SU(2) gauge theory for intertwined orders and hole pockets in the cuprate pseudogap phase

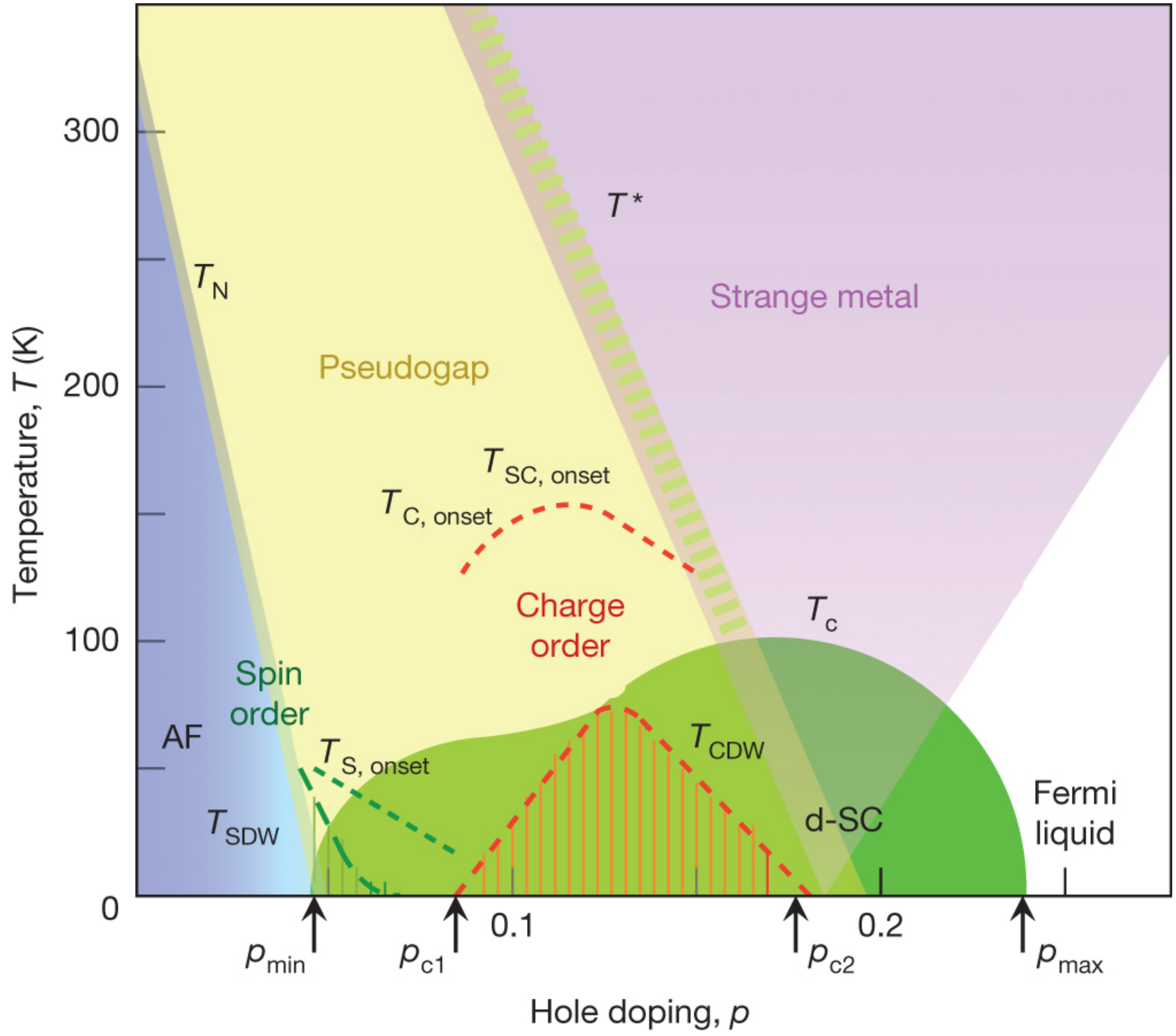
Rutgers University
April 21, 2026

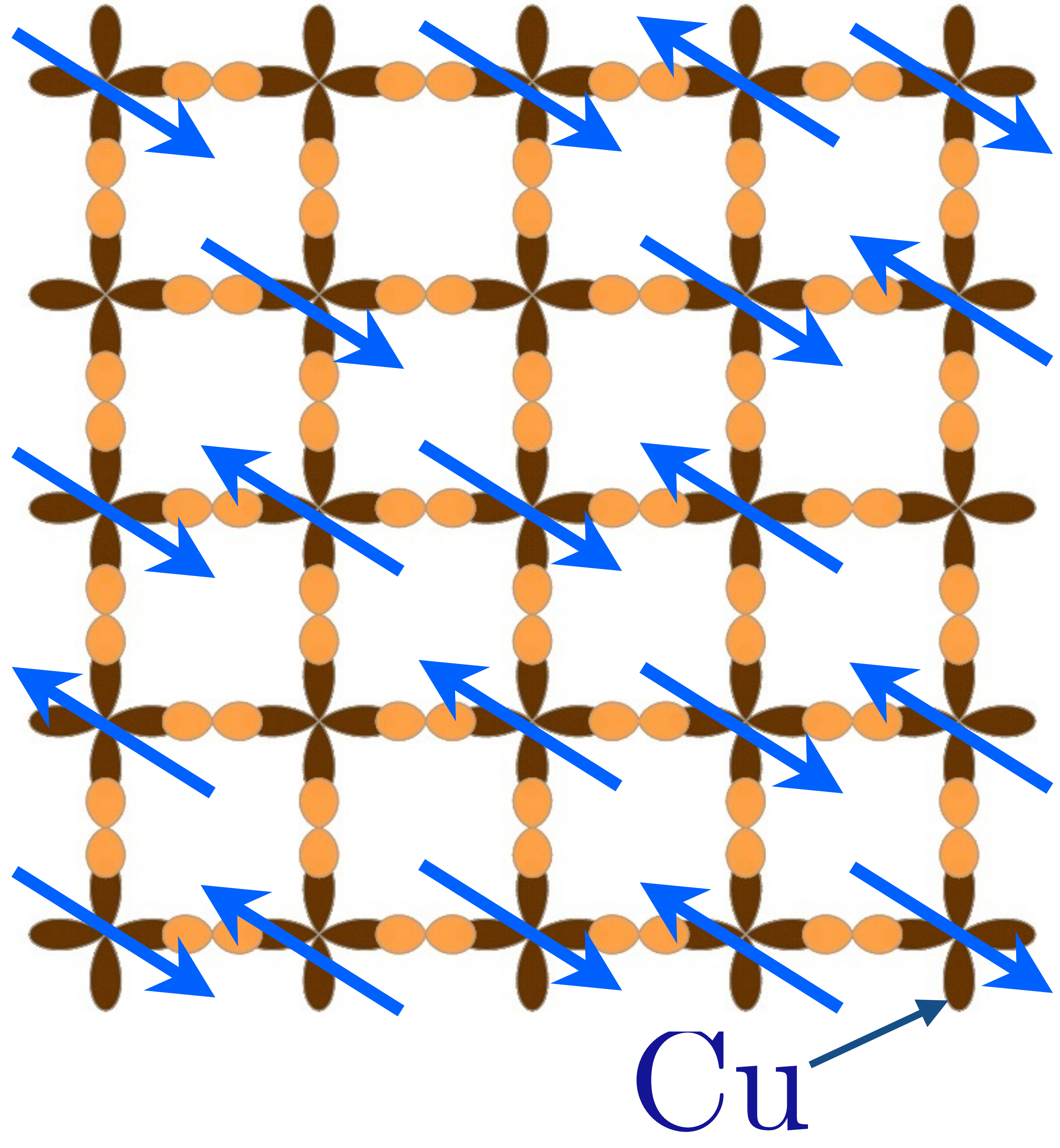
Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



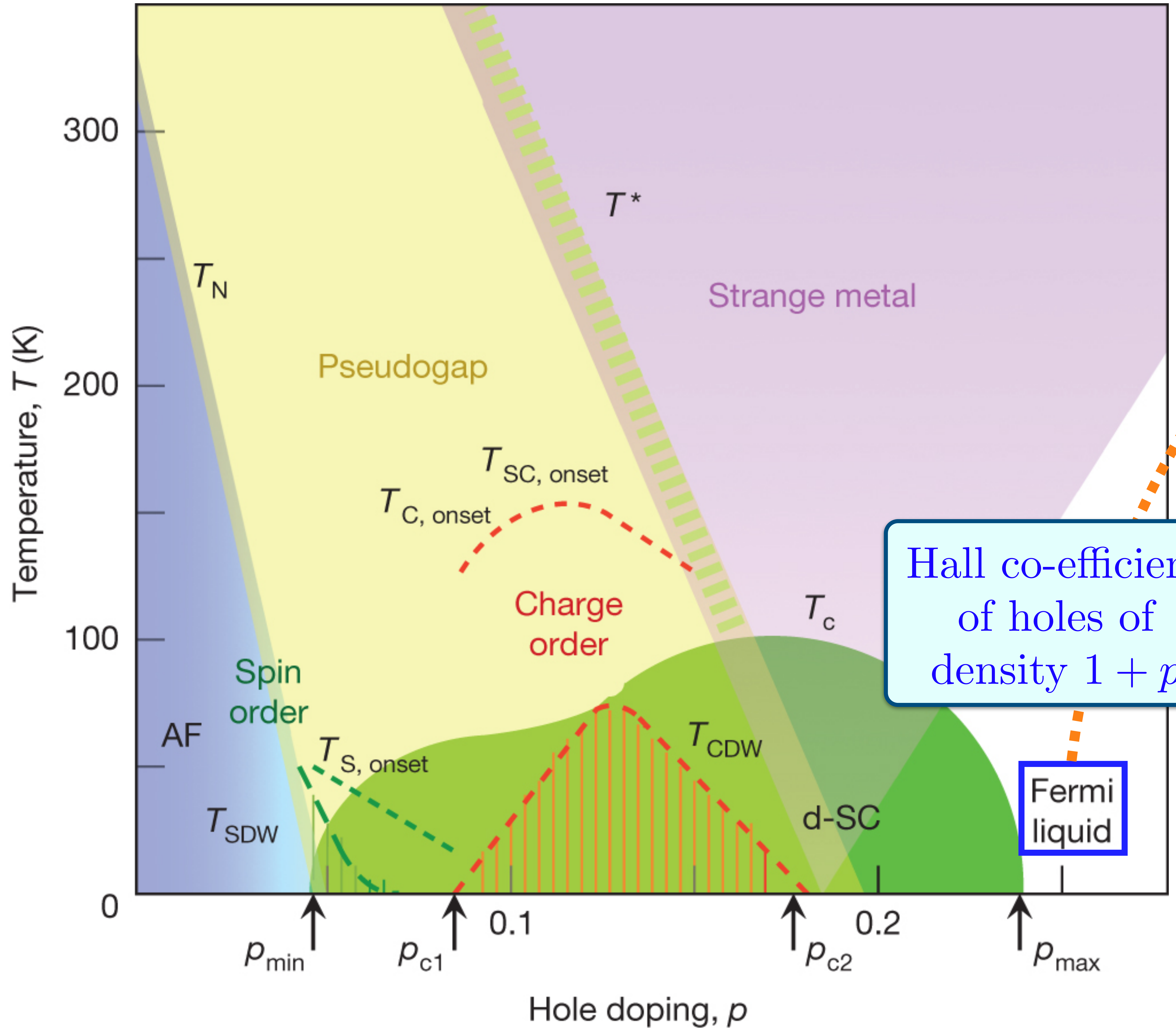


d-SC obtained upon doping AF with density p holes.

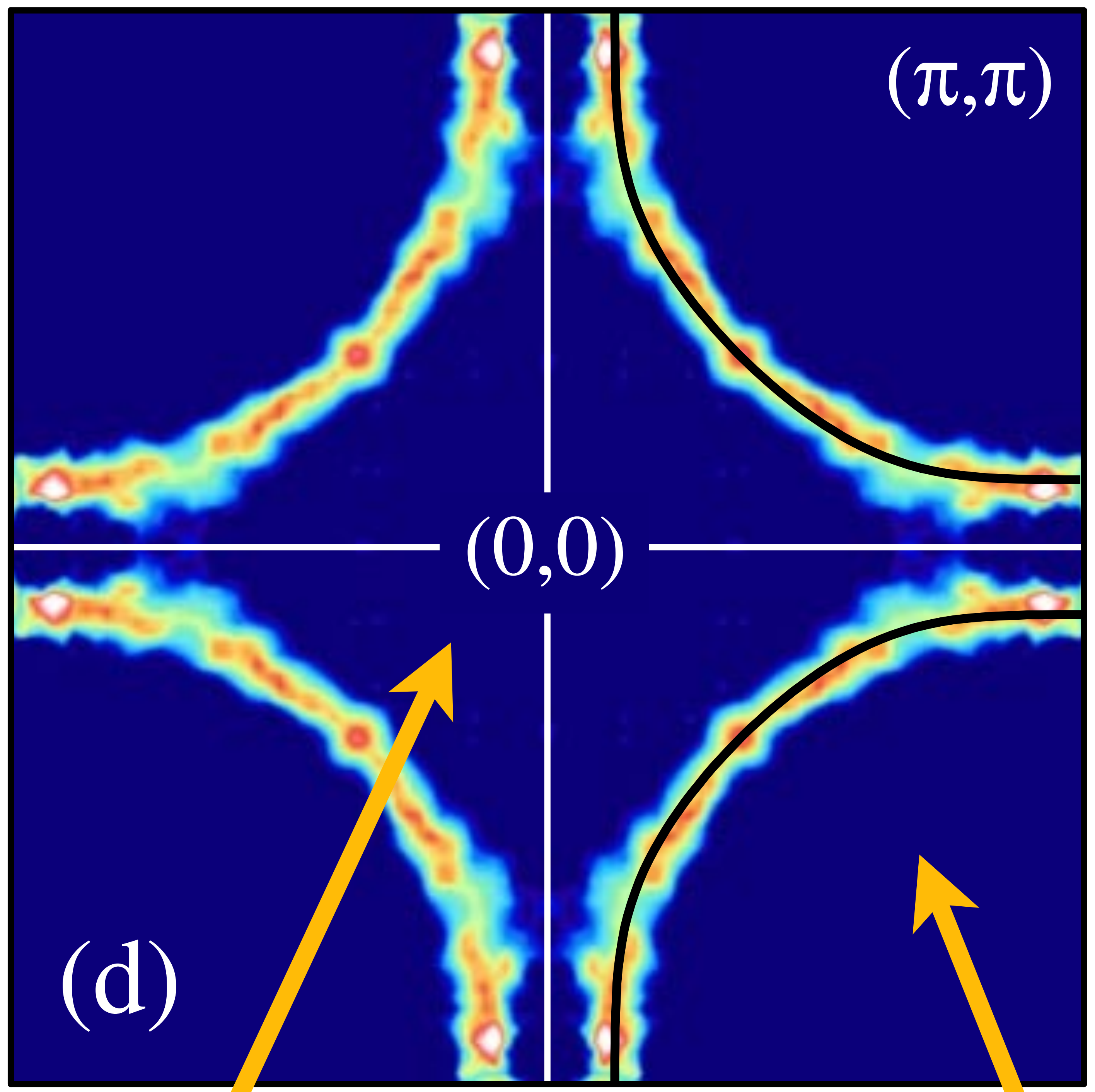
Hole density relative to the filled band $\rho = 1 + p$.

Electron density relative to the empty band $\rho_e = 1 - p$.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



M. Platé *et al.*, PRL **95**, 077001 (2005)

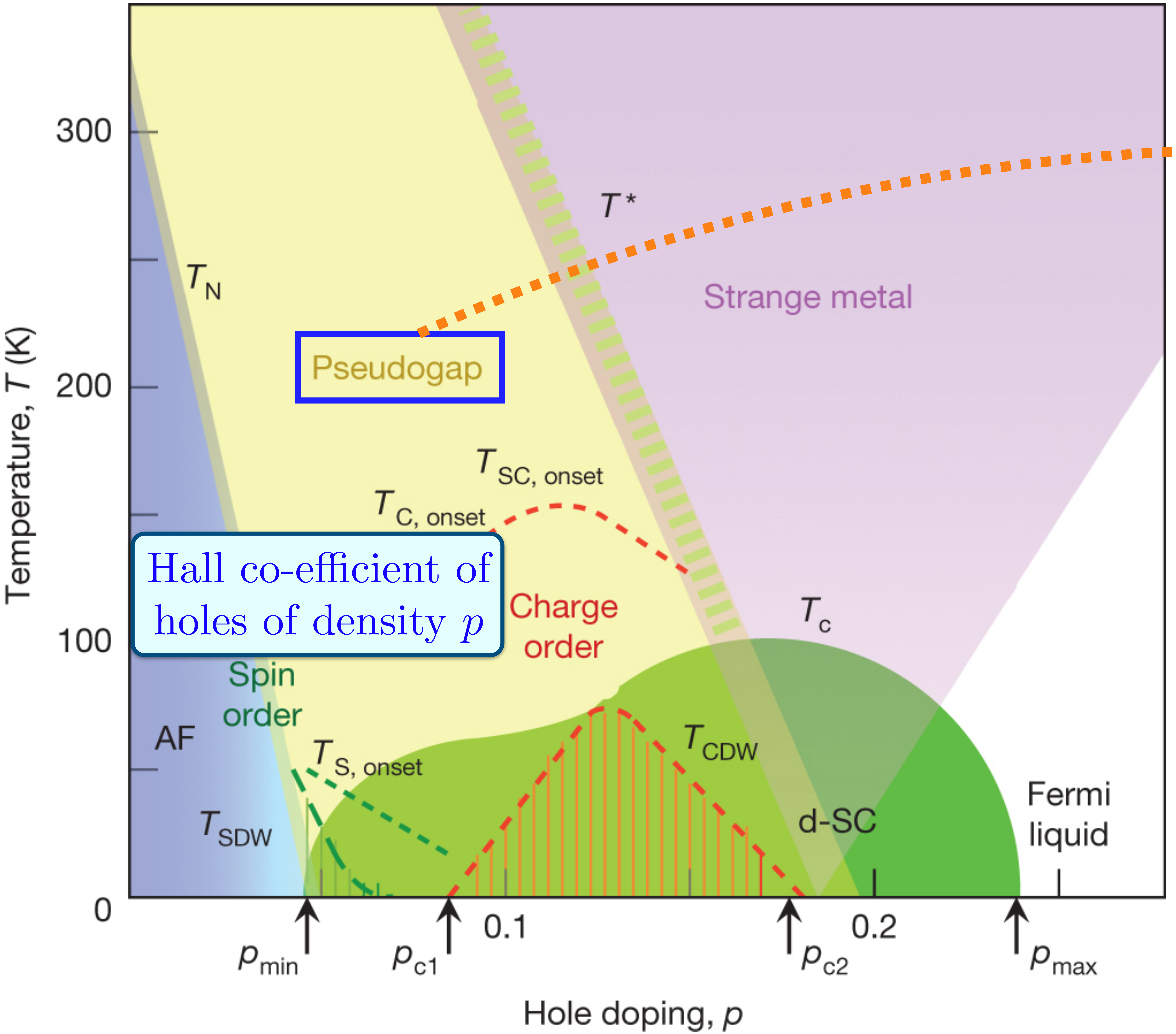


Hall co-efficient of holes of density $1+p$

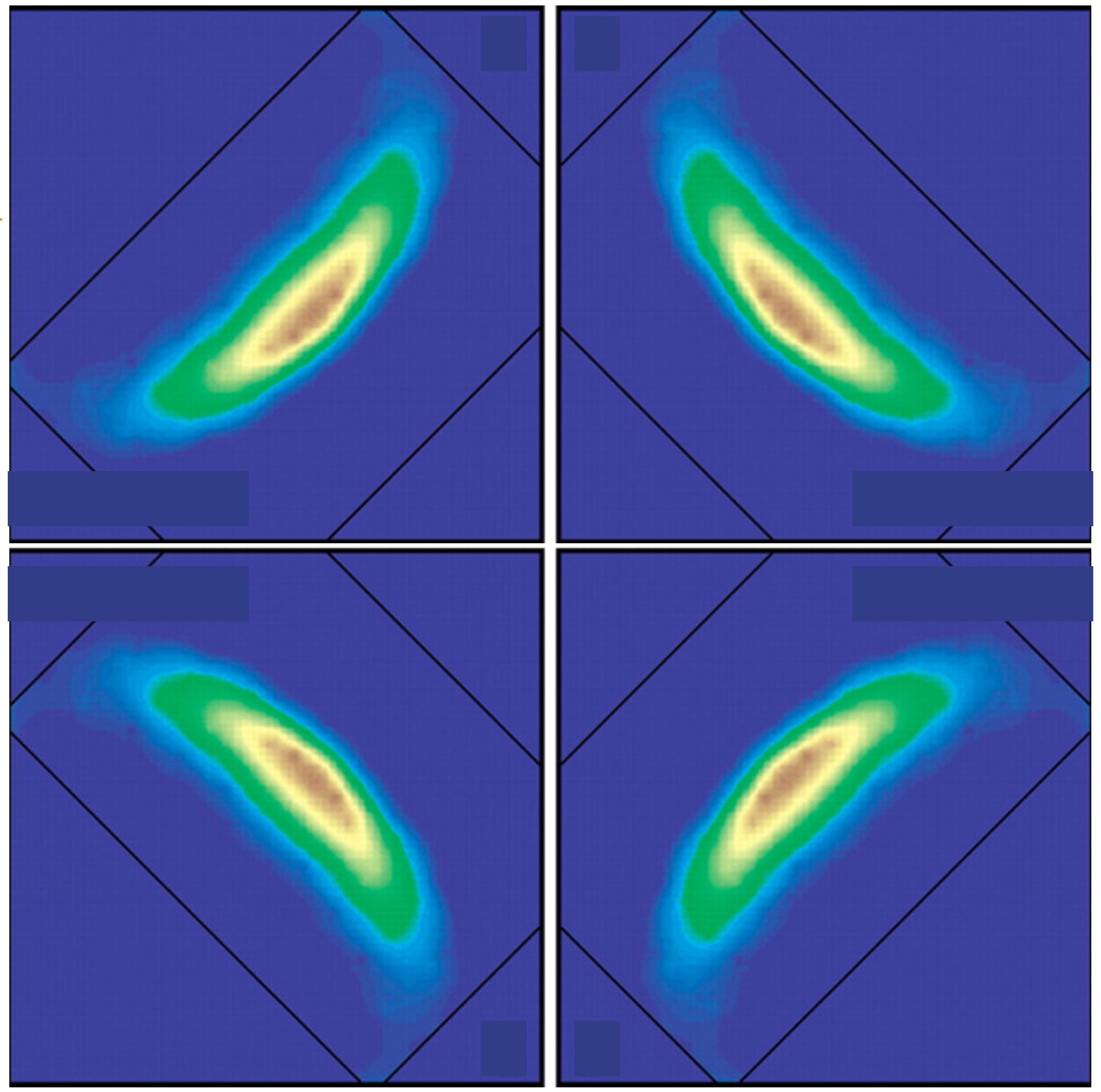
1-p electrons

1+p holes

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

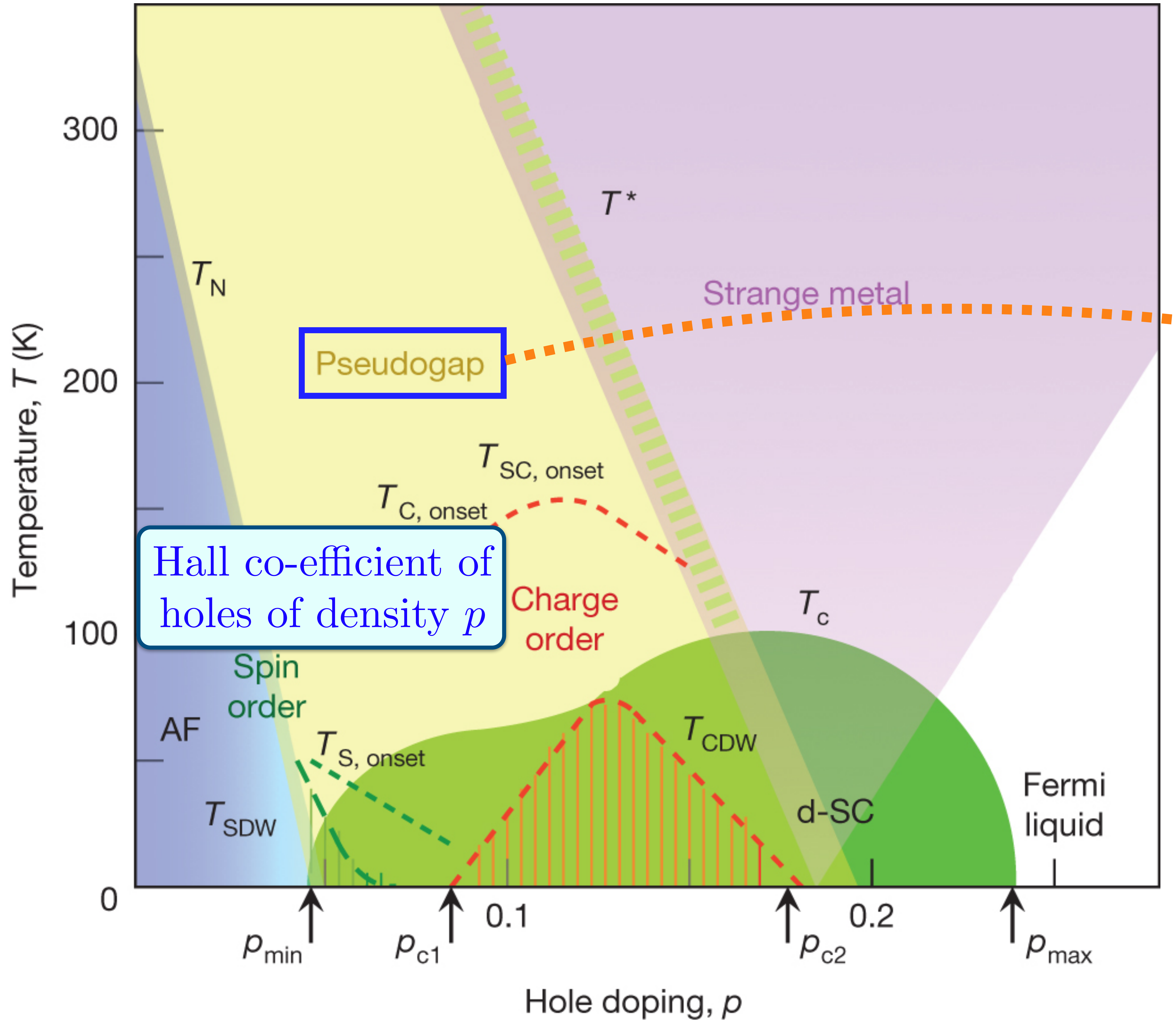


Kyle M. Shen *et al.*, Science **307**, 901 (2005)

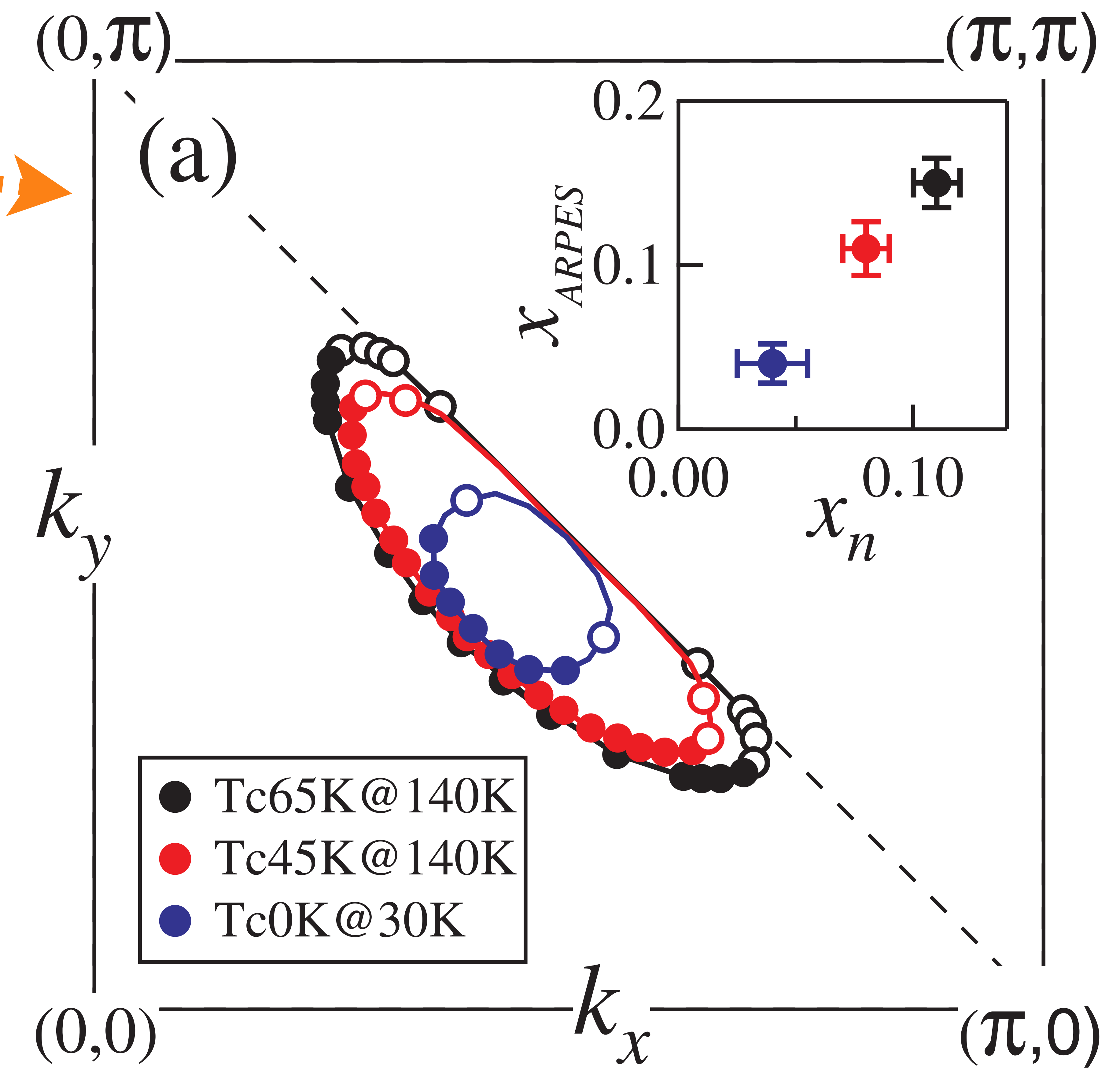


`Fermi arcs' ?

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

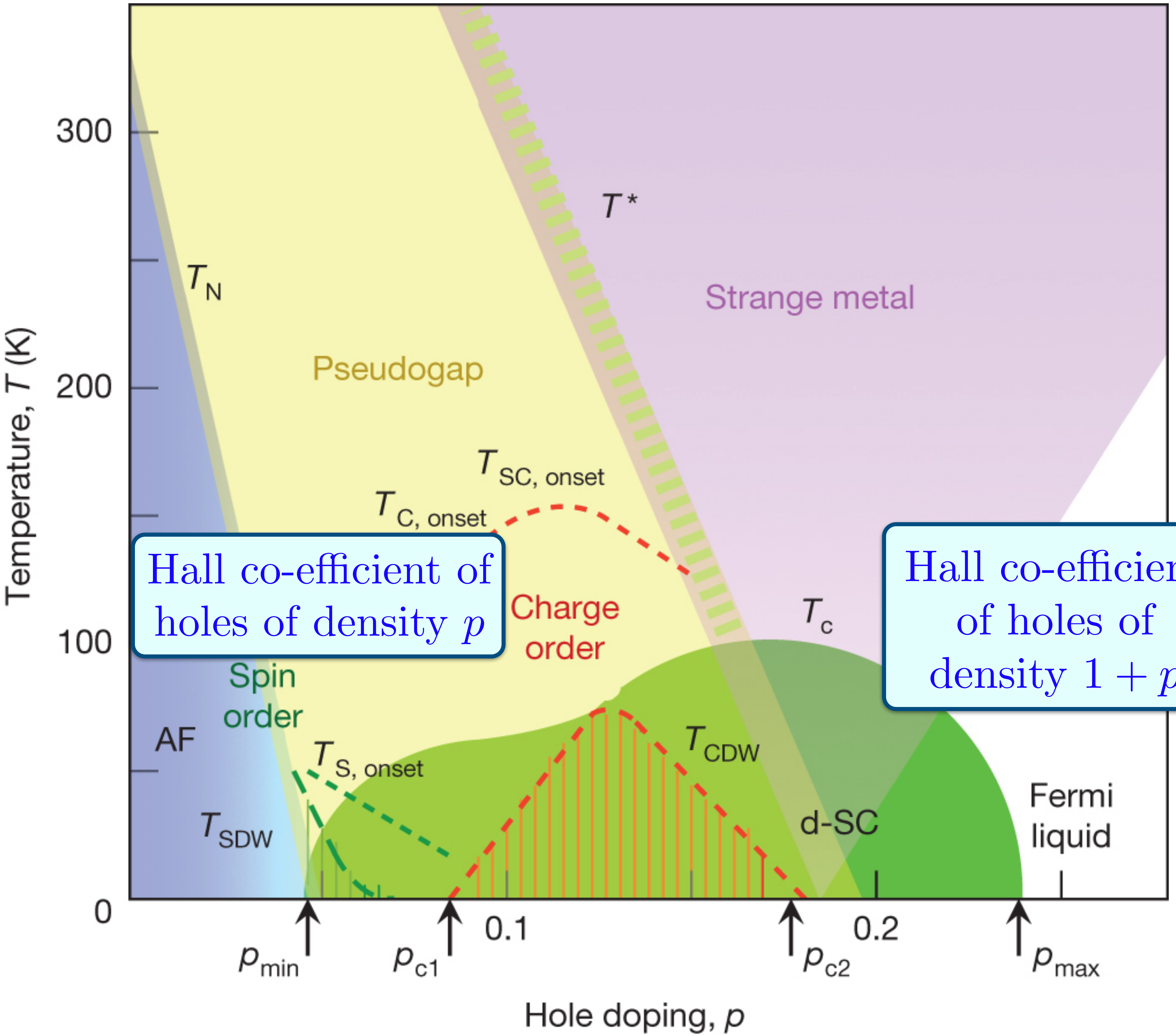


Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).



Or hole pockets ?

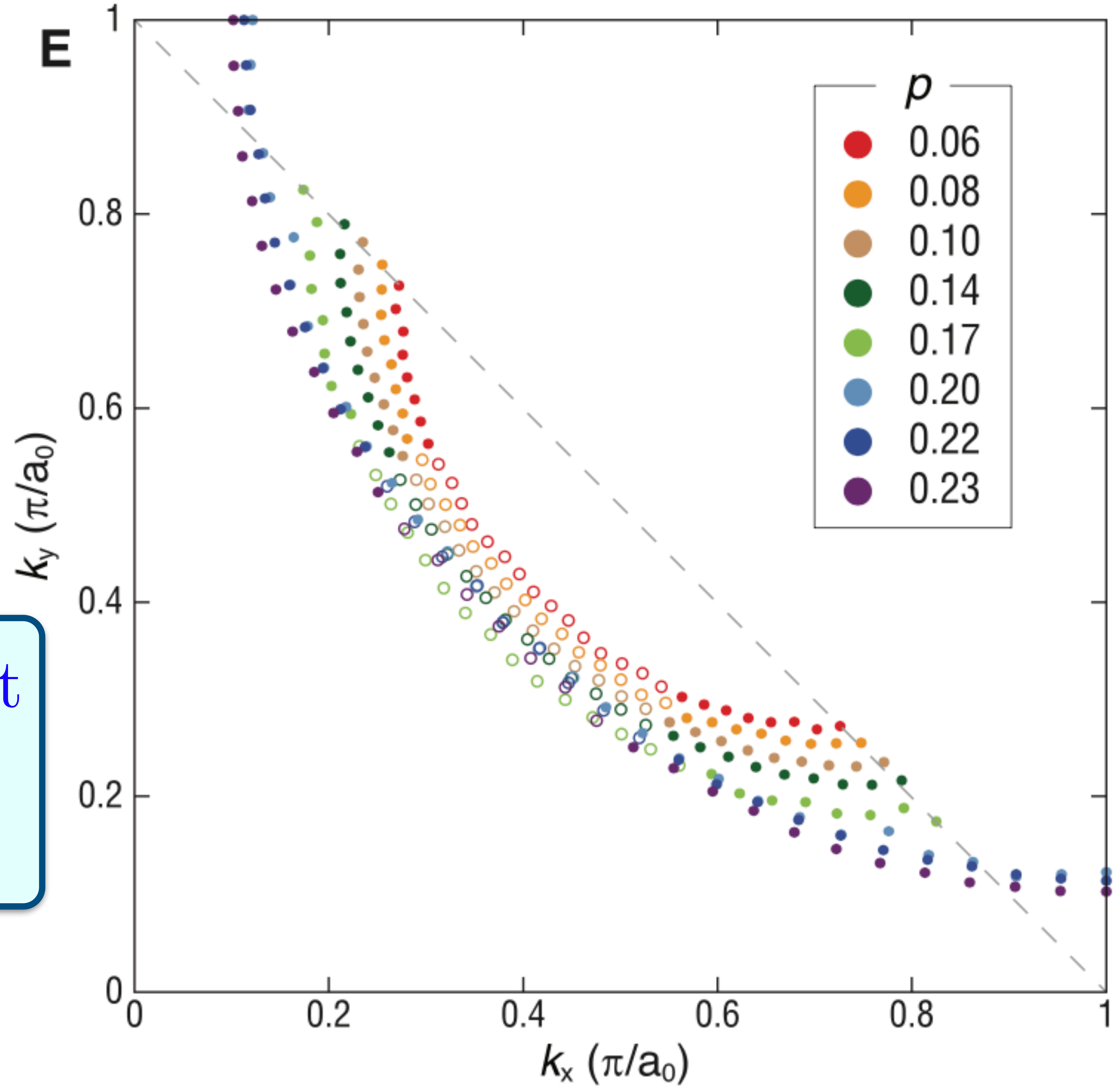
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Hall co-efficient of holes of density p

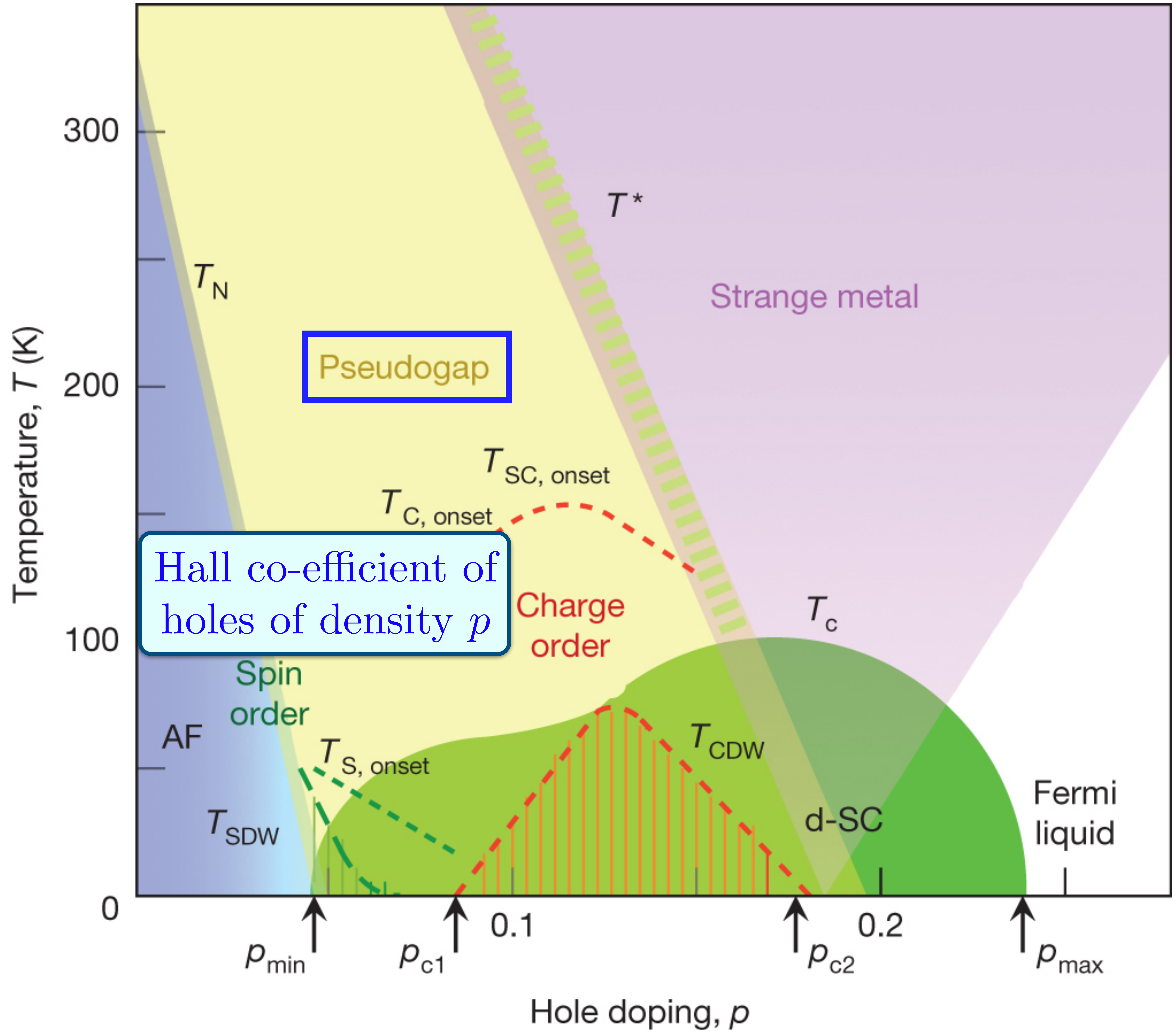
Hall co-efficient of holes of density $1+p$

K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E. -A. Kim, J. C. Davis, *Science* **344**, 612 (2014)



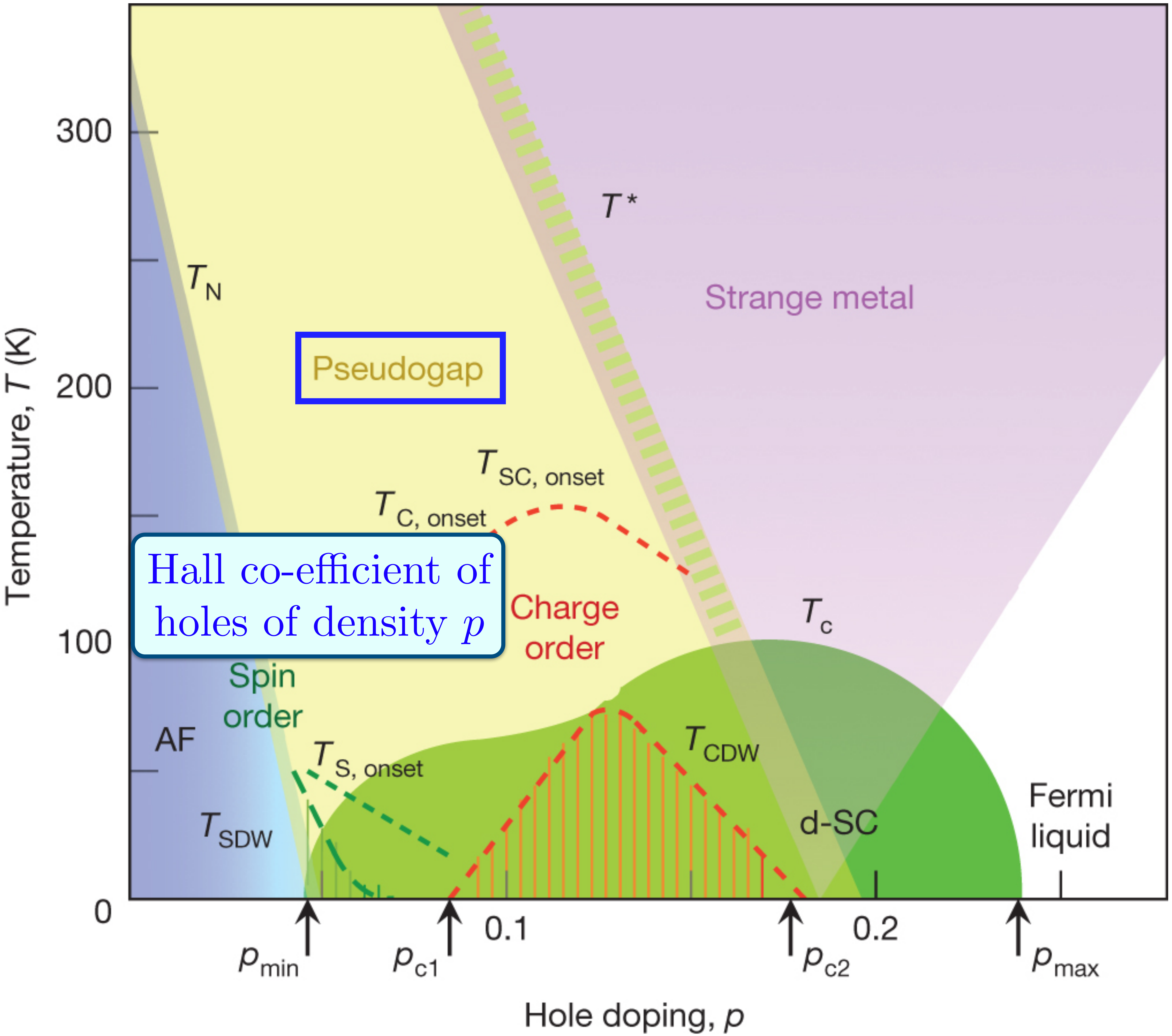
Also
Yang He, Yi Yin, M. Zech, Anjan Soumyanarayanan, M. M. Yee, Tess Williams, M. C. Boyer, Kamalesh Chatterjee, W. D. Wise, I. Zeljkovic, Takeshi Kondo, T. Takeuchi, H. Ikuta, Peter Mistark, Robert S. Markiewicz, Arun Bansil, Subir Sachdev, E. W. Hudson, Jennifer. E. Hoffman, *Science* **344**, 608 (2014)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



