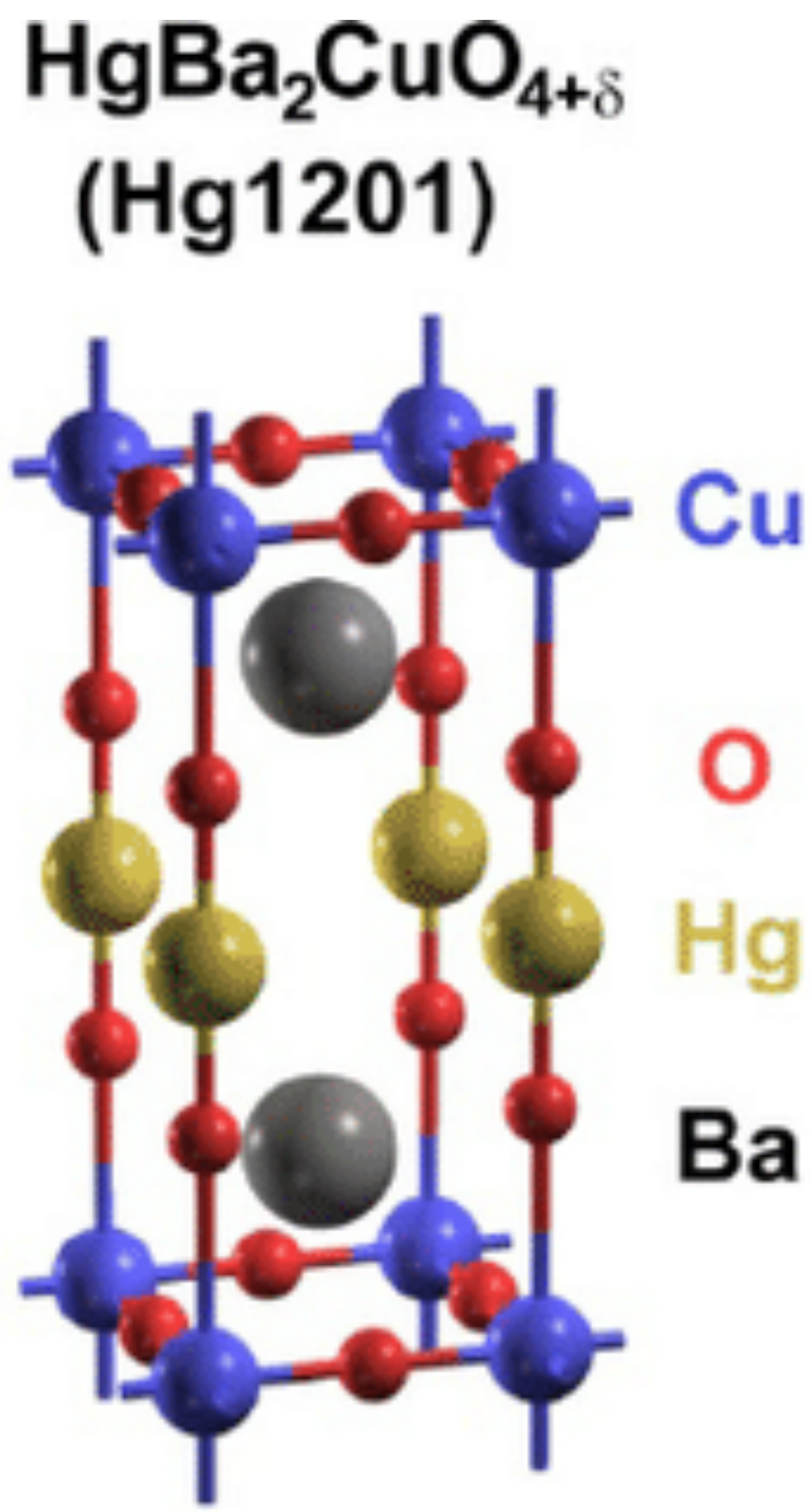
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

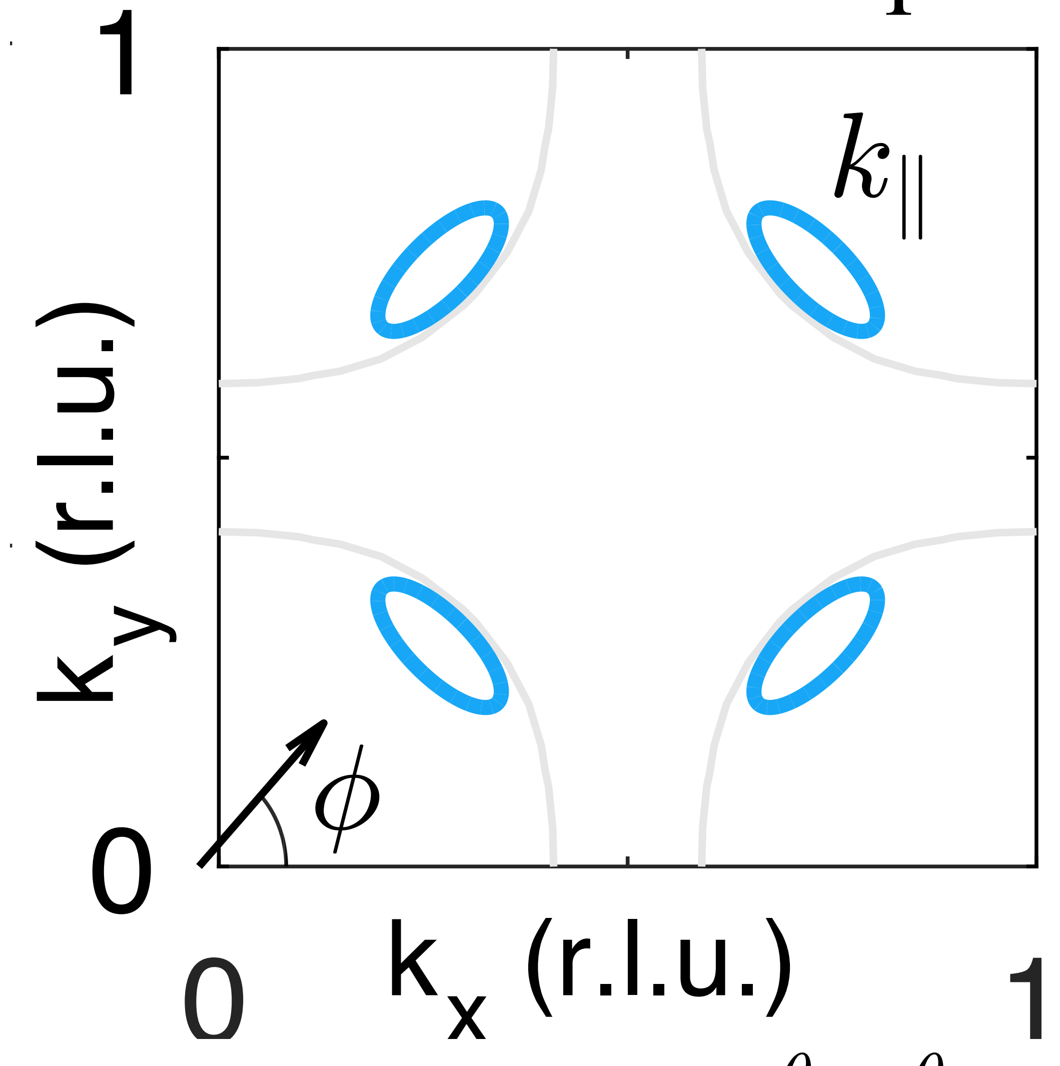
Observation of the Yamaji effect in a cuprate superconductor

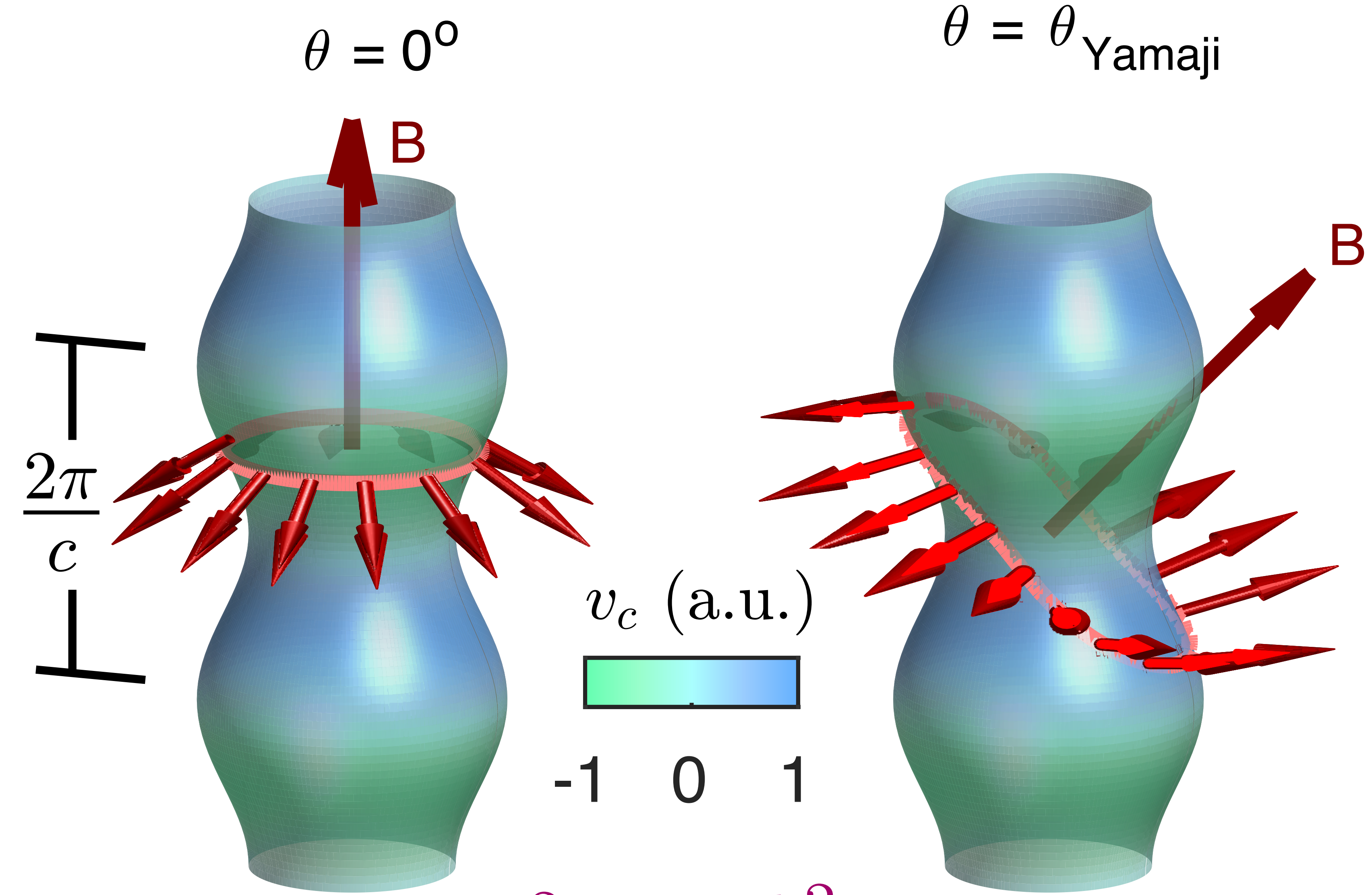
Mun K. Chan[1] ✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025