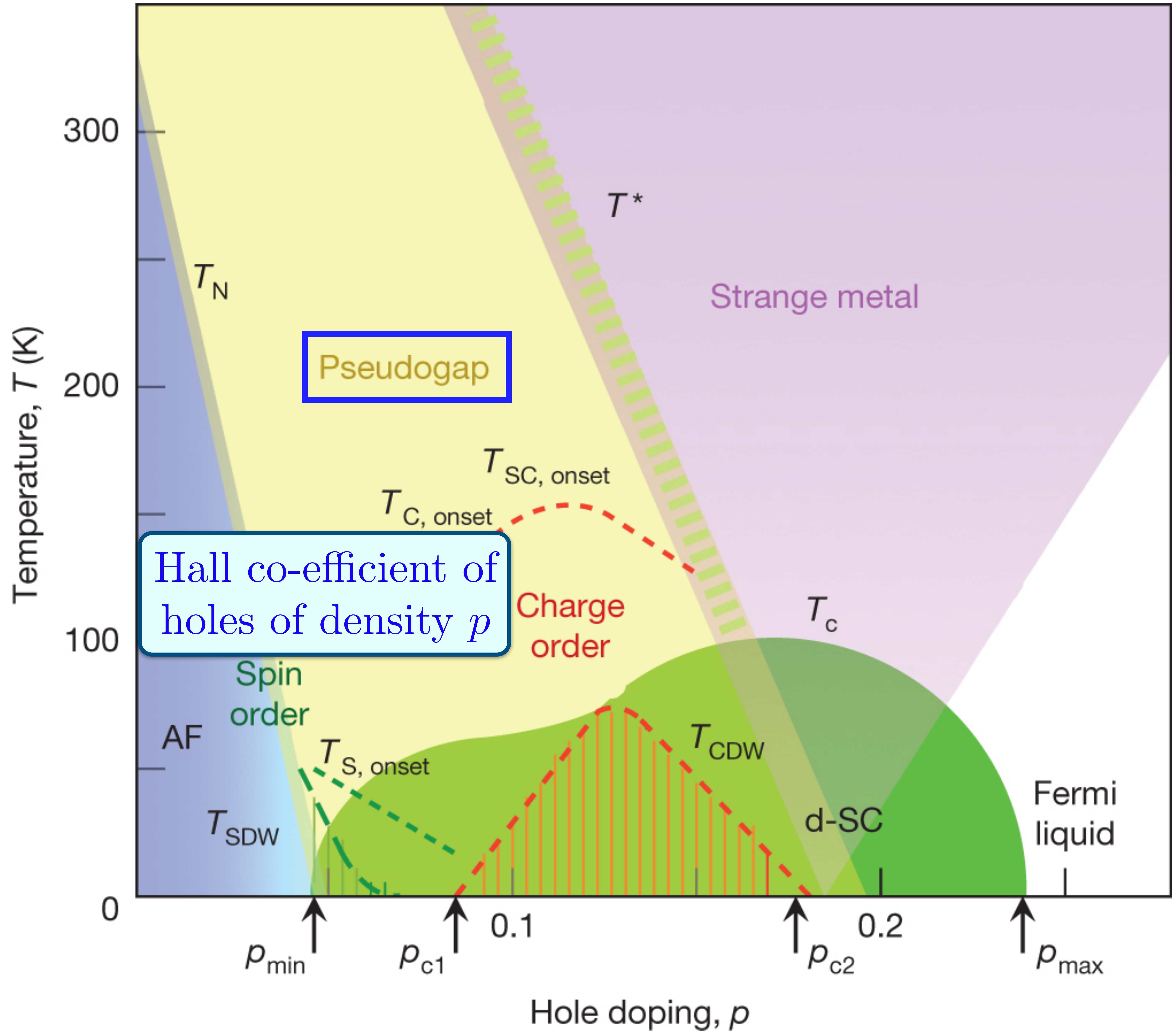
Many theories with fluctuating and intertwined AFM, d-SC and charge orders.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



I argue that a better starting point is a novel quantum ground state with no broken symmetry: the Fractionalized Fermi liquid (FL*)

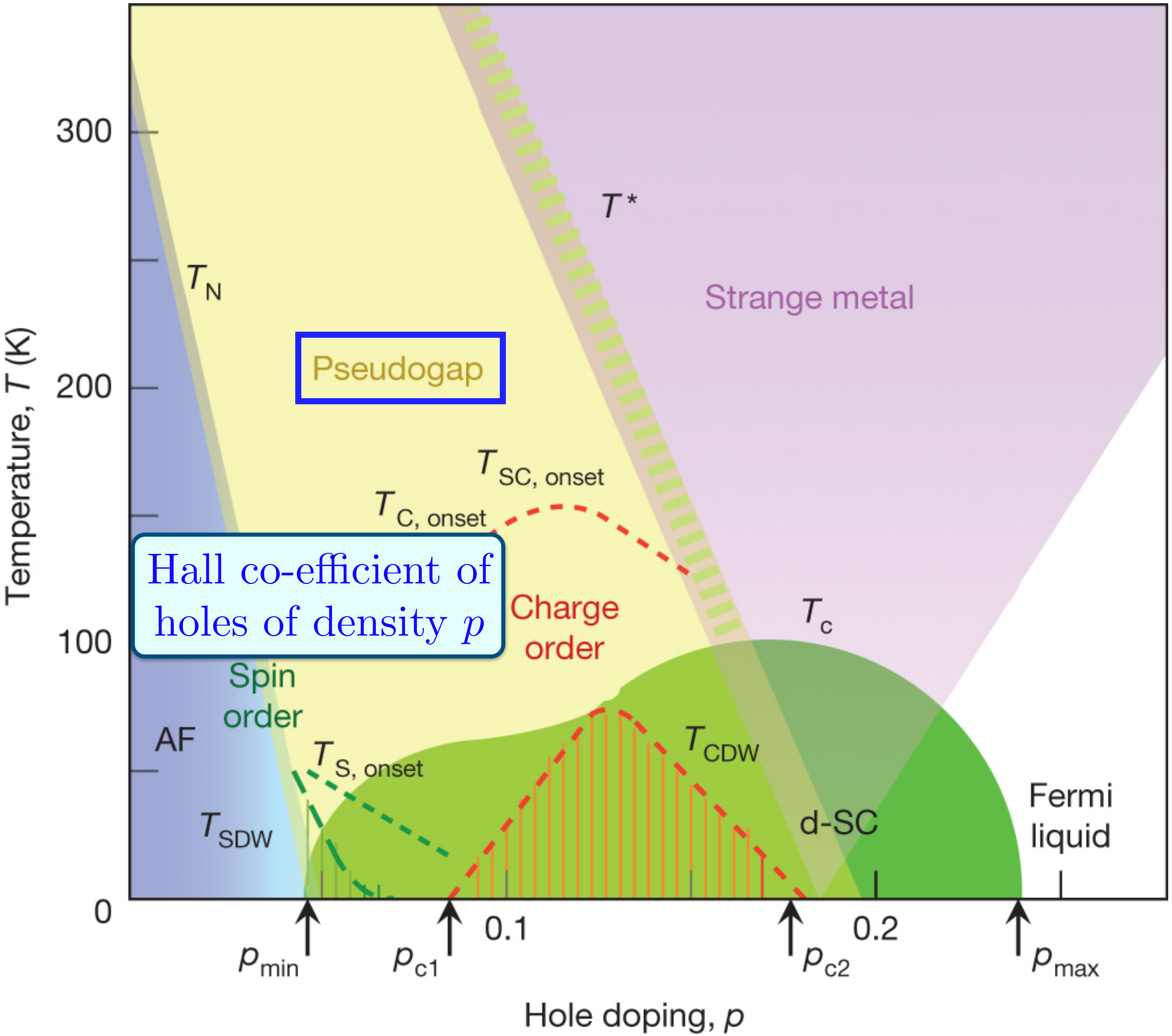
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



I argue that a better starting point is a novel quantum ground state with no broken symmetry: the Fractionalized Fermi liquid (FL*)

1. Explains Hall effect

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



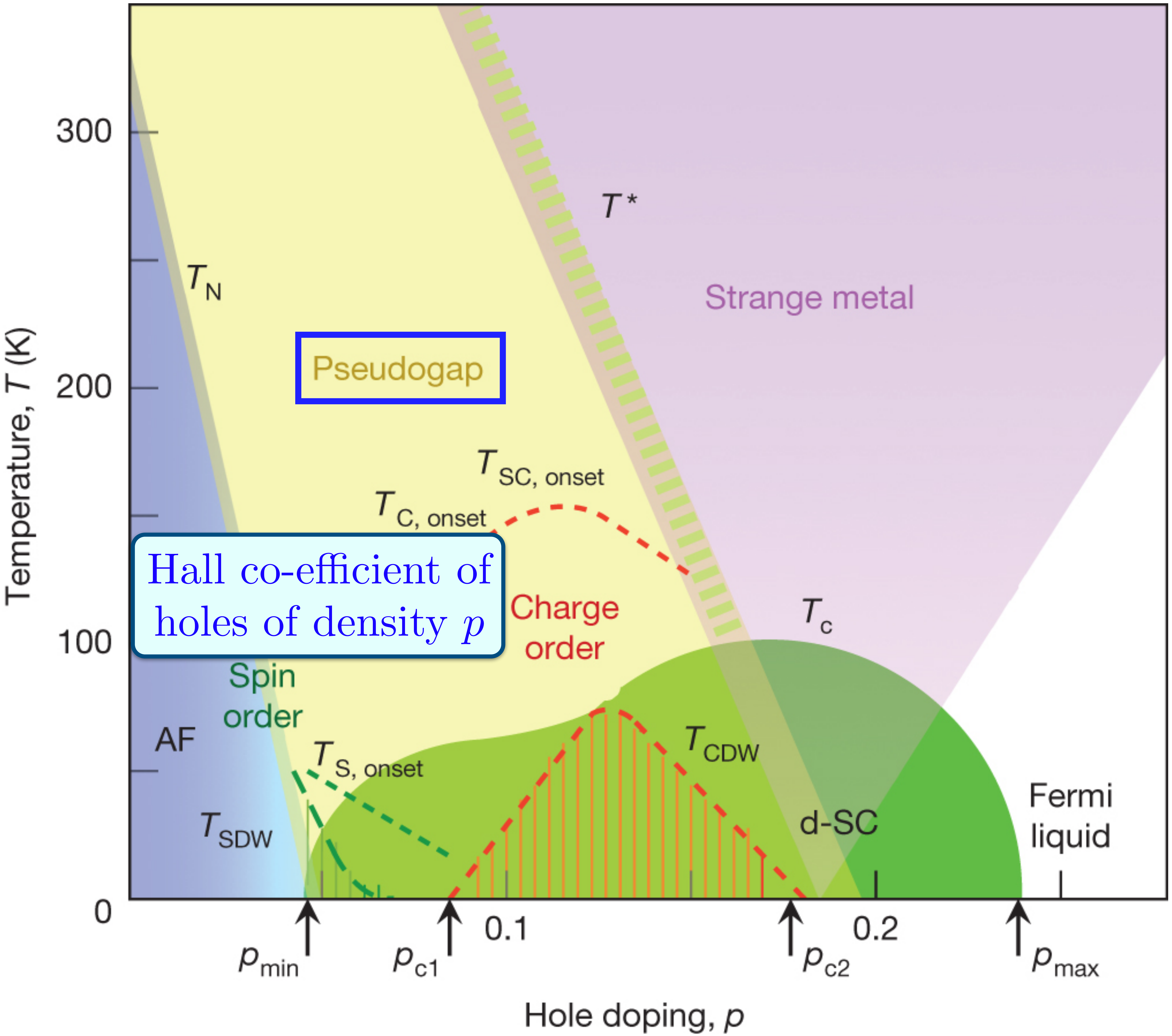
I argue that a better starting point is a novel quantum ground state with no broken symmetry: the Fractionalized Fermi liquid (FL*)

1. Explains Hall effect

2. Recent angle-dependent magnetoresistance (ADMR) experiments support hole pockets with coherent interlayer tunneling of quasiparticles.

Fang et al., *Nature Physics* **18**, 558 (2022)
Chan et al., *Nature Physics* **21**, 1753 (2025)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



I argue that a better starting point is a novel quantum ground state with no broken symmetry: the Fractionalized Fermi liquid (FL*)

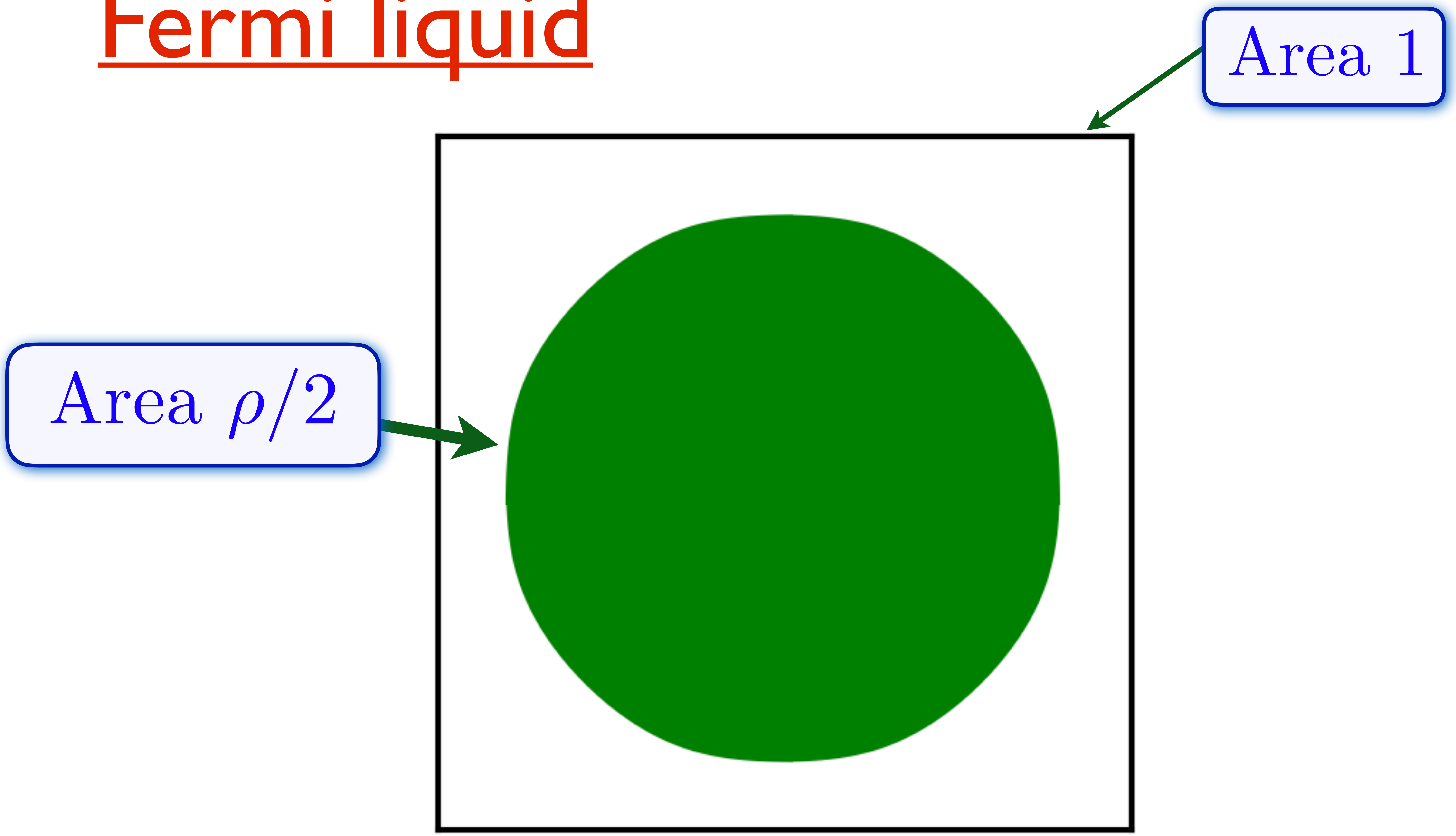
Fluctuations about this state are described by a SU(2) gauge theory which yields a *fractionalized* description of intertwined orders: the order parameters are gauge-invariant composites of a *Higgs field B*

Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density ρ

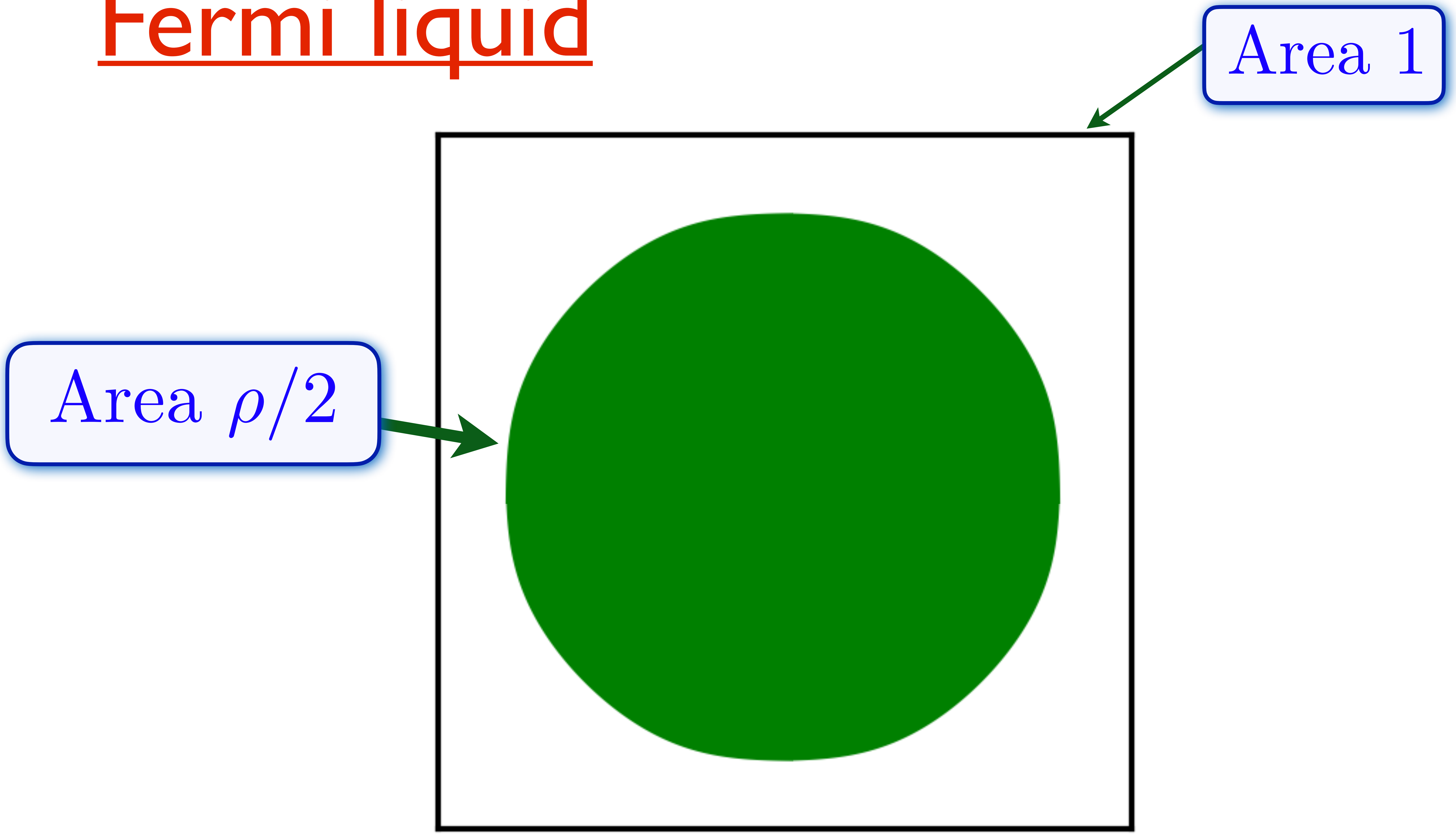


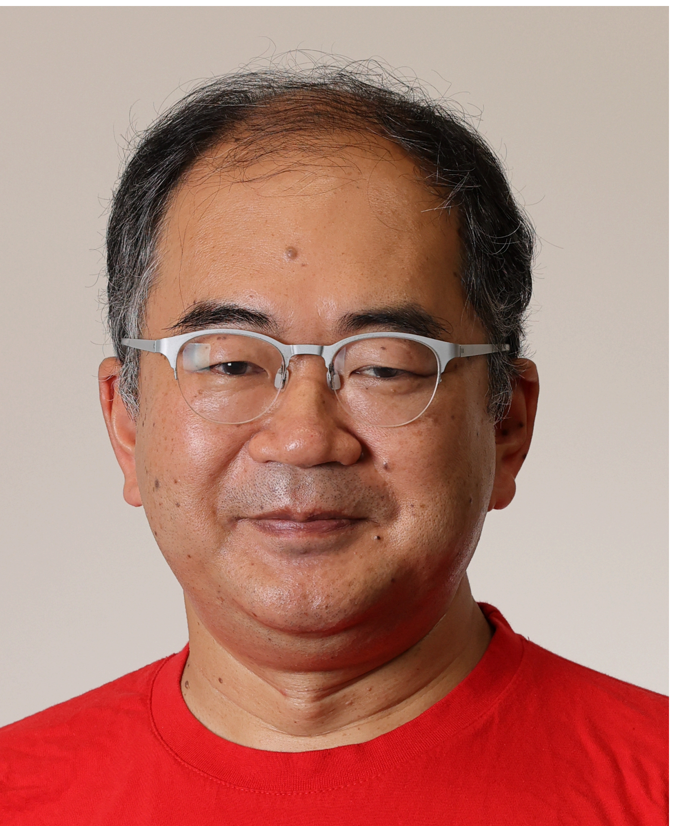
Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Fermi liquid

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density ρ





Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Oshikawa, 2000: Area constrained by an anomaly-argument of global U(1) and translations

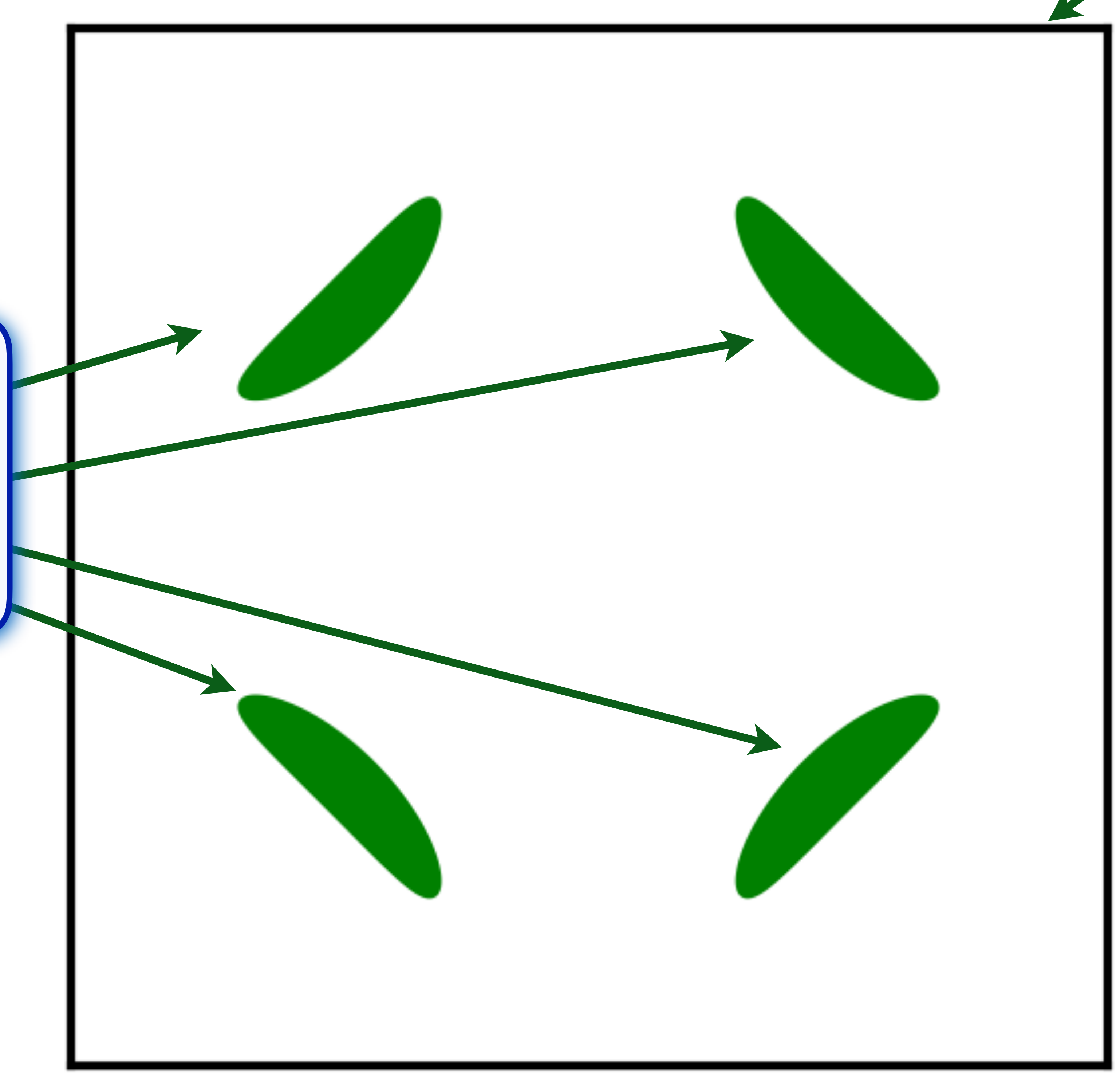
Fractionalized Fermi liquid (FL*)

Area 1

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density $\rho - 1$

Total area
$(\rho - 1)/2$



No
broken
symmetry.

Area per
pocket
$= p/8$

Oshikawa anomaly-argument is satisfied by
the sum of spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

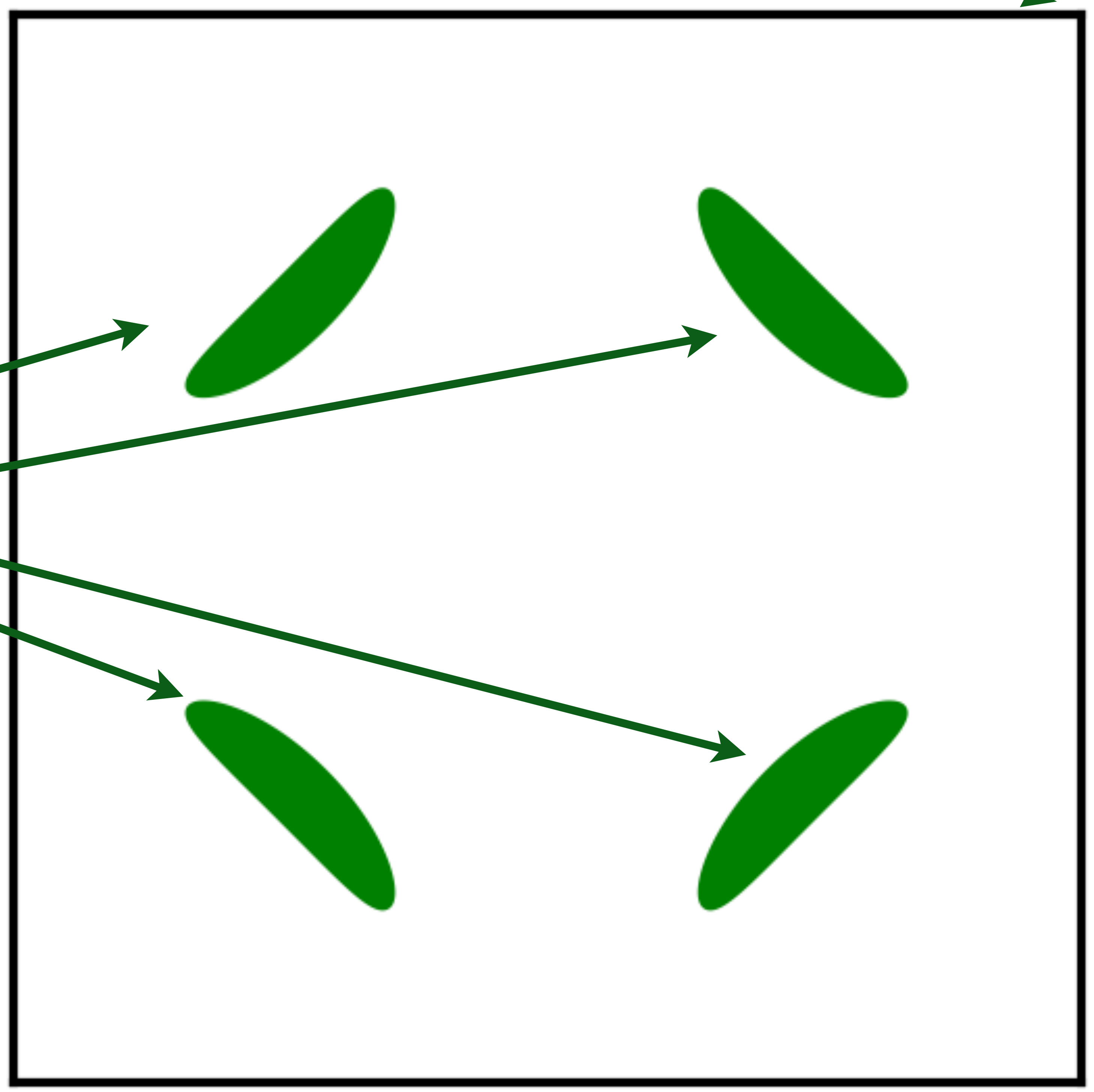
Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density $\rho - 1$

Total area
$(\rho - 1)/2$

Area 1



The density deficit (1) in the area is **quantized** by rigid structure of the spin liquid.

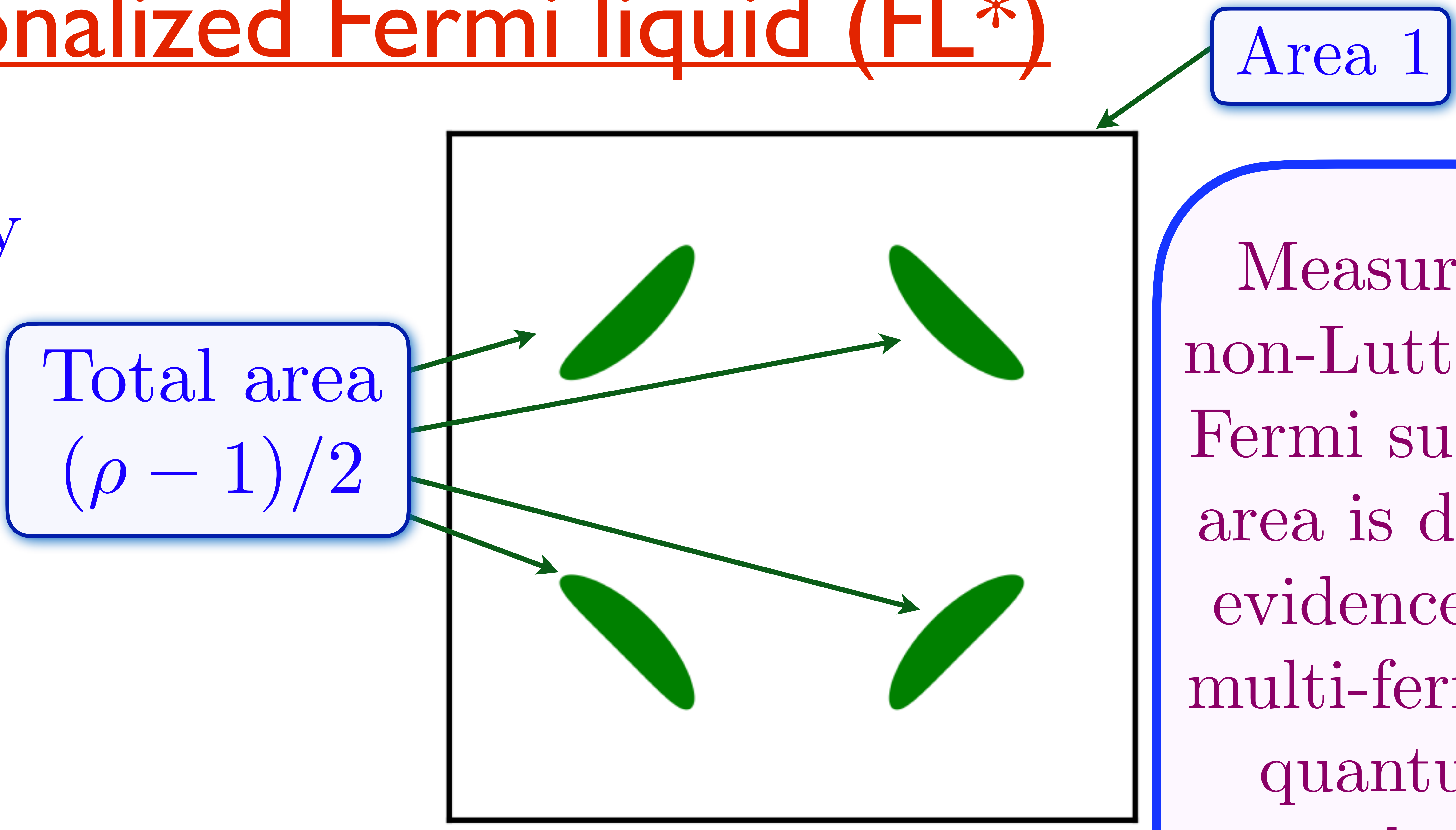
Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density $\rho - 1$

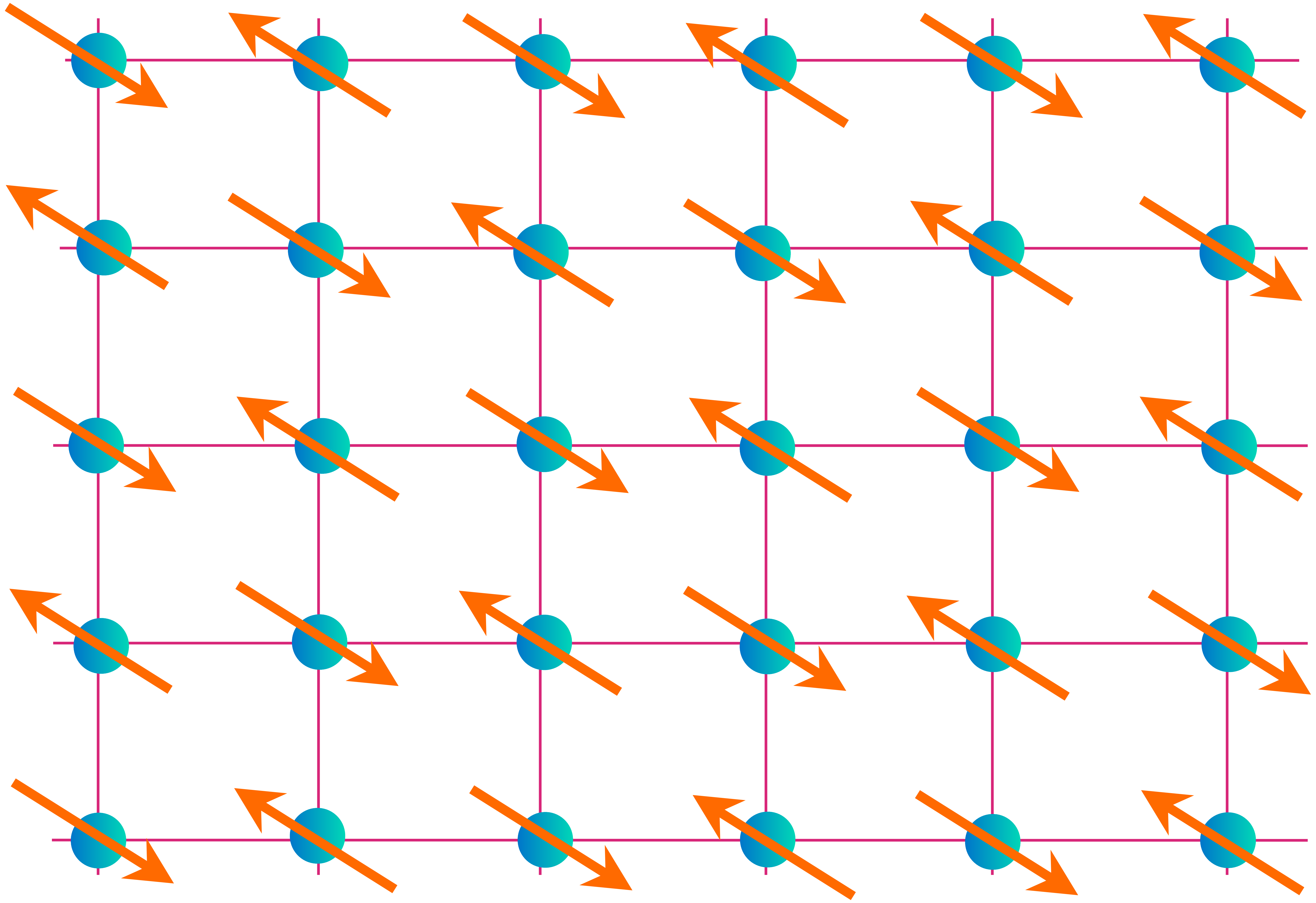


Measuring non-Luttinger Fermi surface area is direct evidence for multi-fermion quantum entanglement.

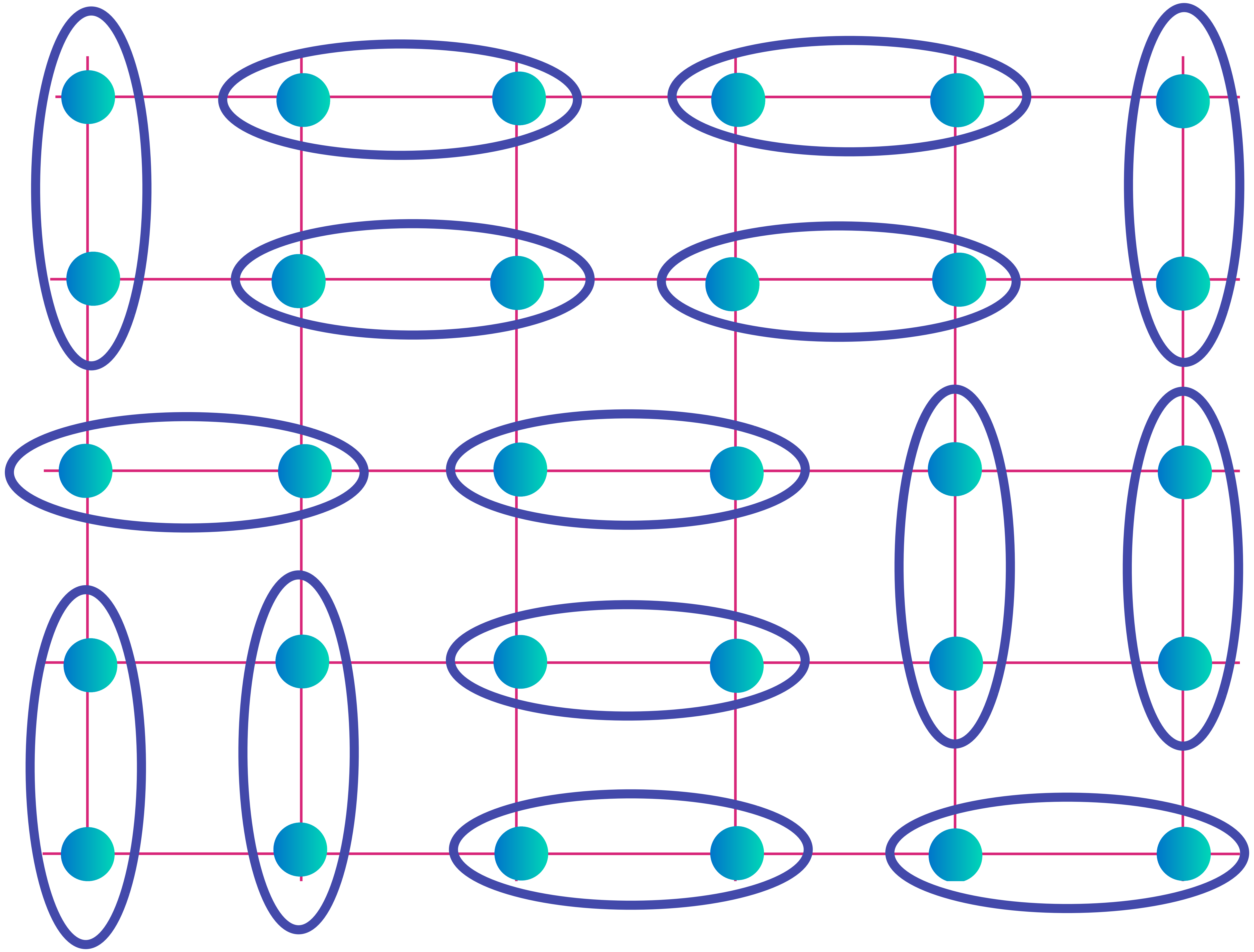
Oshikawa anomaly-argument is satisfied by
the sum of spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Antiferromagnet

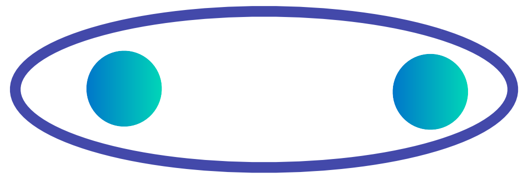


Anderson's Resonating Valence Bond (1972,1987)

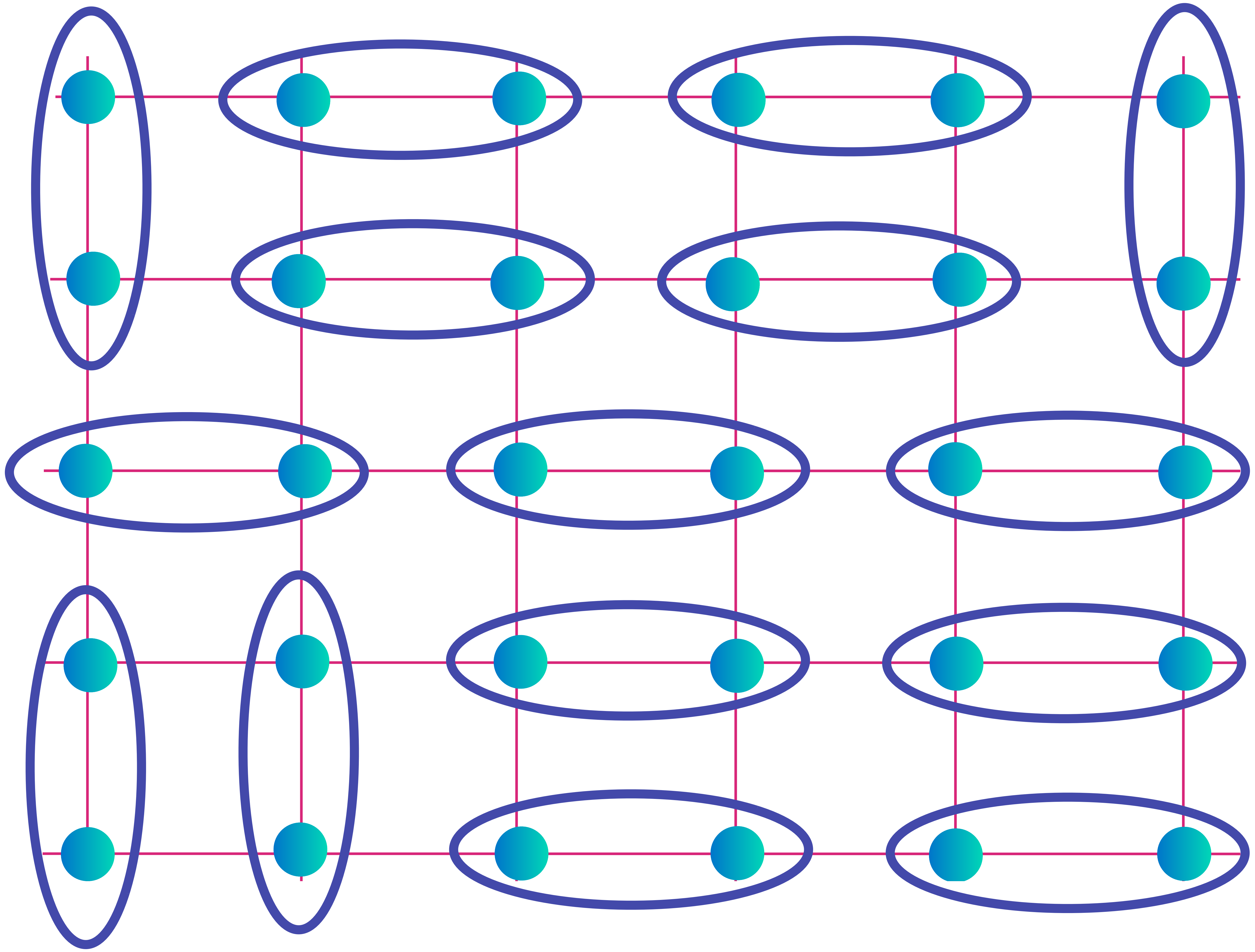


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

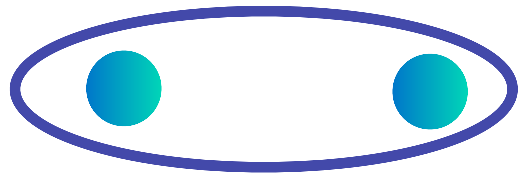
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)

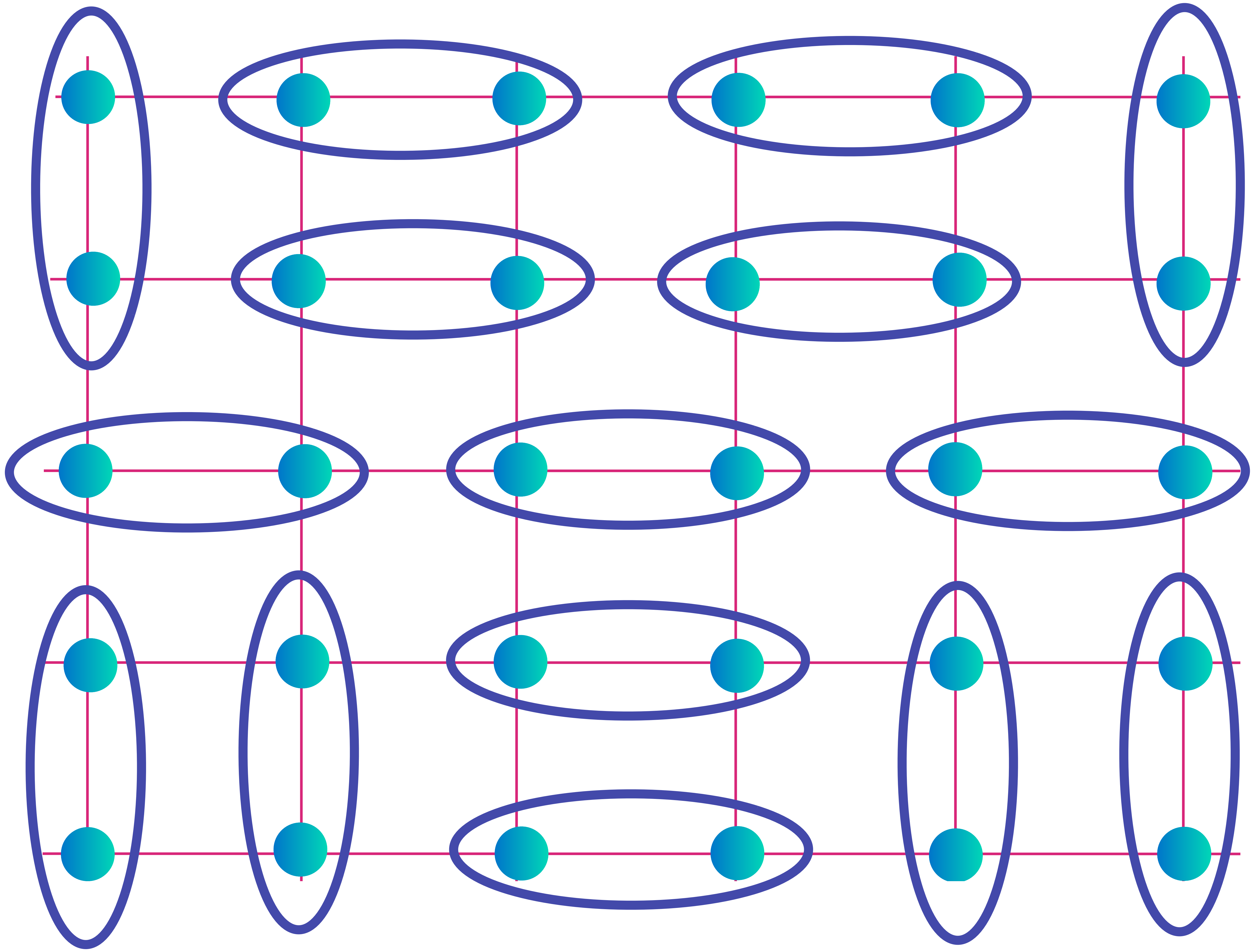


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

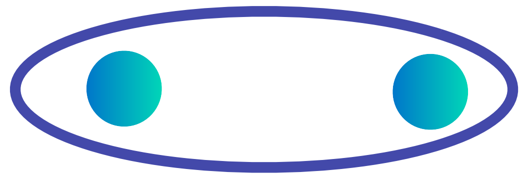
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)

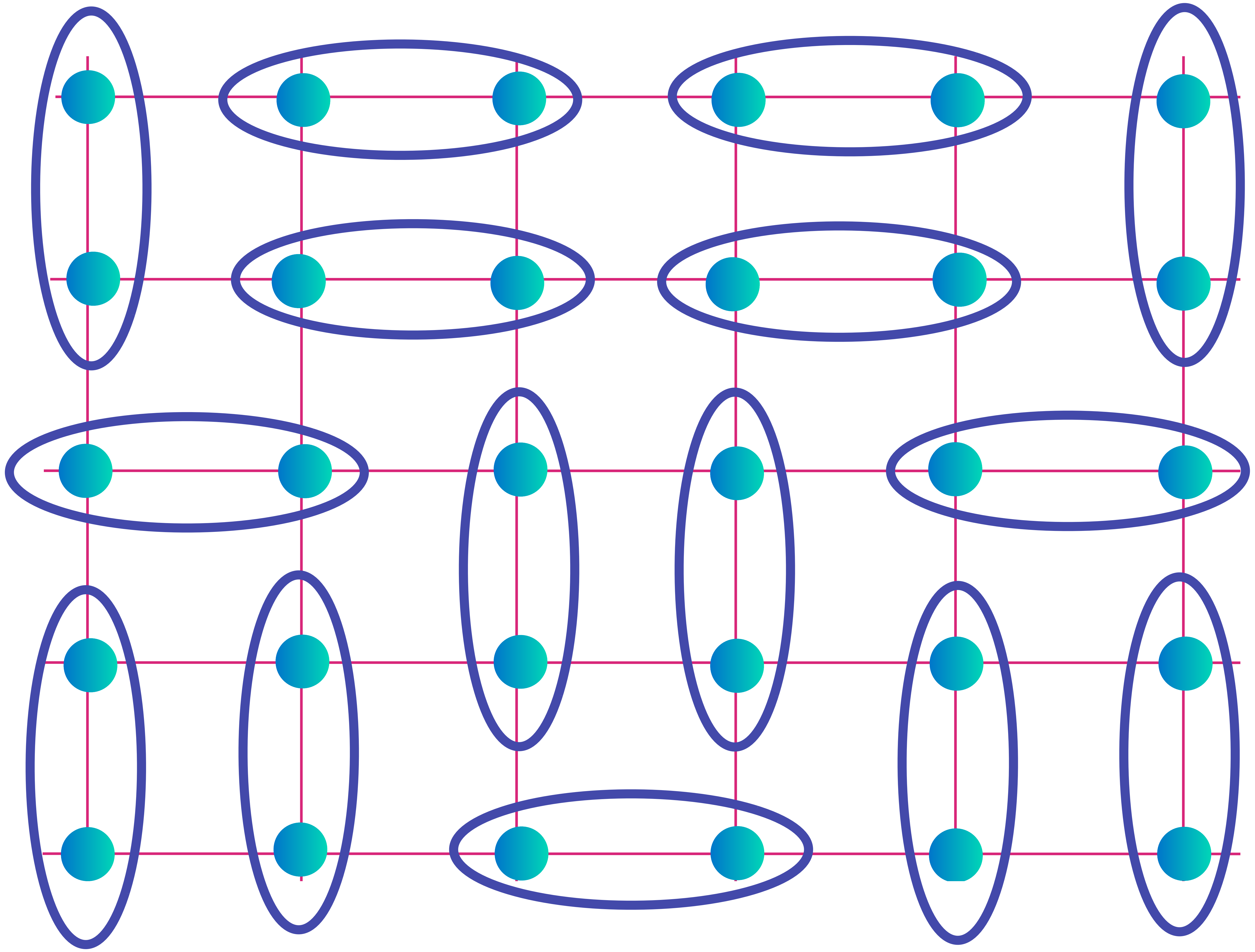


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

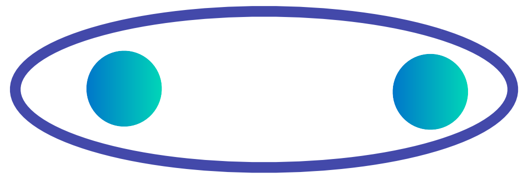
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)

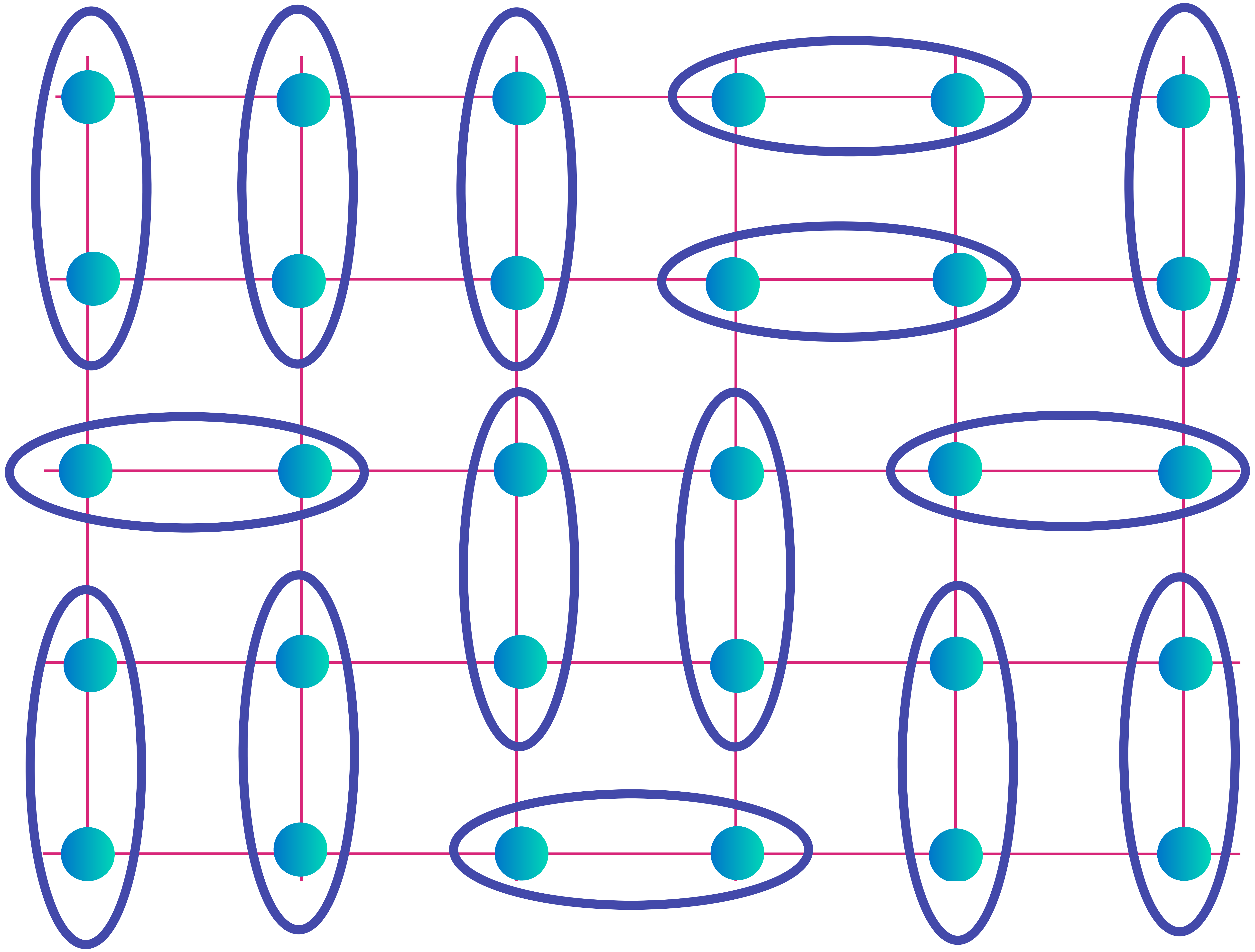


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

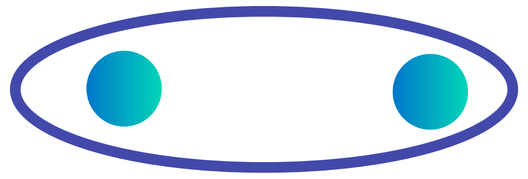
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)

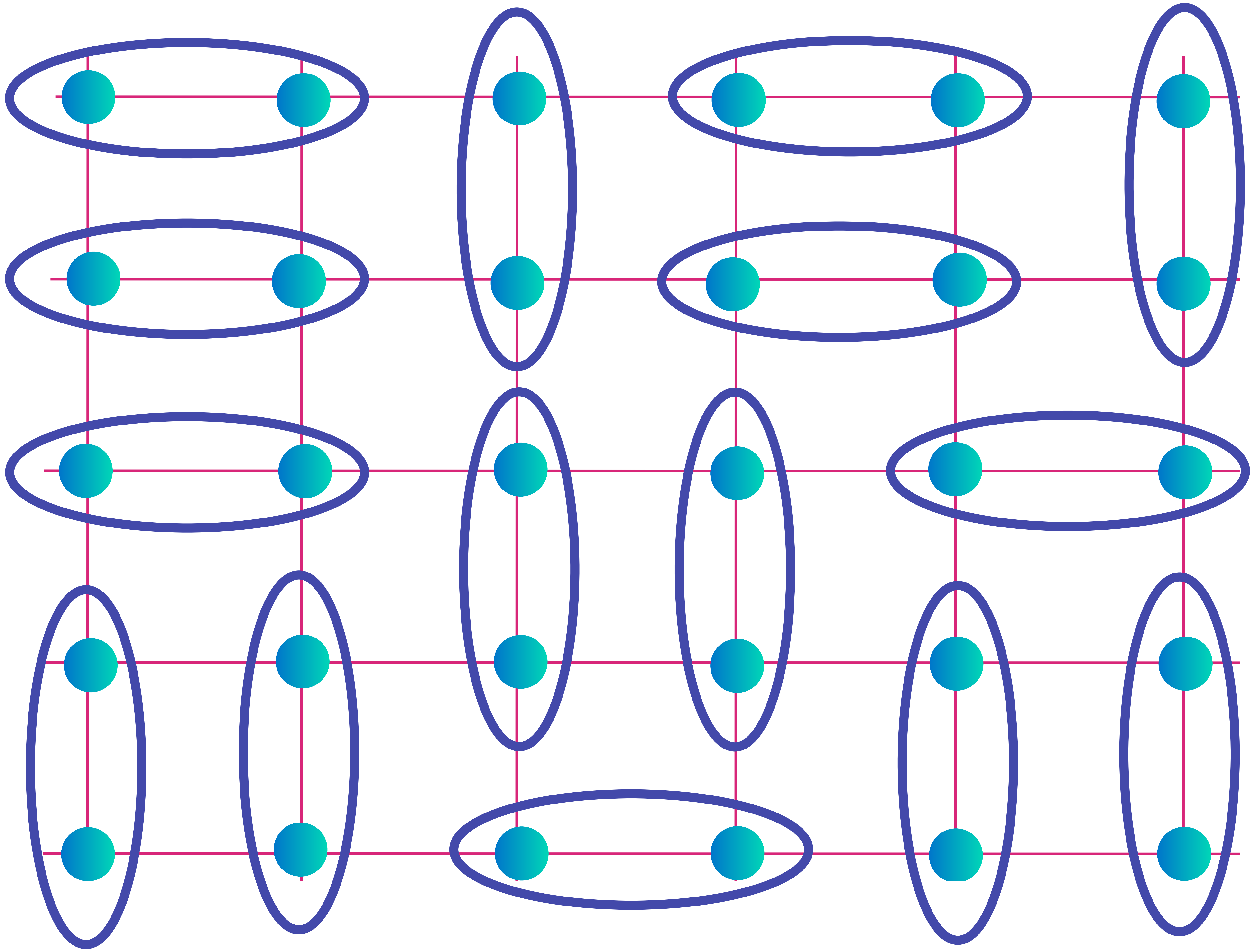


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

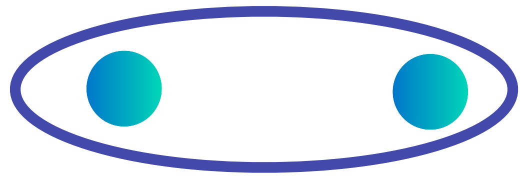
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)

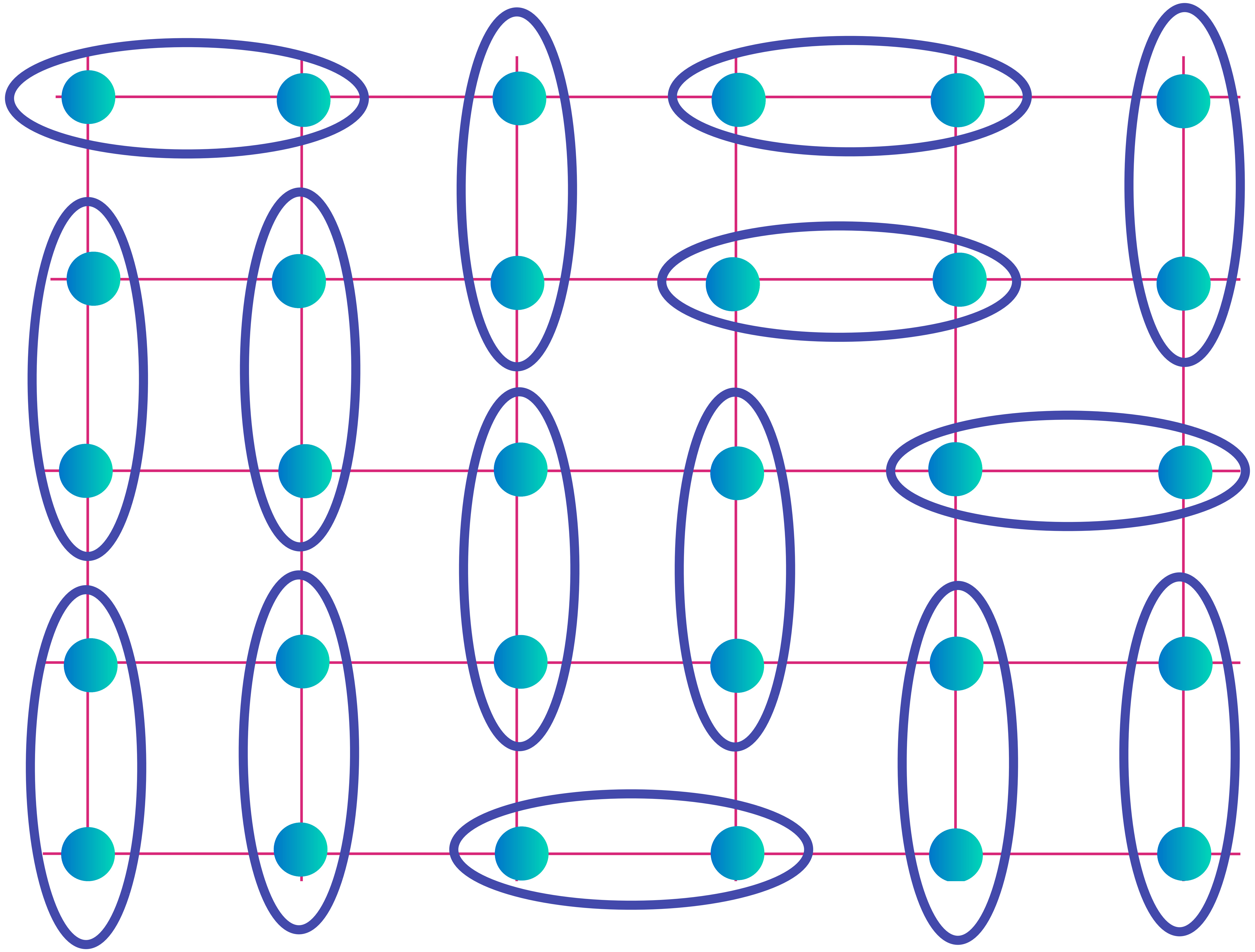


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

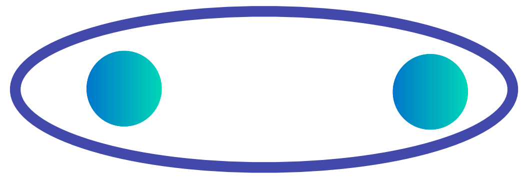
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)



$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

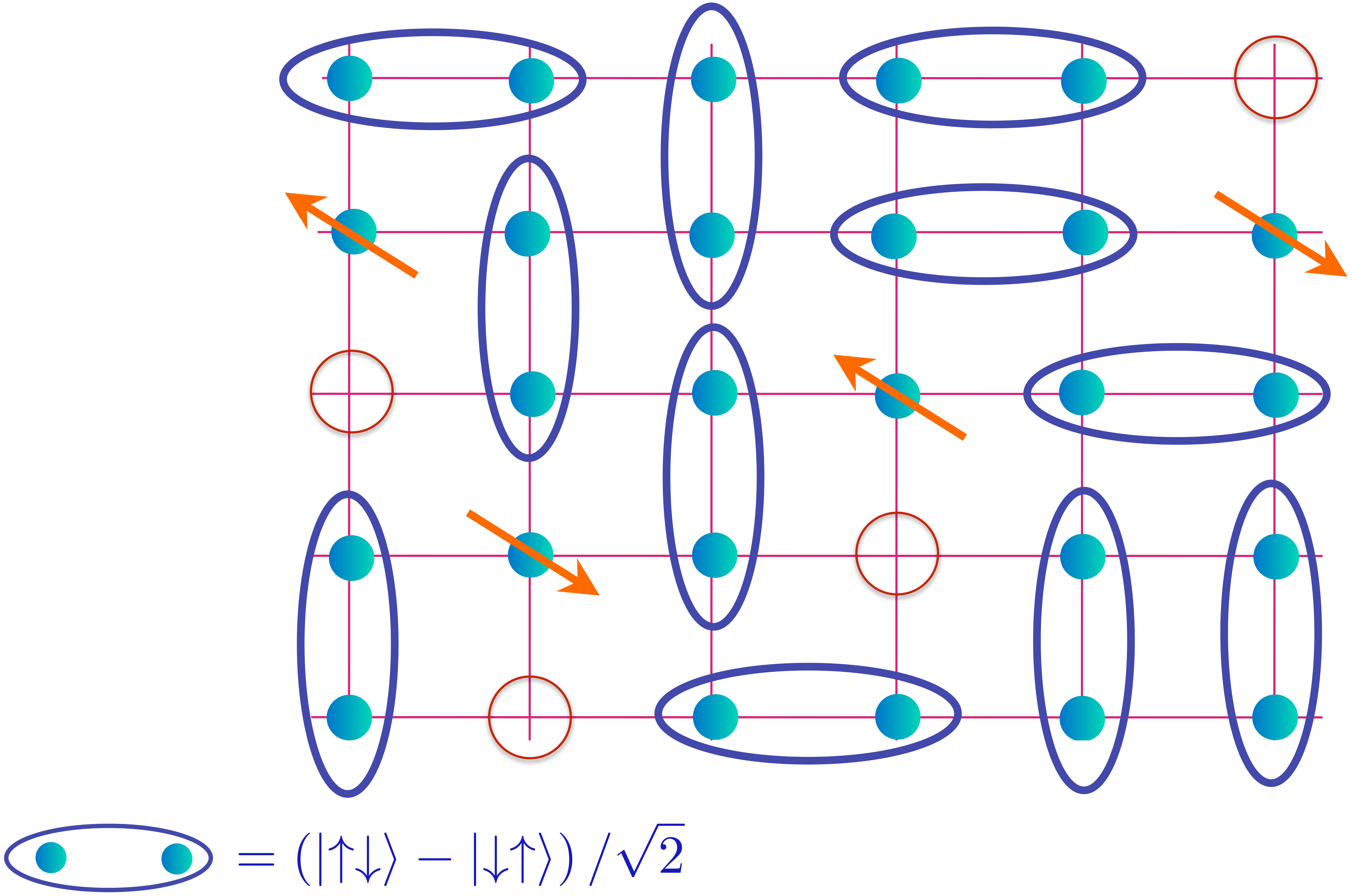
$\mathcal{D} \rightarrow$ dimer covering of lattice

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Quantum Entanglement of an infinite number of spins (bosons)!

To obtain a (super)conductor we have to remove a density p of electrons

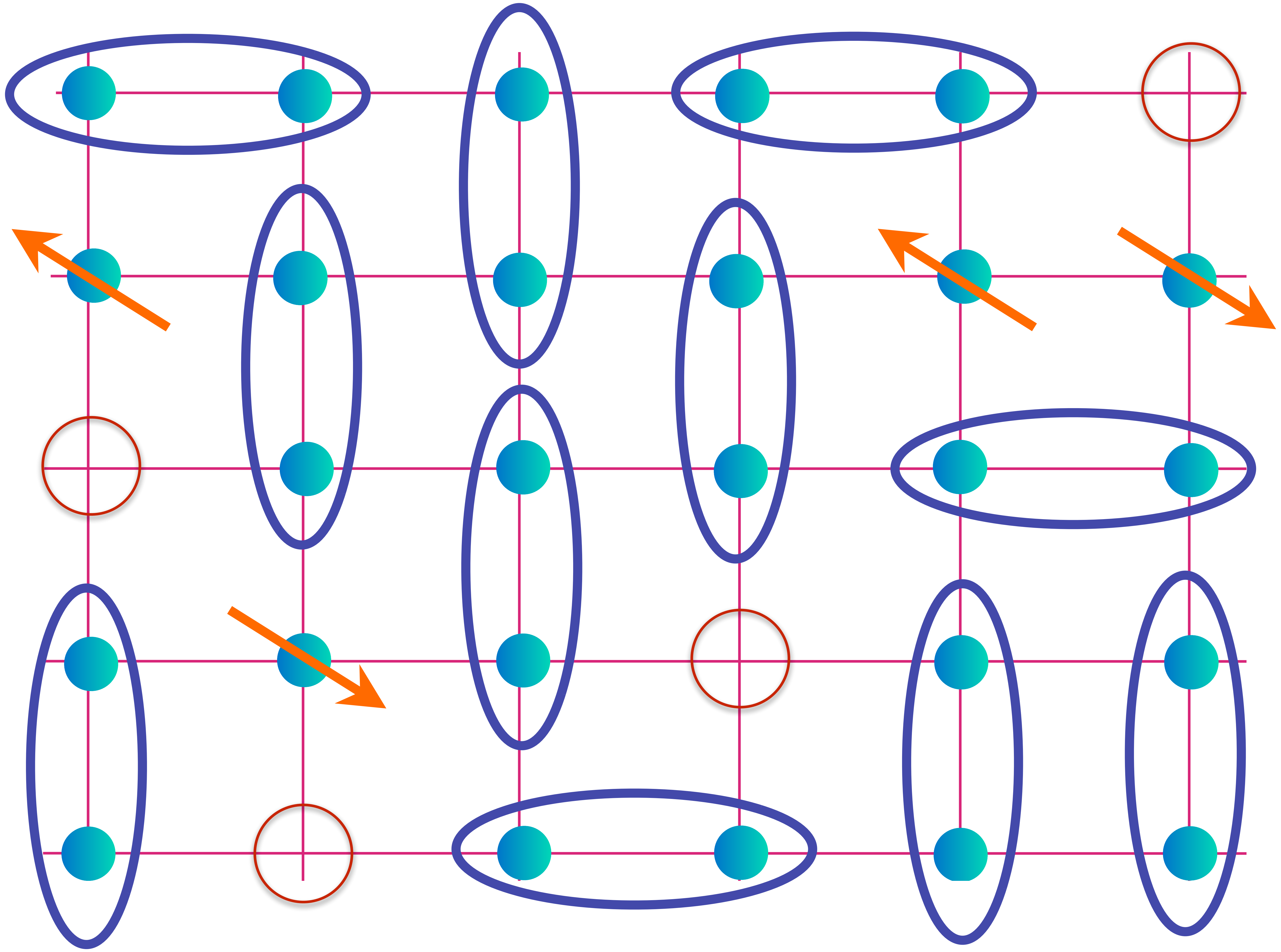
Energy cost to create spinon $\sim J$



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

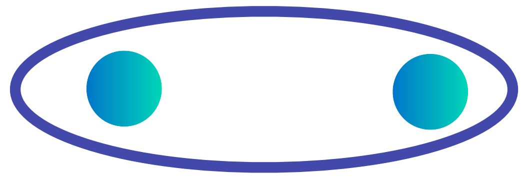
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

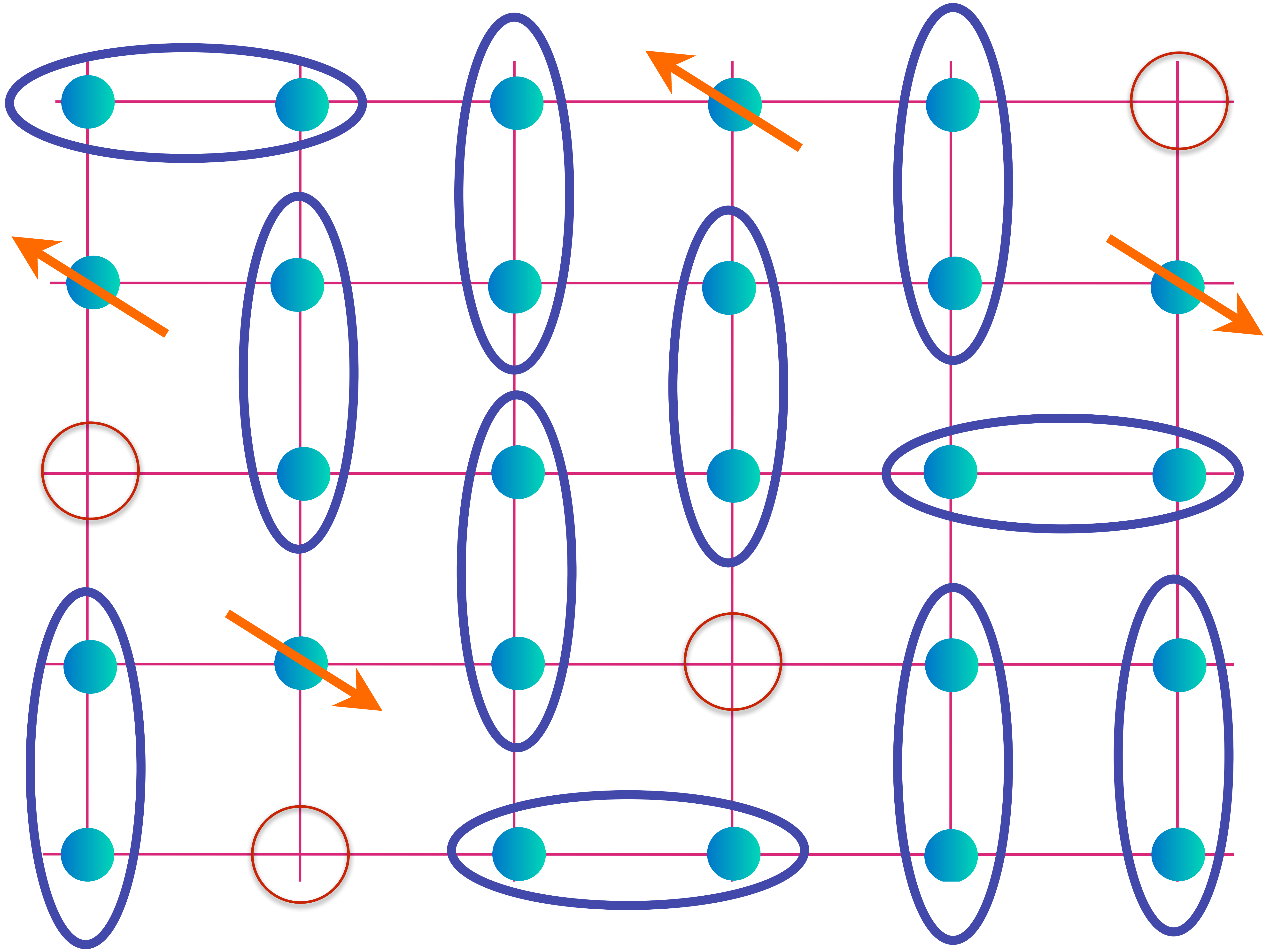


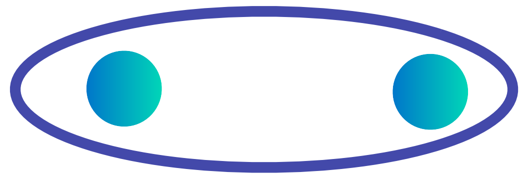
Holons
(charge e, spin 0)
and
spinons
(charge 0, spin $1/2$)
can move
independently

Holons cannot tunnel between layers

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

To obtain a (super)conductor we have to remove a density p of electrons



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

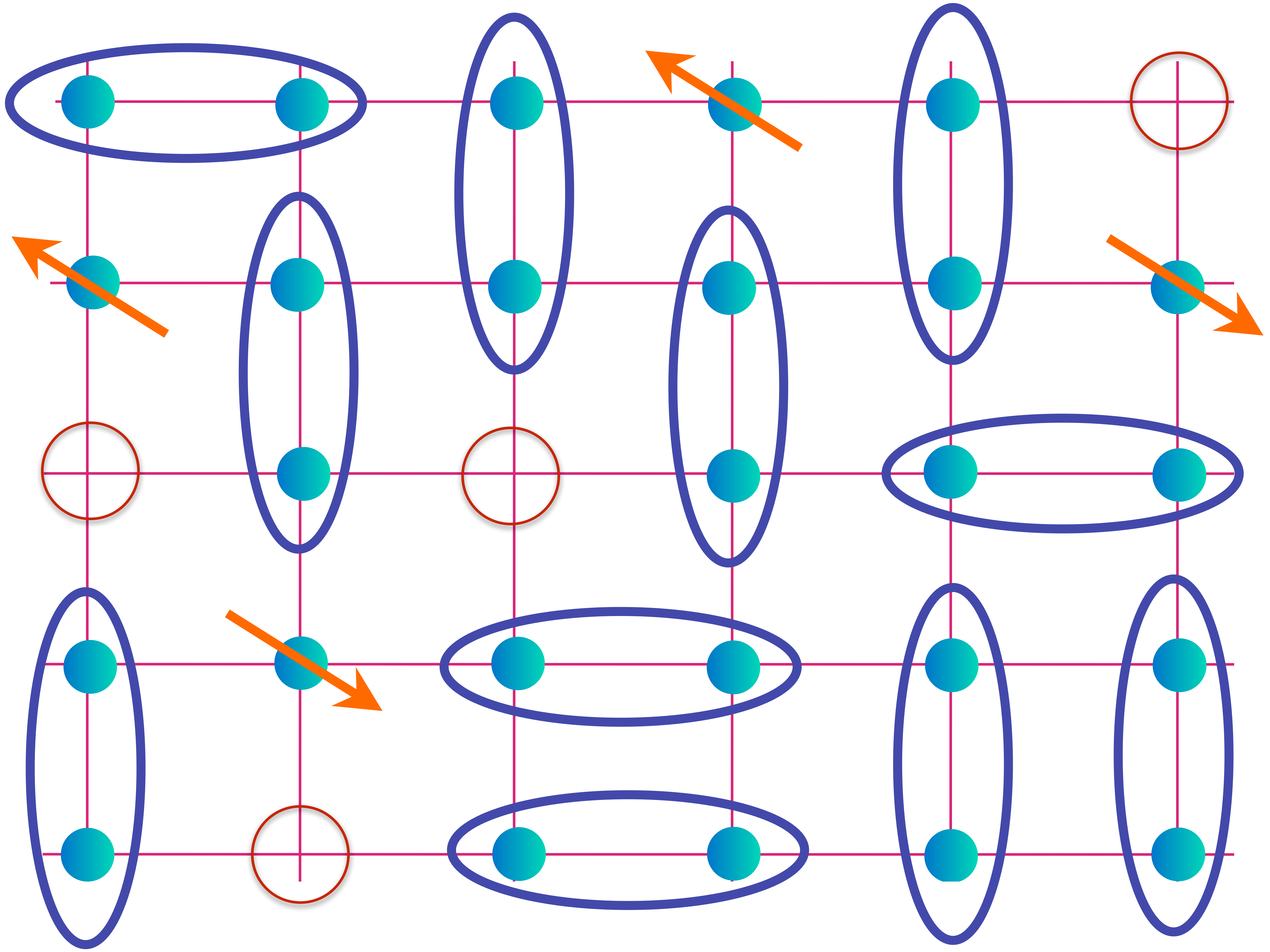
Energy cost to create spinon $\sim J$

Holons (charge e, spin 0) and spinons (charge 0, spin 1/2) can move independently

Holons cannot tunnel between layers

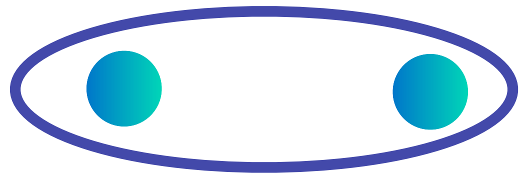
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



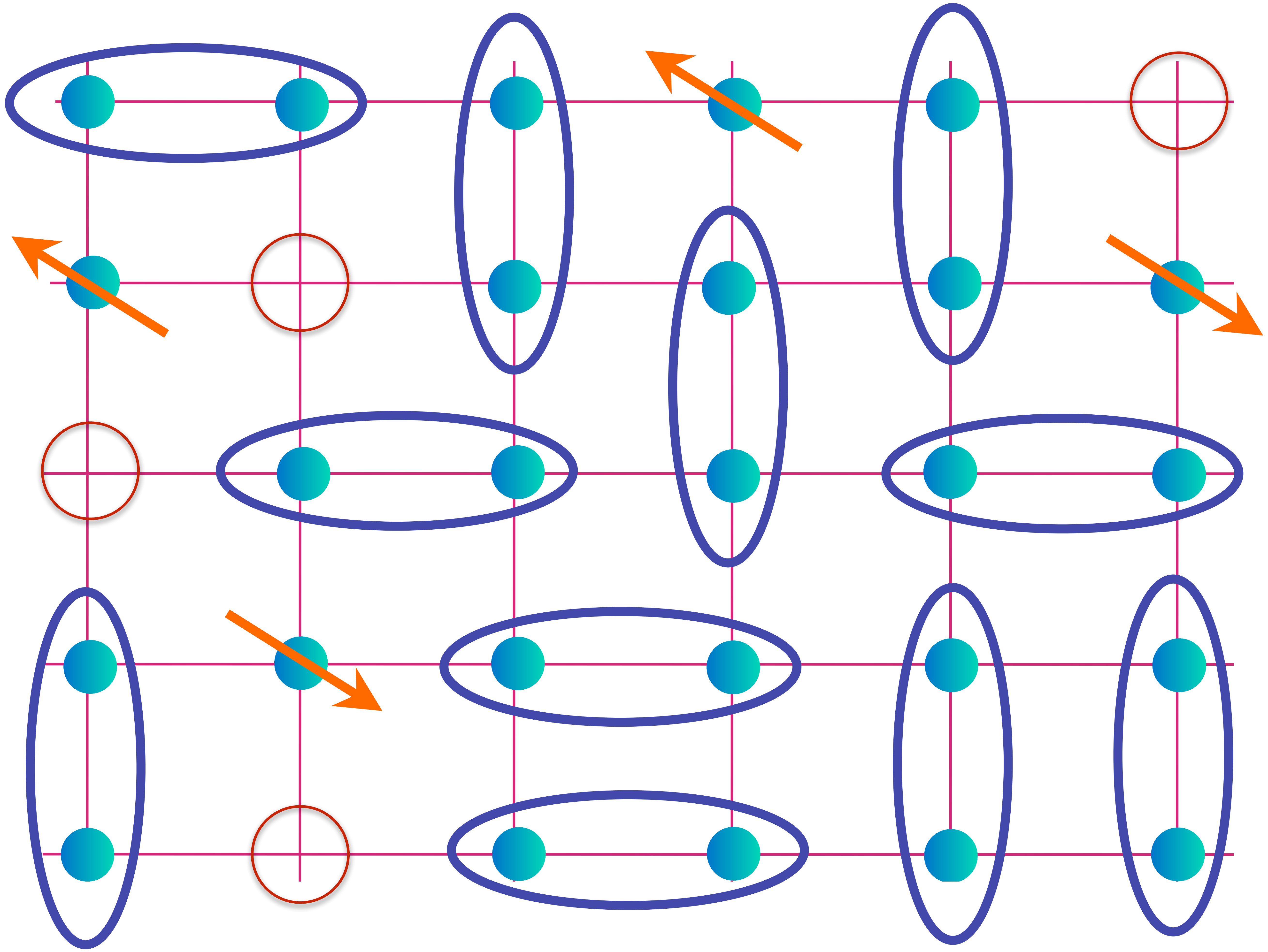
Holons (charge e, spin 0) and spinons (charge 0, spin 1/2) can move independently

Holons cannot tunnel between layers

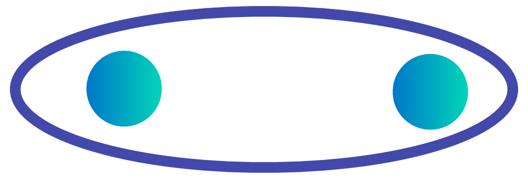
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



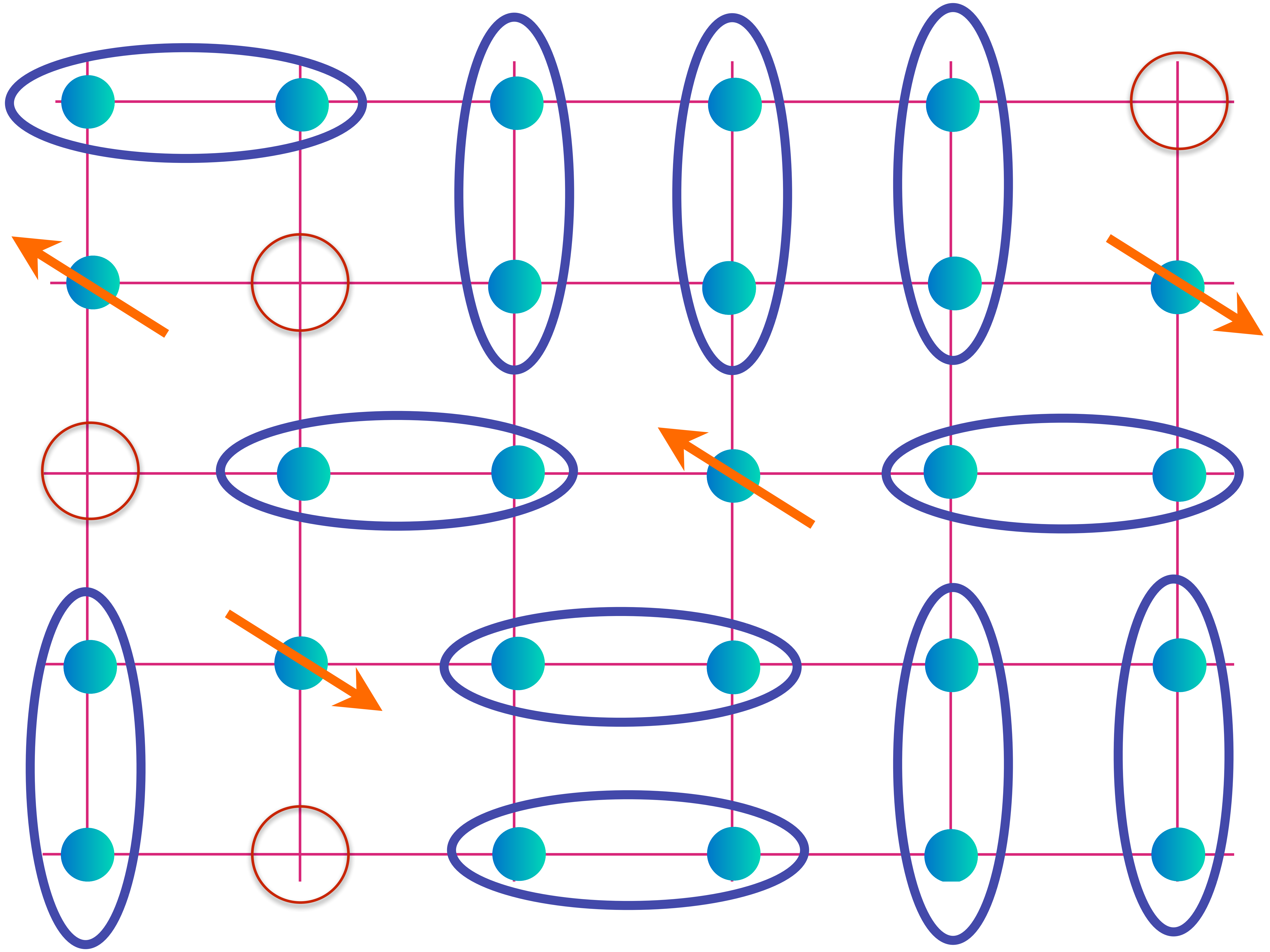
Holons (charge e, spin 0) and spinons (charge 0, spin 1/2) can move independently

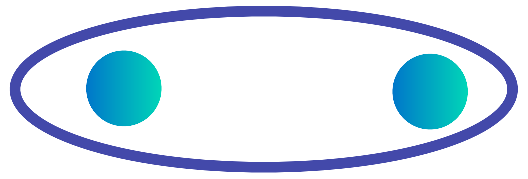
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\,/\sqrt{2}$

Holons cannot tunnel between layers

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



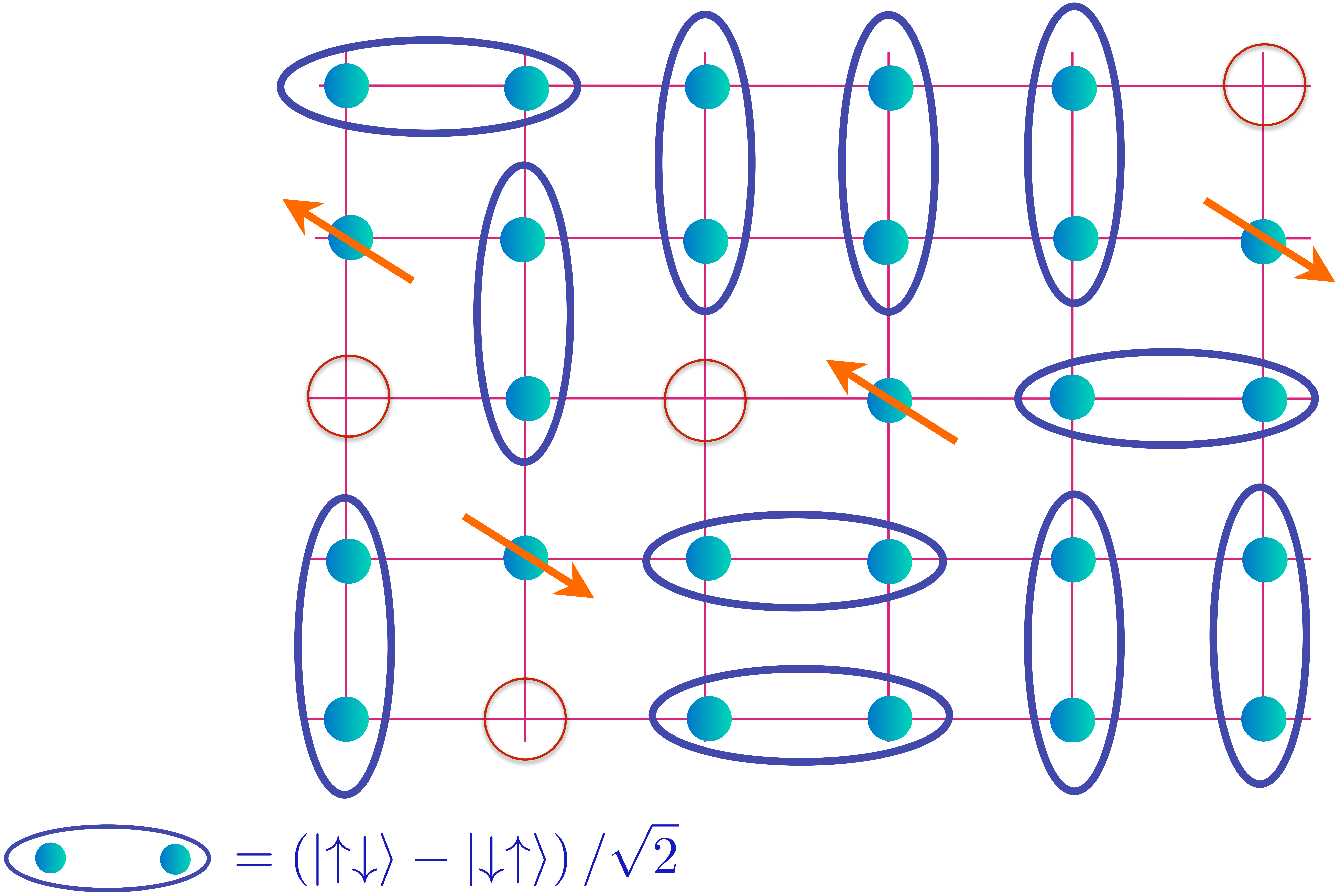
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Holons (charge e, spin 0) and spinons (charge 0, spin 1/2) can move independently