$$\epsilon(\mathbf{k}) = \frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2} - 2t_\perp \cos(k_z c)$$

At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K. Yamaji JPSJ **58**, 1520 (1989)

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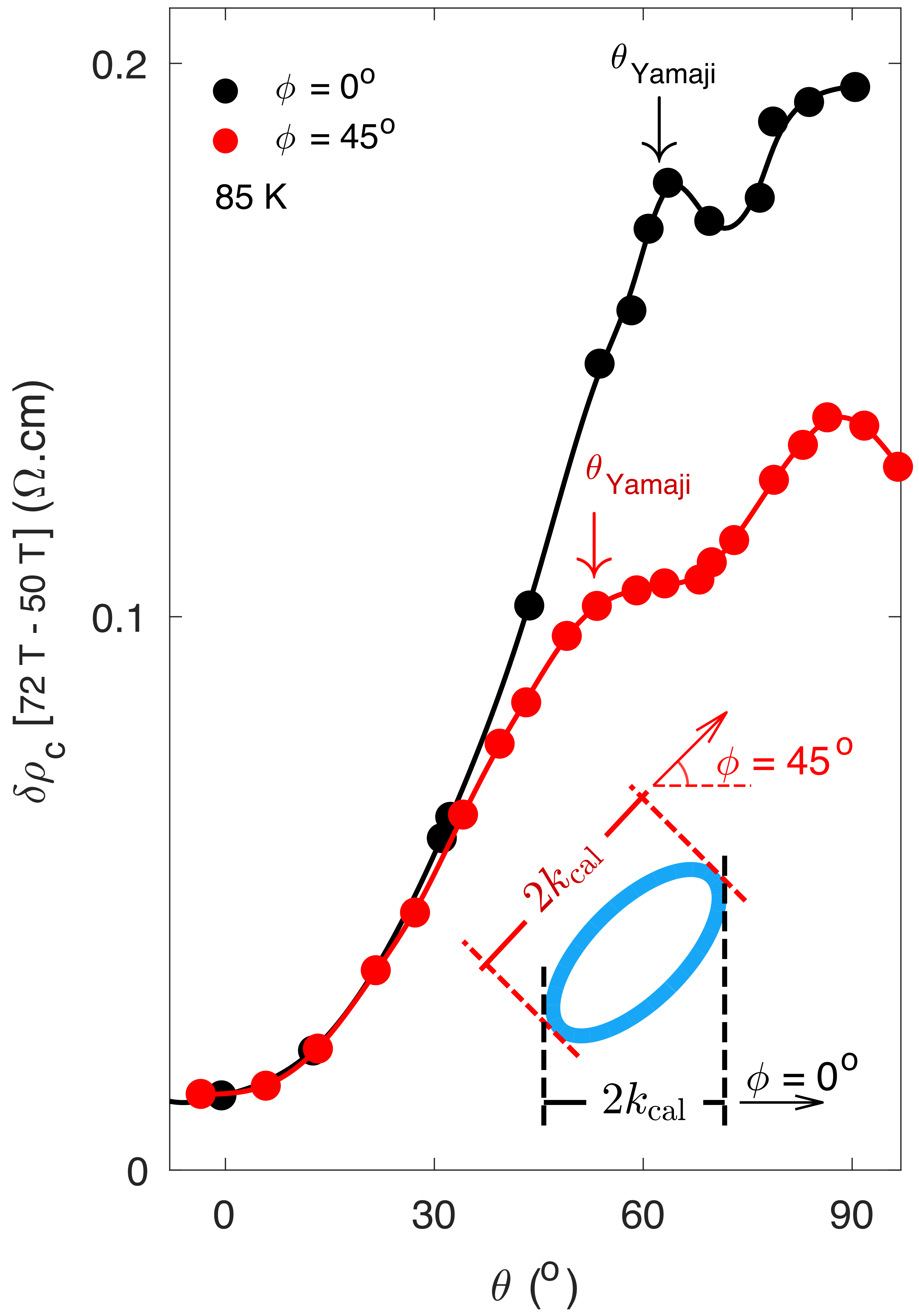
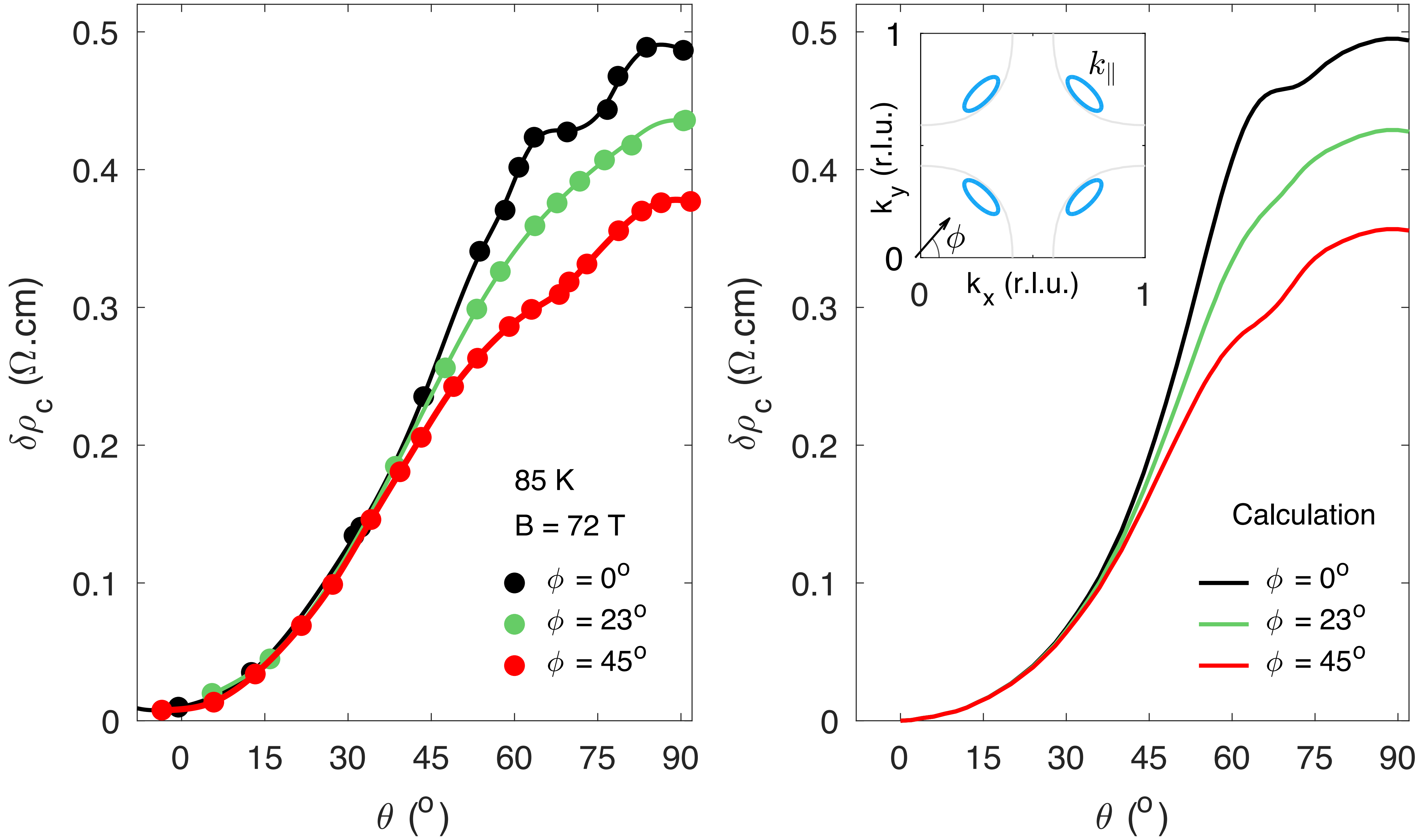




Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

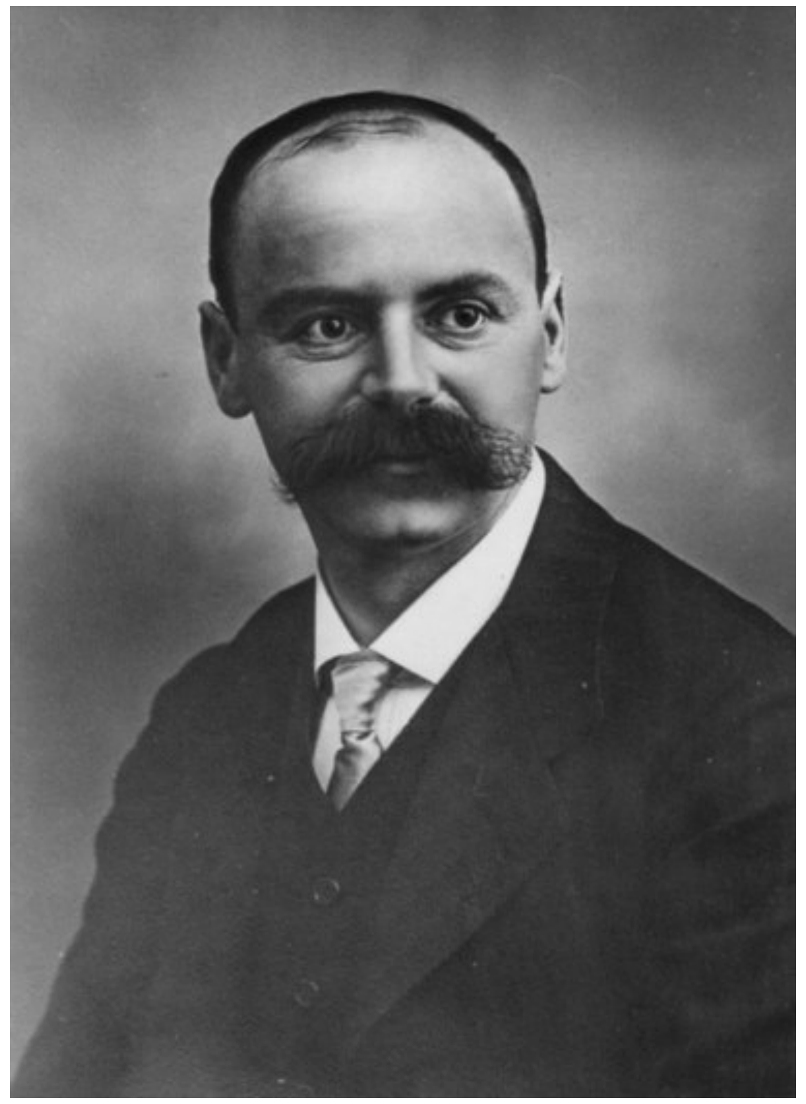
(p/8 also in Yang-Rice-Zjang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Planckian dynamics of black holes and the SYK model

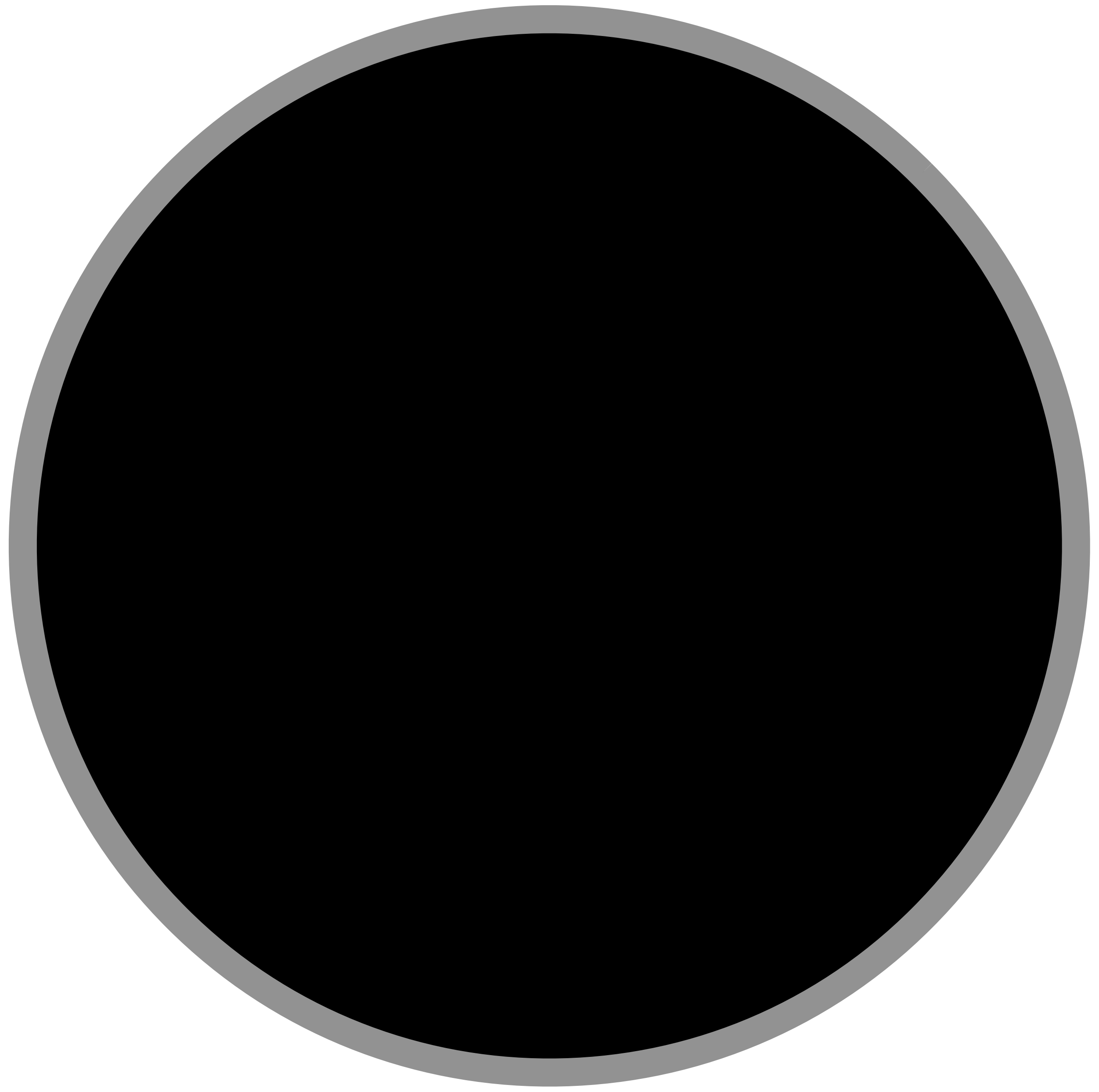
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

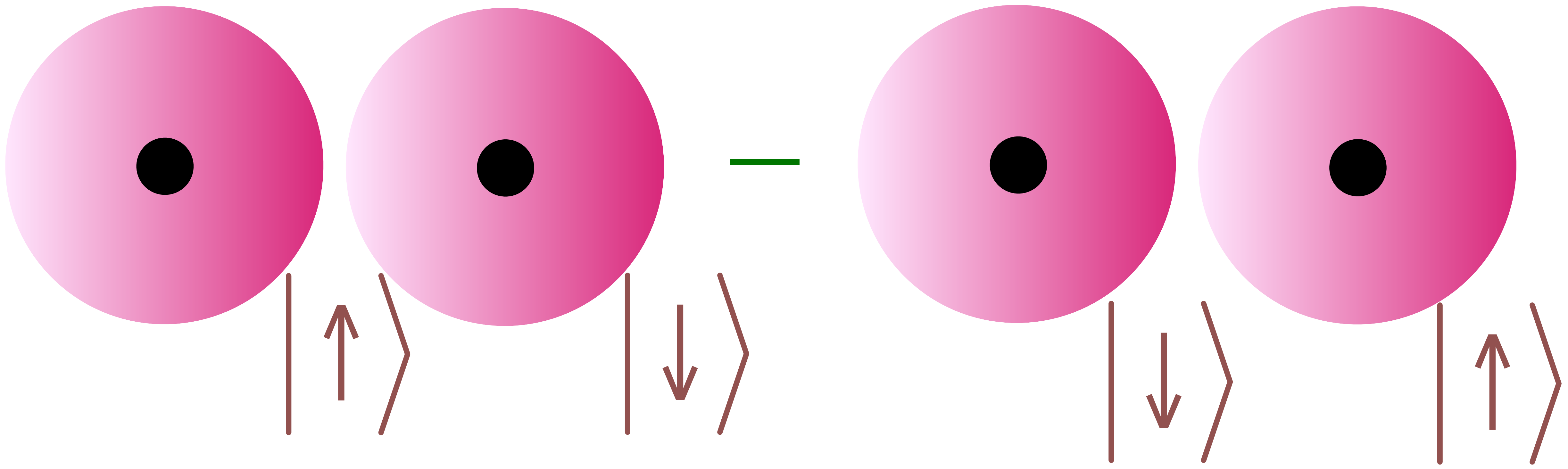
Horizon radius $R = \dfrac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\, mm$!