Holons cannot tunnel between layers

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



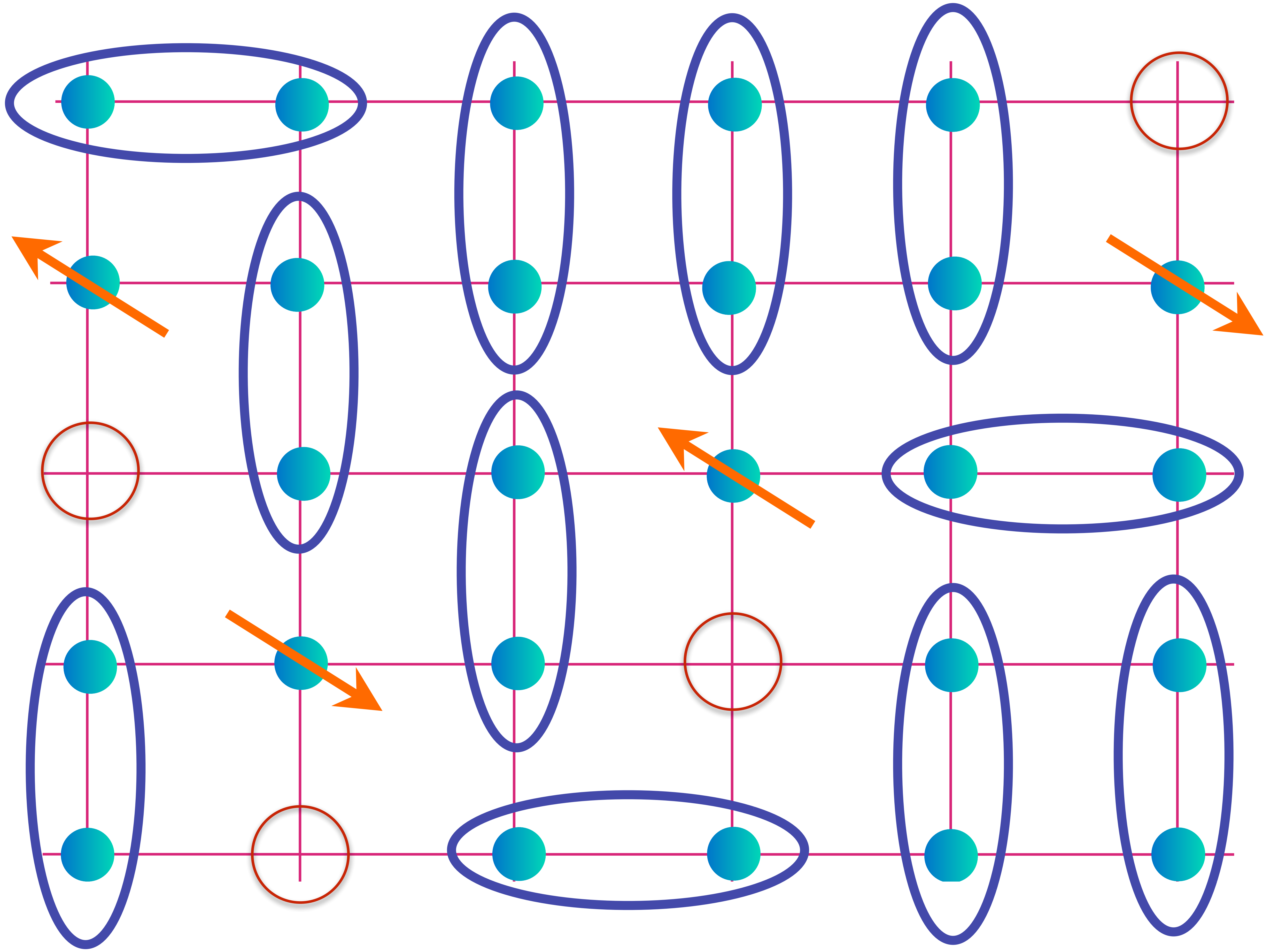
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Holons
(charge e, spin 0)
and
spinons
(charge 0, spin 1/2)
can move
independently

Holons cannot tunnel between layers

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

Holons (charge e, spin 0) and spinons (charge 0, spin 1/2) can move independently

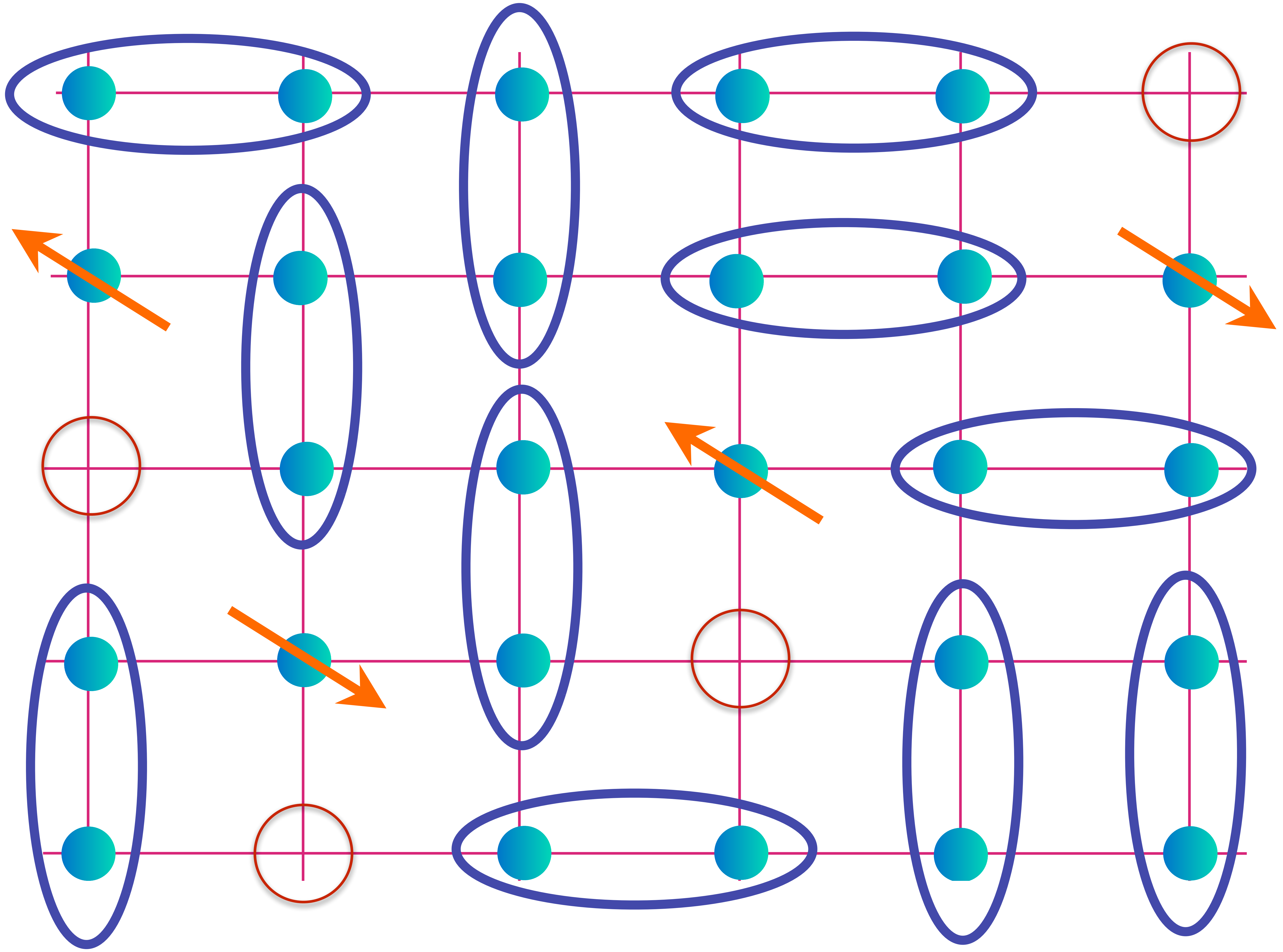
Holons cannot tunnel between layers

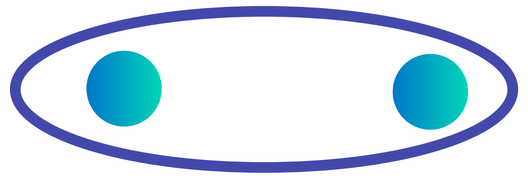
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

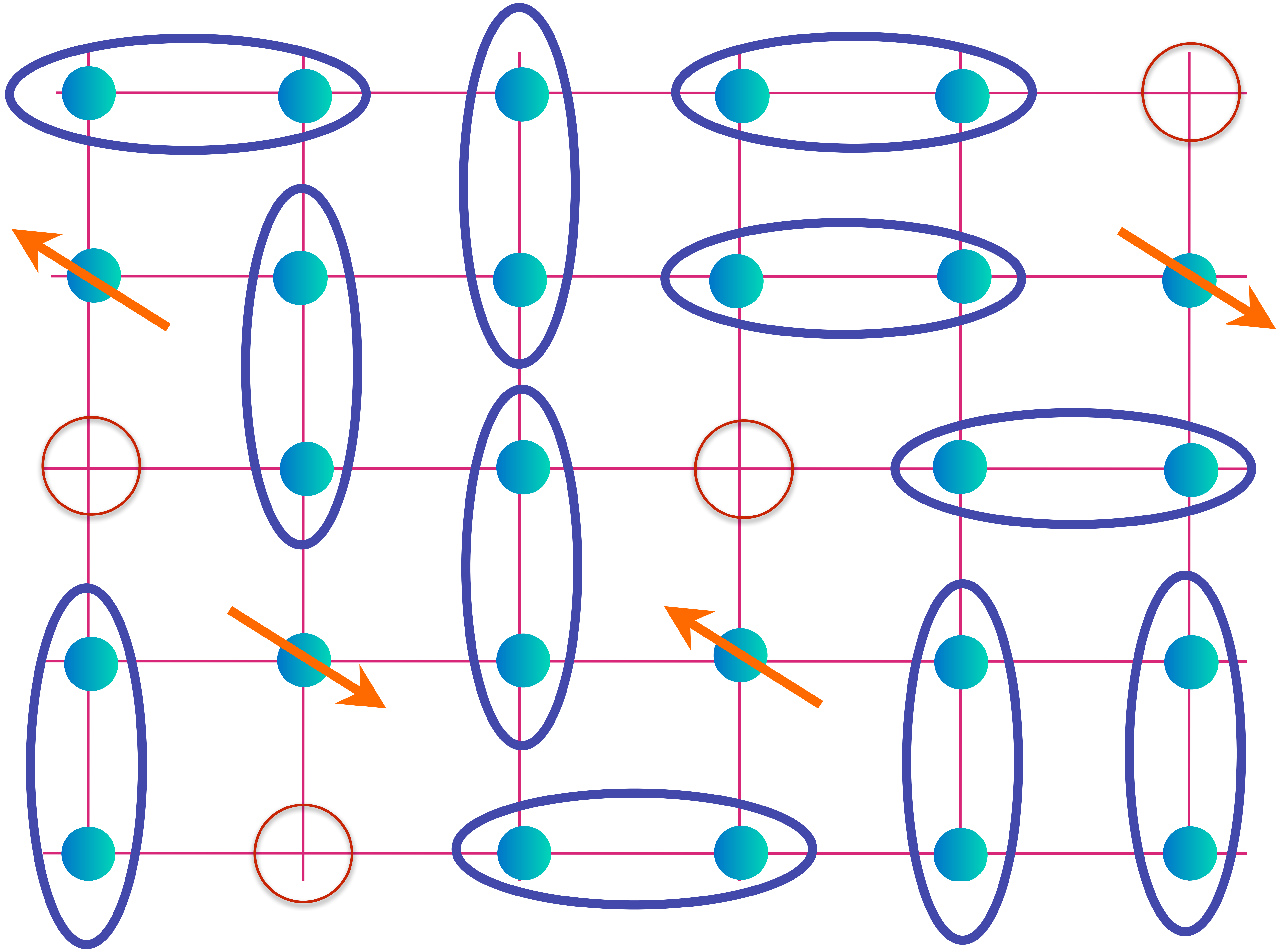
Energy gained by bound state $\sim t$.

But the holons and spinons can gain energy by resonating with each other



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

To obtain a (super)conductor we have to remove a density p of electrons



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Energy cost to create spinon $\sim J$

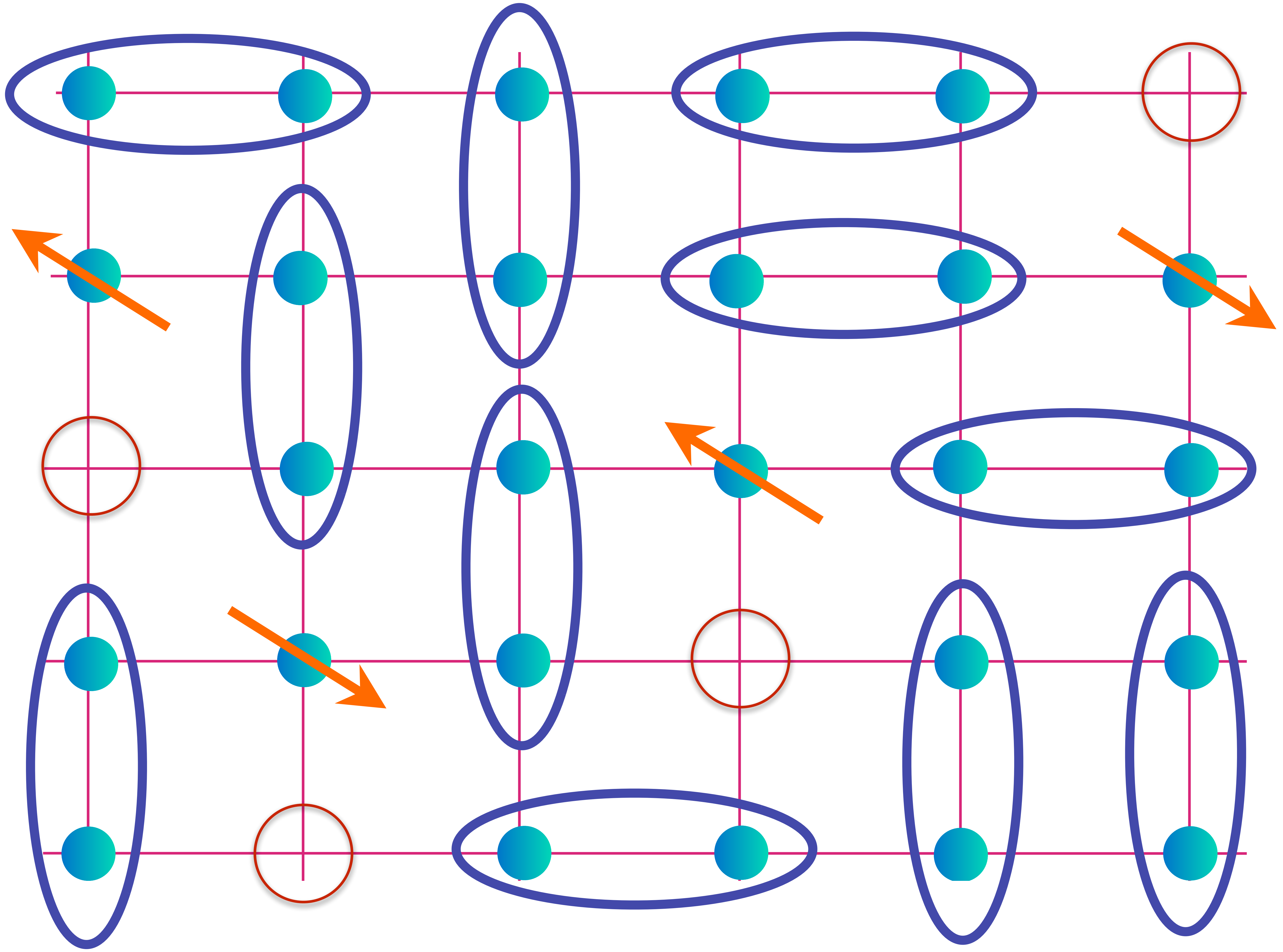
Energy gained by bound state $\sim t$.

But the holons and spinons can gain energy by resonating with each other

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



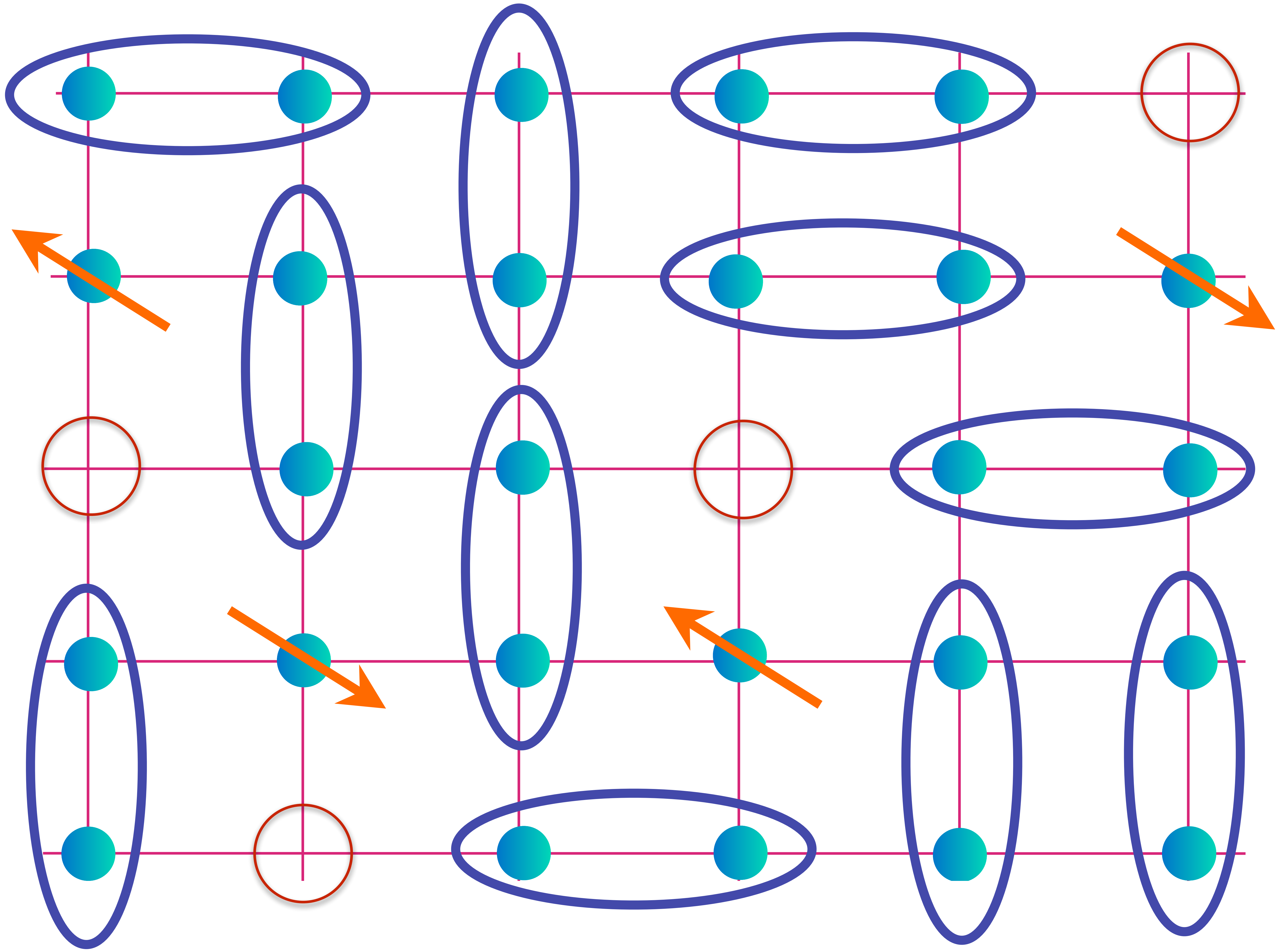
But the holons and spinons can gain energy by resonating with each other

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

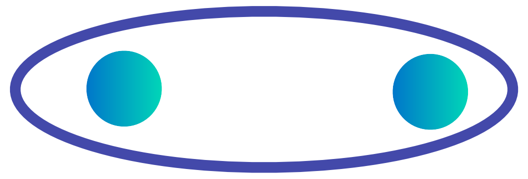
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



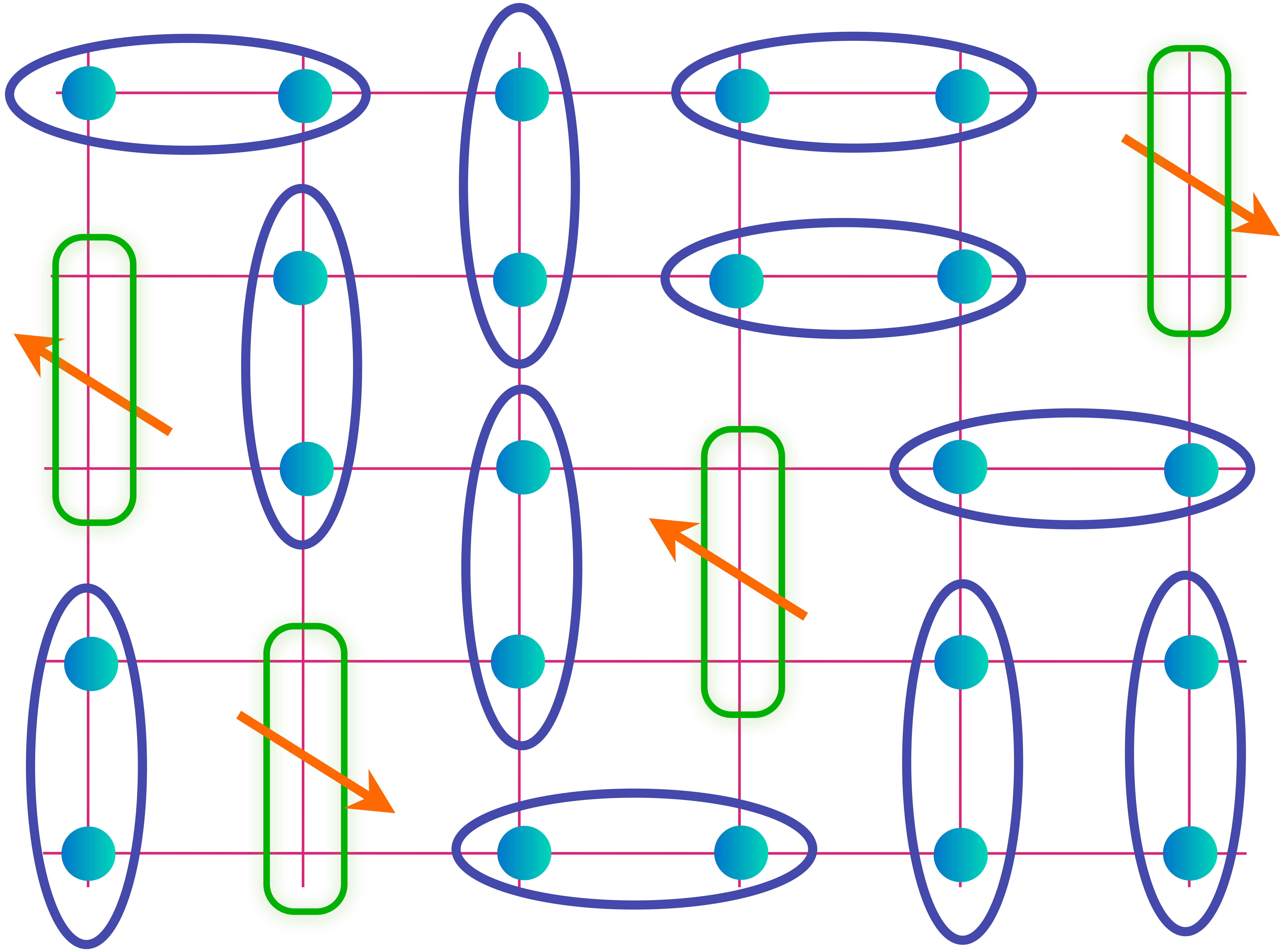
But the holons and spinons can gain energy by resonating with each other

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

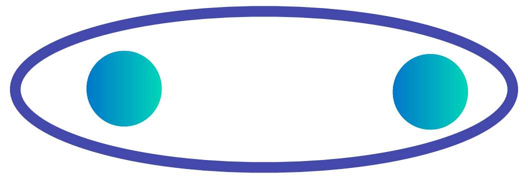
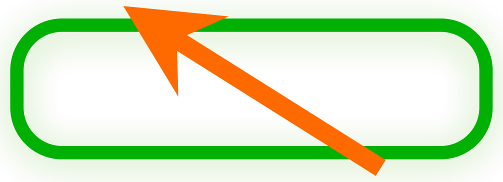
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



But the holons and spinons can gain energy by resonating with each other

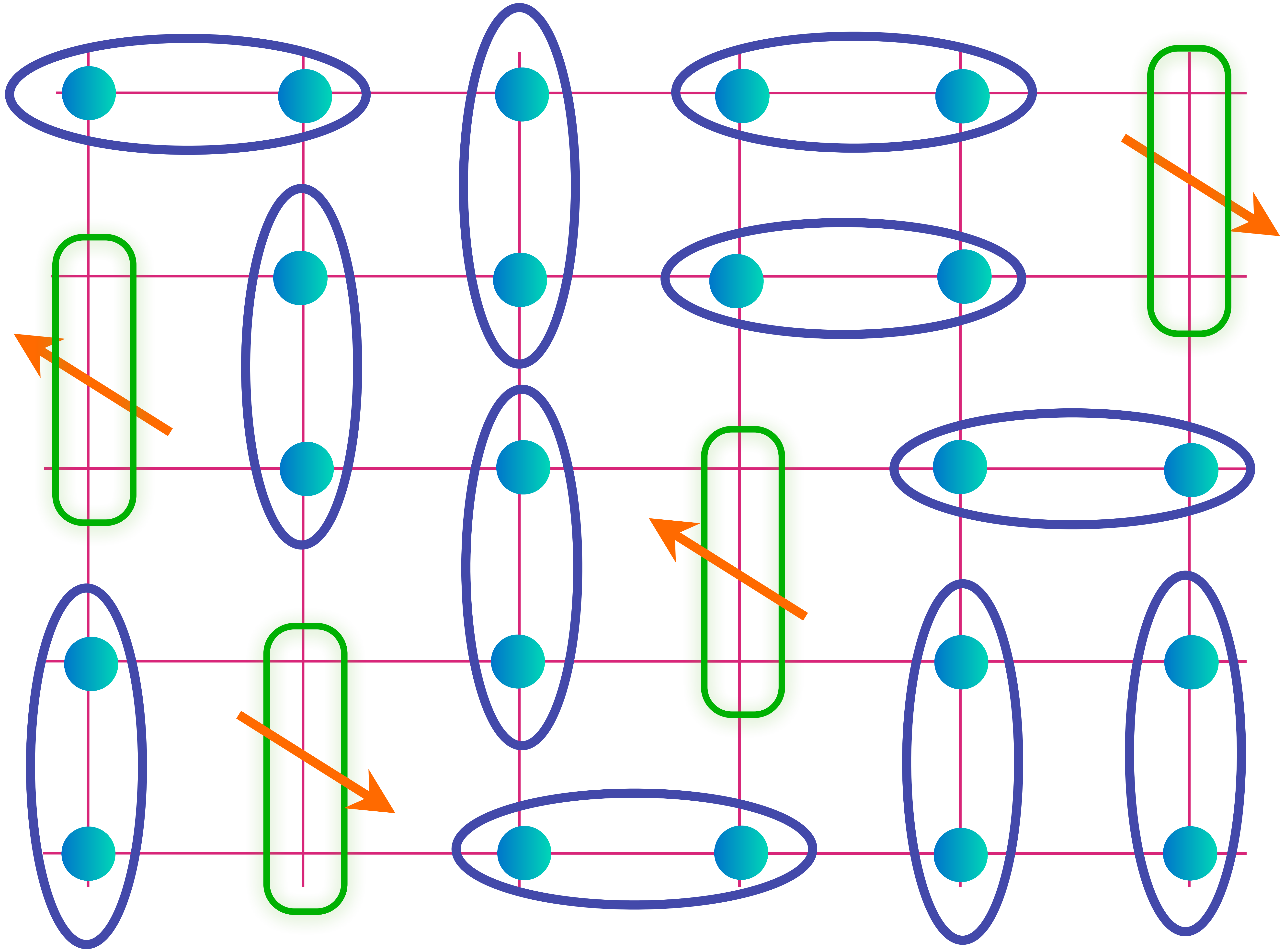
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

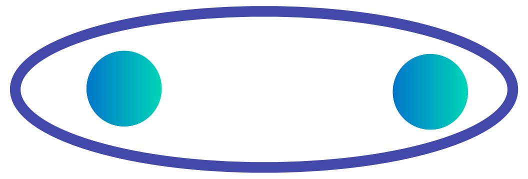
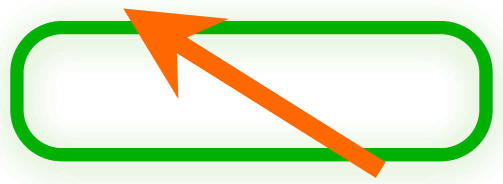
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



FL*:
Fermi gas of holon-spinon bound states

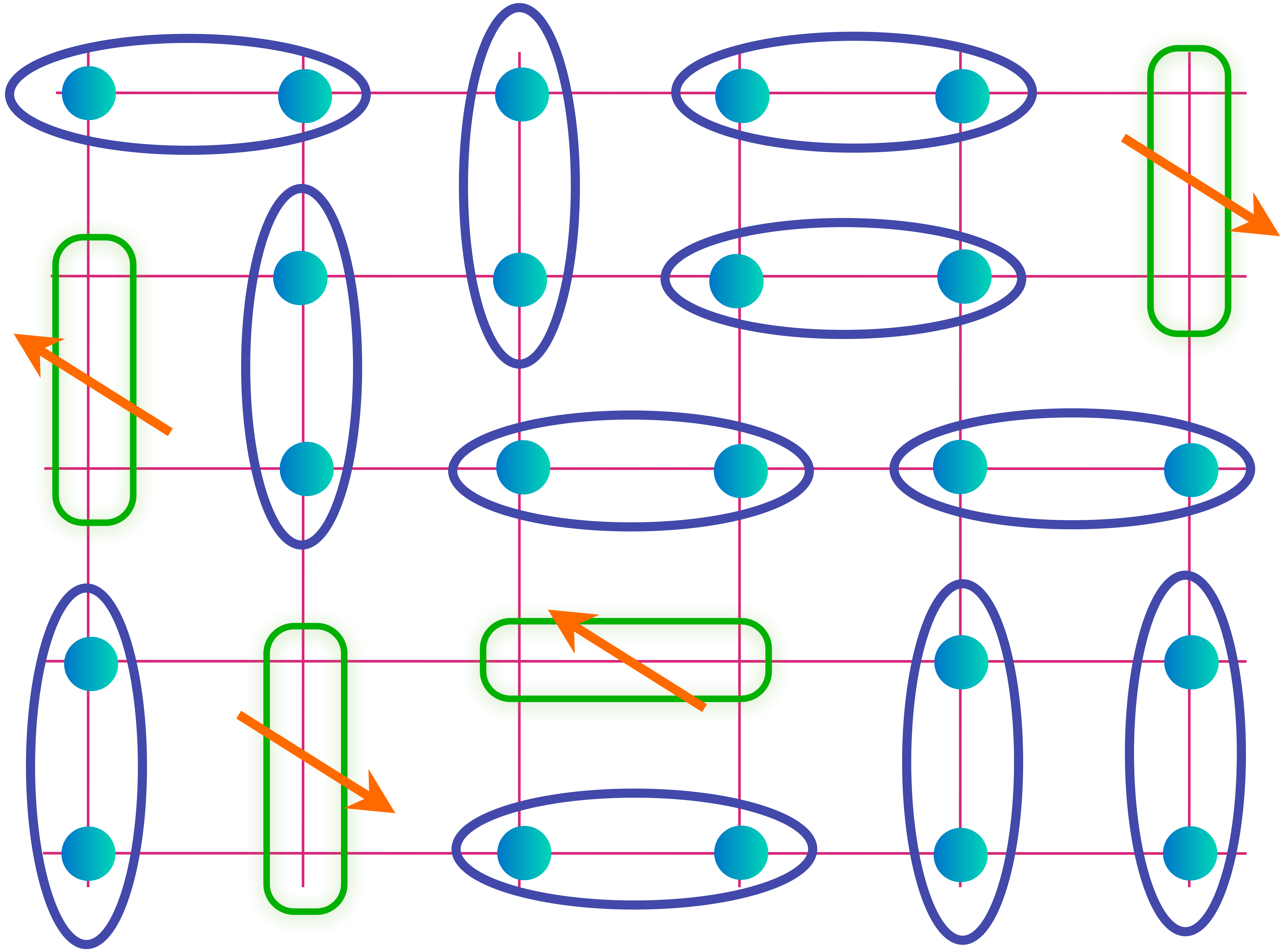
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



FL*:
Fermi gas of holon-spinon bound states

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

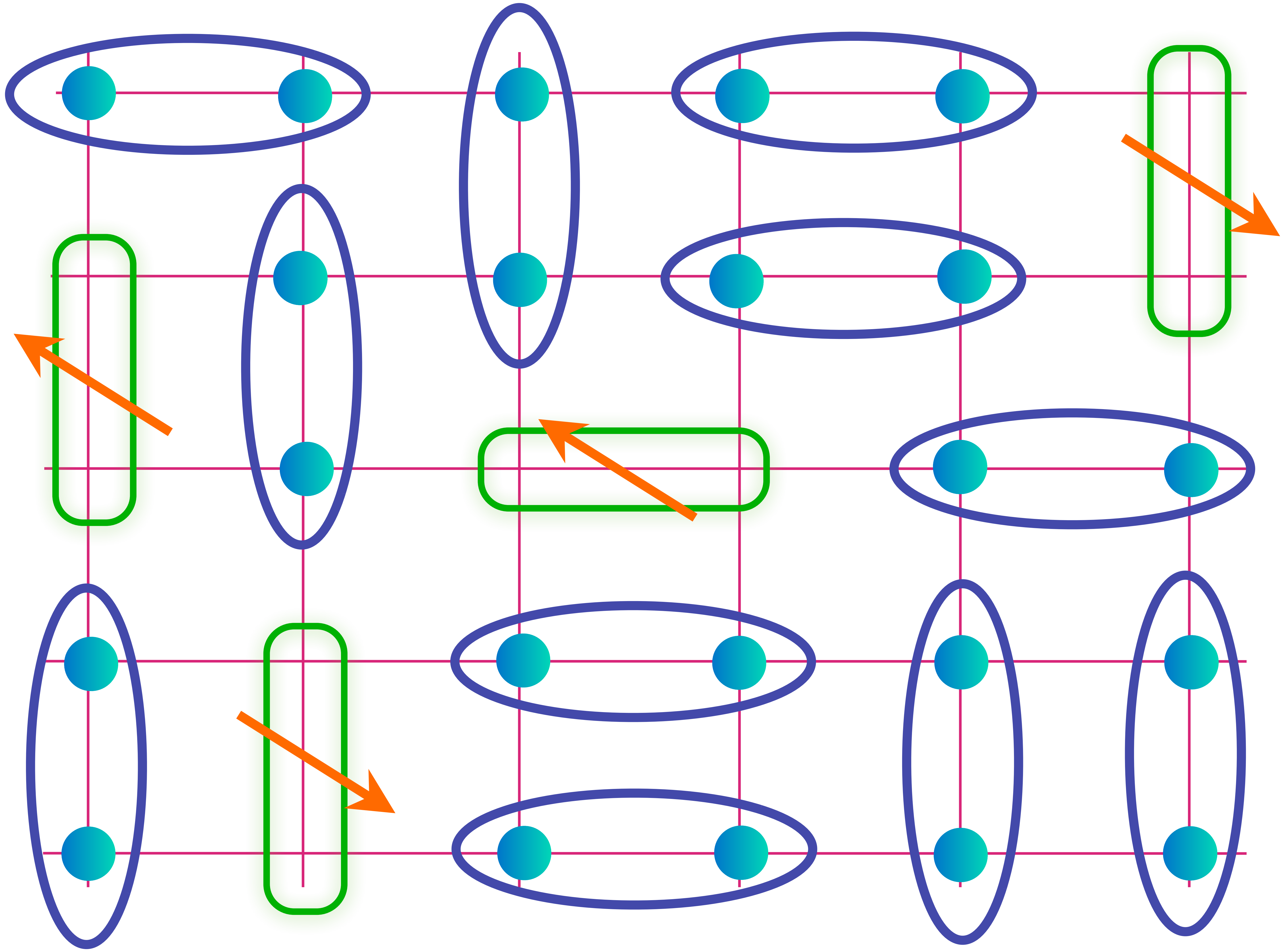
$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



FL*: Fermi gas of holon-spinon bound states

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

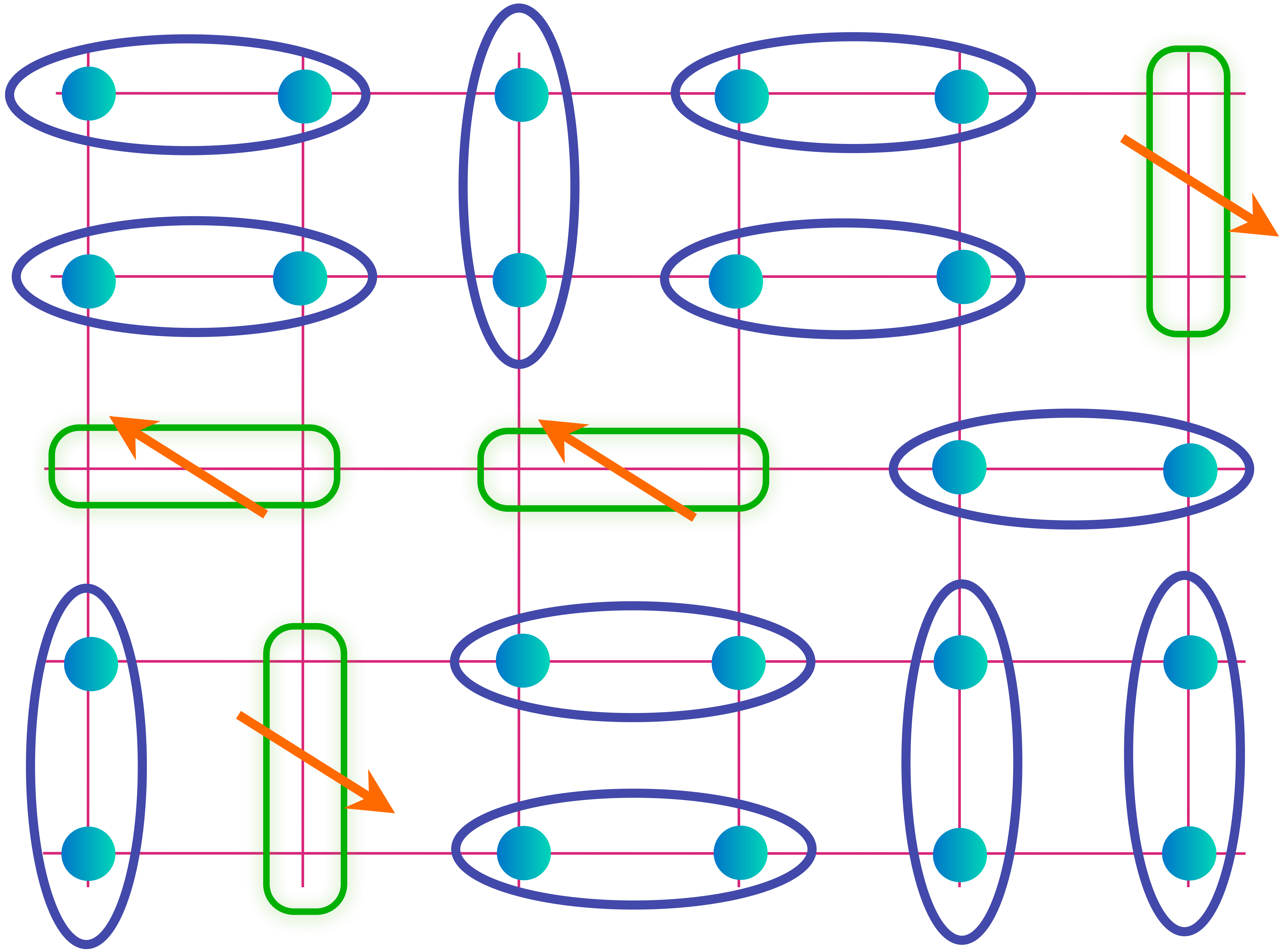
$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

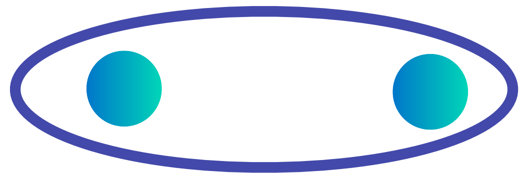
To obtain a (super)conductor we have to remove a density p of electrons

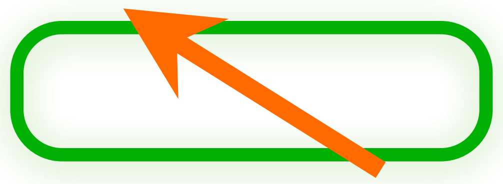
Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



FL*: Fermi gas of holon-spinon bound states

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

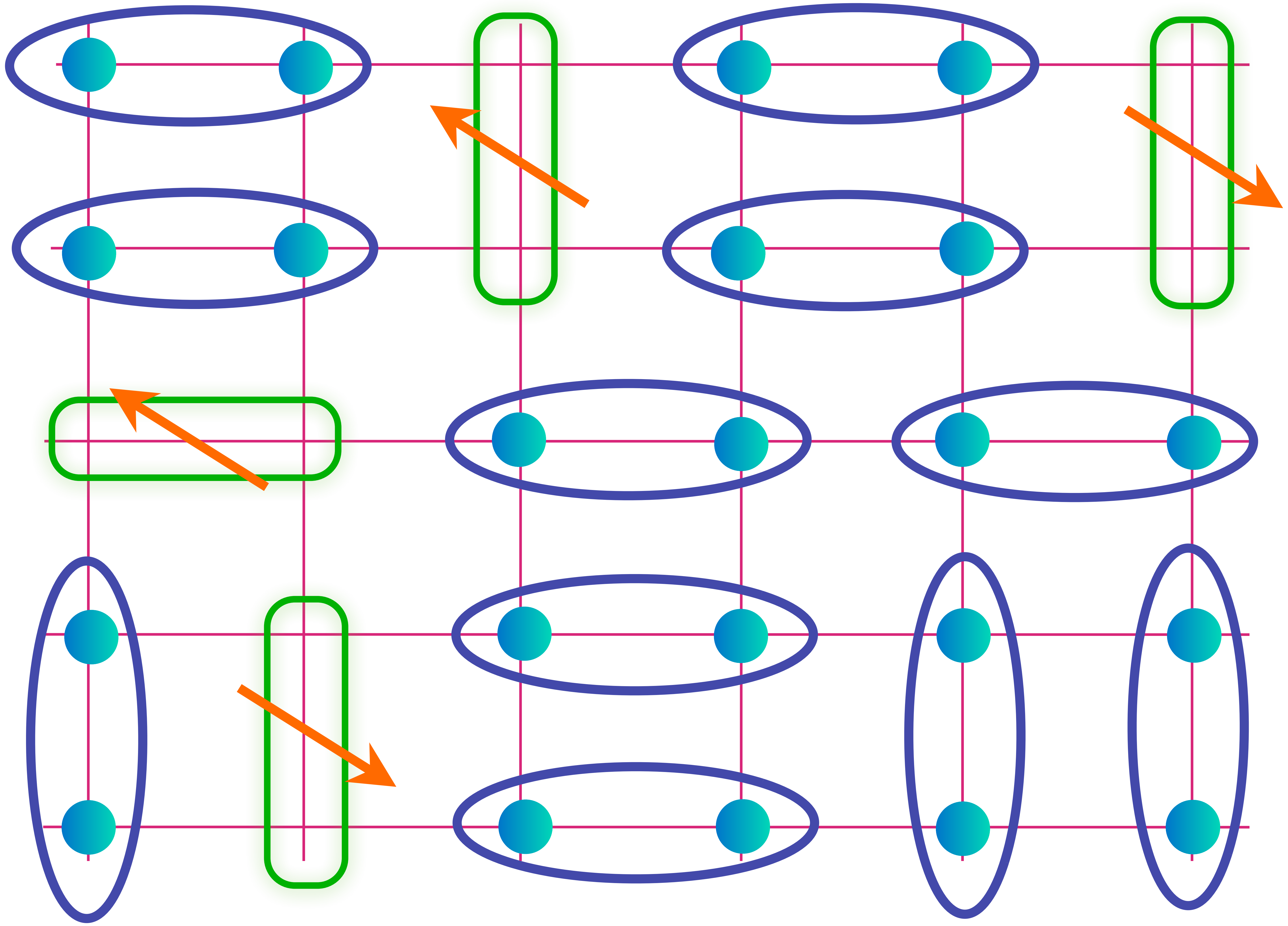
 $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

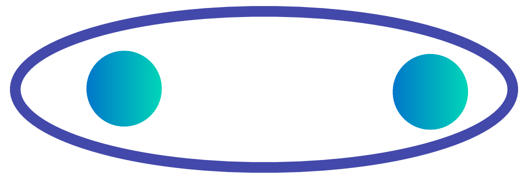
To obtain a (super)conductor we have to remove a density p of electrons

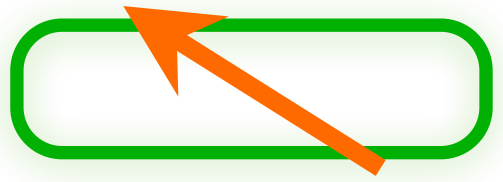
Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



FL*:
Fermi gas of holon-spinon bound states

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

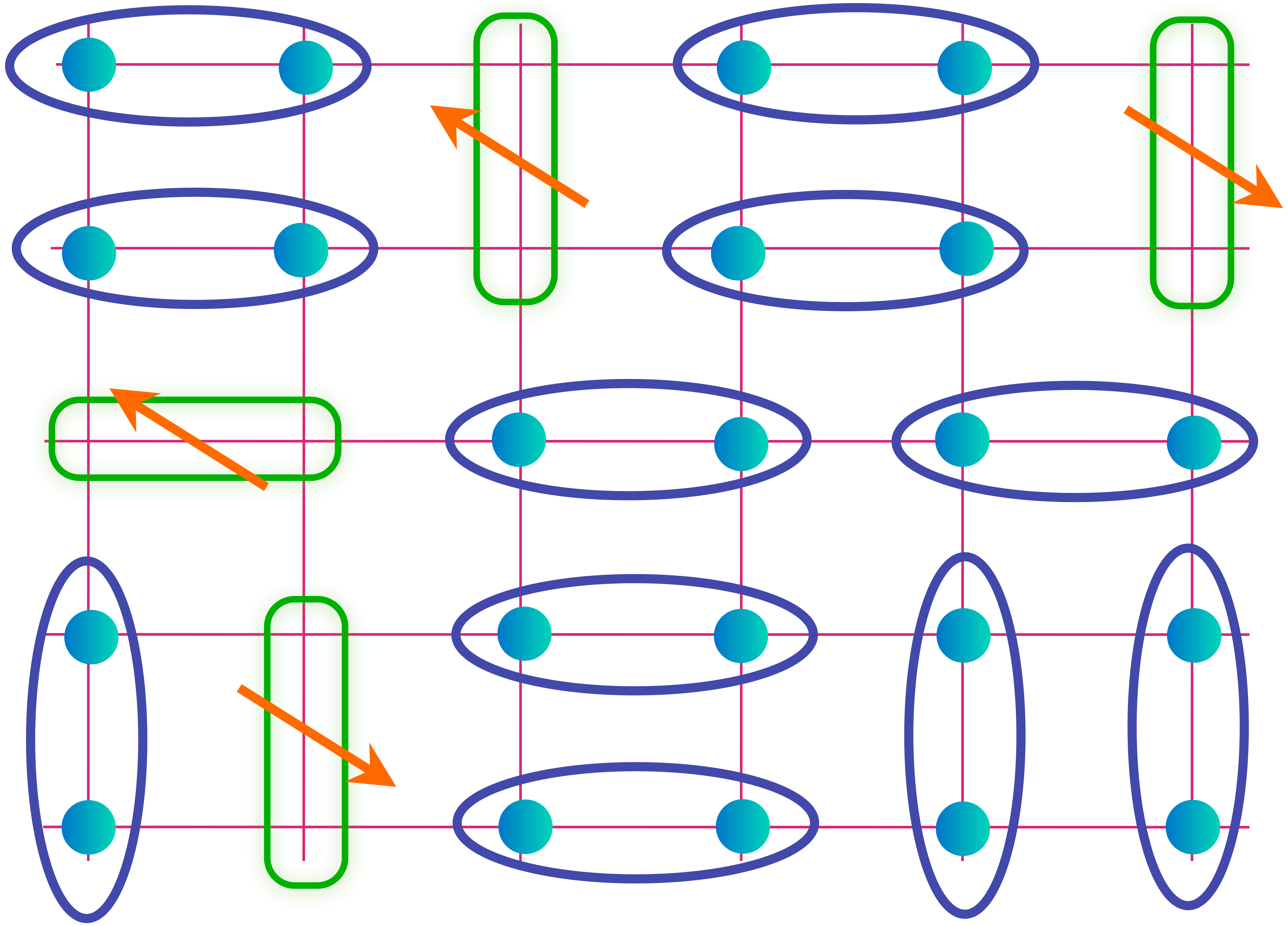
 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

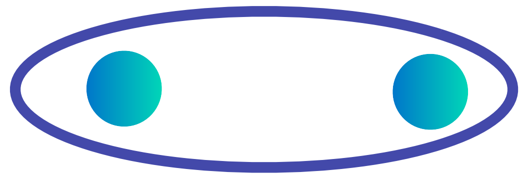
To obtain a (super)conductor we have to remove a density p of electrons

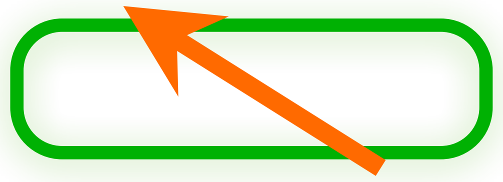
Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



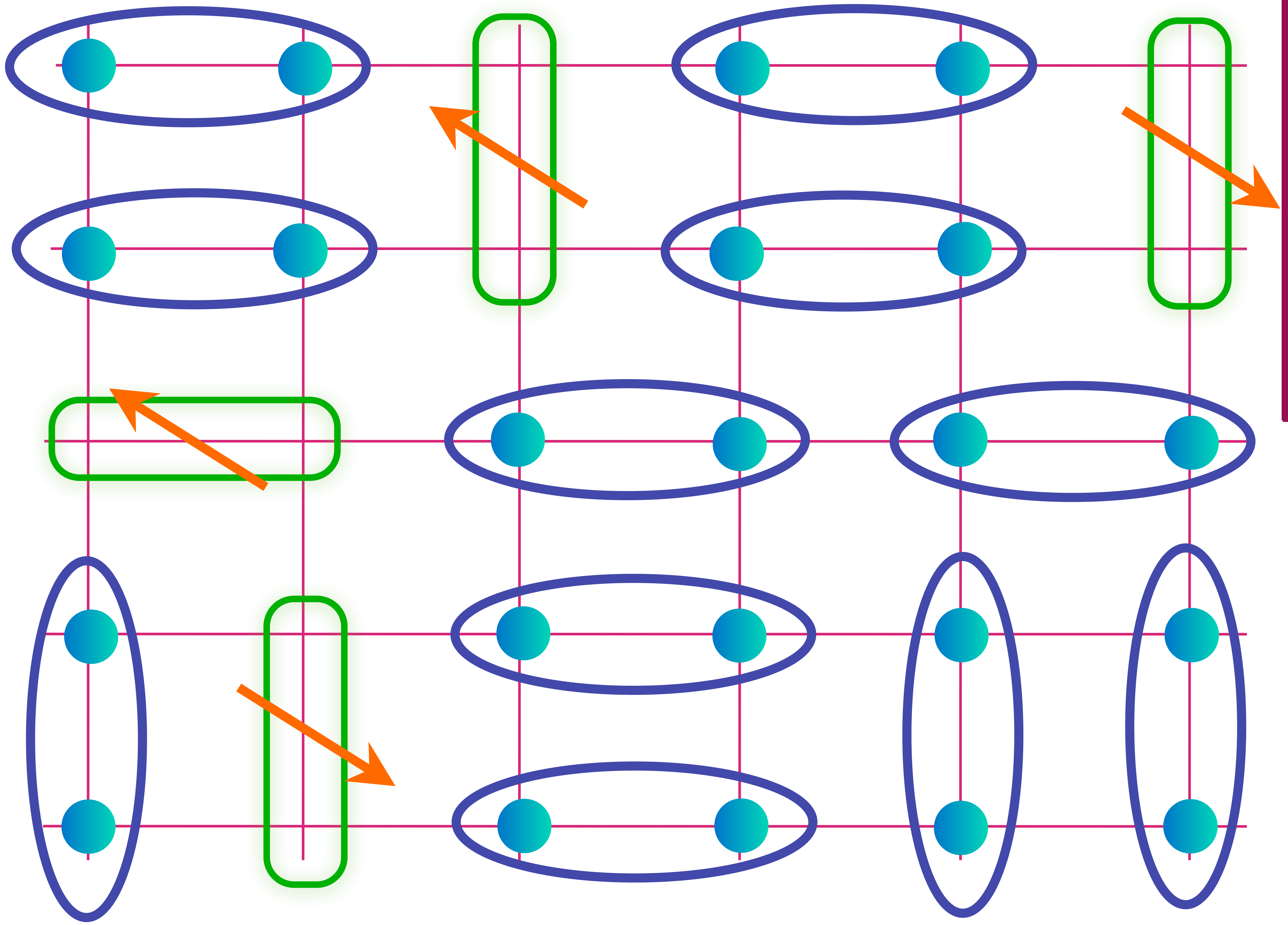
Fermi surface?

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

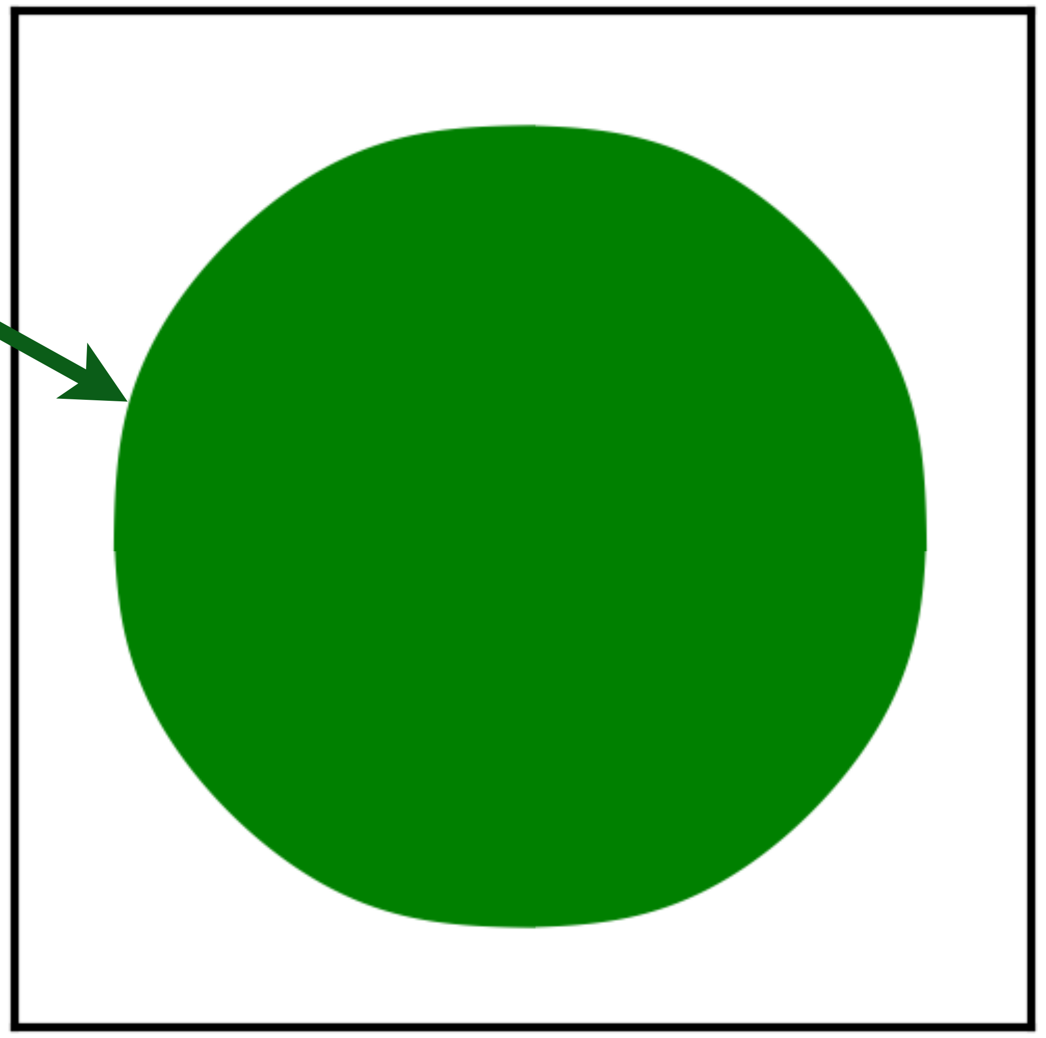
 $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

To obtain a (super)conductor we have to remove a density p of electrons

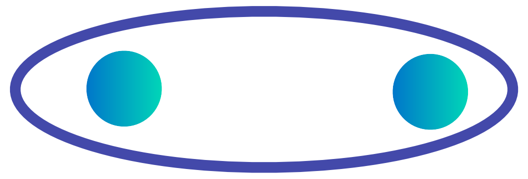


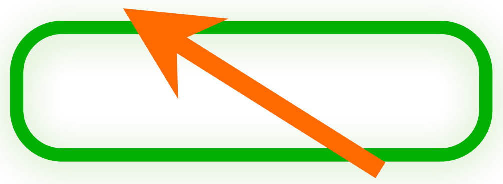
Luttinger area $(1+p)/2$



Count *all* electrons $= 1 - p$. Holes in a filled band $= 1 + p$.

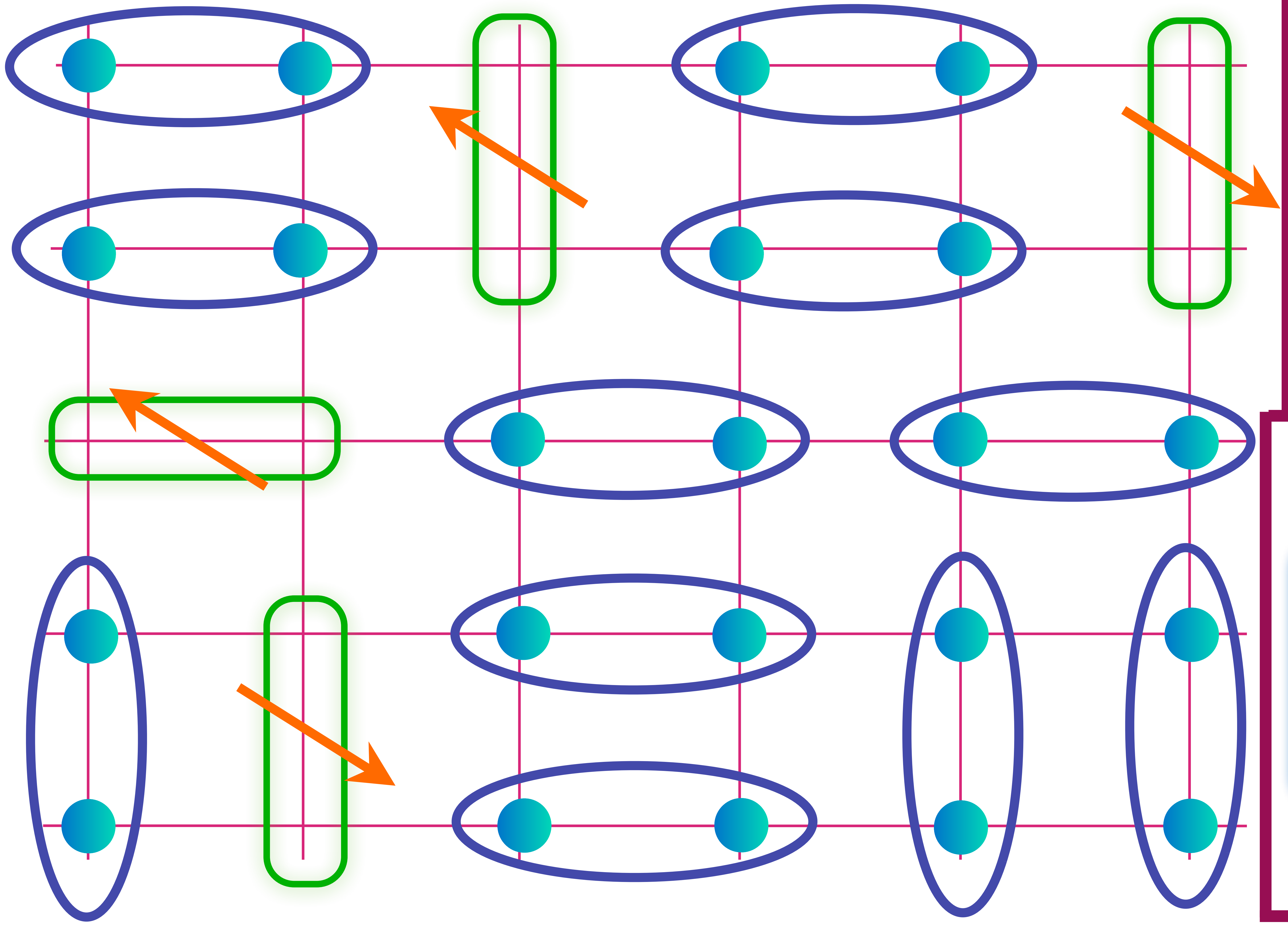
FL*: Fermi gas of holon-spinon bound states

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

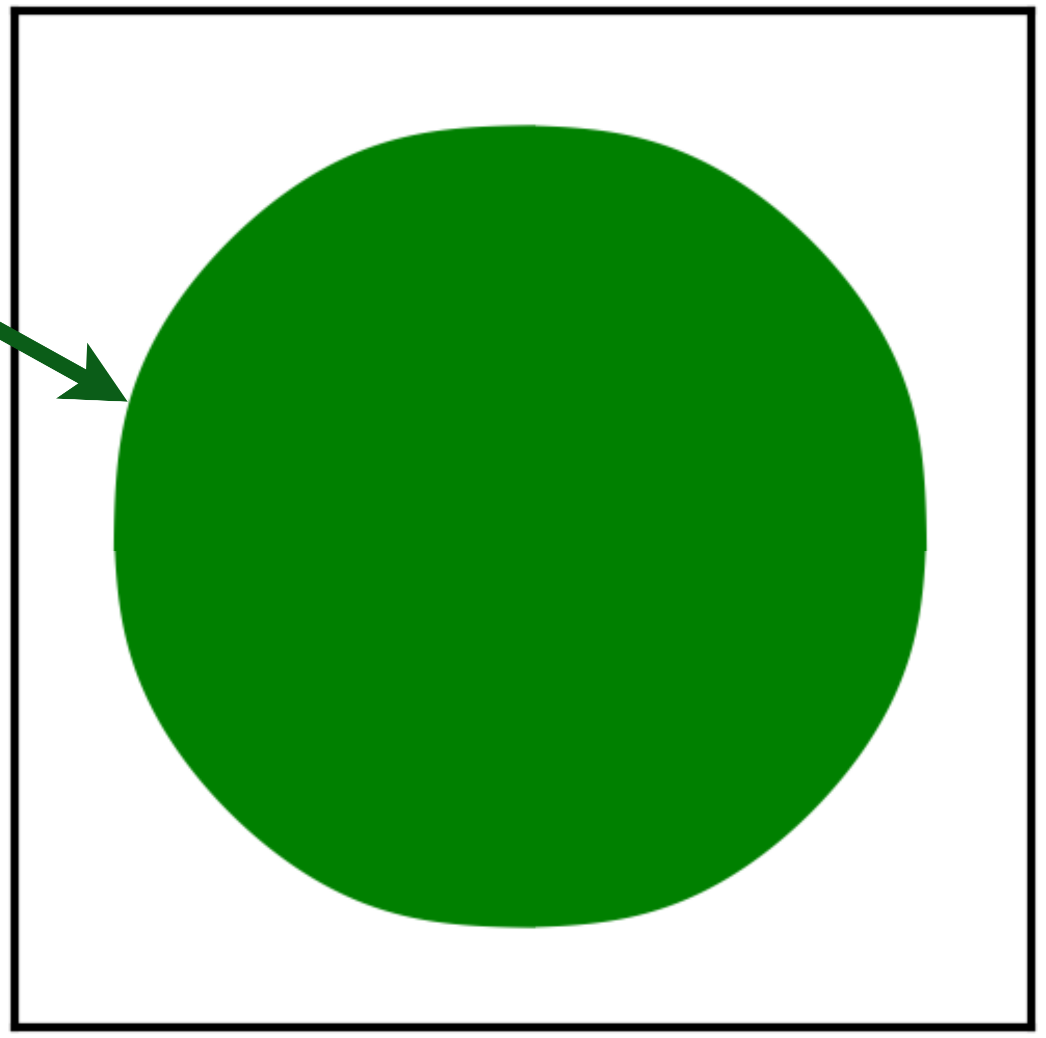
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

To obtain a (super)conductor we have to remove a density p of electrons



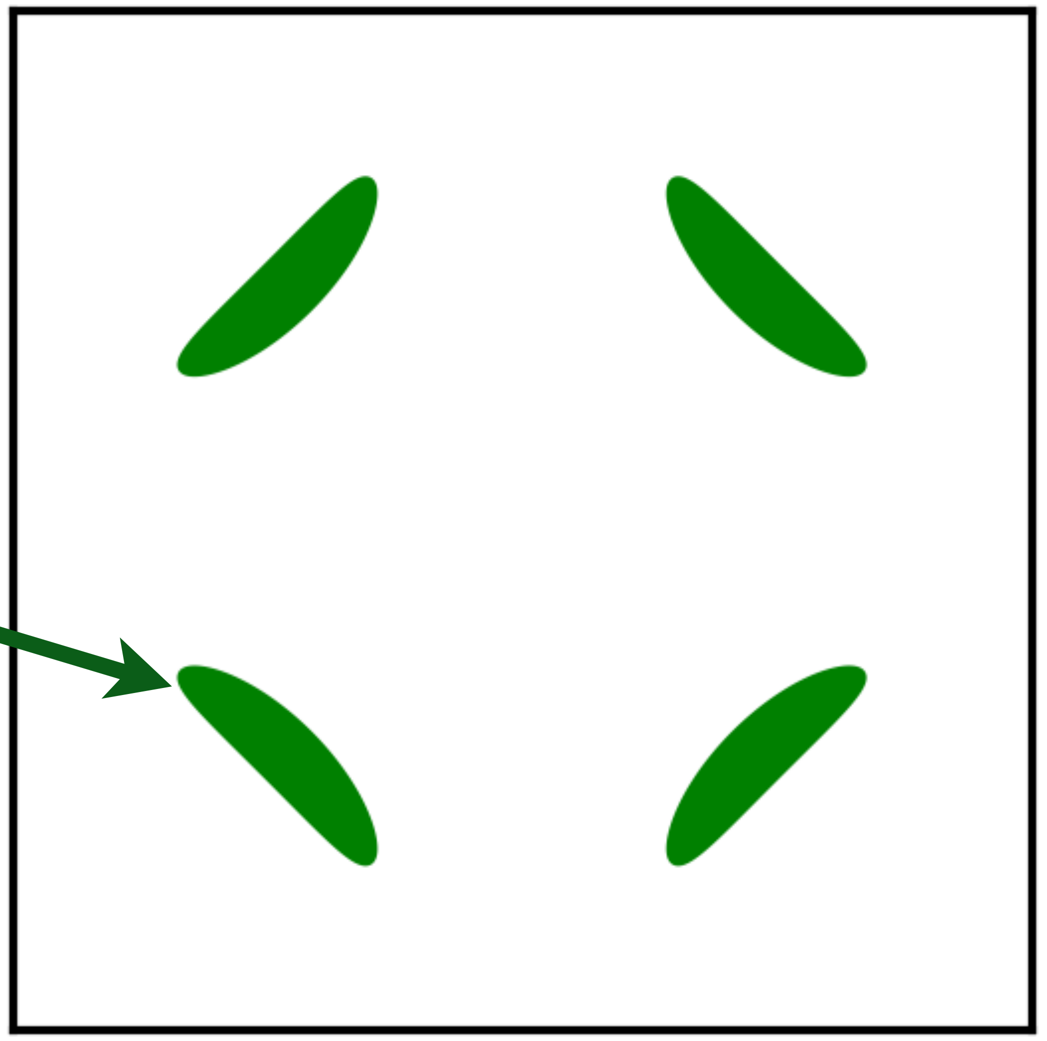
Luttinger area $(1+p)/2$

Count *all* electrons $= 1 - p$. Holes in a filled band $= 1 + p$.



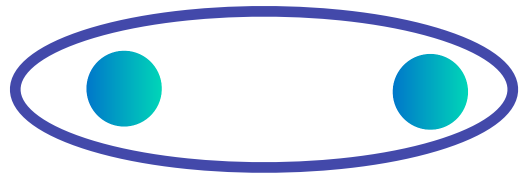
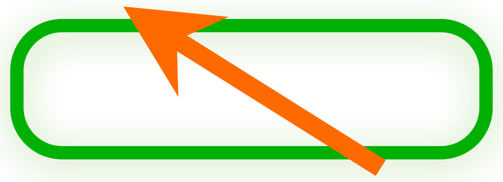
FL

Non-Luttinger area $p/8$



FL*

Count only green dimers

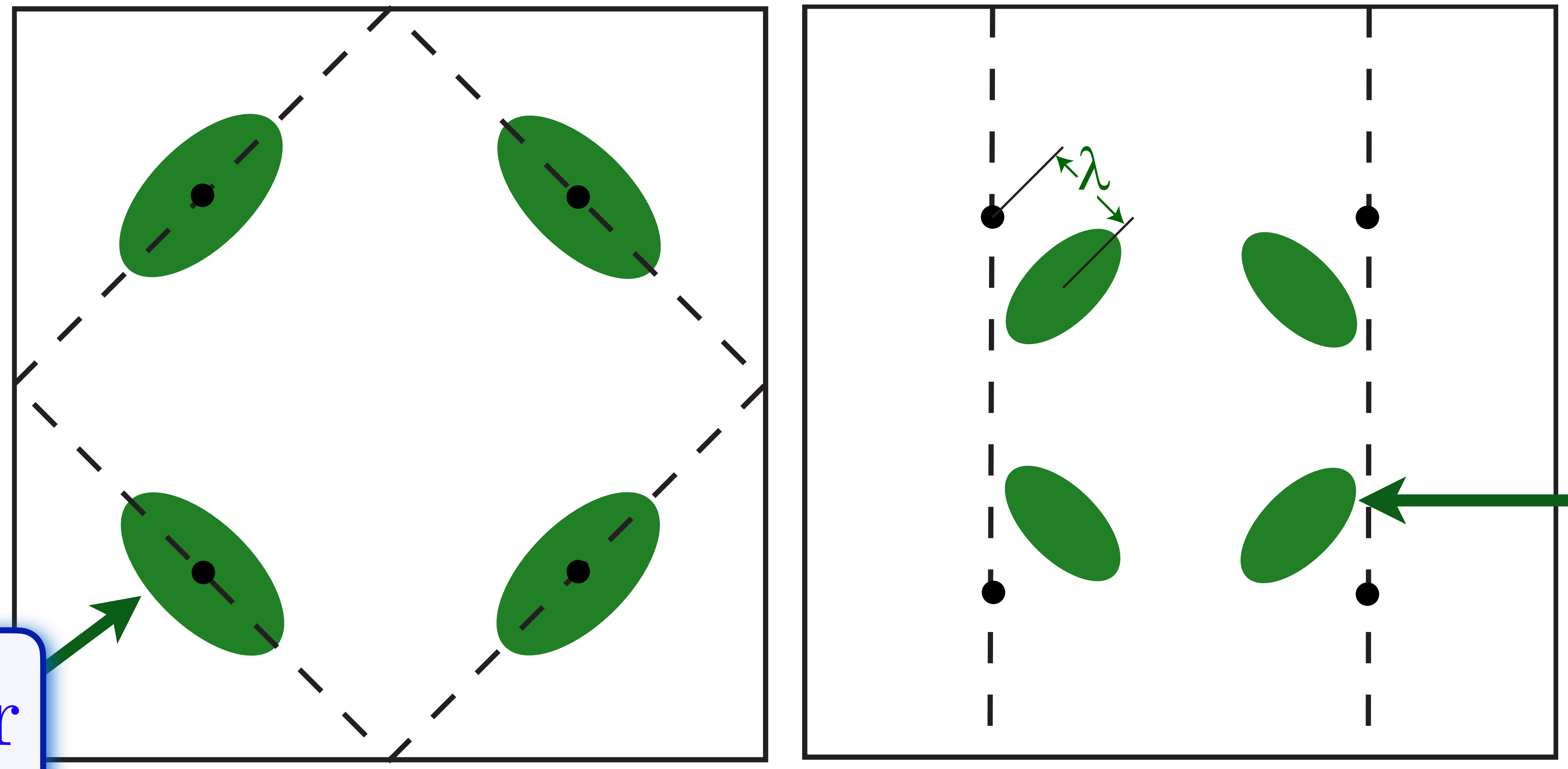
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



Luttinger area $p/4$

AF

Coherent inter-layer transport requires inter-layer spin correlations

*FL**

Coherent inter-layer transport

Non-Luttinger area $p/8$

Note factor of 2 between thermally disordered antiferromagnetism and "quantum disordered" antiferromagnetism in FL*.

Observation of the Yamaji effect in the cuprate pseudogap

See also:

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

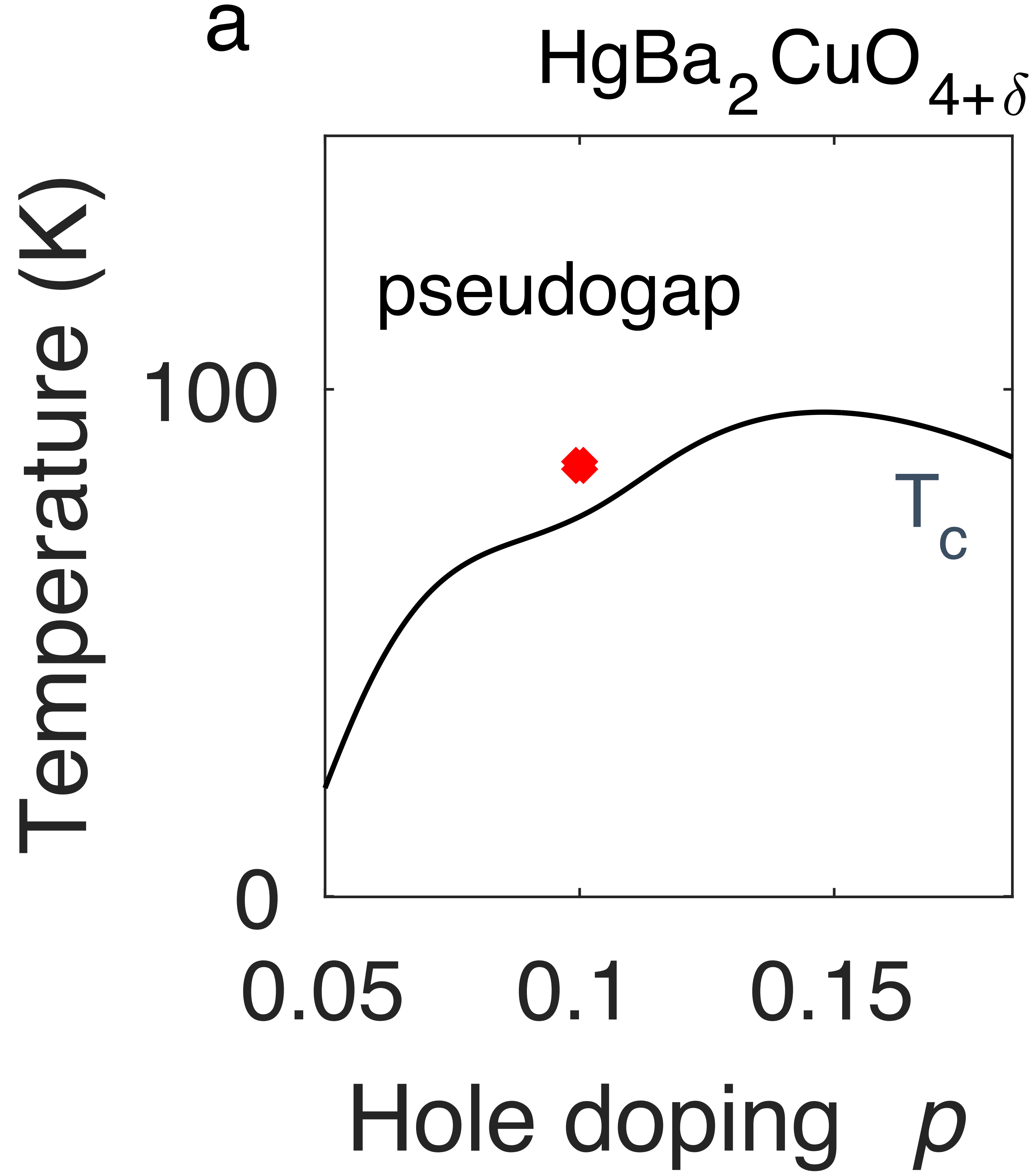
Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$

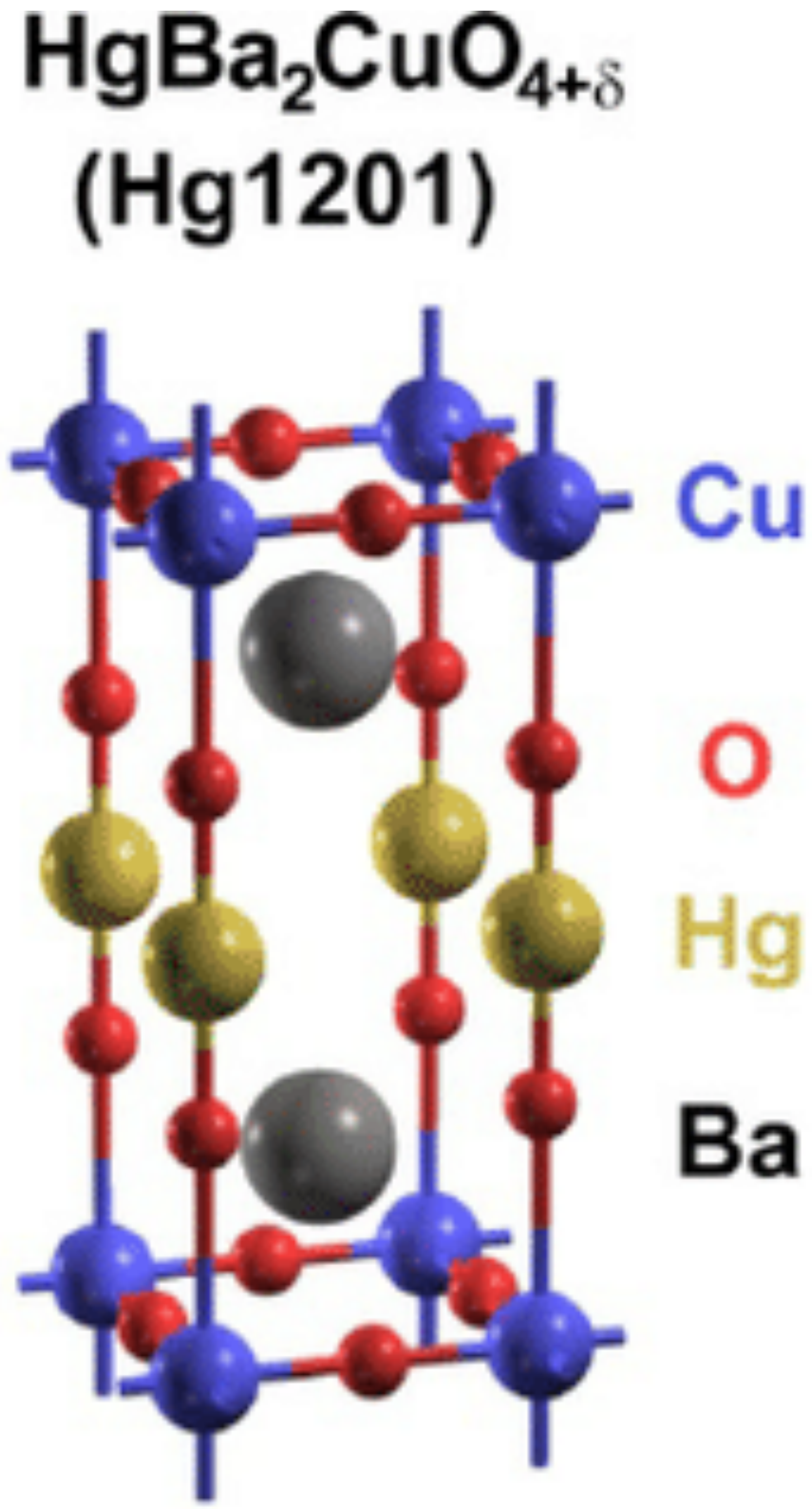
Observation of the Yamaji effect in a cuprate superconductor

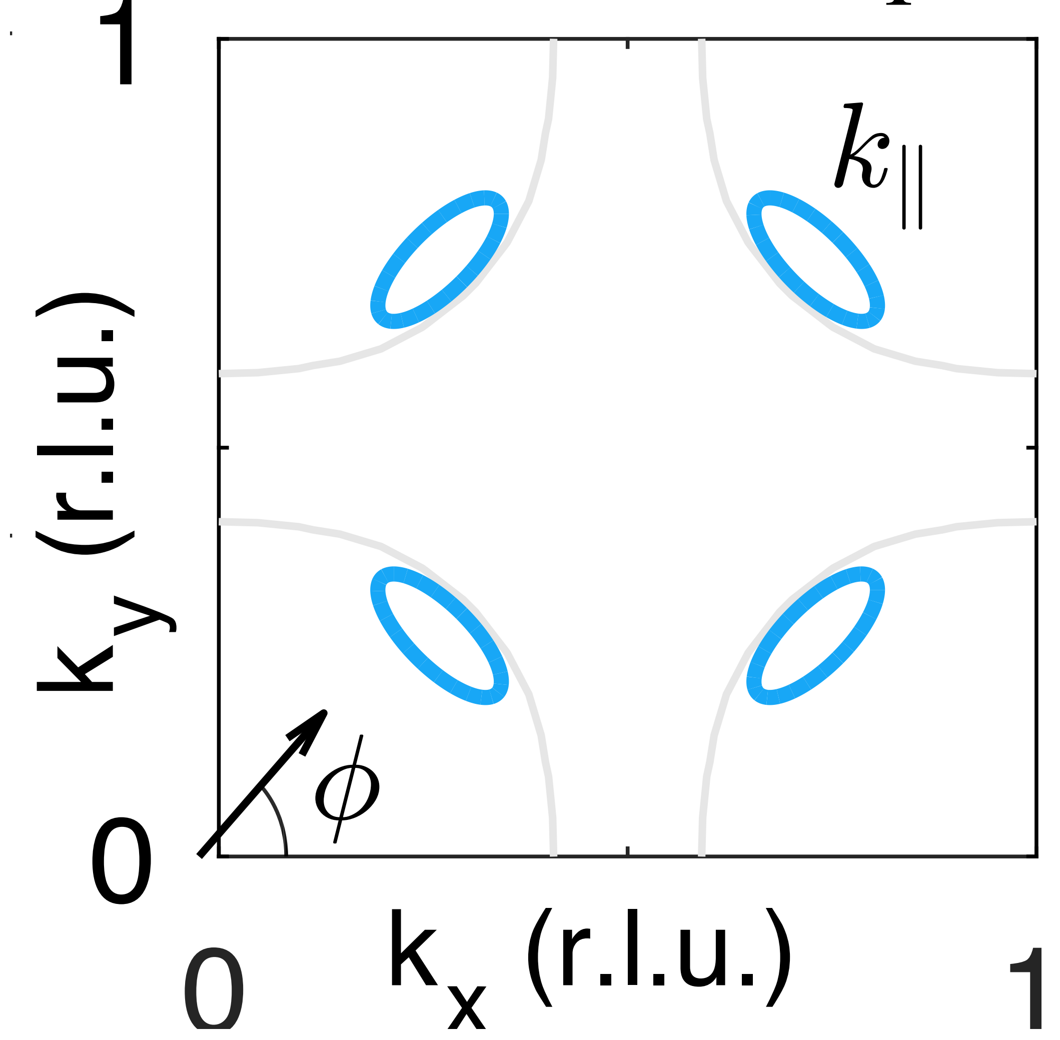
Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

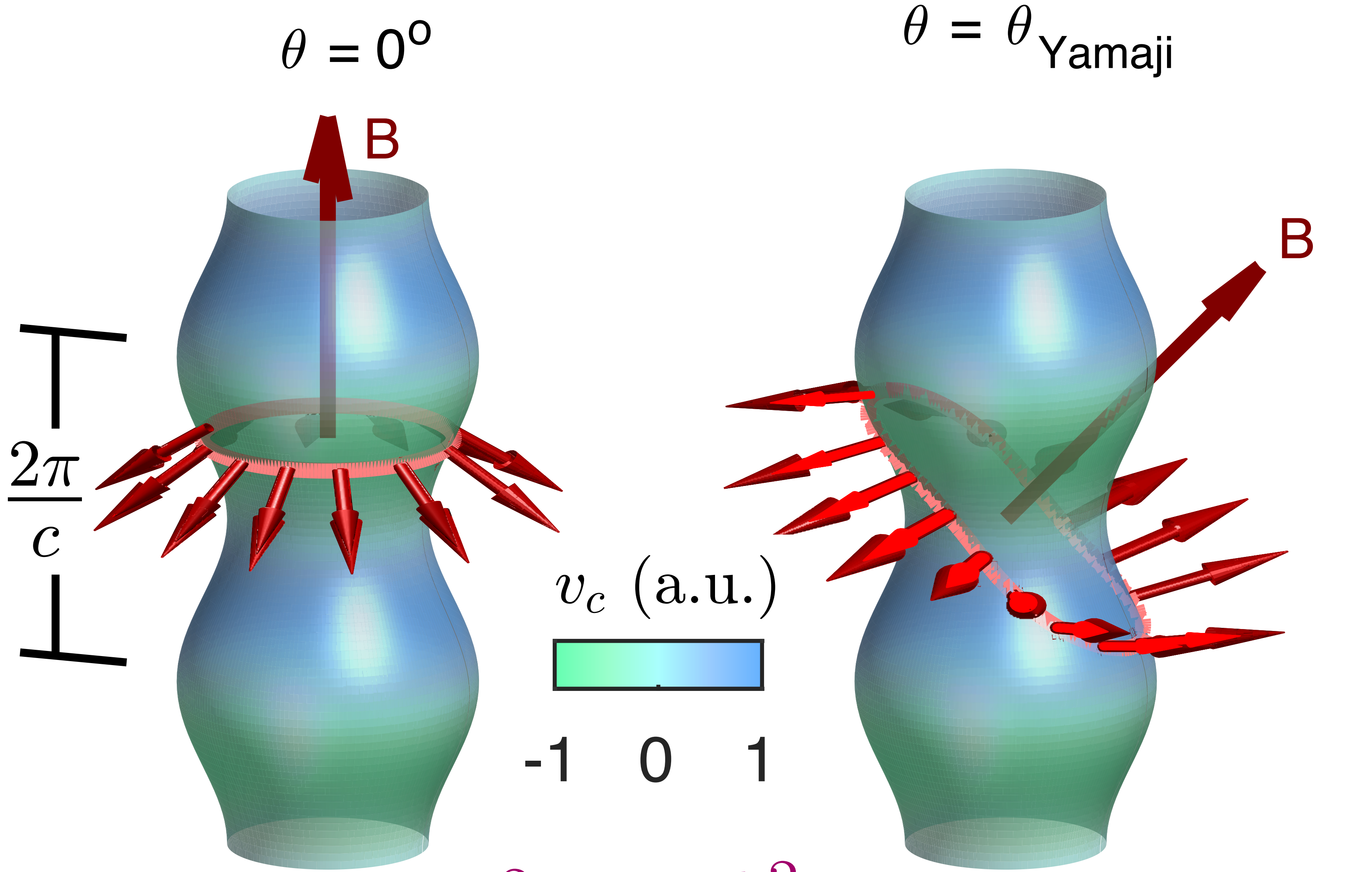
nature physics
21, 1753 (2025)
Published online: 16 September 2025

a









$$\epsilon(\mathbf{k}) = \frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2} - 2t_\perp \cos(k_z c)$$

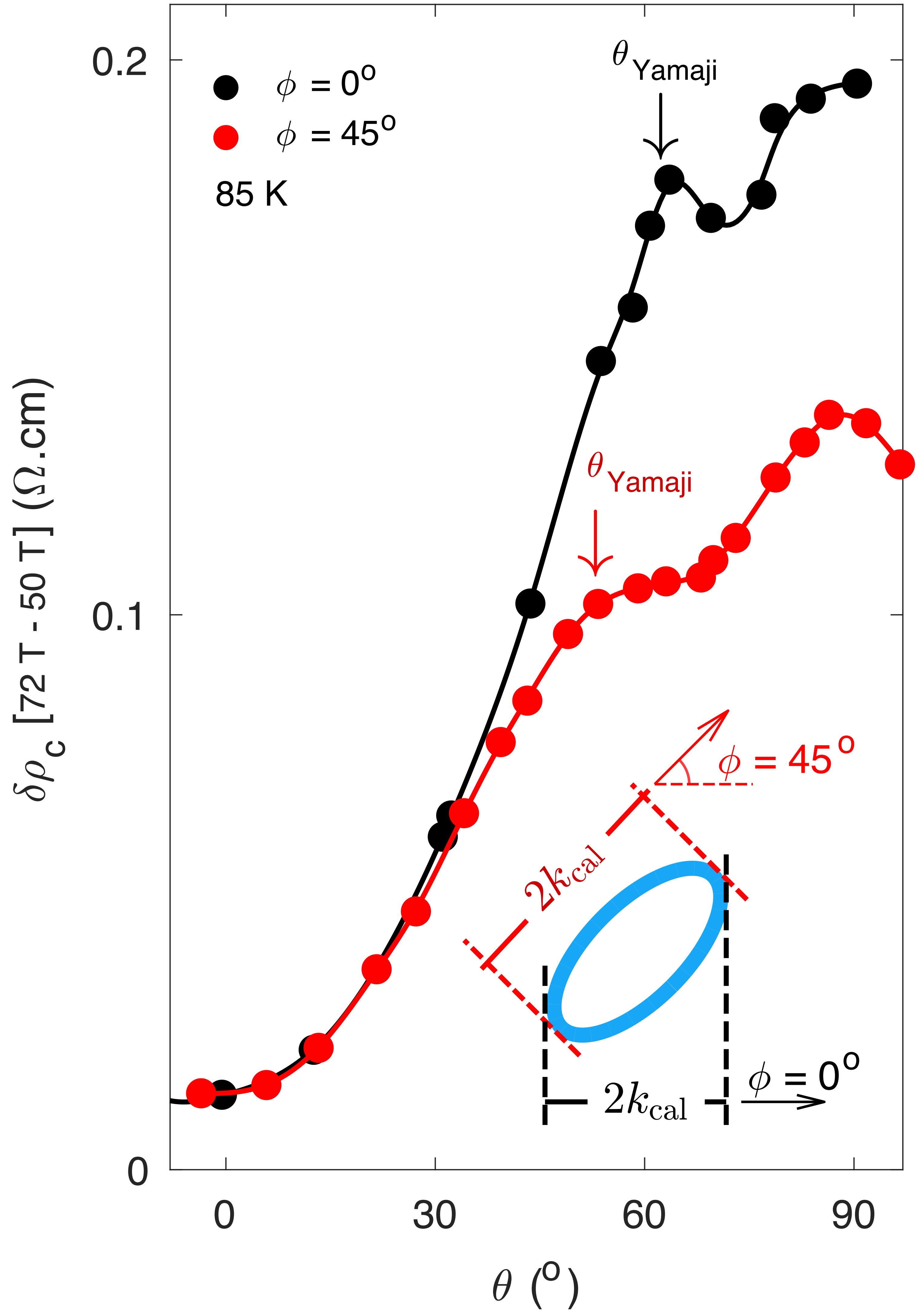
At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