Quantum Entanglement across a black hole horizon



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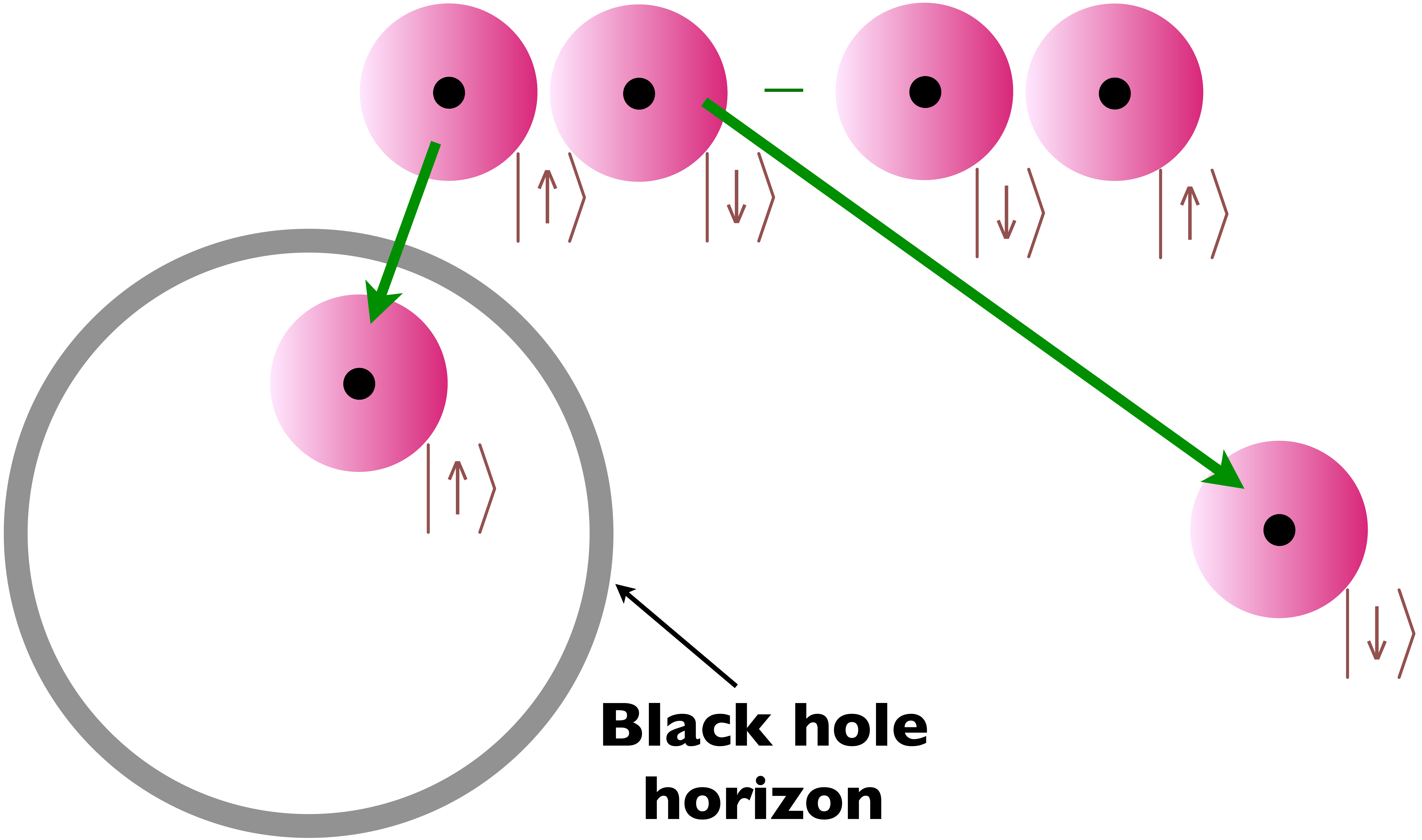

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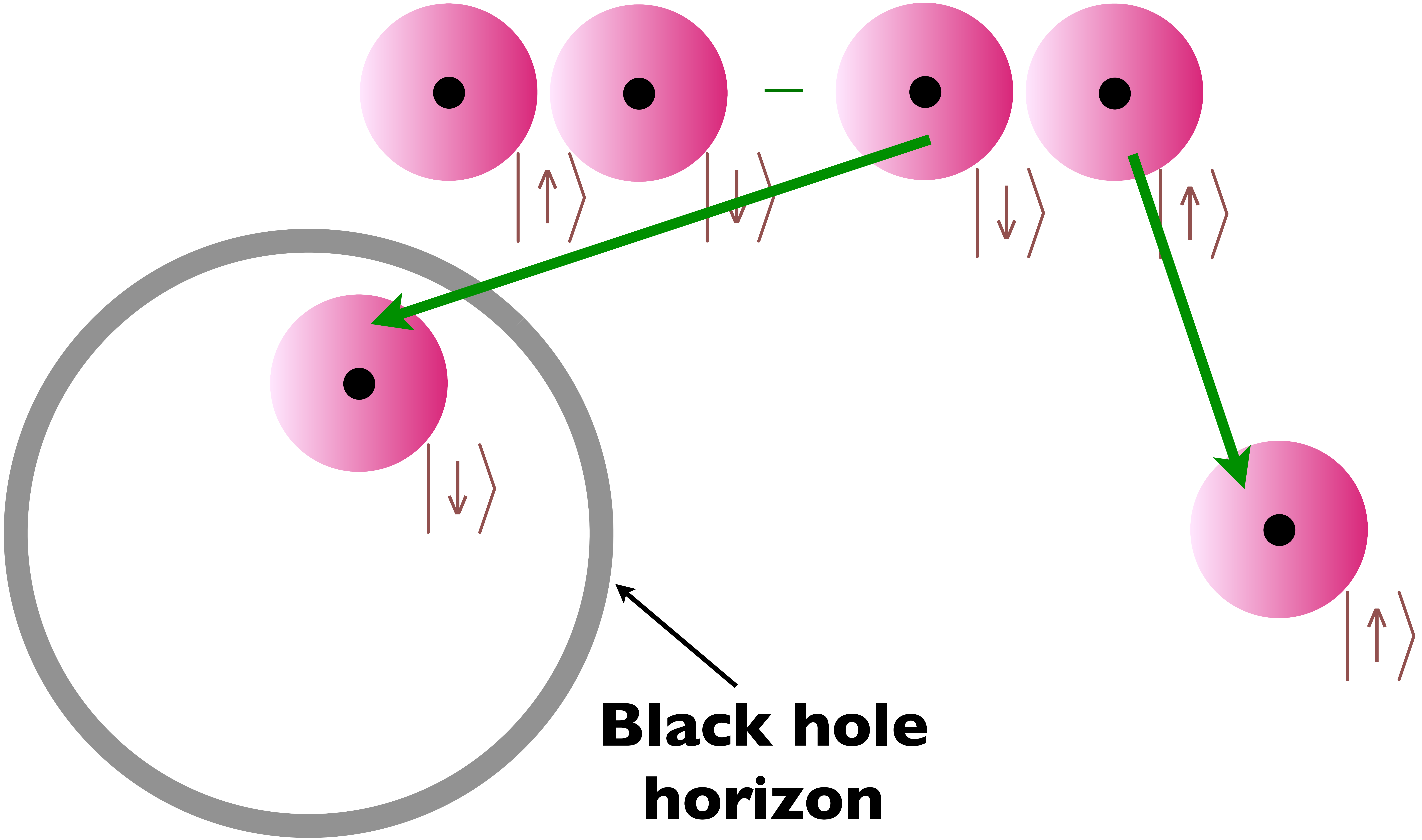
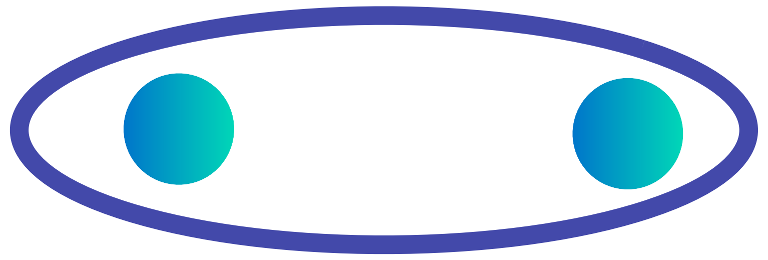
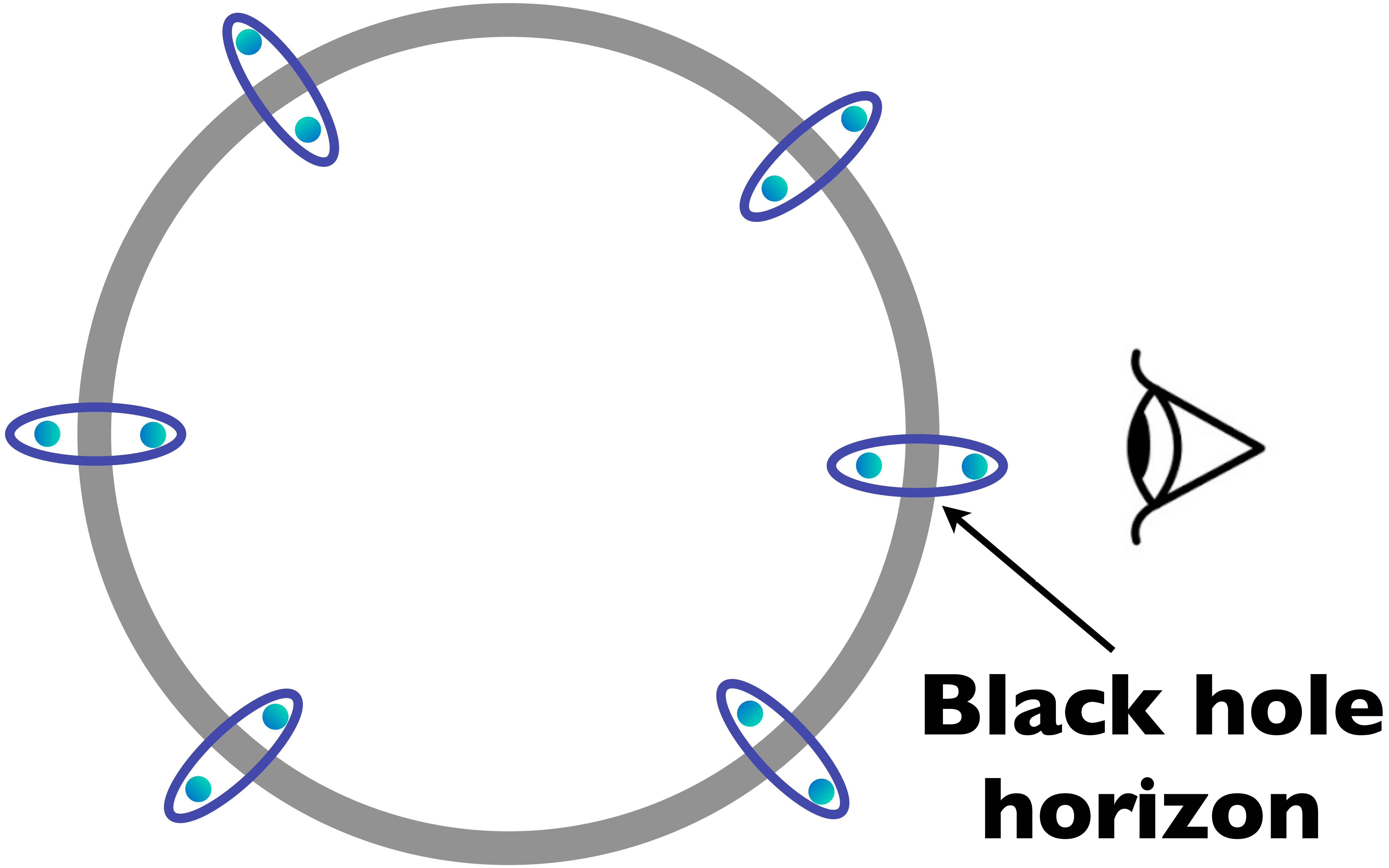


Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



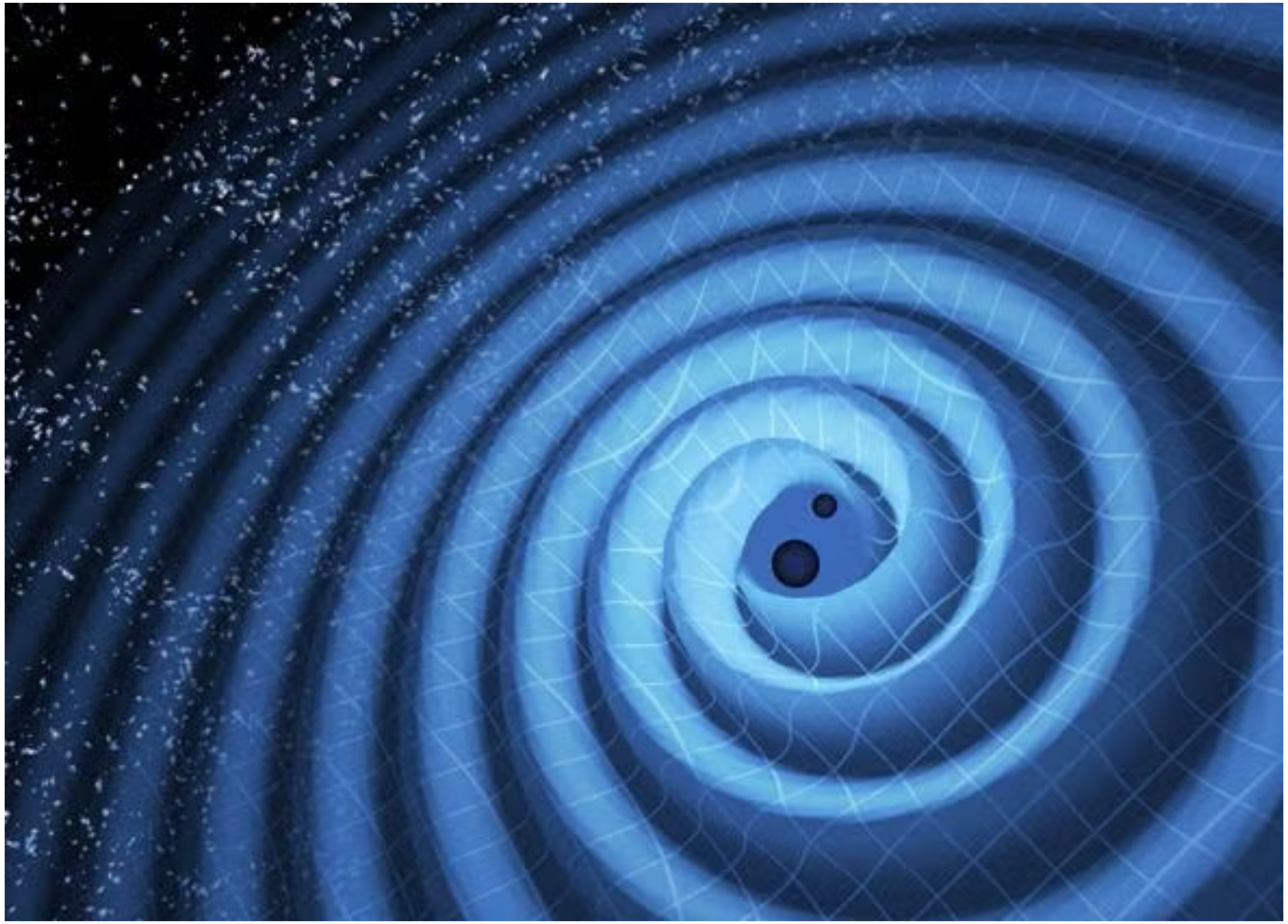
By computations *outside* the black hole, Hawking obtained the black hole entropy

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon



Artwork depicting gravitational waves emanating from two black holes coalescing.
LIGO/T. Pyle

$$\tau_{\rm ring-down} \sim \frac{\hbar}{k_B T_H}$$

Planckian dynamics of quasi-normal modes!

C.V. Vishveshwara
Nature **227**, 936 (1970)

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

is the Hawking temperature of the black hole

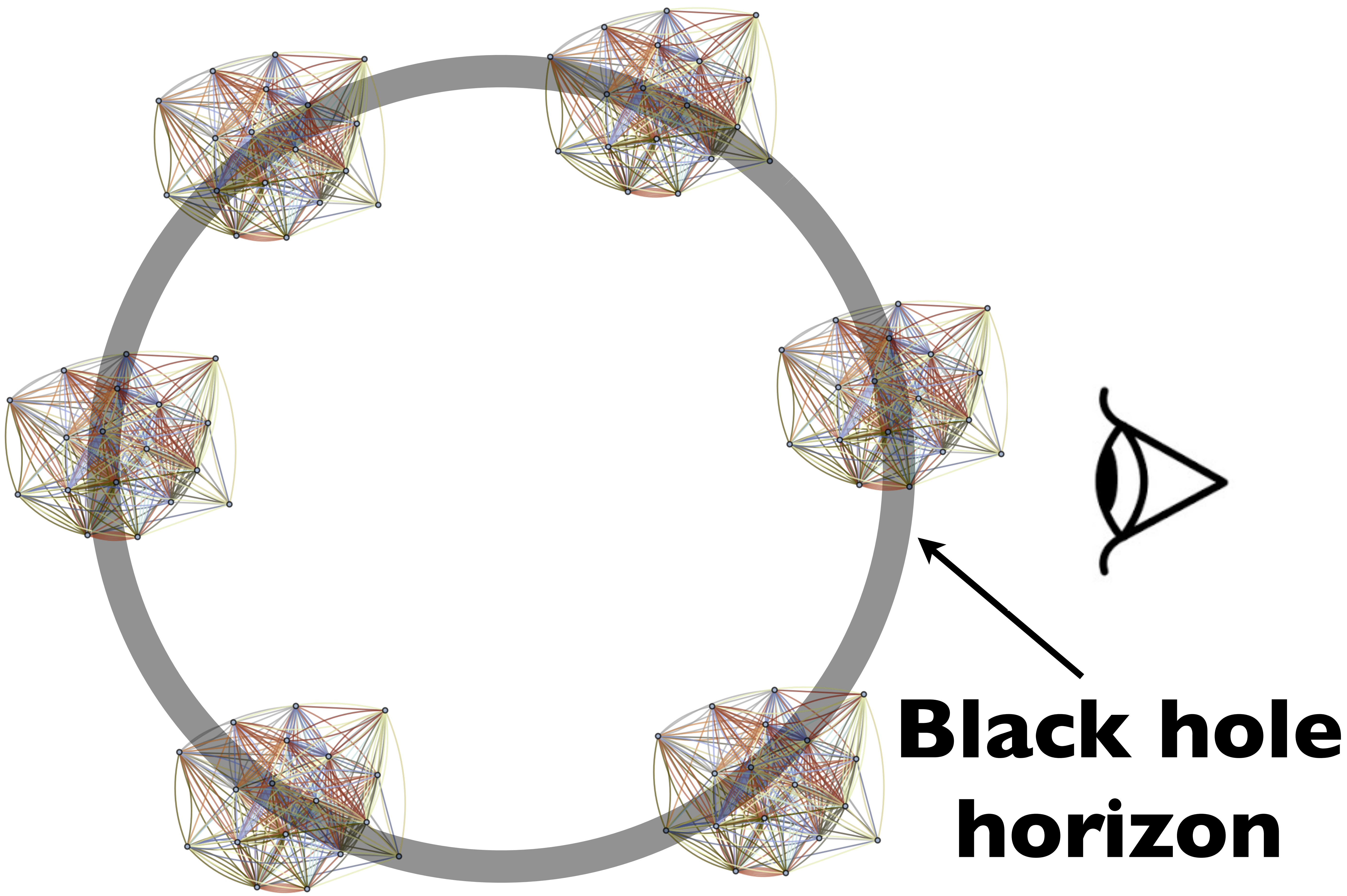
S. Sachdev, PRL **105**, 151602 (2010)