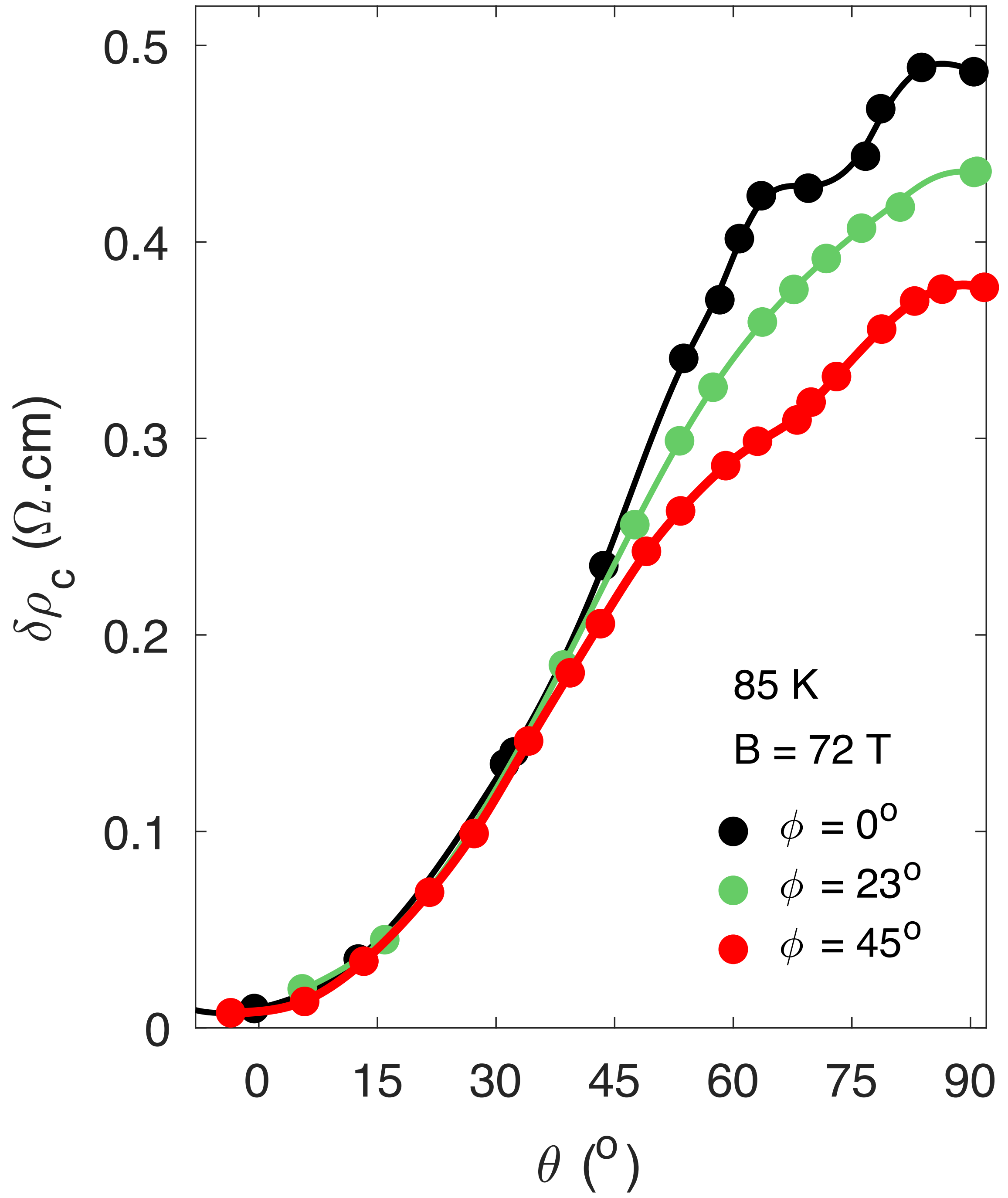
K. Yamaji JPSJ **58**, 1520 (1989)

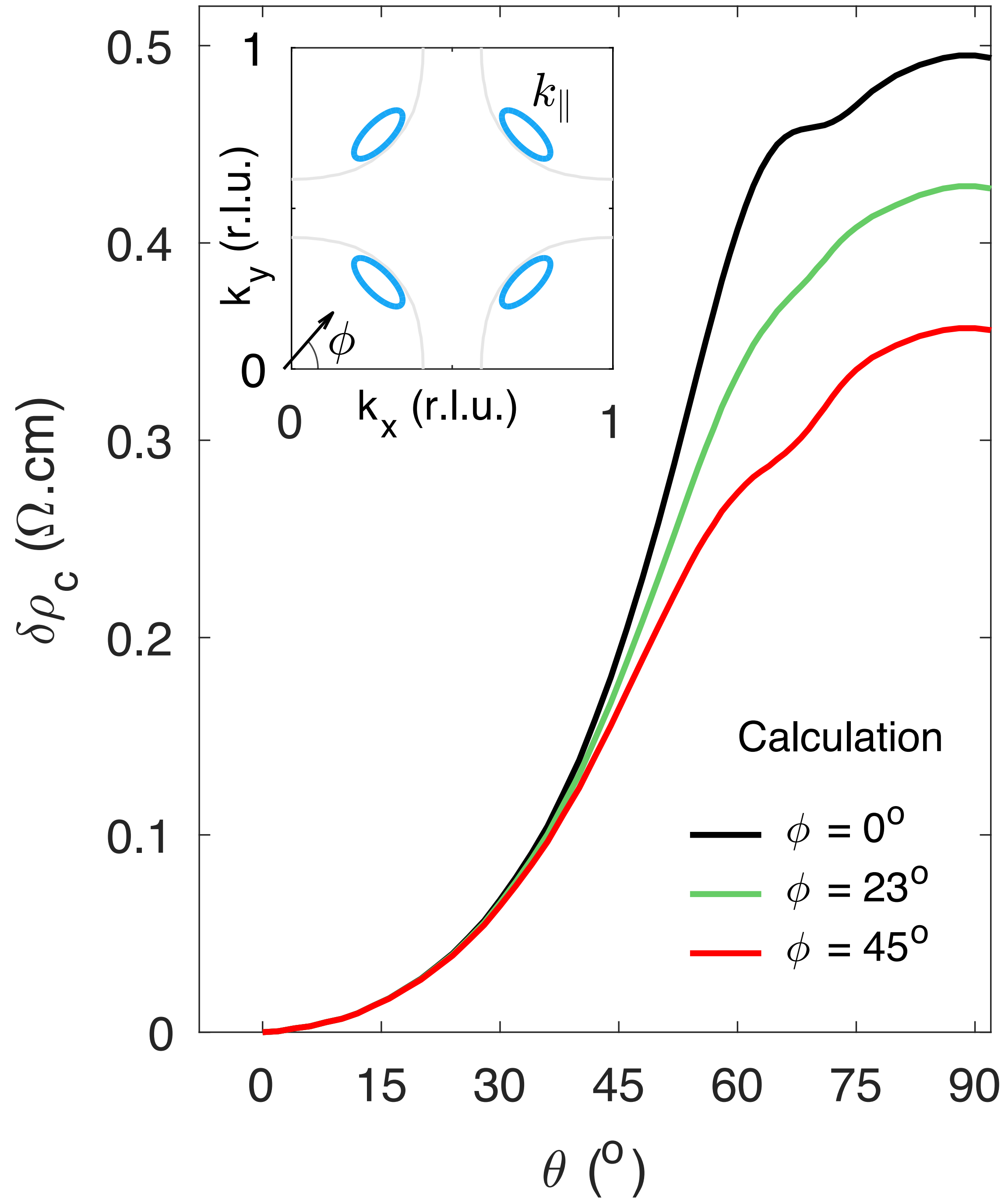
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025







Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

(p/8 also in Yang-Rice-Zjang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

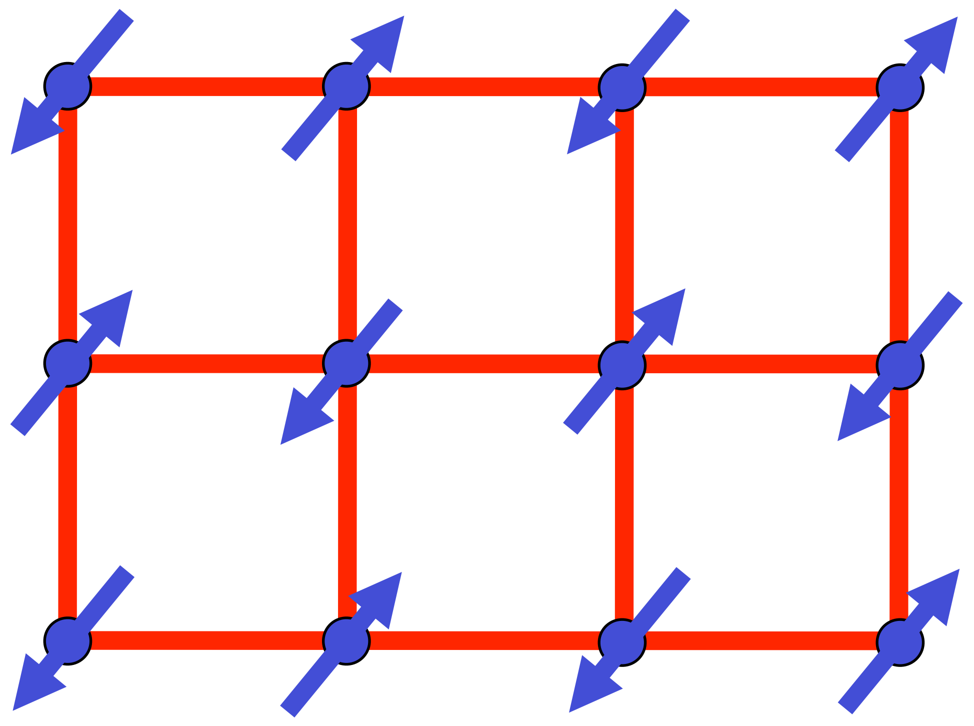
Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Critical quantum spin liquid on the square lattice

S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ bosonic spinons $\boldsymbol{S} \sim b_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_\beta$

Mean field theory with $\langle \varepsilon_{\alpha\beta} b_{i\alpha} b_{j\beta} \rangle \neq 0$
D.P. Arovas and A. Auerbach, PRB **38**, 316 (1988).



$\langle b_\alpha \rangle \neq 0$:
Néel order

$\langle b_\alpha \rangle = 0$:
Spin gap

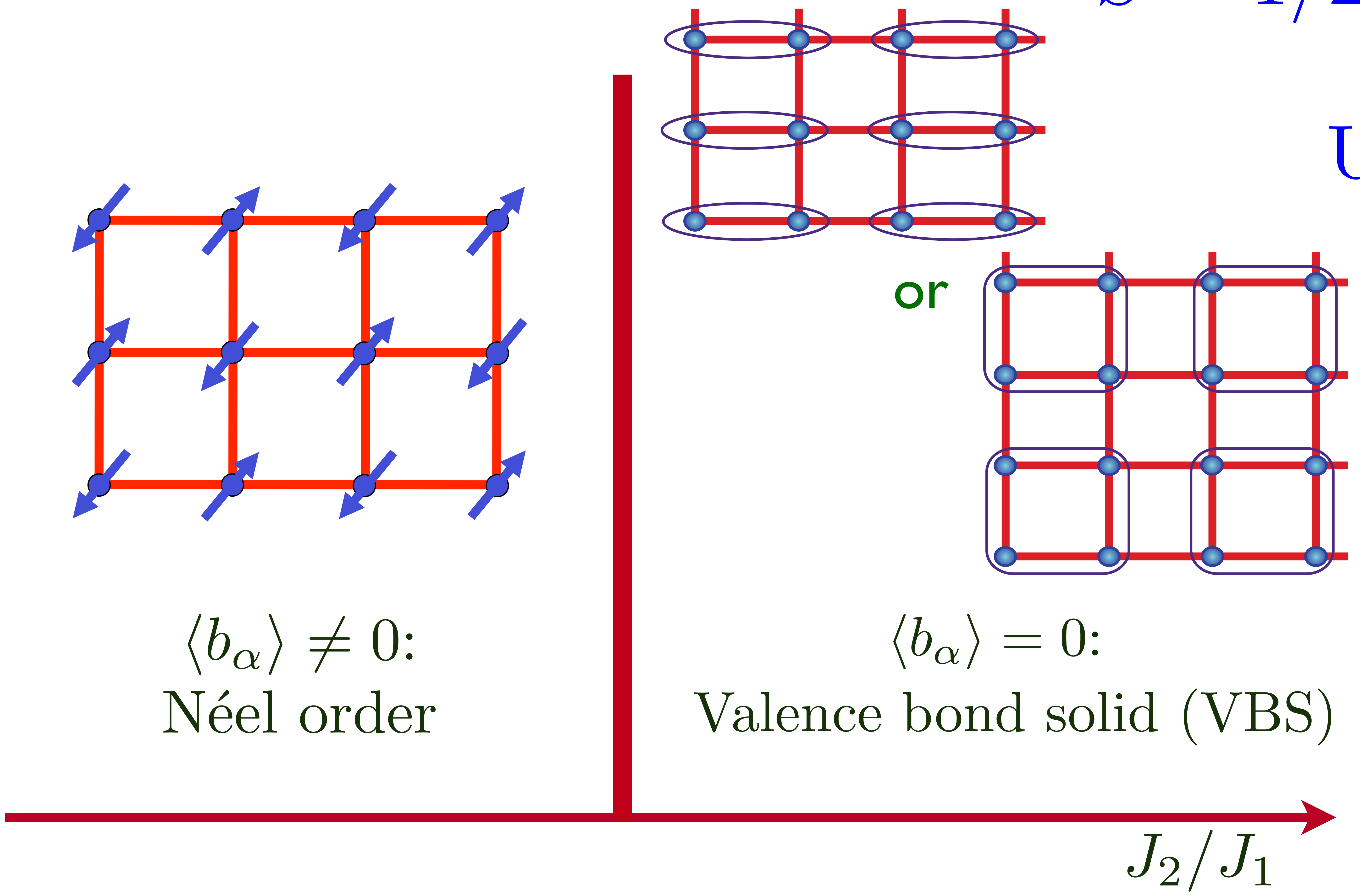
S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ bosonic spinons $\boldsymbol{S} \sim b^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} b_\beta$

U(1) gauge symmetry: $b \to b e^{i\theta}$

$\mathbb{CP}^1$ U(1) gauge theory.

't Hooft/Lieb-Schultz-Mattis anomalies realized by monopole Berry phases.



$$\mathcal{L}_z = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

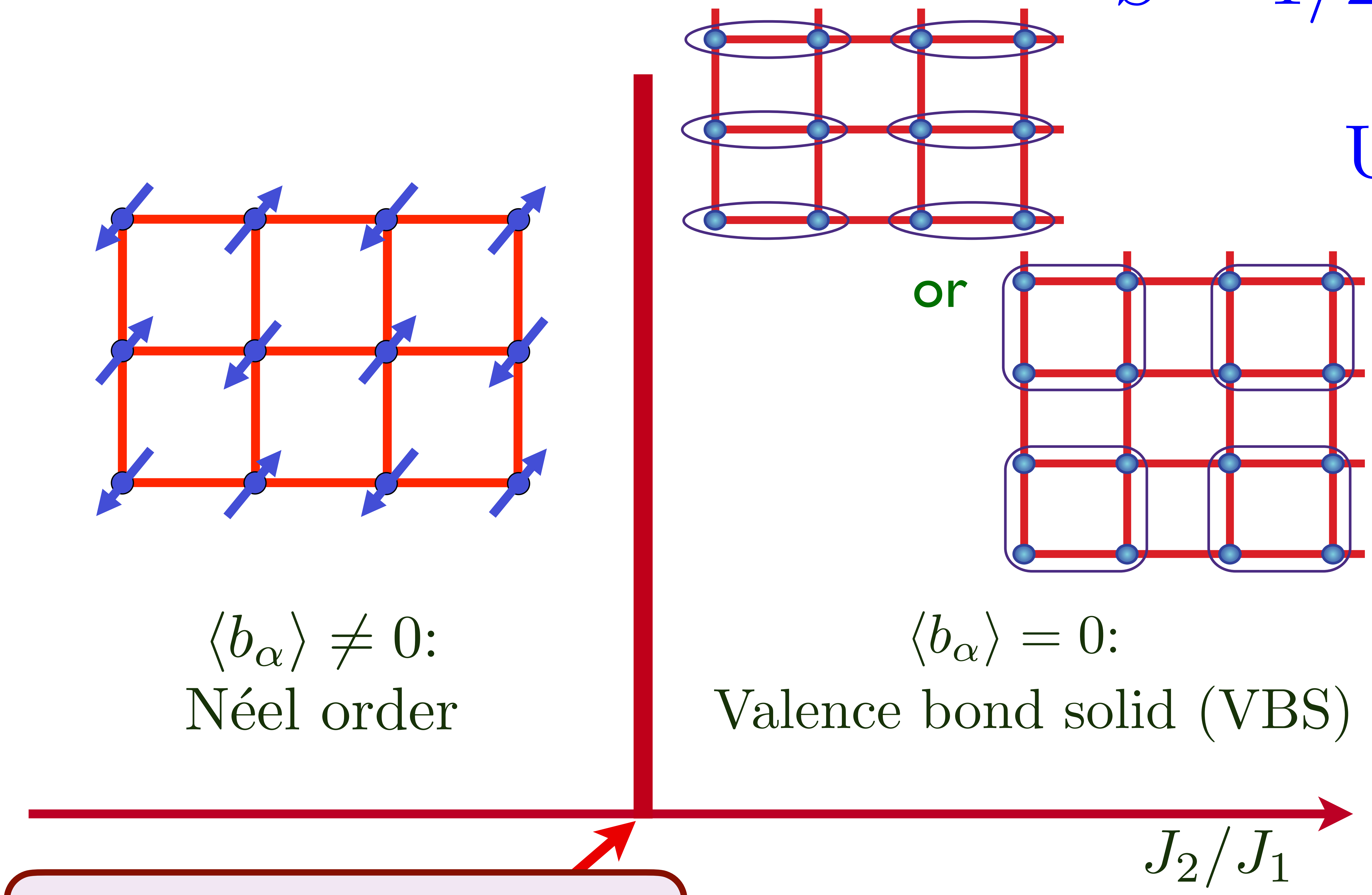
S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ bosonic spinons $\boldsymbol{S} \sim b_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_\beta$

U(1) gauge symmetry: $b \to b e^{i\theta}$

$\mathbb{CP}^1$ U(1) gauge theory.

't Hooft/Lieb-Schultz-Mattis anomalies realized by monopole Berry phases.



Critical spin liquid without quasiparticles?

$$\mathcal{L}_z = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science **303**, 1490 (2004)

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

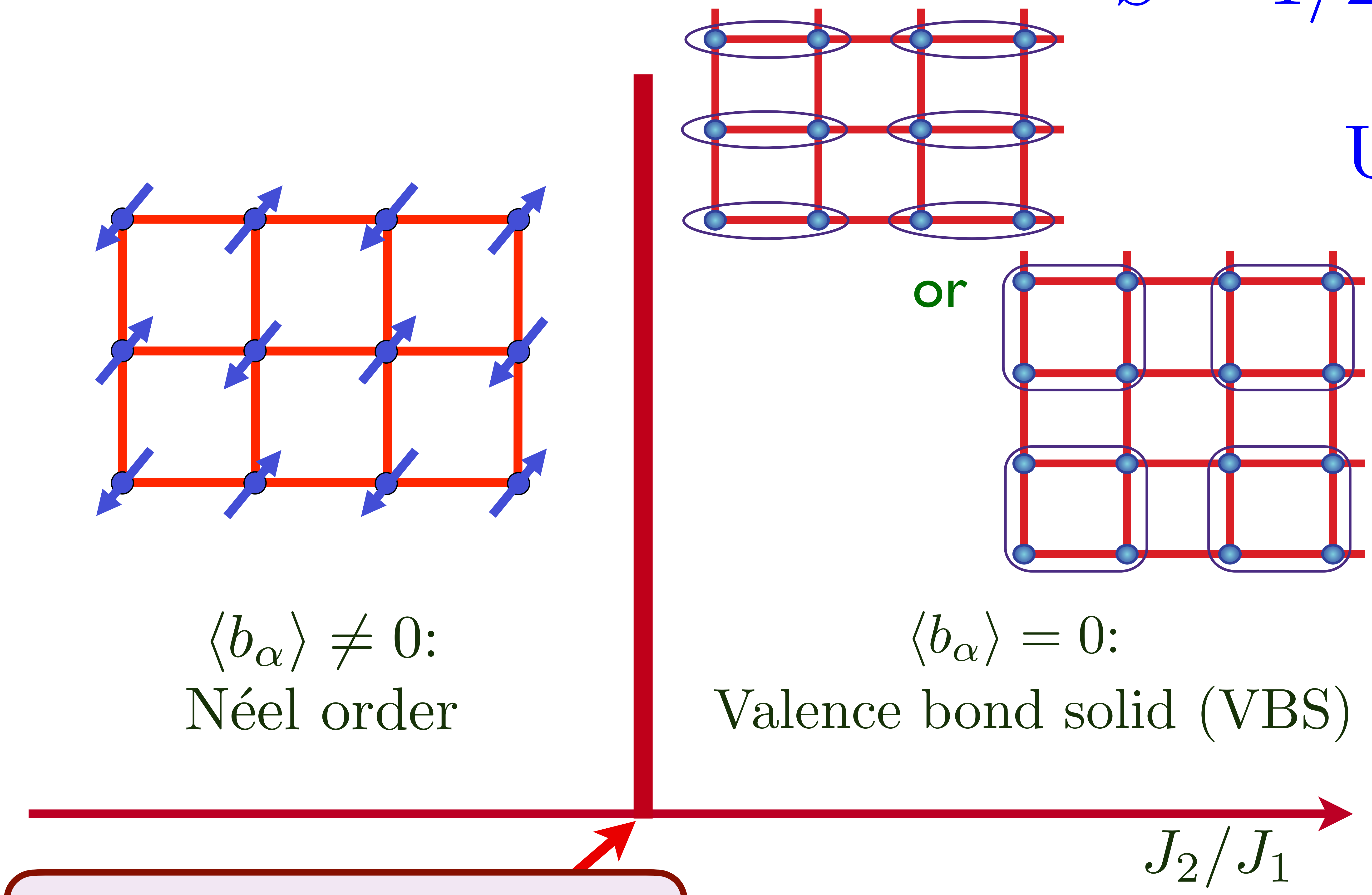
S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ bosonic spinons $\boldsymbol{S} \sim b^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} b_\beta$

U(1) gauge symmetry: $b \to b e^{i\theta}$

$\mathbb{CP}^1$ U(1) gauge theory.

't Hooft/Lieb-Schultz-Mattis
anomalies realized by
monopole Berry phases.



Consistent with many
numerical studies

Critical spin liquid
without quasiparticles?

$$\mathcal{L}_z = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science **303**, 1490 (2004)

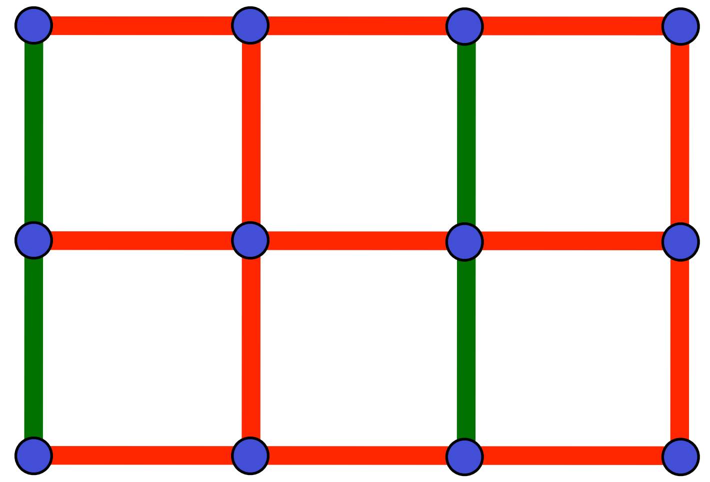
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ fermionic spinons $\boldsymbol{S} \sim f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

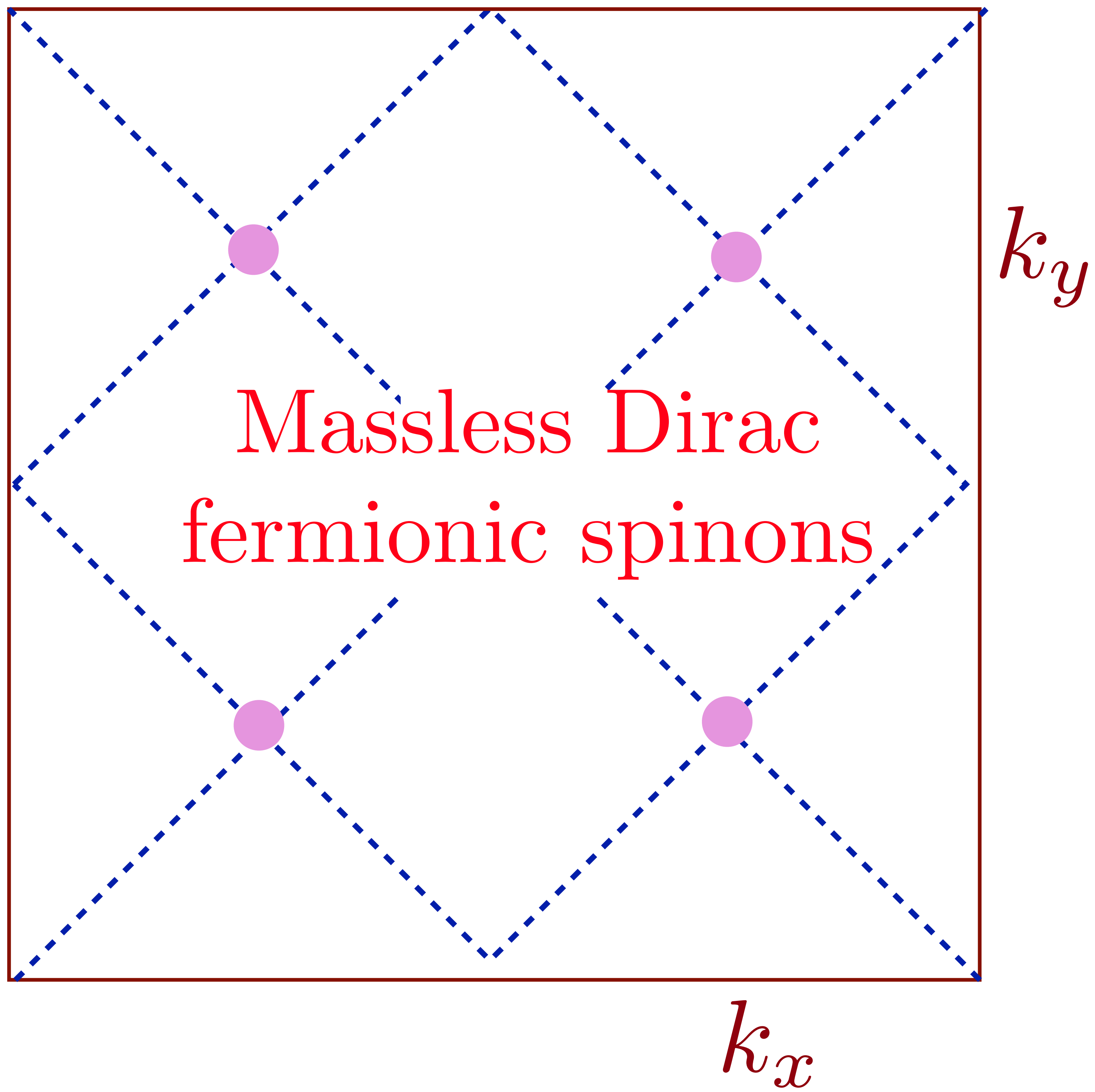
G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987); G. Kotliar PRB **37**, 3664 (1988)
I. Affleck, J.B. Marston, PRB **37**, 3774 (1988); F.C. Zhang, C. Gros, T.M. Rice, H. Shiba Supercond. Sci. Tech. **1**, 36 (1988)


$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(f_{\boldsymbol{i}\alpha}^\dagger f_{\boldsymbol{j}\alpha} - f_{\boldsymbol{j}\alpha}^\dagger f_{\boldsymbol{i}\alpha} \right)$$



$e_{\boldsymbol{ij}} = 1$

$e_{\boldsymbol{ij}} = -1$

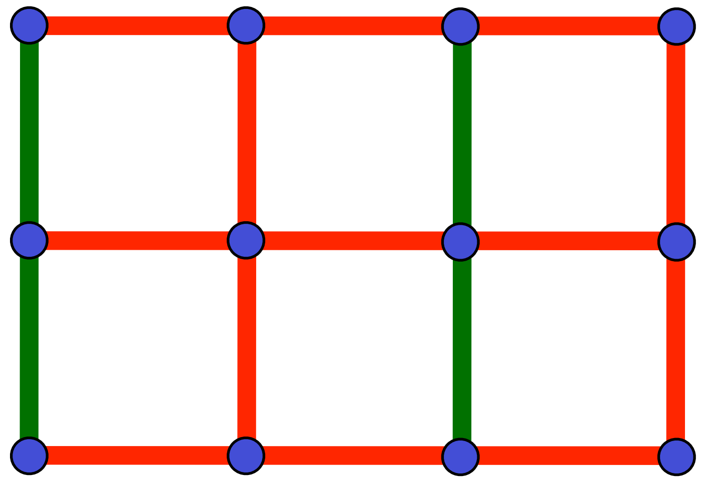


S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ fermionic spinons $\boldsymbol{S} \sim f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi_{\boldsymbol{i}}^\dagger U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi_{\boldsymbol{j}}^\dagger U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f_{\boldsymbol{i}\downarrow}^\dagger \end{pmatrix}$$

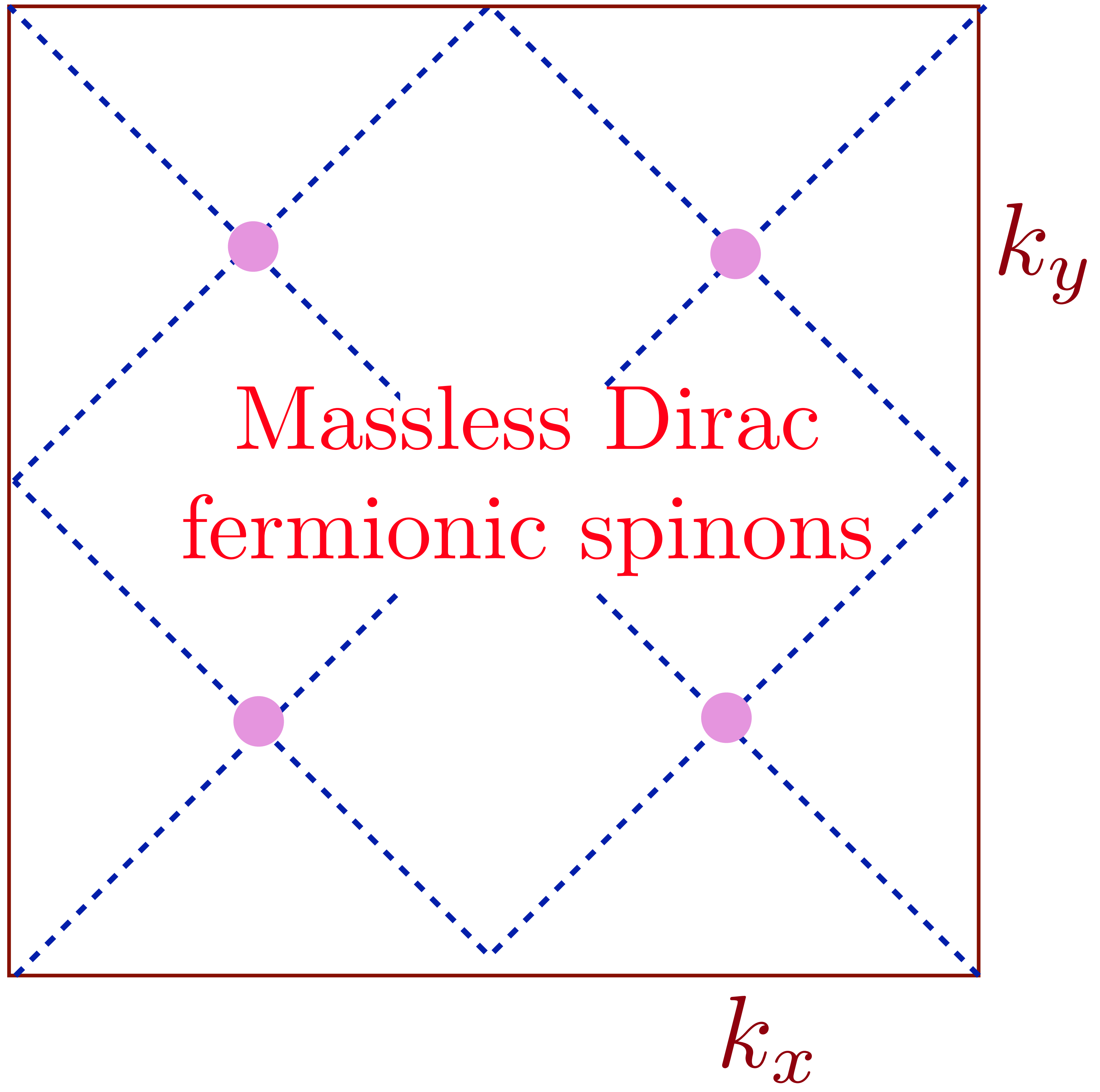


$e_{\boldsymbol{ij}} = 1$

$e_{\boldsymbol{ij}} = -1$

$$\boxed{\mathcal{L} = i\overline{\psi}\gamma_\mu D_\mu \psi.}$$

J_2/J_1

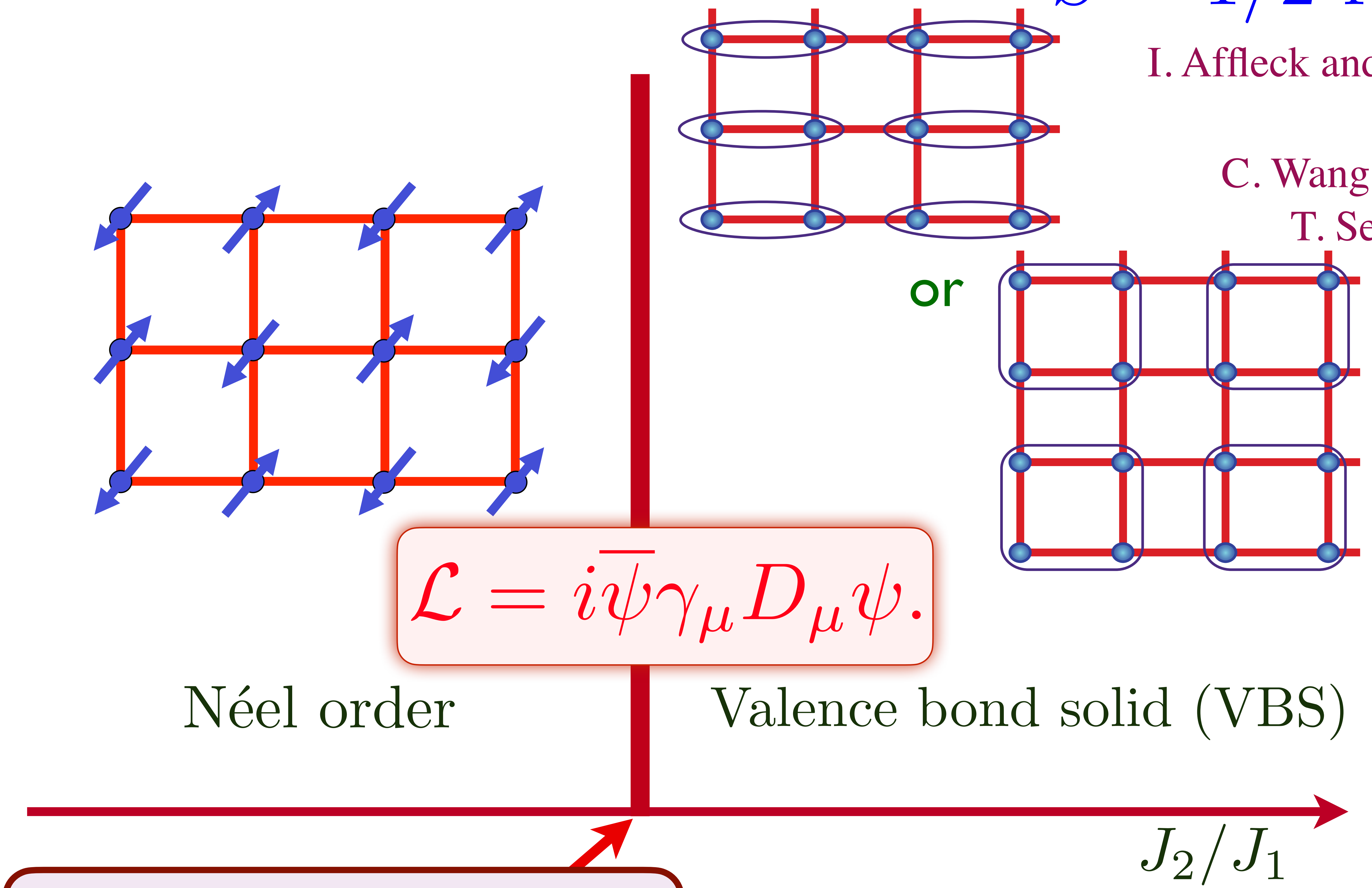


$N_f = 2$ SU(2) QCD

S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ fermionic spinons $\boldsymbol{S} \sim f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

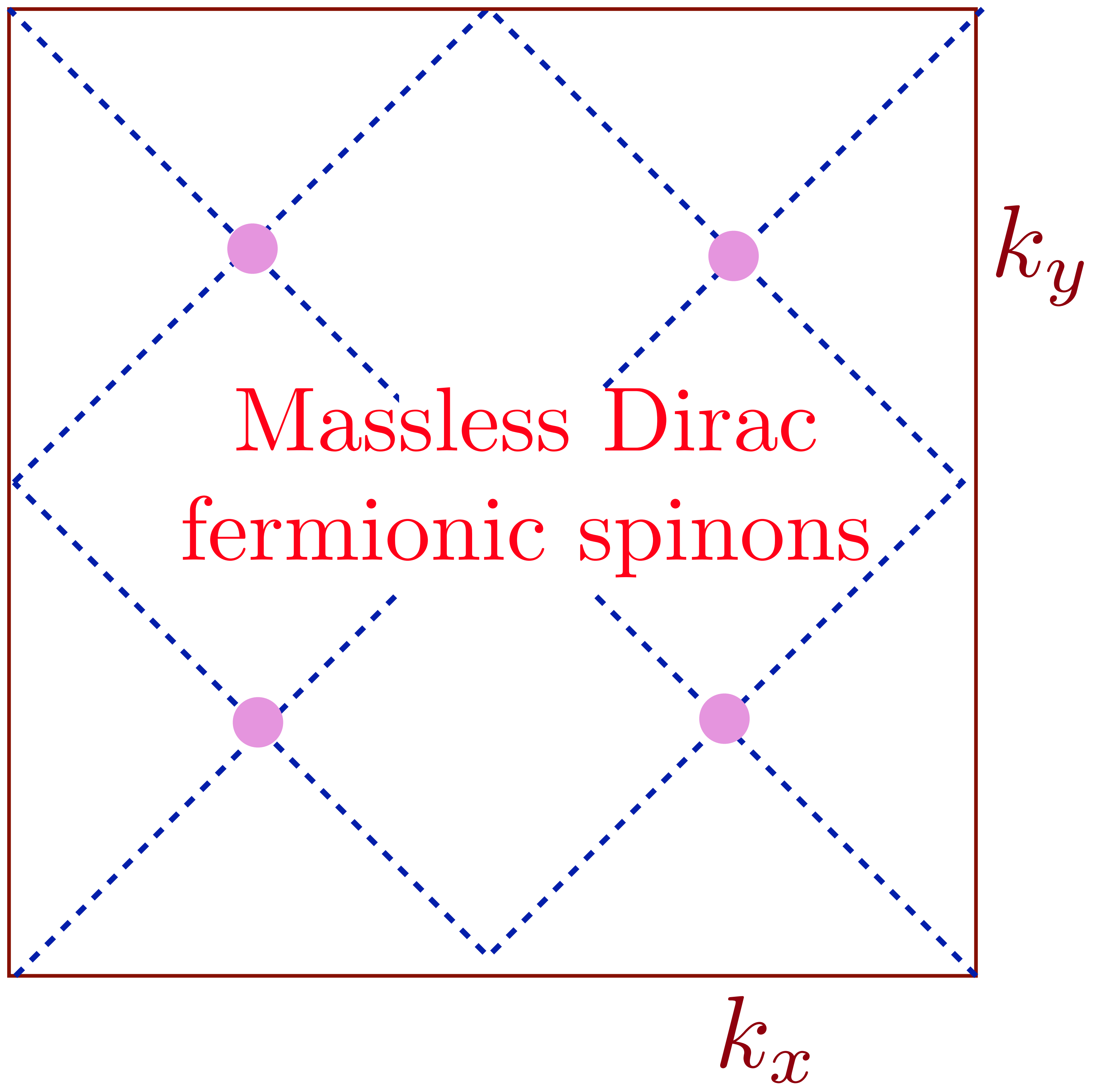
I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)
Ying Ran, MIT Ph.D. thesis (2007)
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)



$$\mathcal{L} = i\overline{\psi}\gamma_\mu D_\mu \psi.$$

Critical spin liquid without quasiparticles?

$N_f = 2$ SU(2) QCD



Confining instability to Néel and VBS, as in $\mathbb{CP}^1$ theory of Read+SS

S=1/2 square lattice

Bosonic spinons:
$\mathbb{CP}^1$ U(1) gauge theory

N. Read and S. Sachdev, PRL **62**, 1694 (1989)

Nearly-critical S=1/2 square lattice antiferromagnet without quasiparticles

SU(2) gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

Obtained from a saddle-point of fermionic spinons moving in π-flux.

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); C. Wang, A. Nahum, M. A. Metlitski, C. Xu, T. Senthil, *Phys. Rev. X* **7**, 031051 (2017); Zheng Zhou, Liangdong Hu, Wei Zhu, Yin-Chen He, PRX **14**, 021044 (2024); S. M. Chester N. Su, PRL **132**, 111601 (2024). B.-B. Chen, X. Zhang, Y. Wang, K. Sun, Z. Y. Meng, PRL **132**, 246503 (2024); J. Takahashi, H. Shao, B. Zhao, W. Guo, A. W. Sandvik, arXiv:2405.06607.

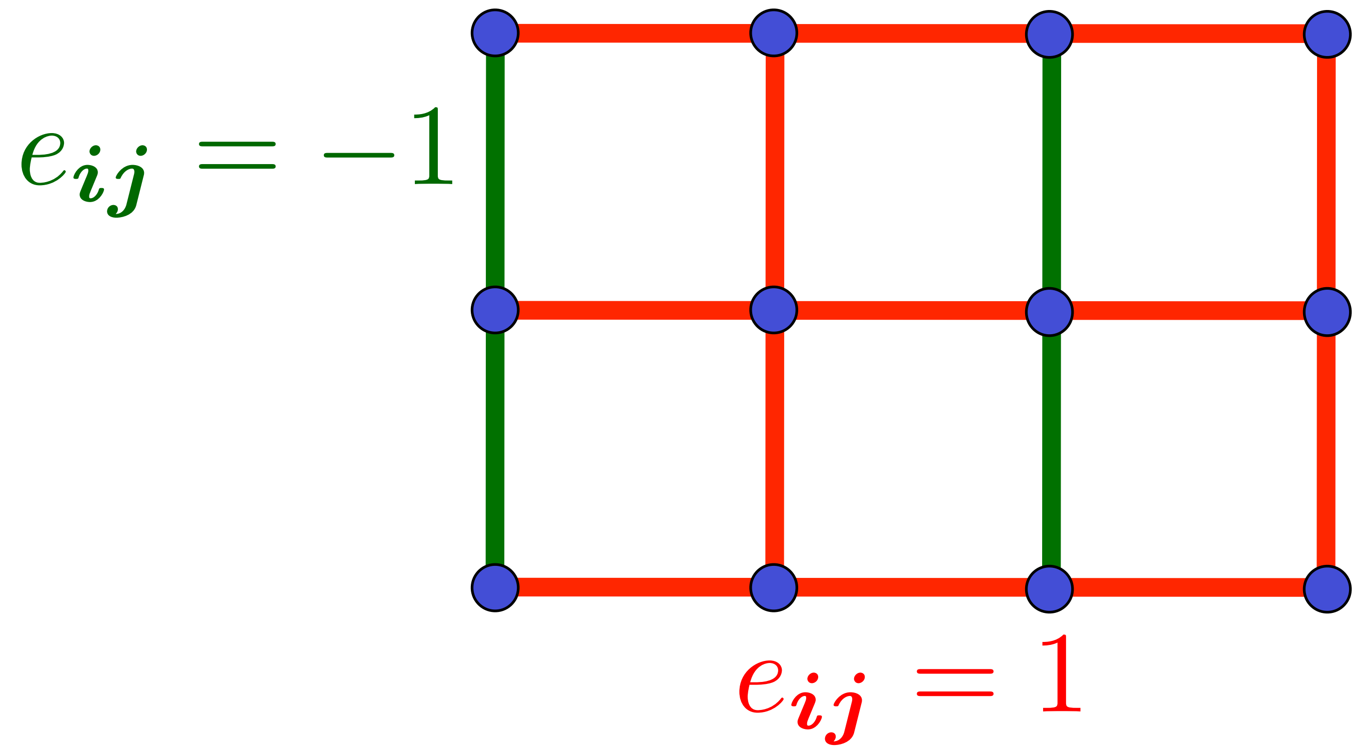
Fractionalized Intertwined Orders

Adding charge fluctuations to the π-flux spin liquid

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$\mathcal{H}_f = iJ \sum_{\langle ij \rangle} e_{\boldsymbol{ij}} \left(f^\dagger_{\boldsymbol{i}\alpha} f_{\boldsymbol{j}\alpha} - f^\dagger_{\boldsymbol{j}\alpha} f_{\boldsymbol{i}\alpha} \right) \Rightarrow \boxed{iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_i U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right)} \; ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

$\mathcal{H}_f$ is invariant under SU(2) rotations in spin and SU(2) rotations in Nambu space; $U_{\boldsymbol{ij}}$ is the SU(2) gauge field.



$$\boxed{T_x T_y = -T_y T_x}$$

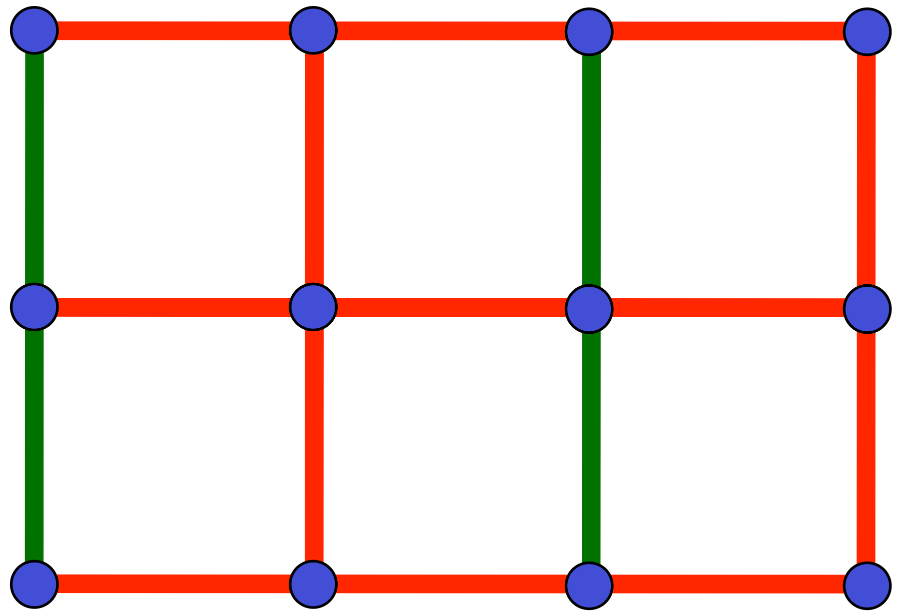
M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Adding charge fluctuations to the π-flux spin liquid

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$\mathcal{H}_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f^\dagger_{i\alpha} f_{j\alpha} - f^\dagger_{j\alpha} f_{i\alpha} \right) \Rightarrow \boxed{iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_i U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right)} \; ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

$\mathcal{H}_f$ is invariant under SU(2) rotations in spin and SU(2) rotations in Nambu space; $U_{\boldsymbol{ij}}$ is the SU(2) gauge field.

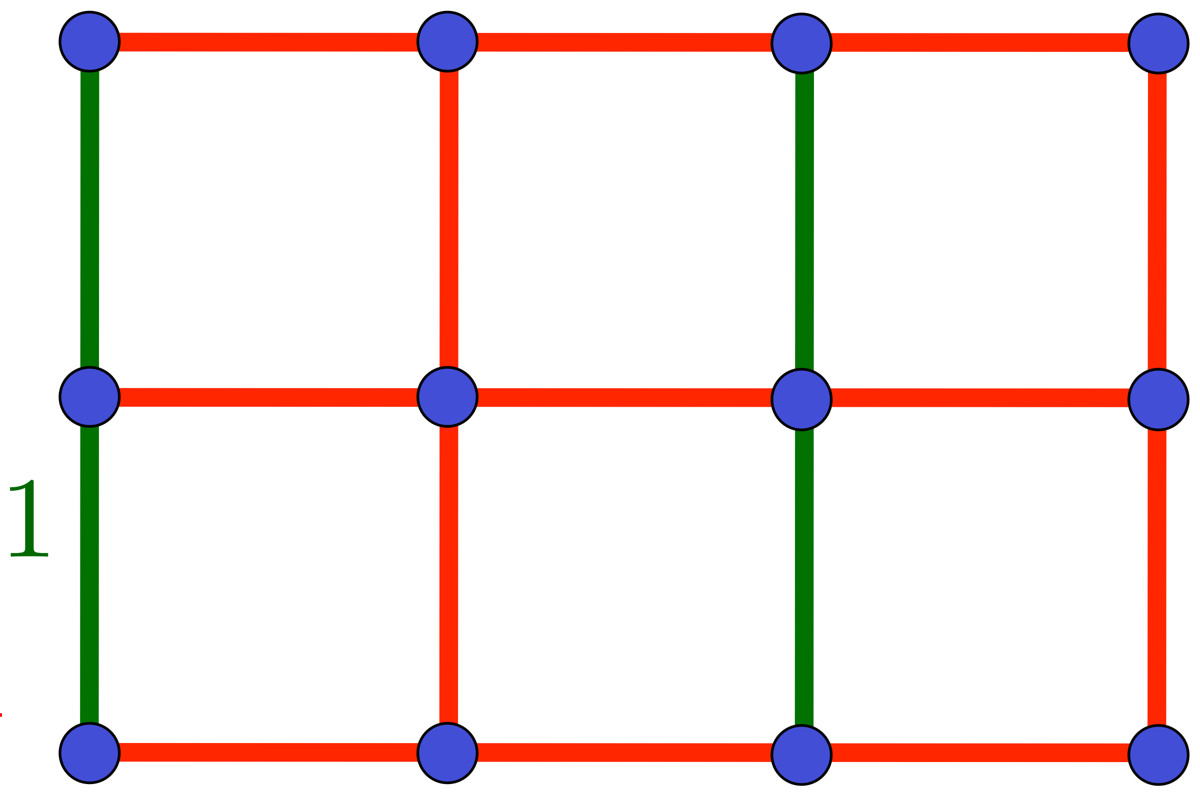


$$\boxed{T_x T_y = -T_y T_x}$$

- Introduce a charge e, SU(2) fundamental boson $B_{\boldsymbol{i}}$ such that the composite of $B_{\boldsymbol{i}}$ and $\Psi_{\boldsymbol{i}}$ is an electron. The projective symmetries require:

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

f_α and B both move in π-flux

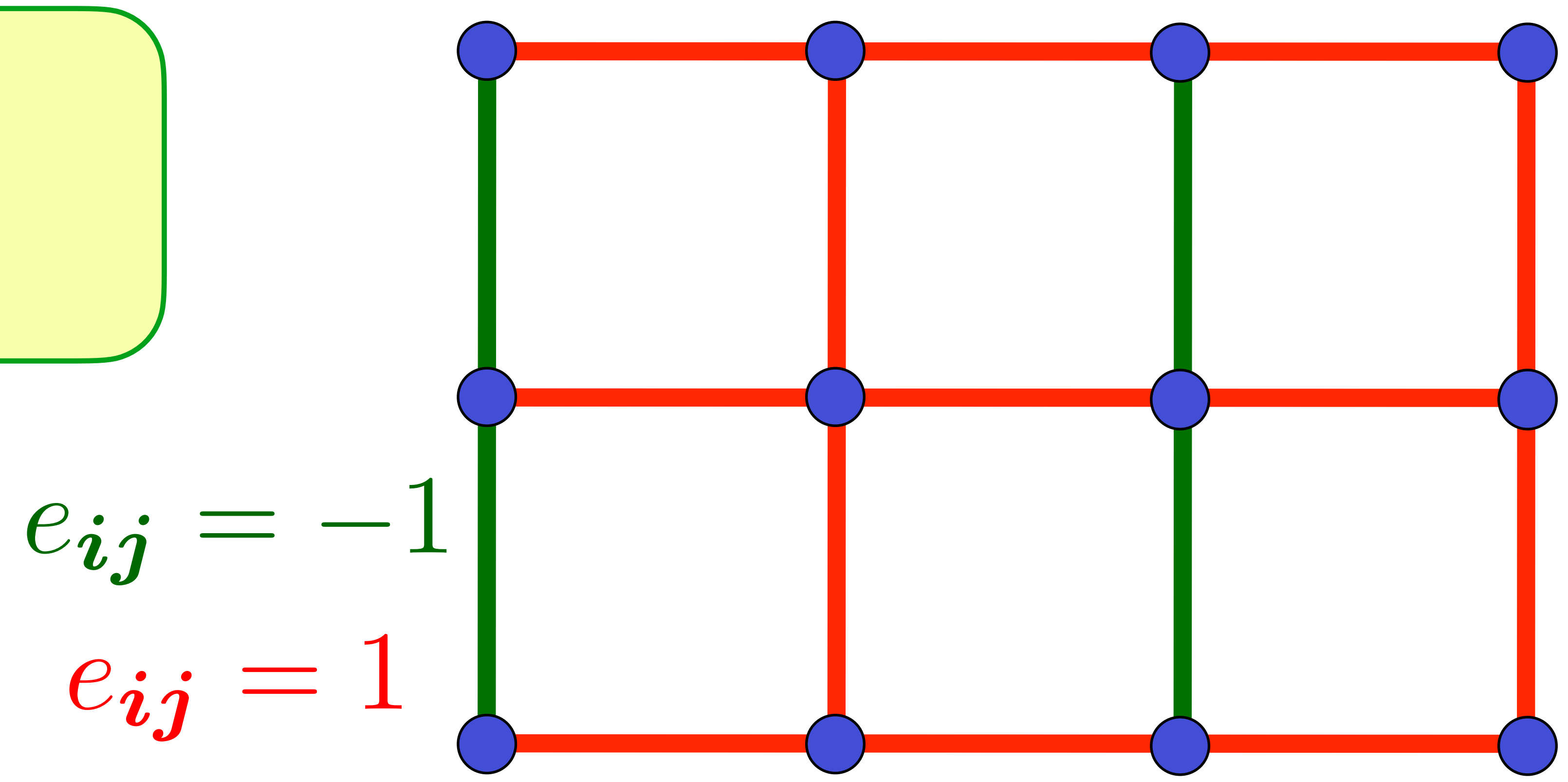


Symmetry	f_α	B_a
T_x	$(-1)^y f_\alpha$	$(-1)^y B_a$
T_y	f_α	B_a
P_x	$(-1)^x f_\alpha$	$(-1)^x B_a$
P_y	$(-1)^y f_\alpha$	$(-1)^y B_a$
P_{xy}	$(-1)^{xy} f_\alpha$	$(-1)^{xy} B_a$
$\mathcal{T}$	$(-1)^{x+y} \varepsilon_{\alpha\beta} f_\beta$	$(-1)^{x+y} B_a$

Projective transformations of the f spinons and B chargons on lattice sites $\boldsymbol{i} = (x, y)$ under the symmetries $T_x : (x, y) \to (x+1, y)$; $T_y : (x, y) \to (x, y+1)$; $P_x : (x, y) \to (-x, y)$; $P_y : (x, y) \to (x, -y)$; $P_{xy} : (x, y) \to (y, x)$; and time-reversal $\mathcal{T}$. The indices α, β refer to global SU(2) spin, while the index $a = 1, 2$ refers to gauge SU(2).

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

f_α and B both move in π-flux



Symmetry	f_α	B_a
T_x	$(-1)^y f_\alpha$	$(-1)^y B_a$
T_y	f_α	B_a
P_x	$(-1)^x f_\alpha$	$(-1)^x B_a$
P_y	$(-1)^y f_\alpha$	$(-1)^y B_a$
P_{xy}	$(-1)^{xy} f_\alpha$	$(-1)^{xy} B_a$
$\mathcal{T}$	$(-1)^{x+y} \varepsilon_{\alpha\beta} f_\beta$	$(-1)^{x+y} B_a$

Projective transformations of the f spinons and B chargons on lattice sites $\boldsymbol{i} = (x, y)$ under the symmetries $T_x : (x,y) \to (x+1, y)$; $T_y : (x,y) \to (x, y+1)$; $P_x : (x,y) \to (-x, y)$; $P_y : (x,y) \to (x, -y)$; $P_{xy} : (x,y) \to (y,x)$; and time-reversal $\mathcal{T}$. The indices α, β refer to global SU(2) spin, while the index $a = 1, 2$ refers to gauge SU(2).

Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim$

$$\Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$$

site charge density: $\left\langle c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B^\dagger_{\boldsymbol{i}} B_{\boldsymbol{i}}$

bond density: $\left\langle c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + c^\dagger_{\boldsymbol{j}\alpha} c_{\boldsymbol{i}\alpha} \right\rangle$

$$\sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B^\dagger_{\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$$

bond current: $i \left\langle c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - c^\dagger_{\boldsymbol{j}\alpha} c_{\boldsymbol{i}\alpha} \right\rangle$

$$\sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B^\dagger_{\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Energy functional for B and U: $\mathcal{E}[B,U] = \mathcal{E}_2[B,U] + \mathcal{E}_4[B,U] + \mathcal{E}_{YM}[U]$

$$\mathcal{E}_2[B,U] = (r + 2\sqrt{2}w) \sum_{\boldsymbol{i}} B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}} + iw \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(B_{\boldsymbol{i}}^\dagger U_{\boldsymbol{ij}} B_{\boldsymbol{j}} - B_{\boldsymbol{j}}^\dagger U_{\boldsymbol{ji}} B_{\boldsymbol{i}} \right)$$

$$\mathcal{E}_4[B,U] = \frac{u}{2} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}} \right) + g \sum_{\langle \boldsymbol{ij} \rangle} |\Delta_{\boldsymbol{ij}}|^2 + J_1 \sum_{\langle \boldsymbol{ij} \rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij} \rangle} J_{\boldsymbol{ij}}^2$$
$$+ V_{11} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}} \right) + V_{22} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}+2\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}-2\hat{\boldsymbol{y}}} \right)$$

$$\mathcal{E}_{YM}[U] = \kappa \sum_{\square} \left[1 - \frac{1}{2} \mathrm{Re}\mathrm{Tr} \prod_{\boldsymbol{ij} \in \square} U_{\boldsymbol{ij}} \right]$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

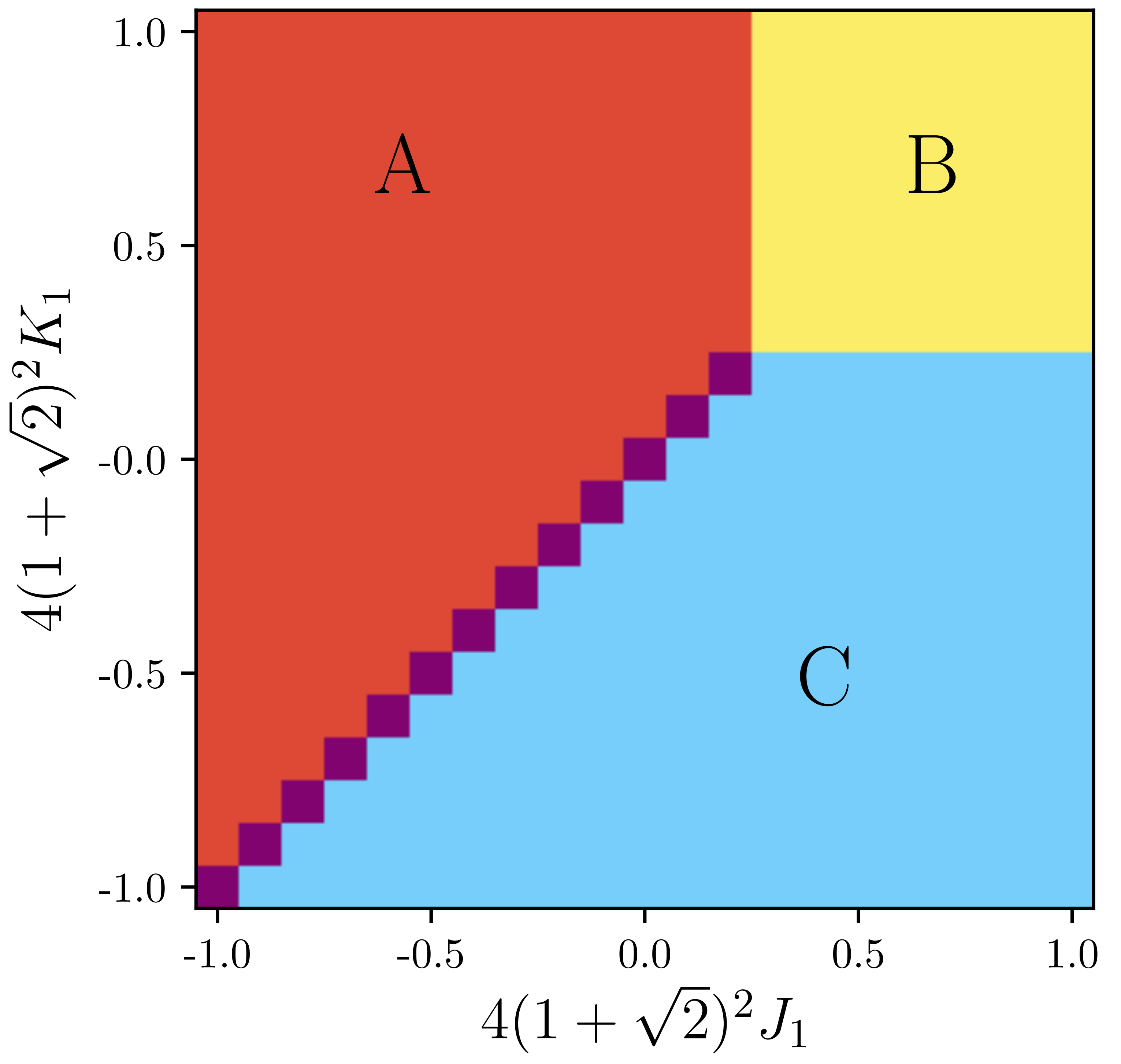
Adding charge fluctuations to the π-flux spin liquid

M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023)

Adding charge fluctuations to the π-flux spin liquid

M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang,
M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023)

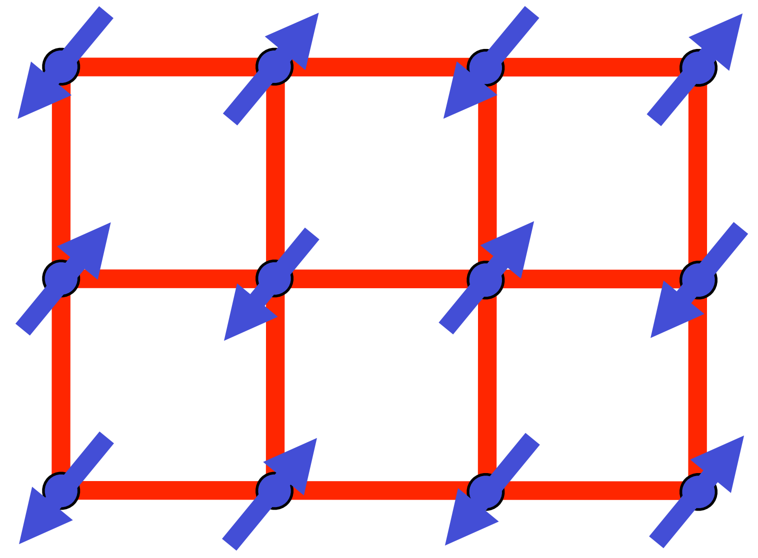
Adding charge fluctuations to the π-flux spin liquid

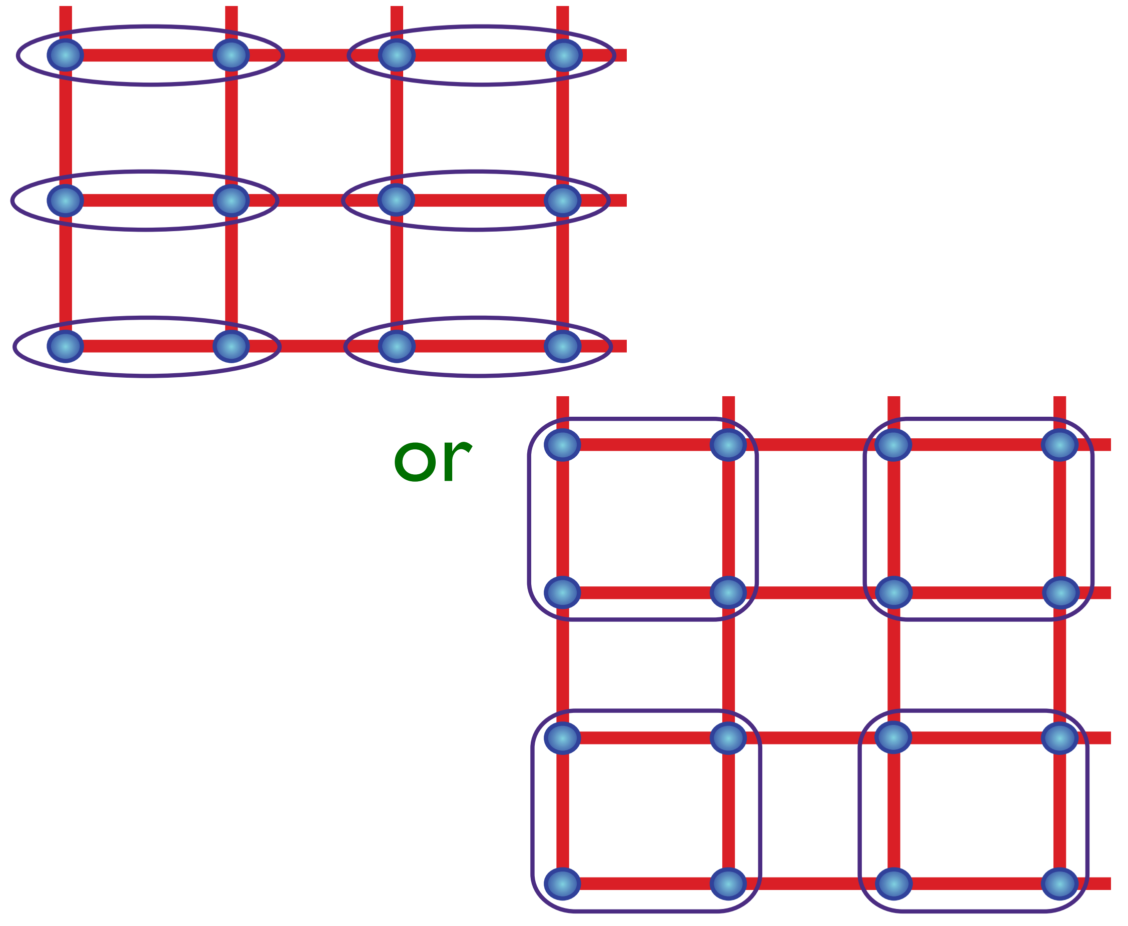


$\langle B \rangle \neq 0$

Including only nearest-neighbor couplings in B

$\langle B \rangle = 0$





Confining phase: Néel order

Confining phase: VBS order

s

From the theory of f_α spinons

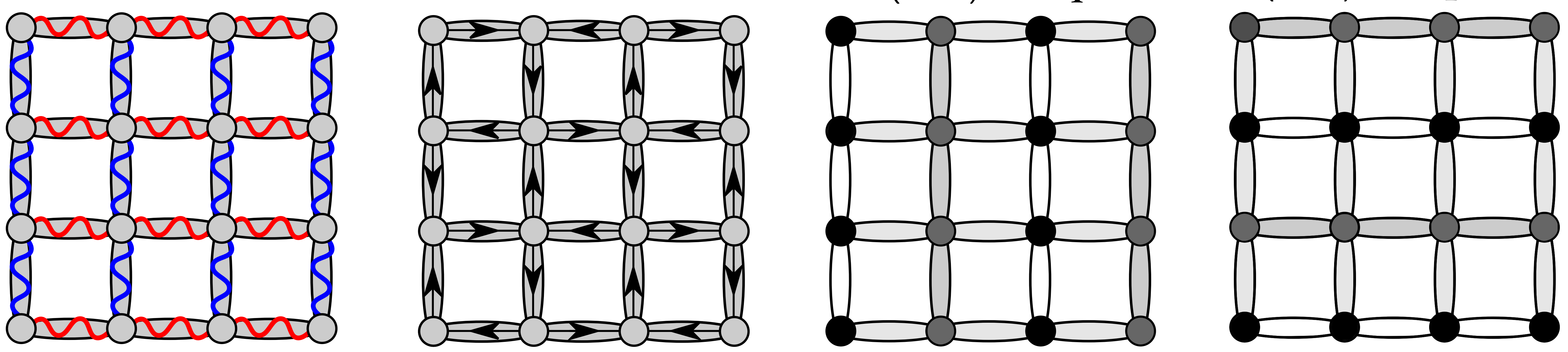
r

Phase B *d*-wave SC

Phase C *d*-density

Phase A (π,0) stripe

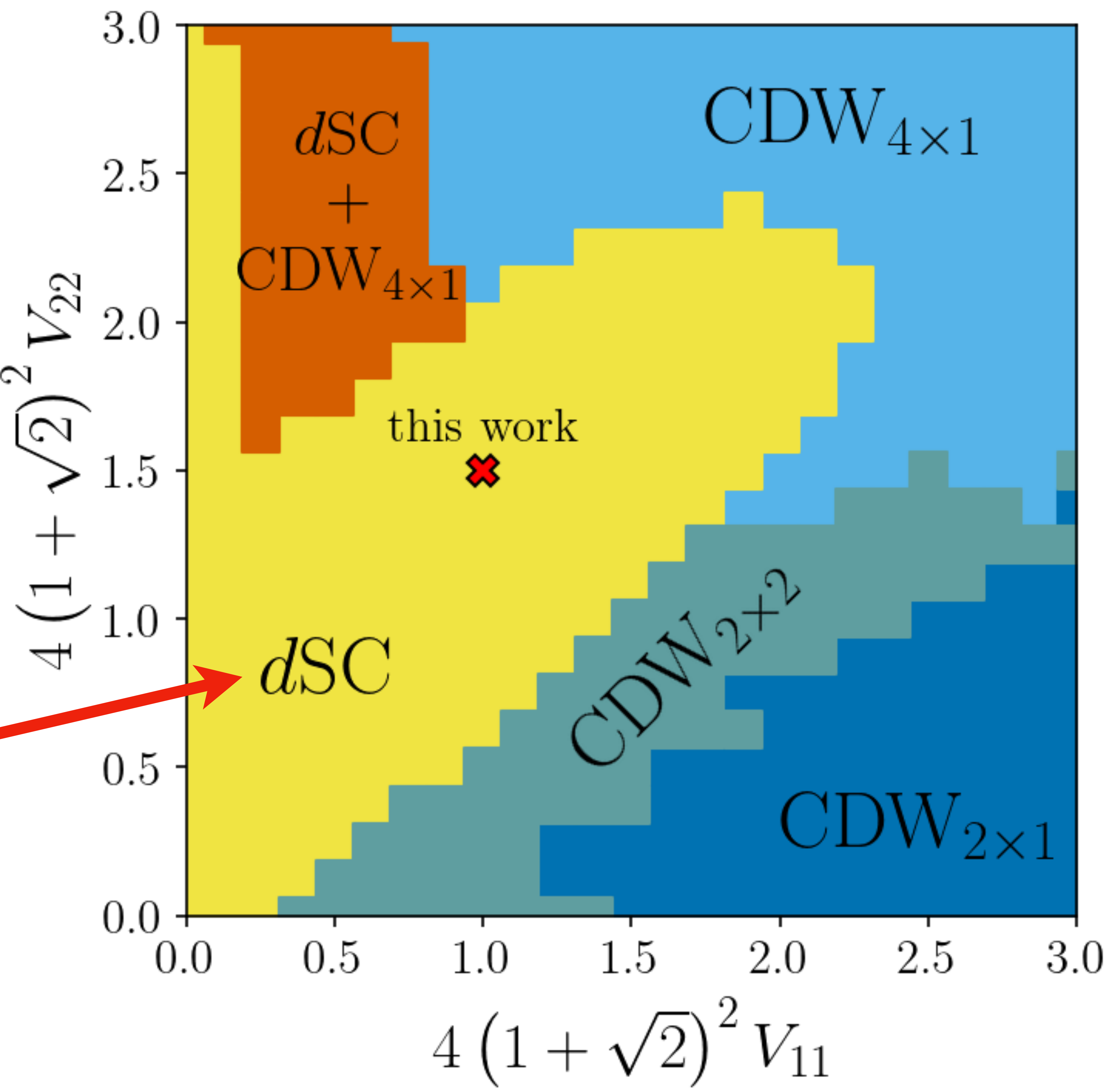
Phase A (0,π) stripe



M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023)

At $T = 0$, minimize $\mathcal{E}[B, U]$.

d-SC with
4 nodal quasiparticles



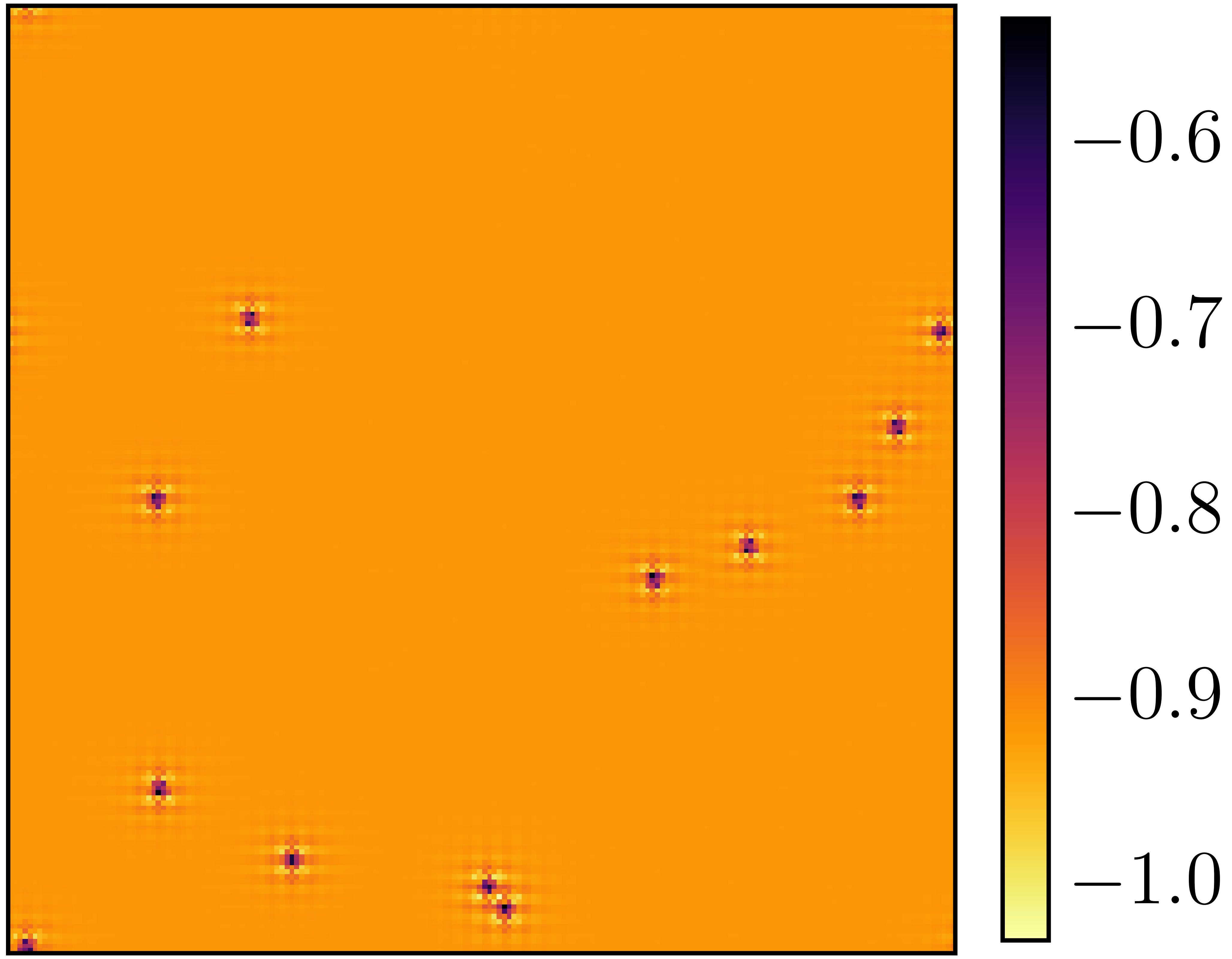
Parameters chosen so that the ground state is a d-wave superconductor,
and second best state is a period-4 stripe.

Monte Carlo at a temperature T

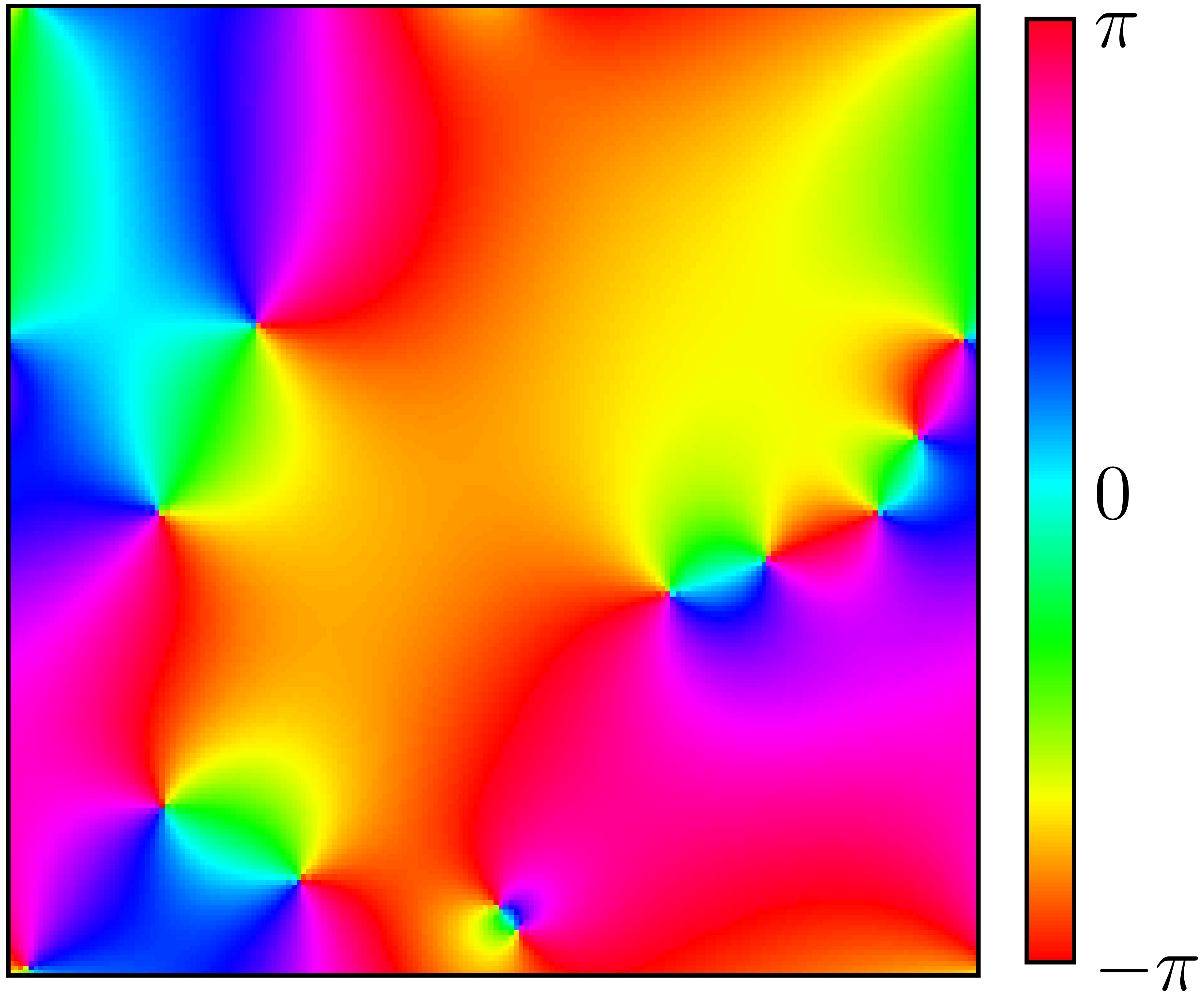
$$\mathcal{Z}_{2+0} = \int \prod_i \mathcal{D}B_{\boldsymbol{i}} \int \prod_{\langle \boldsymbol{ij} \rangle} \mathcal{D}U_{\boldsymbol{ij}} \exp\left[-\mathcal{E}[B,U]/T\right]$$

- Simulation of classical, thermal theory for bosons B, U defined by $\mathcal{Z}_{2+0}$

Monte Carlo at a temperature T

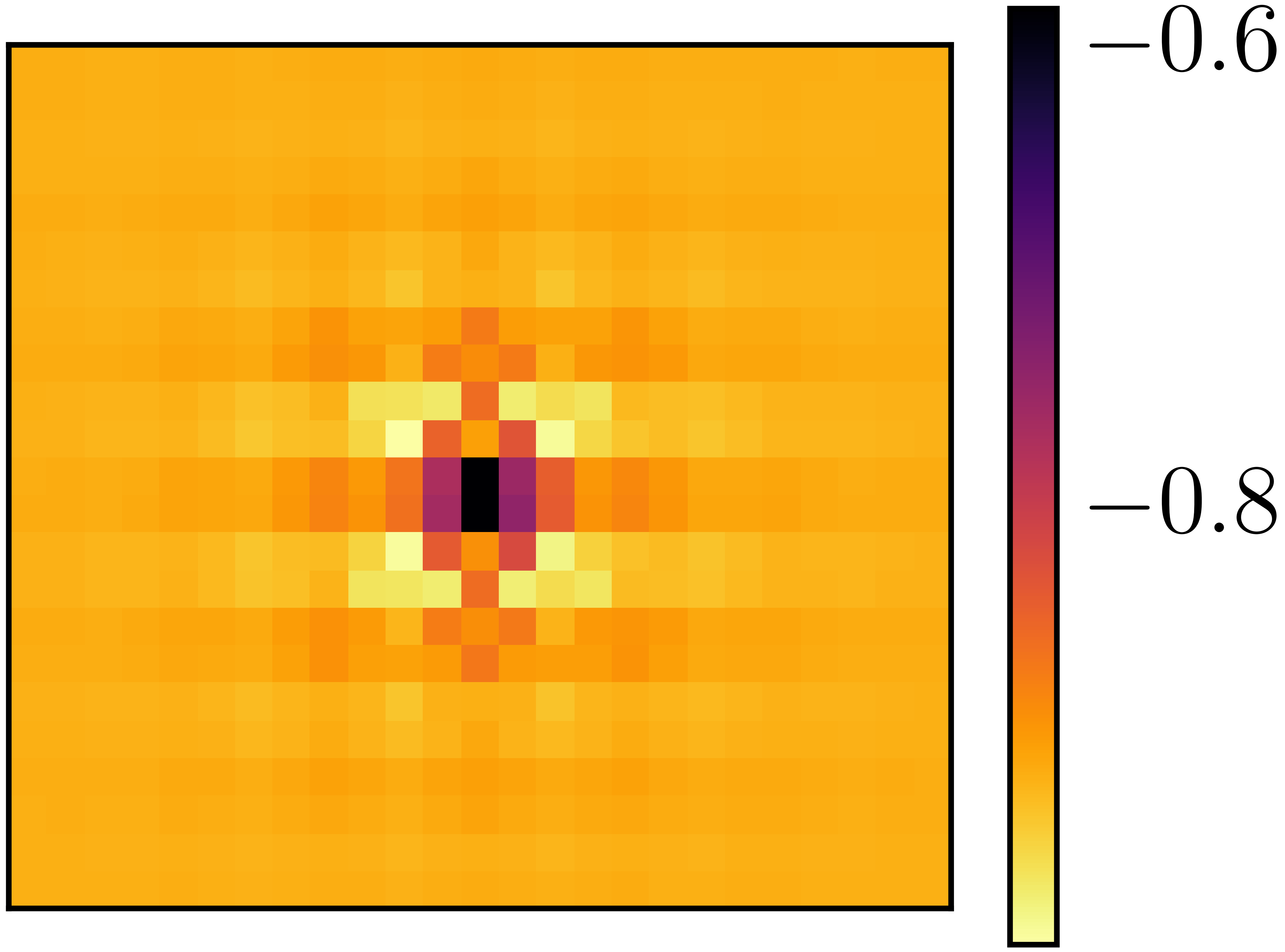


Bond density

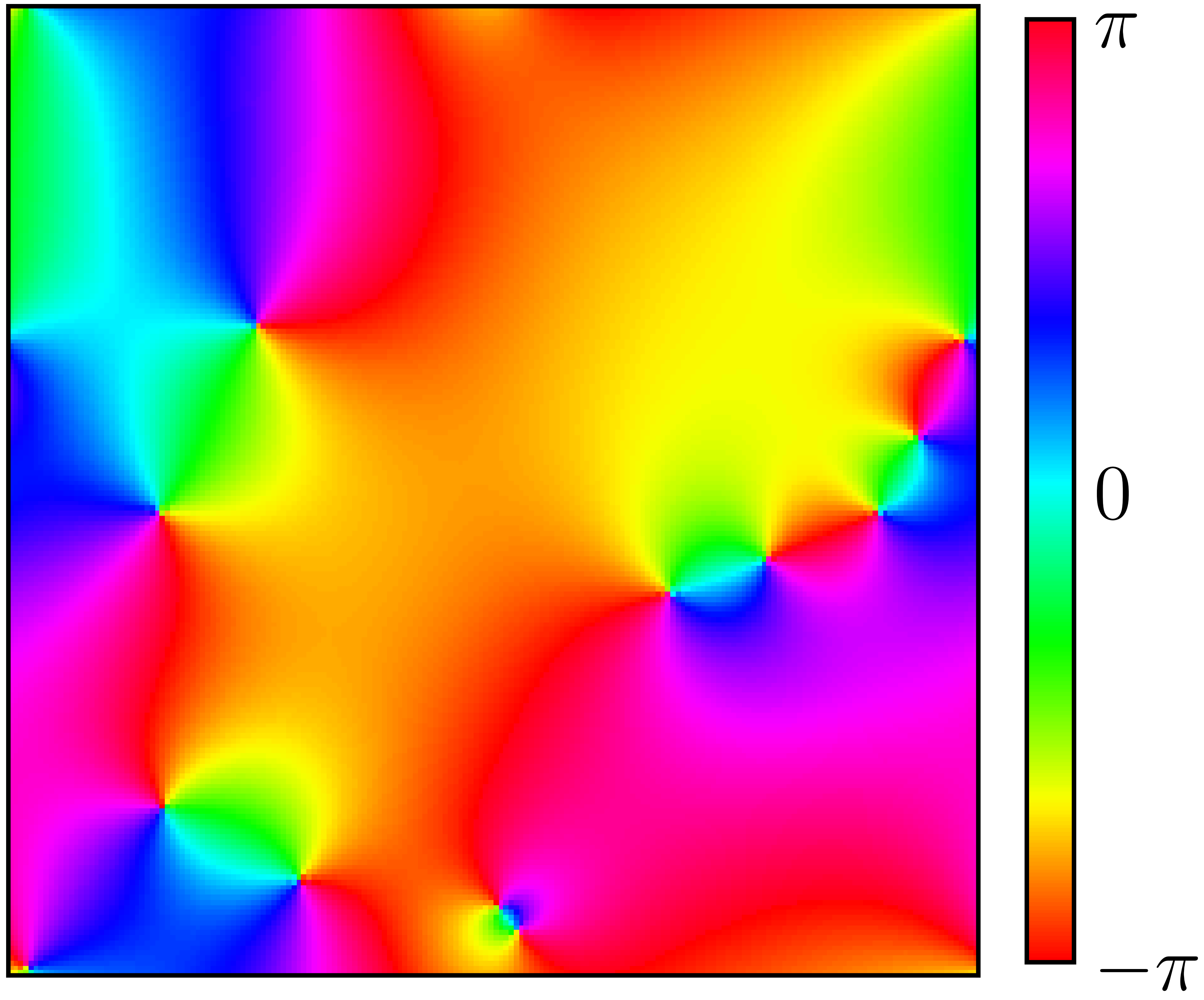


Phase of pairing amplitude

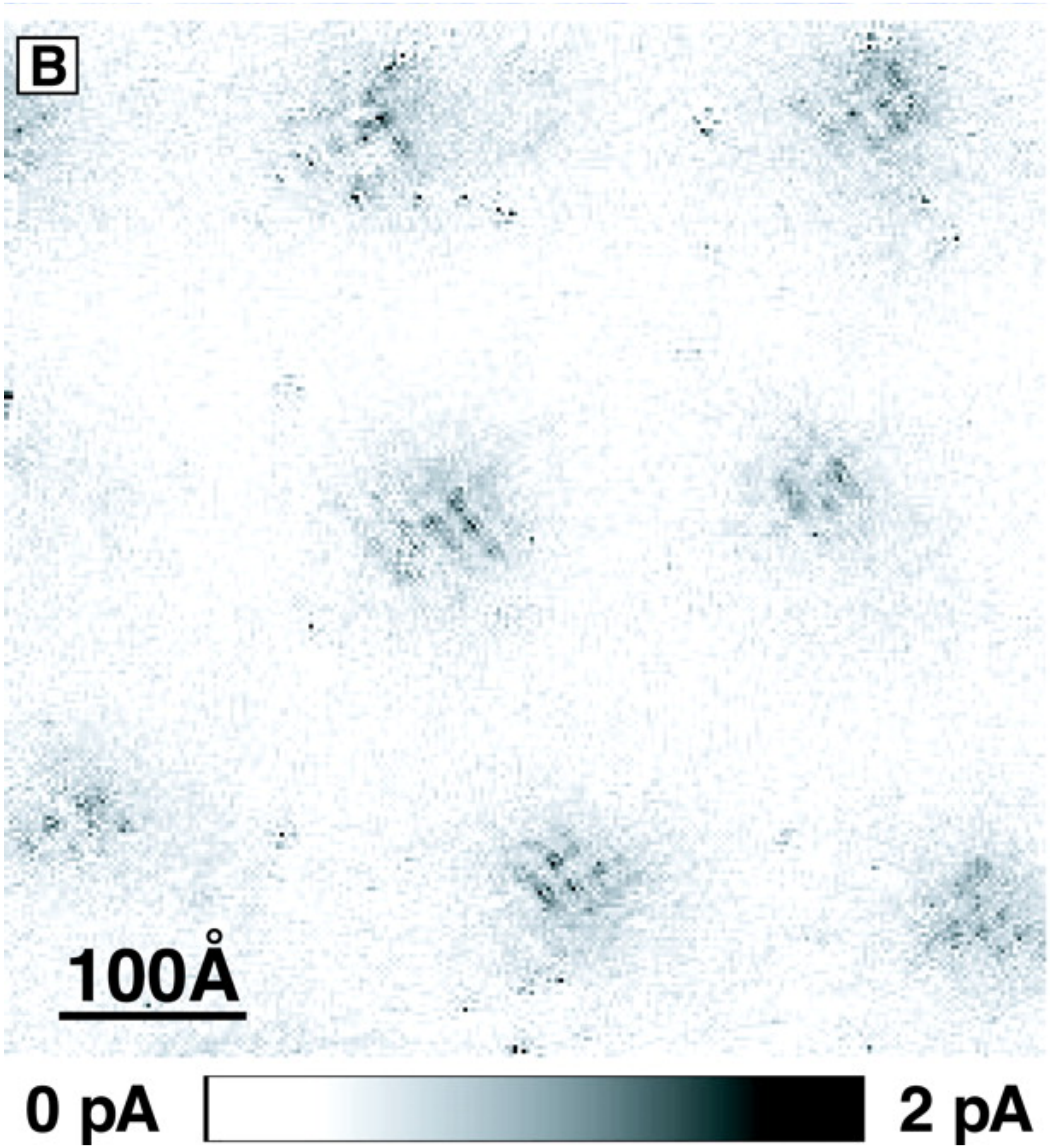
Monte Carlo at a temperature T



Bond density



Phase of pairing amplitude

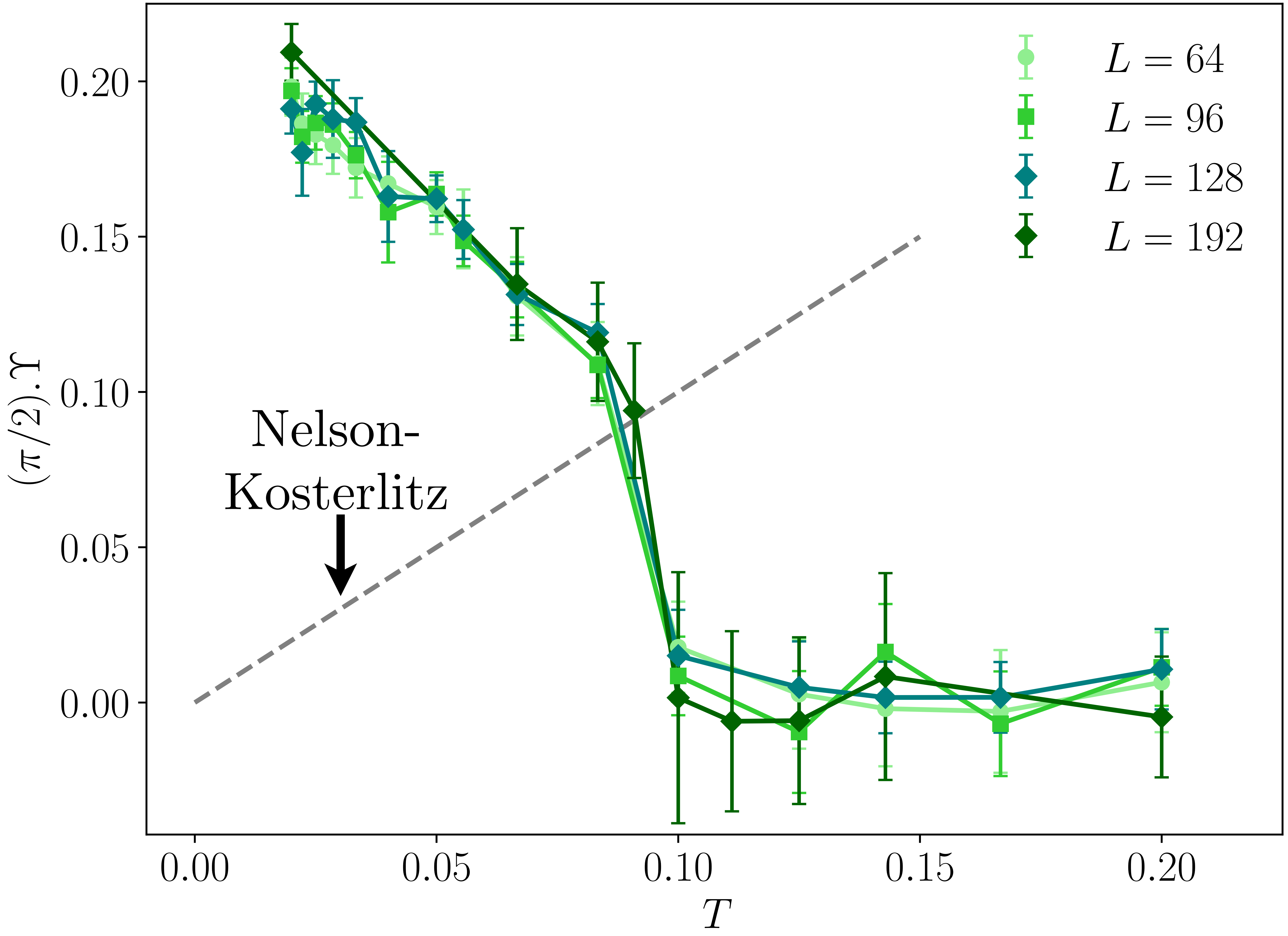


A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman, E. W. Hudson,
K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, J.C. Davis
Science **295**, 466 (2002)