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

S. Sachdev, PRL **105**, 151602 (2010)

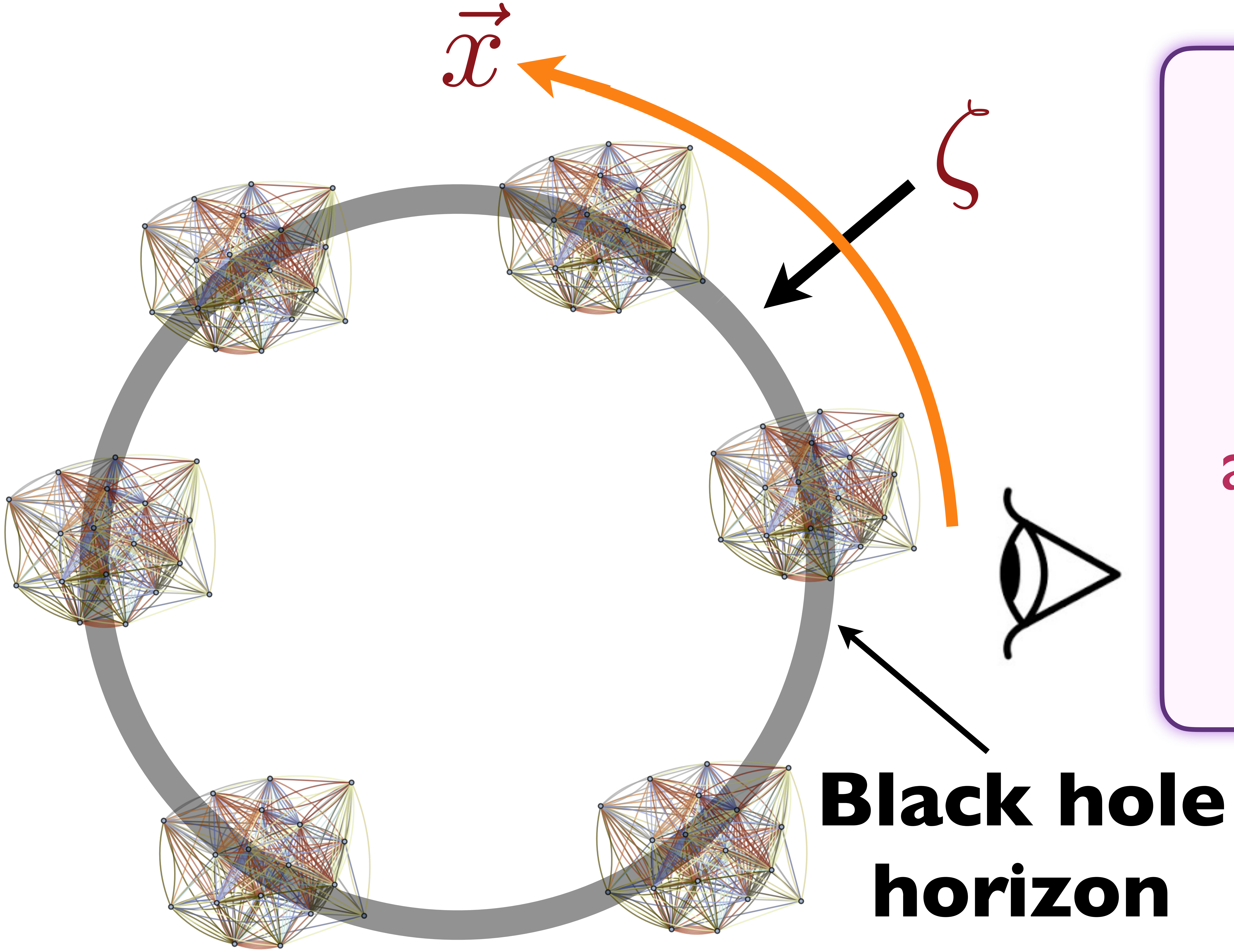
Holographic Metals and the Fractionalized Fermi Liquid



Subir Sachdev

"...This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R_2$ physics of Reissner- Nordström black holes."

Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge

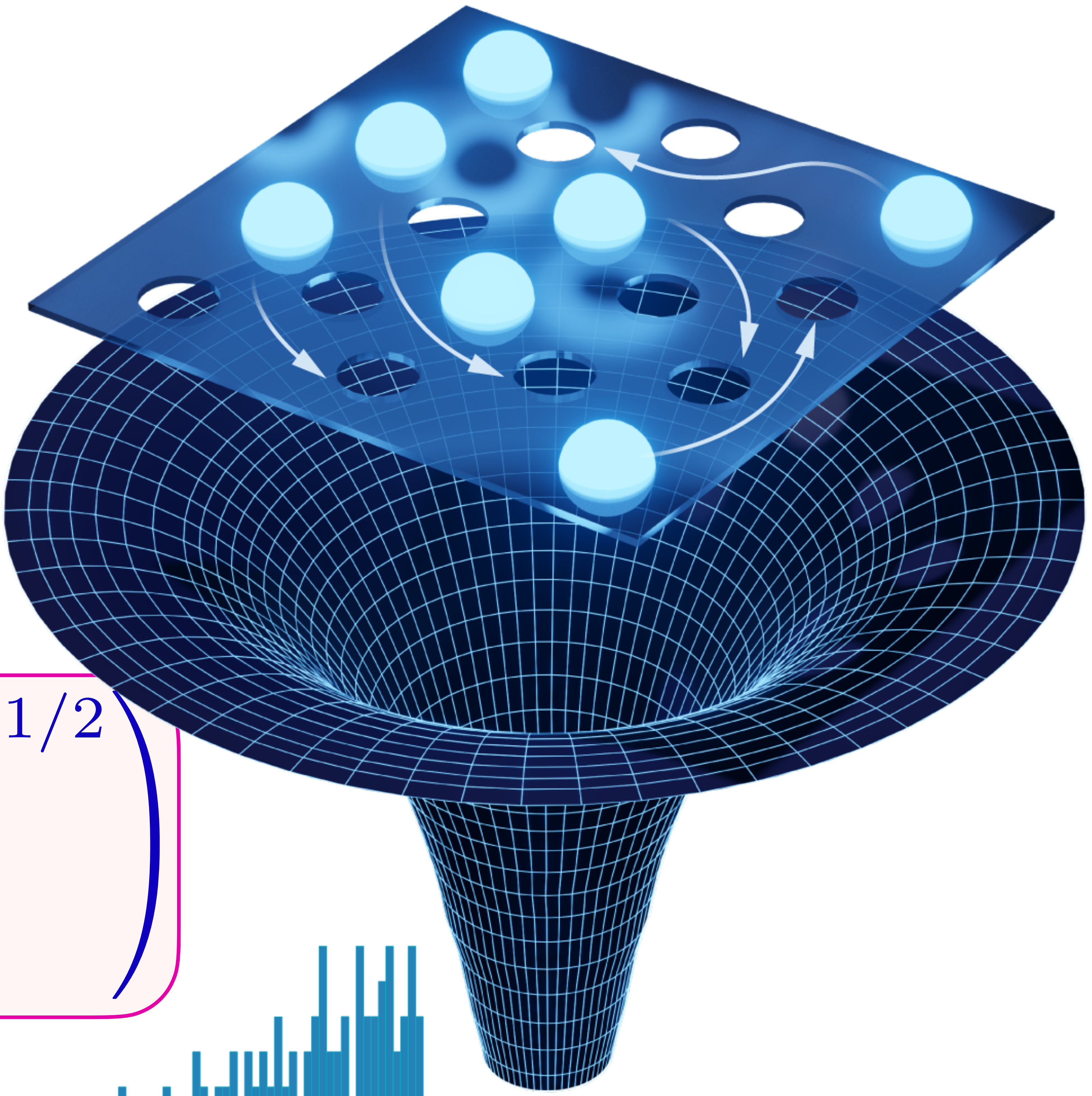


The quantum versions of Maxwell's and Einstein's equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

Kitaev (2015), Maldacena Stanford (2015)

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

D(E) of charged black holes from the SYK model



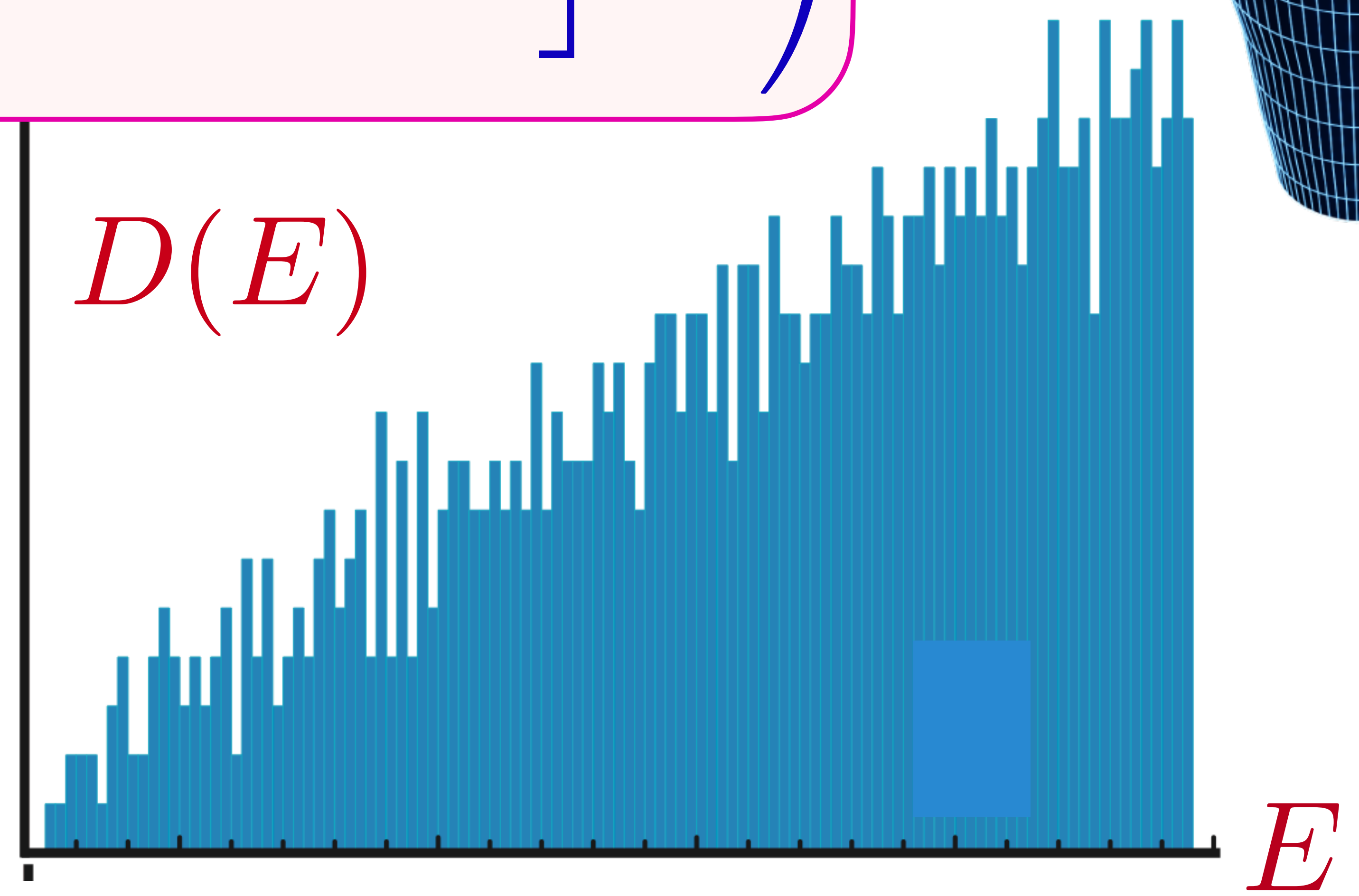
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}{A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



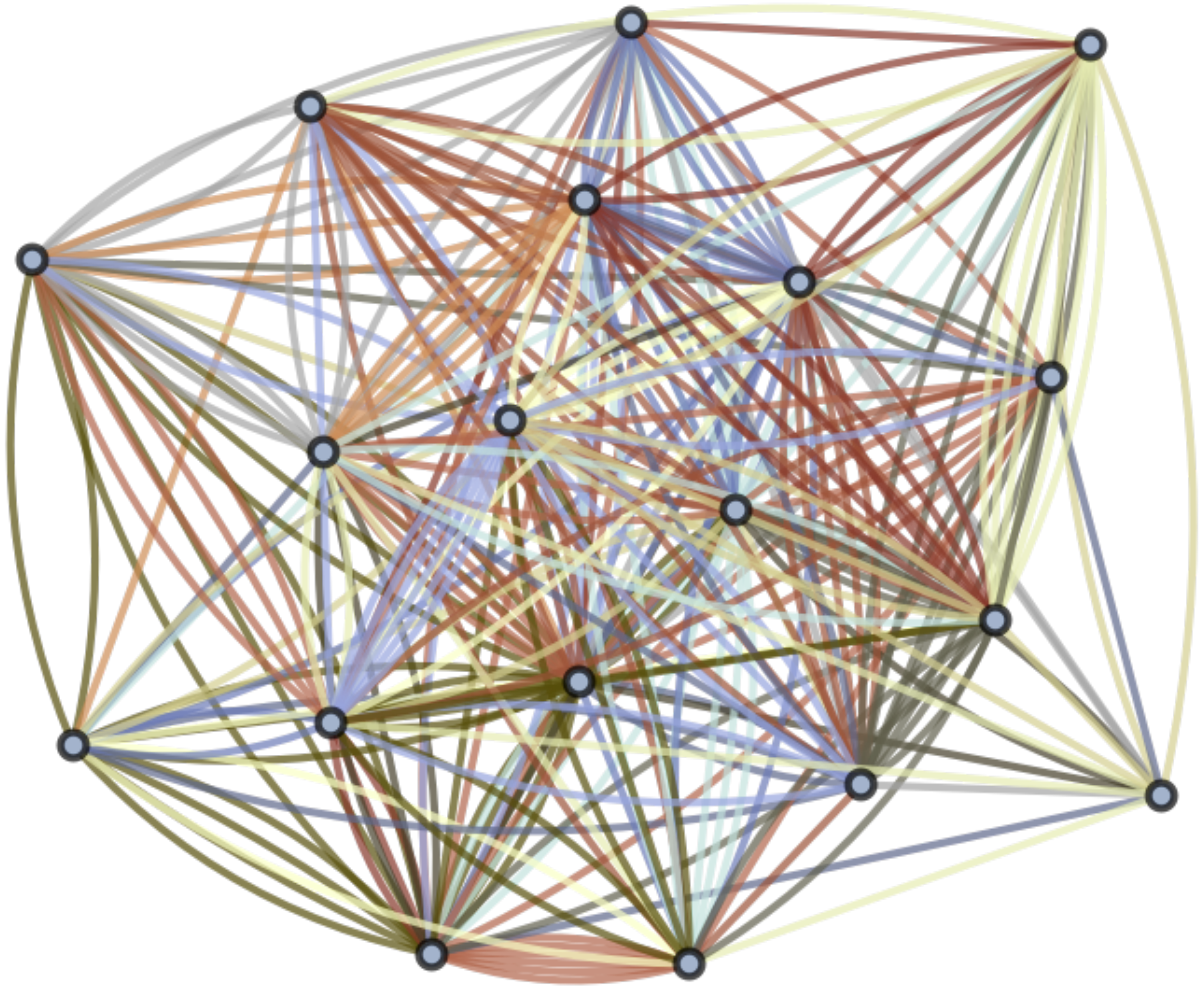
Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

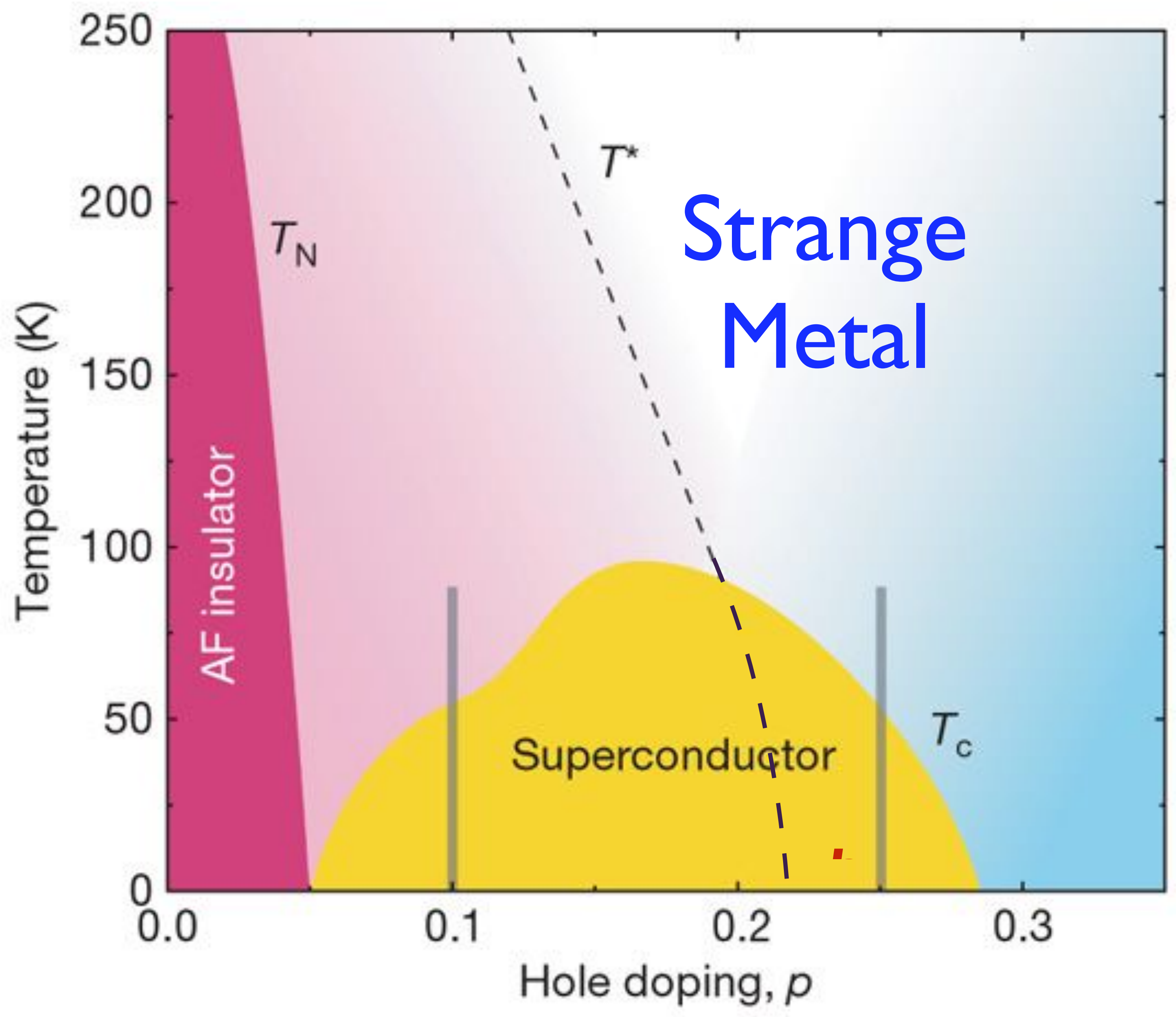
Recap

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



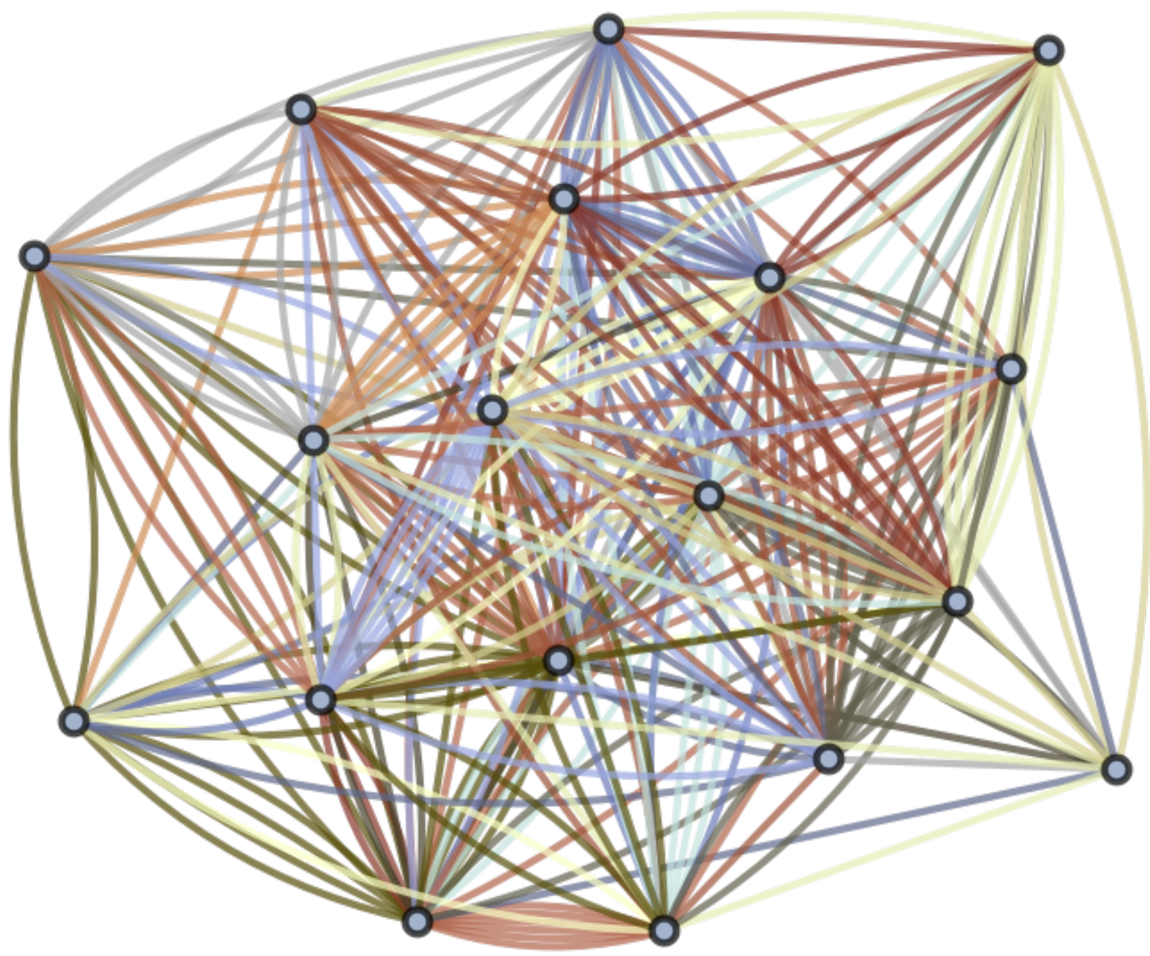
The Sachdev-Ye-Kitaev (SYK) model



The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



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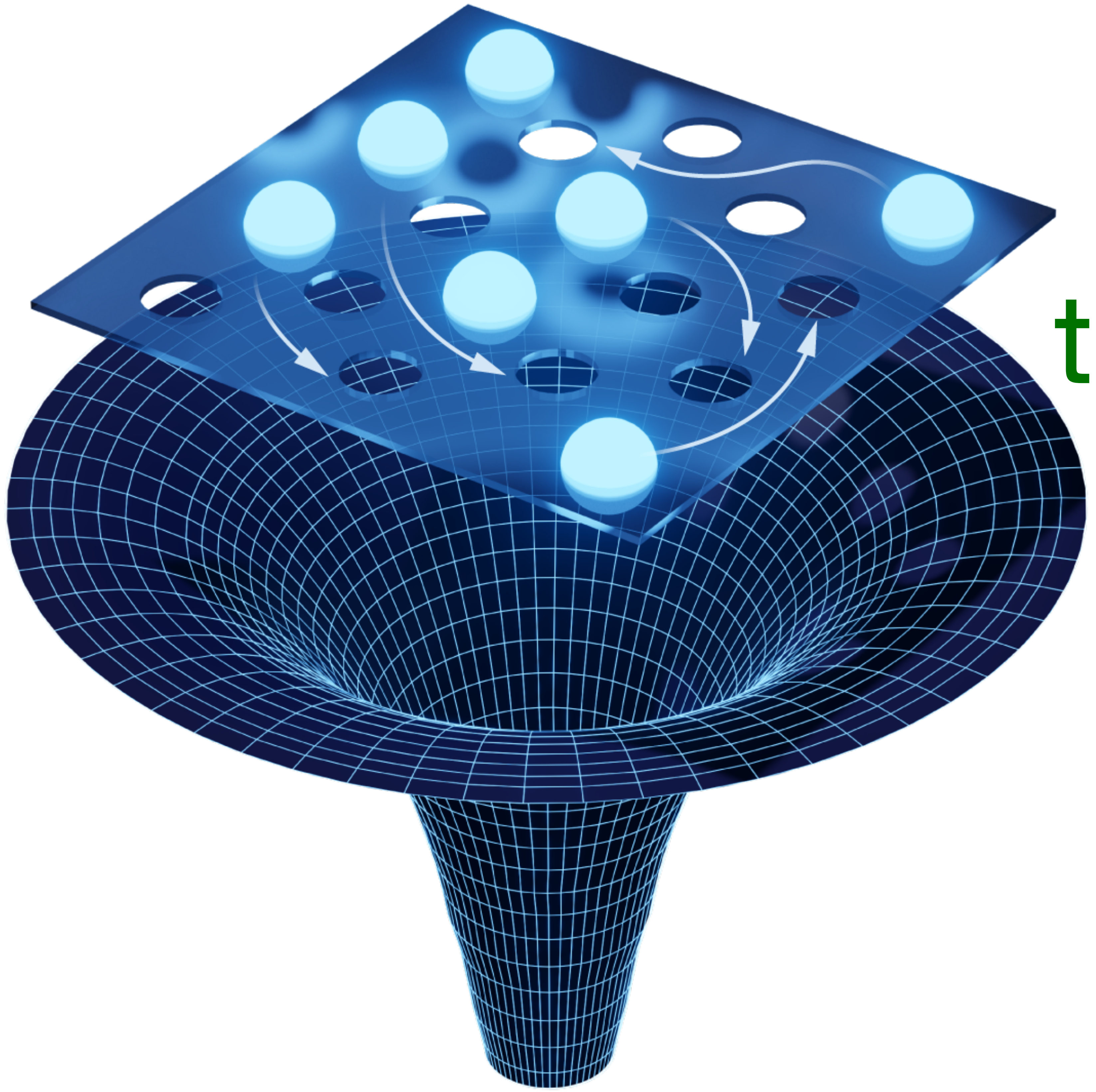

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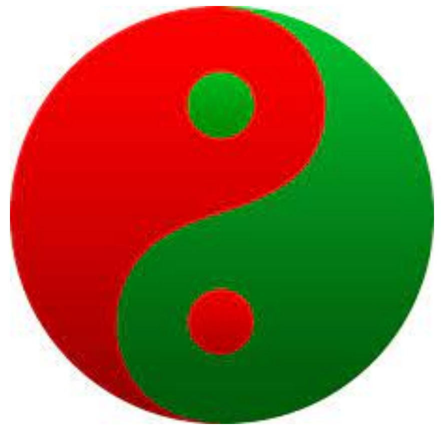
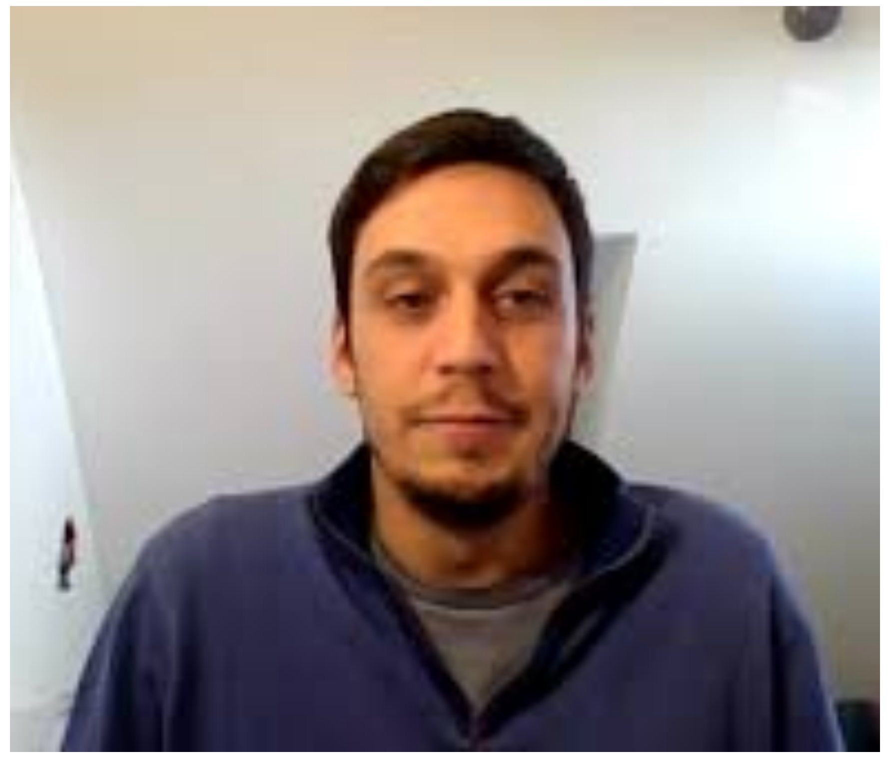
Sachdev, Ye (1993)



In a *dual* set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)



Maine Christos
Caltech

Pietro Bonetti
Stuttgart

Alexander
Nikolaenko

Aavishkar Patel
ICTS, Bengaluru

The Institute of Mathematical Sciences, Chennai

Harshit Pandey

Ravi Shanker

Sayantan Sharma

- *Lectures on insulating and conducting quantum spin liquids*, S. Sachdev, arXiv:2512.23962
- *Fractionalized Fermi liquids and the cuprate phase diagram*, P. M. Bonetti, M. Christos, A. Nikolaenko, A.A. Patel, and S. Sachdev, Reports on Progress in Physics **89** 044501 (2026).
- *Thermal SU(2) lattice gauge theory for intertwined orders and hole pockets in the cuprates*, H. Pandey, M. Christos, P. M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, and S. Sachdev, PNAS in press, arXiv:2507.0533