Monte Carlo at a temperature T

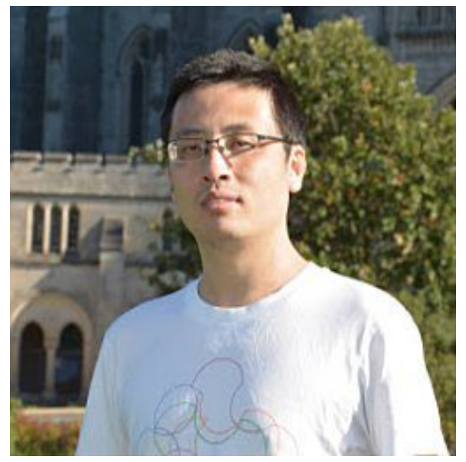
$\Upsilon =$ Helicity Modulus

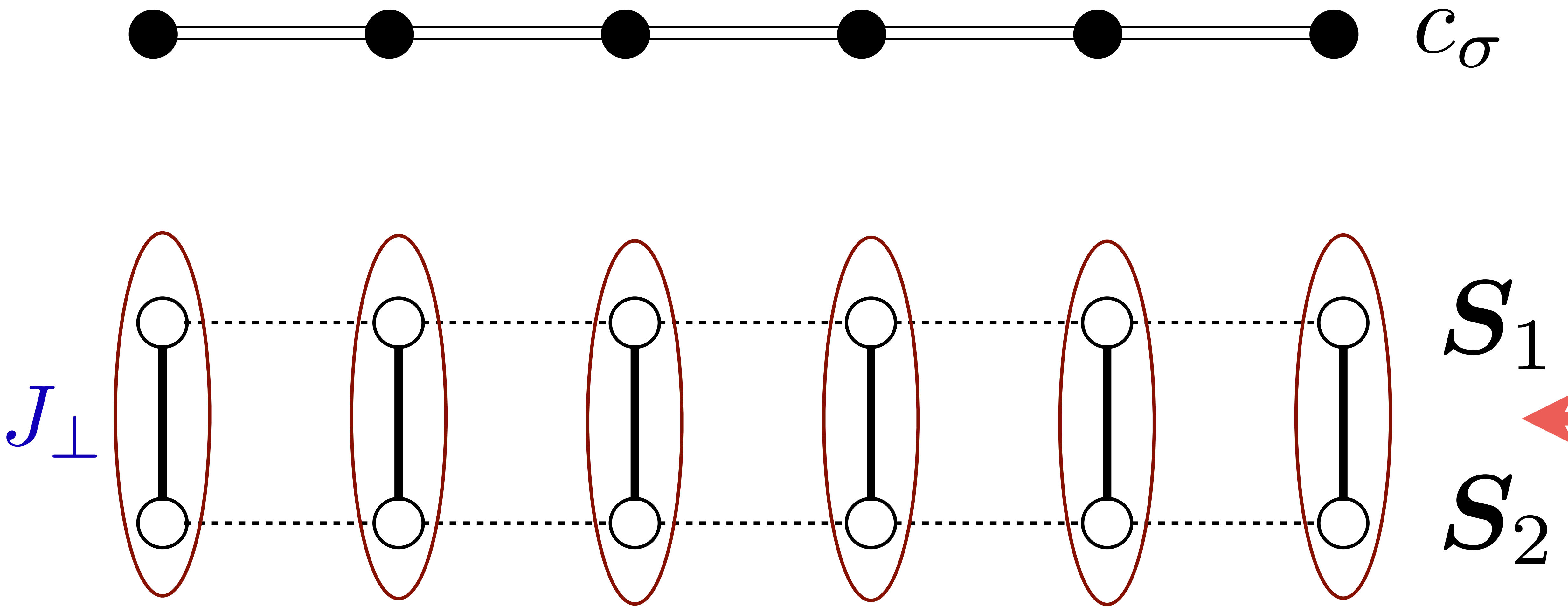


Theory of hole pockets and fractionalized intertwined orders

Ancilla Layer Model of FL* in a single-band Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)



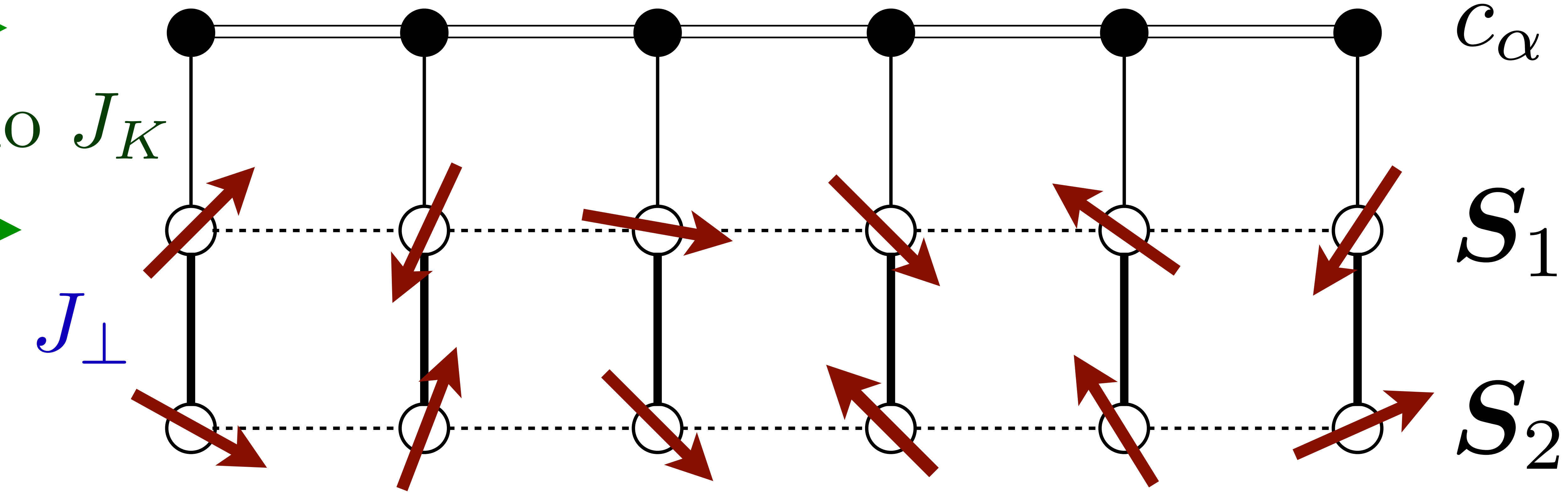


Hubbard model of hole density 1+p

Ancilla qubits in a “trivial” gapped insulator

$$\mathcal{U}\left(\mathcal{H}_{\rm Hubbard} + \mathcal{H}_{\rm trivial\ \ insulator}\right)\mathcal{U}^{-1} = \mathcal{H}_{\rm ancilla}$$

Antiferromagnetic Kondo J_K



Free holes of density 1+p

Schrieffer-Wolff transformation at large $J_\perp$ yields $U \sim J_K^2/J_\perp$

Ancilla Layer Model of FL* in a single-band Hubbard model

$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f_{1\boldsymbol{i}\alpha}^\dagger f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c_{\boldsymbol{i}\alpha}^\dagger f_{1\boldsymbol{i}\alpha} + f_{1\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right)$$

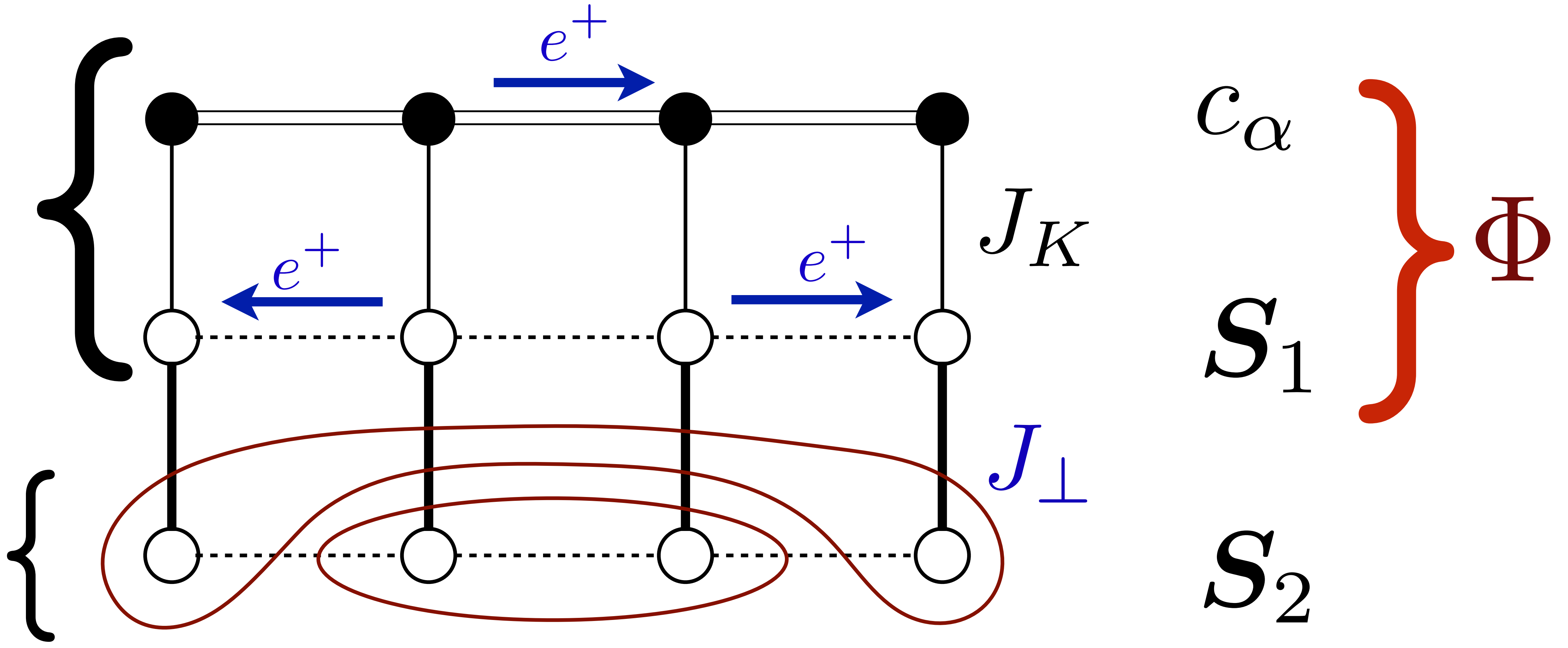
Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f_{1\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p \pmod 2$.
Small Fermi surface!

Your favorite spin liquid



Ancilla Layer Model of FL* in a single-band Hubbard model

$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c^\dagger_{\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} + f^\dagger_{1\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right)$$

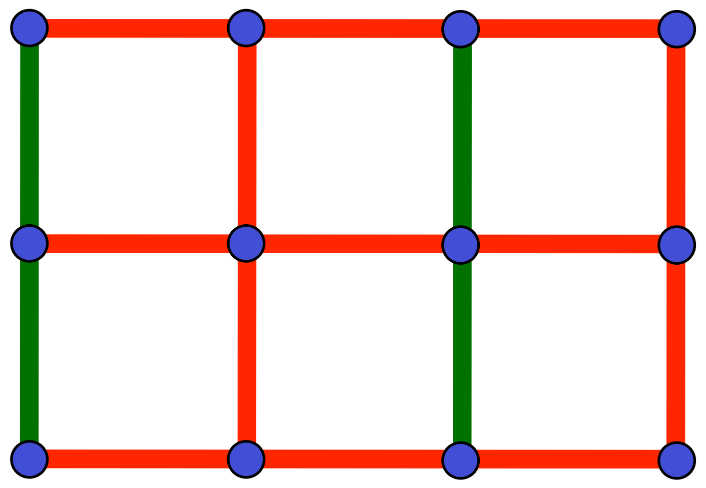
Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_{\boldsymbol{i}} U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) + \mathcal{E}_{YM}[U] \; ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

π-flux $\boldsymbol{S}_2$ spin liquid.

$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$



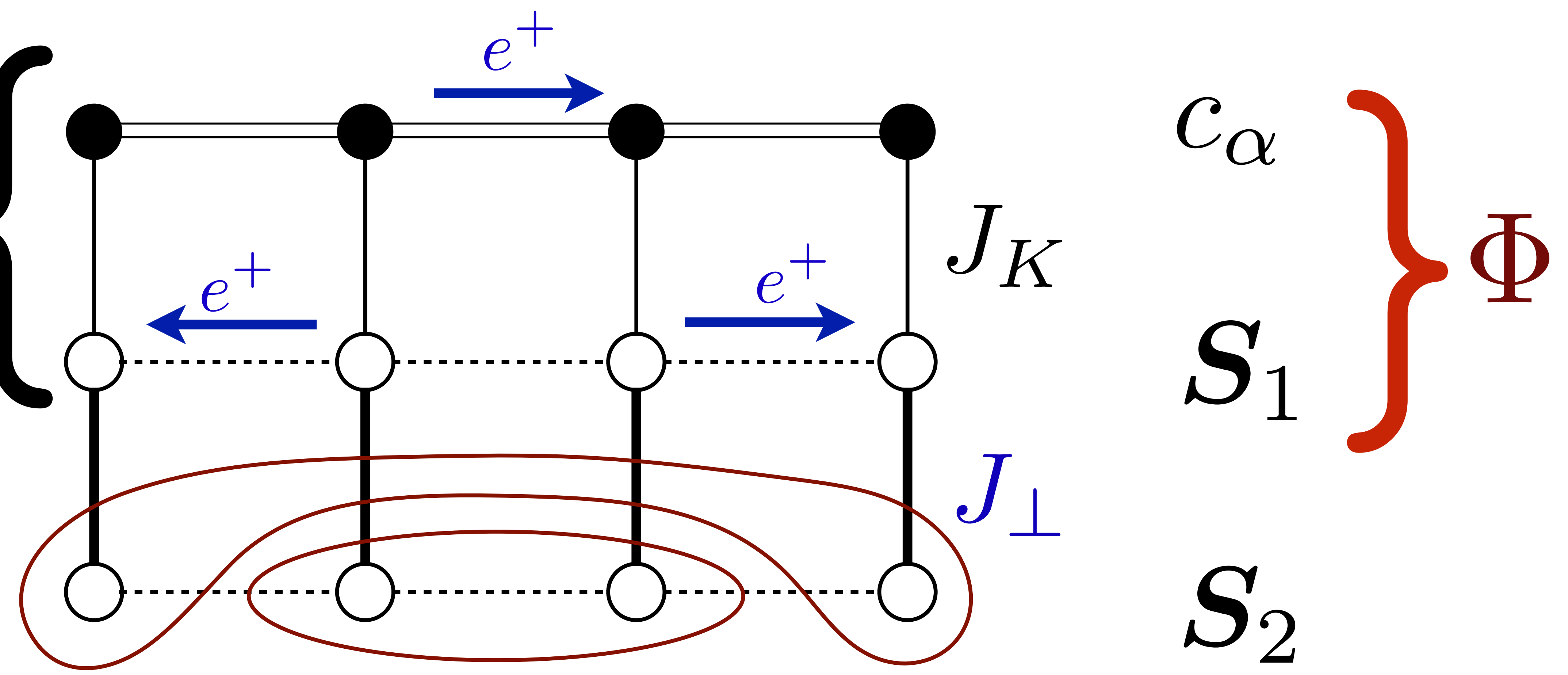
$e_{\boldsymbol{ij}} = 1$

$e_{\boldsymbol{ij}} = -1$

Fermionic spinons f moving in π-flux and an emergent SU(2) gauge field U

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

π-flux spin liquid



Ancilla Layer Model of FL* in a single-band Hubbard model

$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c^\dagger_{\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} + f^\dagger_{1\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right)$$

Heavy Fermi liquid of electrons c, f_1
$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_{\boldsymbol{i}} U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) + \mathcal{E}_{YM}[U] \; ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

π-flux $\boldsymbol{S}_2$ spin liquid.
$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

$$H_{\text{coupling}} = \sum_{\boldsymbol{i}} \left(B^*_{1\boldsymbol{i}} f^\dagger_{1\boldsymbol{i}\alpha} f_{\boldsymbol{i}\alpha} + B^*_{2\boldsymbol{i}} \varepsilon_{\alpha\beta} f^\dagger_{1\boldsymbol{i}\alpha} f^\dagger_{\boldsymbol{i}\beta} + \text{H.c.} \right)$$

Couple Kondo lattice and spin liquid by charge e, SU(2) fundamental Higgs boson B

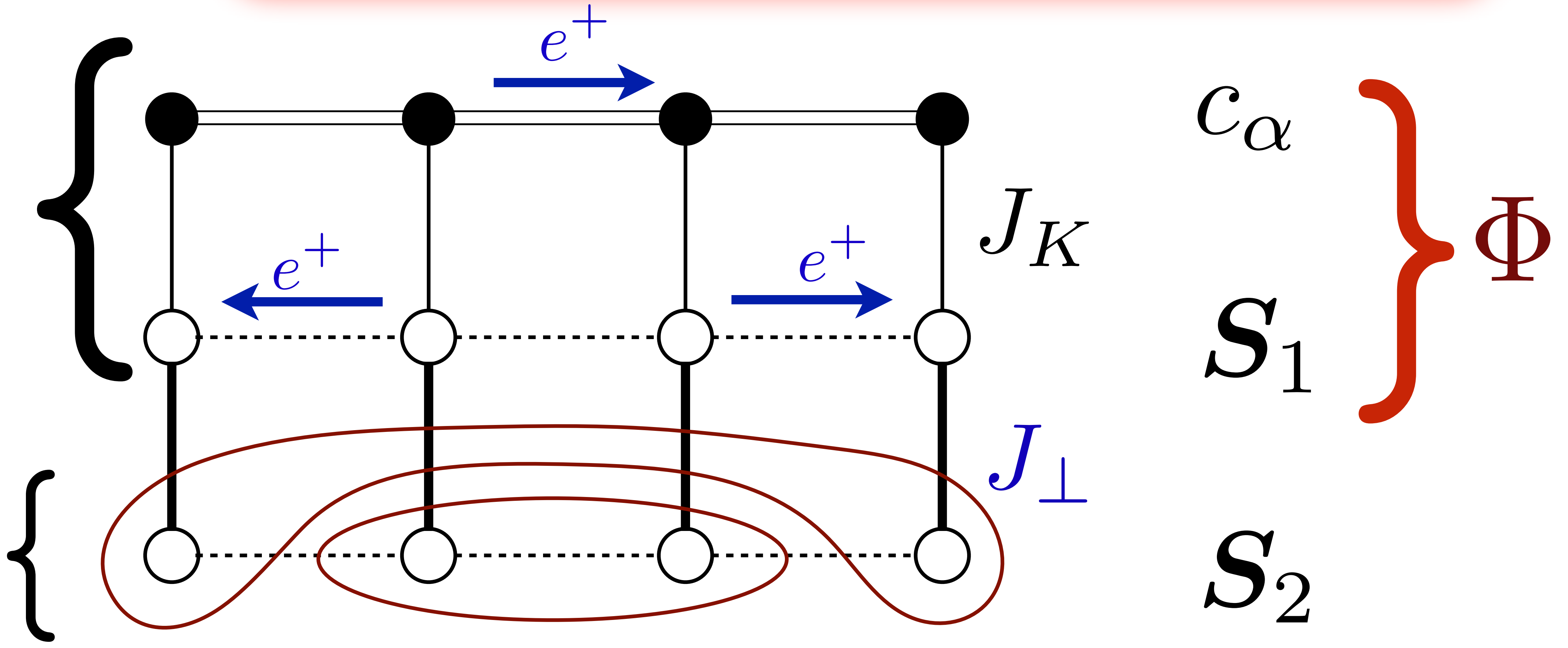
$$V_{\text{Higgs}} = \mathcal{E}_2[B,U] + \mathcal{E}_4[B,U]$$

V_{Higgs} dictated by symmetry of spin liquid

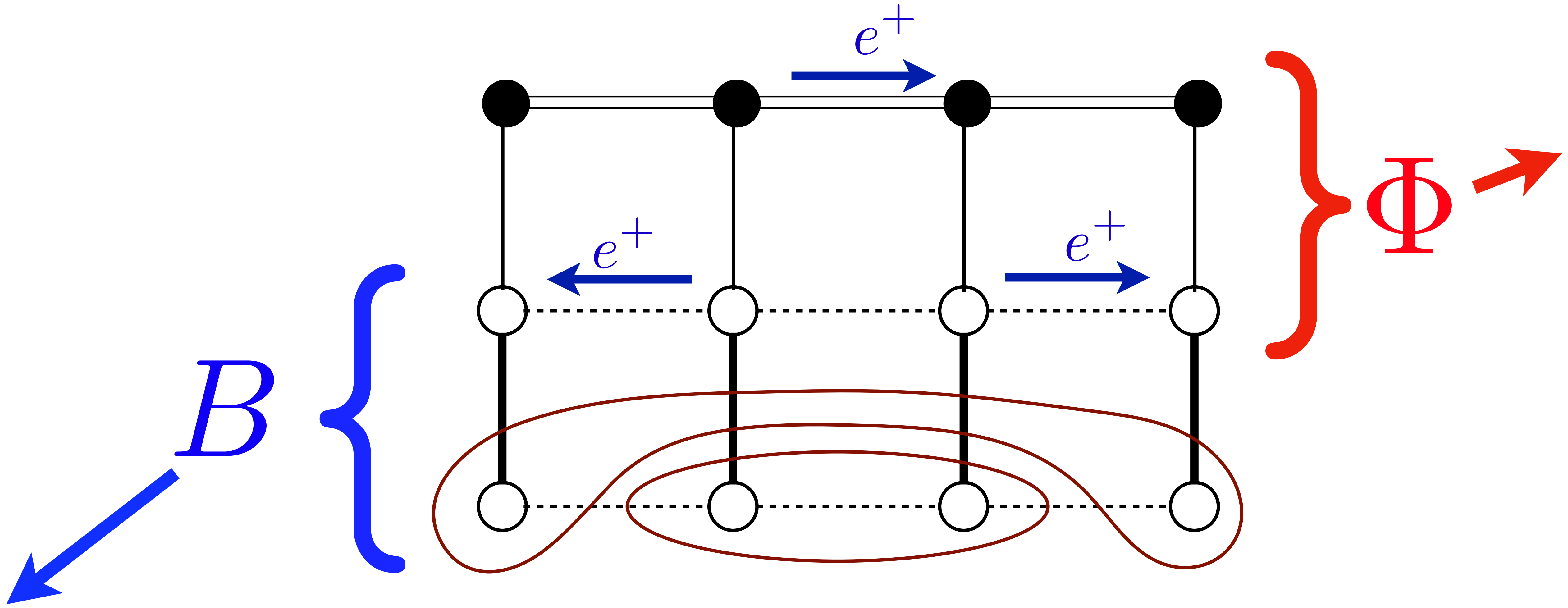
Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p \pmod 2$.
Small Fermi surface!

B

π-flux spin liquid



Ancilla Layer Model of FL* in a single-band Hubbard model



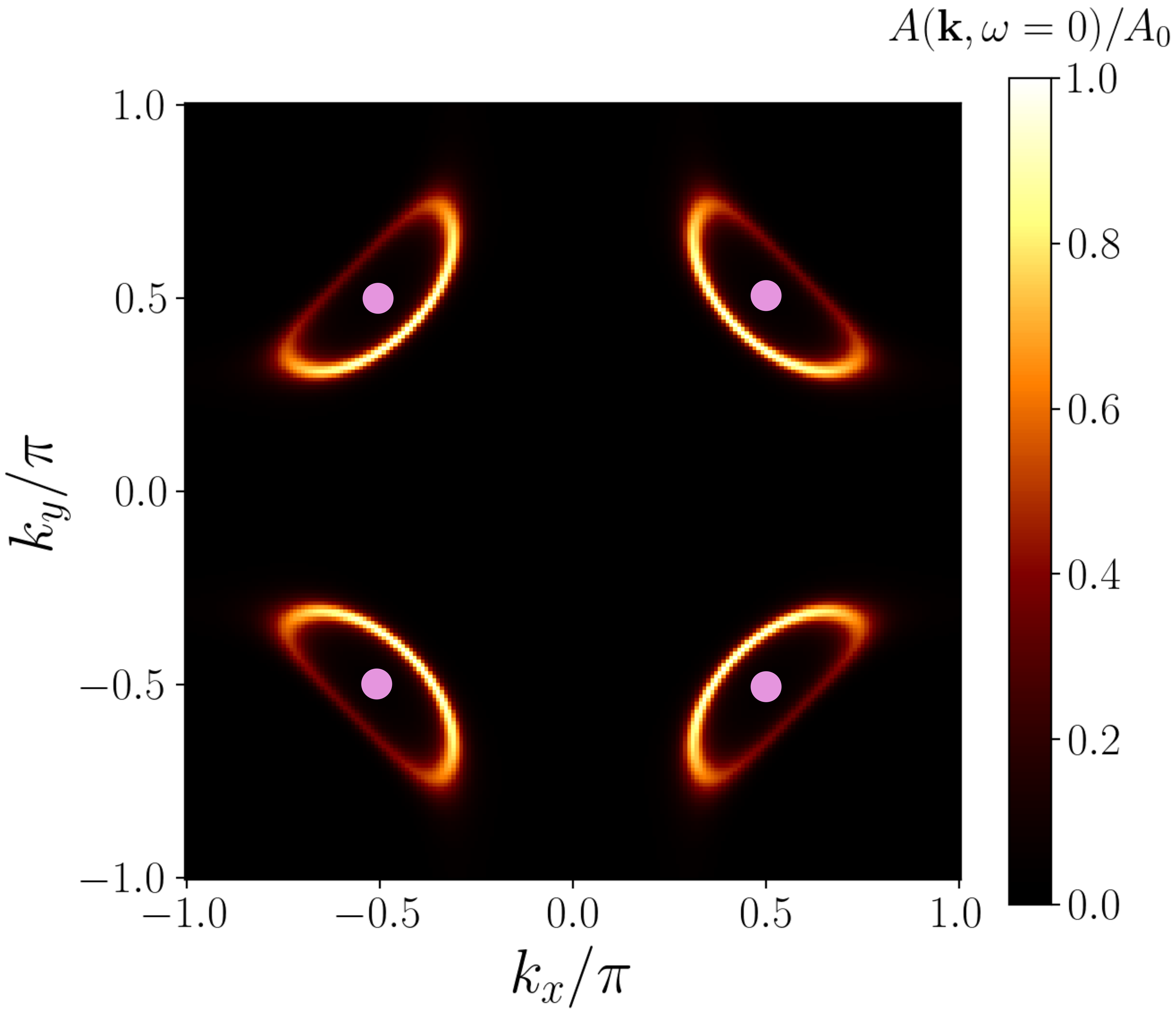
Higgs field Φ determines the pseudogap.
In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.
- Higgs boson B has charge e, and is a SU(2) fundamental.
- Yukawa coupling between c_α, f_α and B.
- B is a **fractionalized order parameter**, whose composites describe numerous superconducting and charge order parameters!

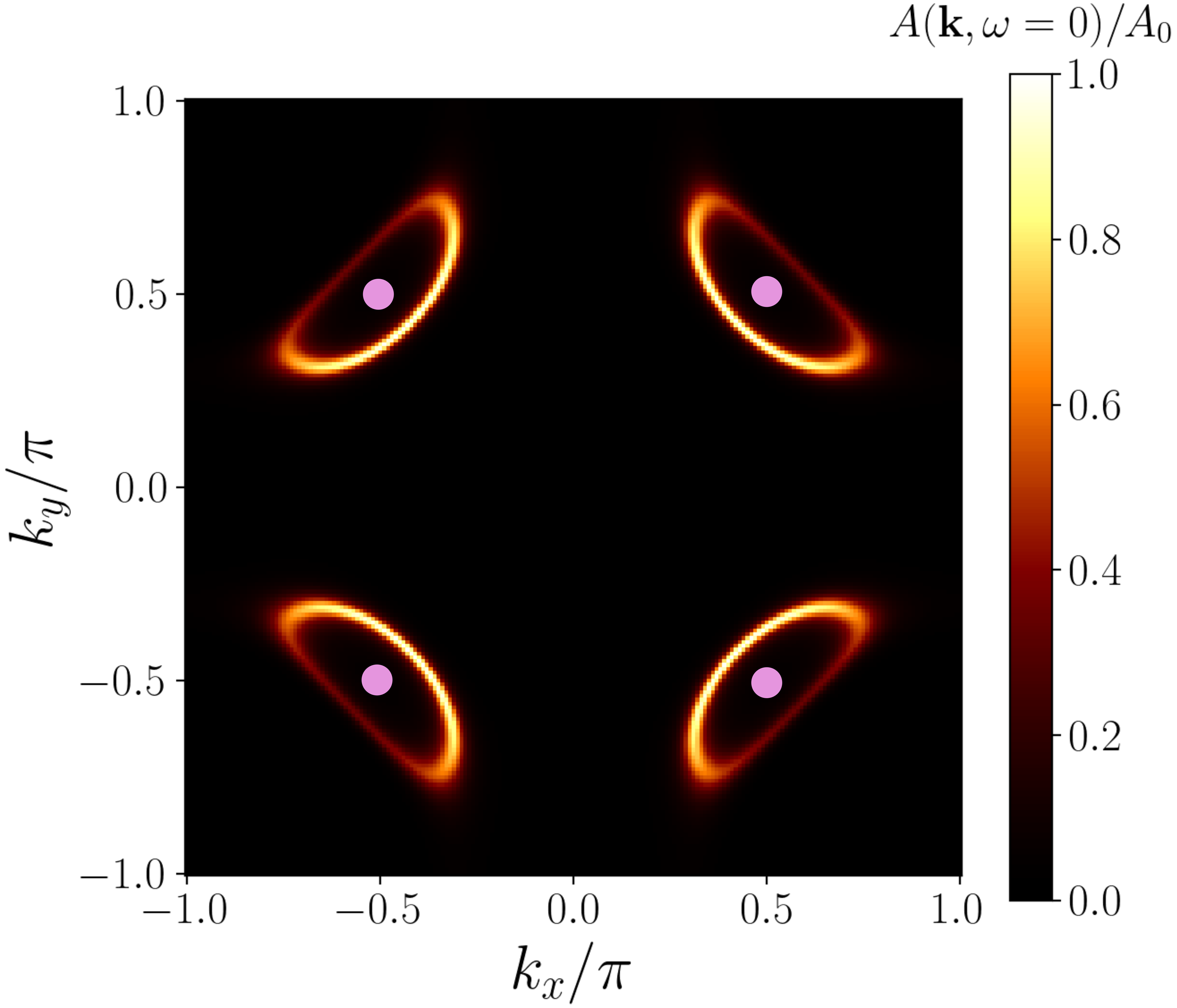
Monte Carlo at a temperature T

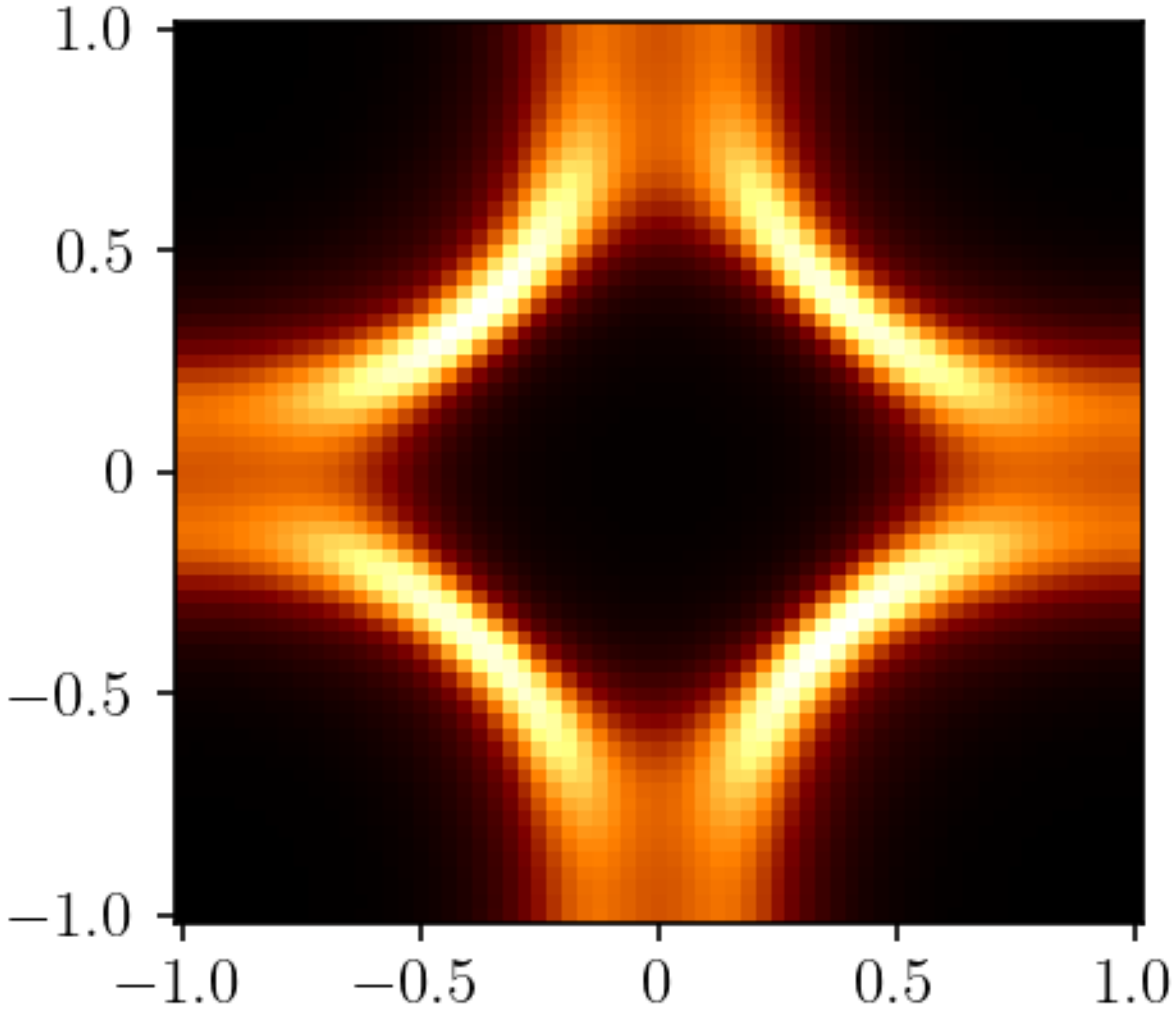
$$\mathcal{Z}_{2+0} = \int \prod_i \mathcal{D}B_{\boldsymbol{i}} \int \prod_{\langle \boldsymbol{ij} \rangle} \mathcal{D}U_{\boldsymbol{ij}} \exp\left[-\mathcal{E}[B,U]/T\right]$$

- Simulation of classical, thermal theory for bosons B, U defined by $\mathcal{Z}_{2+0}$
- Diagonalize 3-layer fermion Hamiltonian for c, f_1, f for each snapshot of B, U, and average.



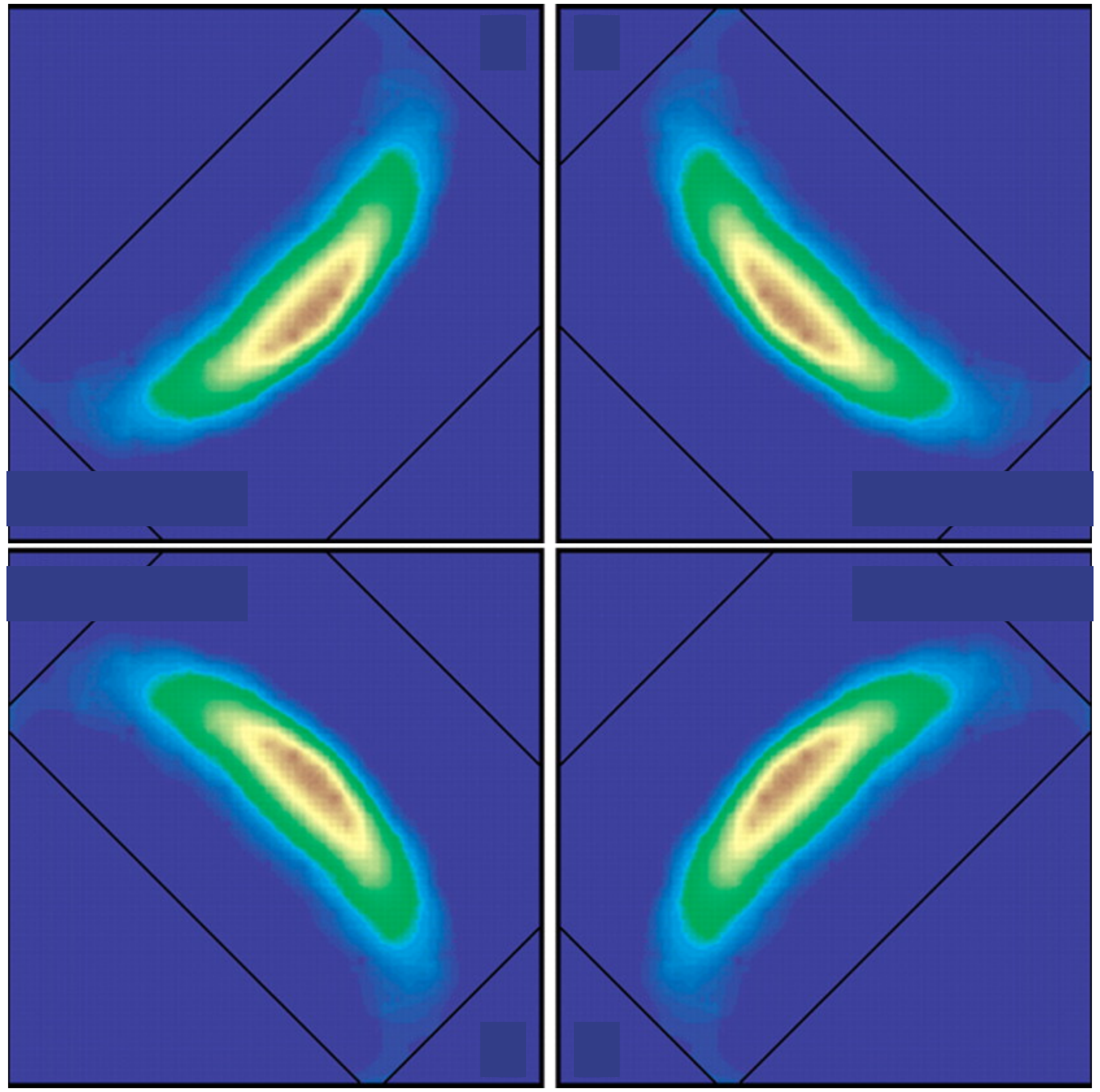
FL* fermionic spectrum with $B = 0$, $U = 1$
4 holes pockets of size $p/8$;
4 nodal spinons





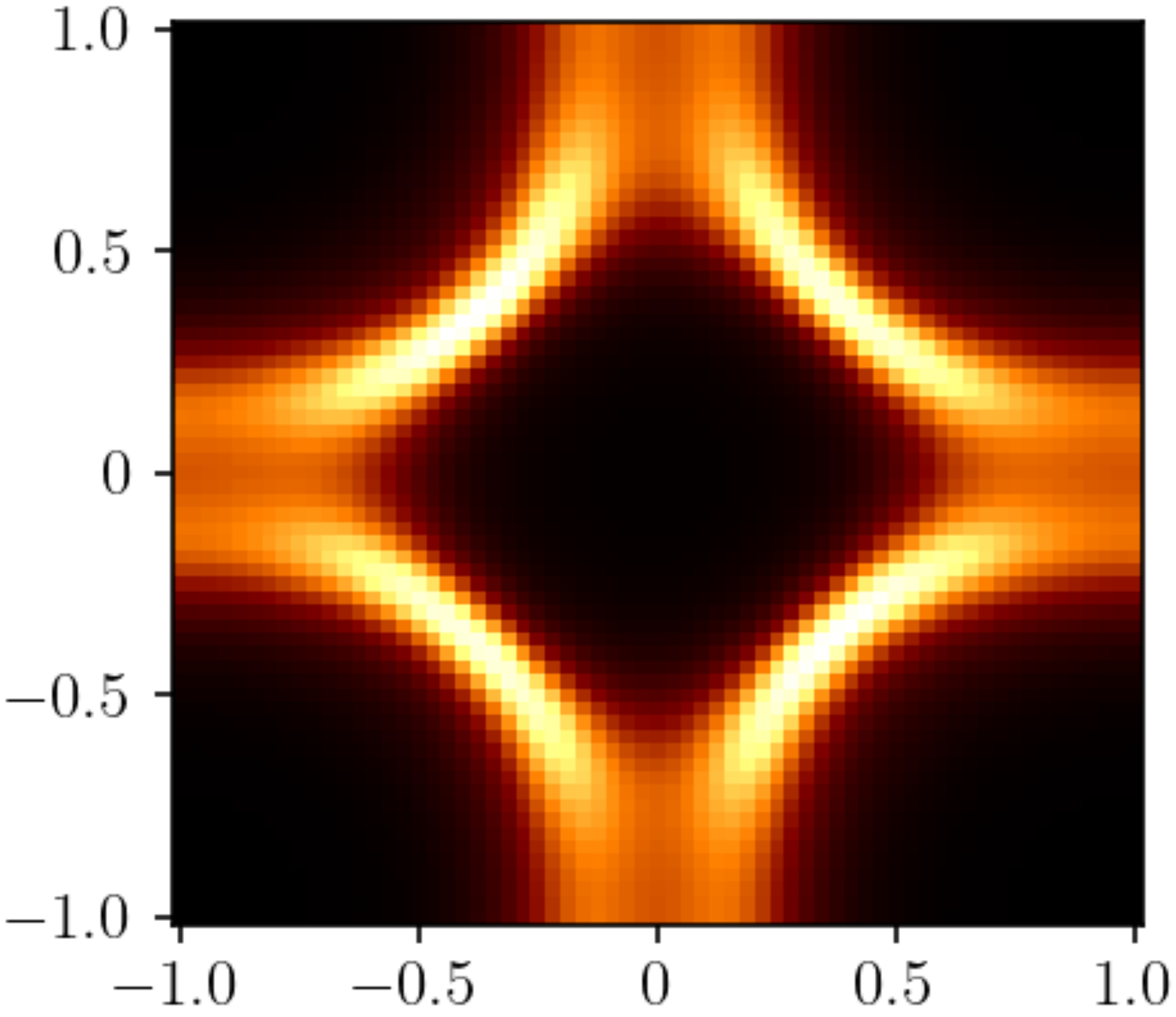
FL* fermionic spectrum with $B = 0$, $U = 1$
4 holes pockets of size $p/8$;
4 nodal spinons

Monte Carlo at a temperature $T > T_{KT}$



Kyle M. Shen, . . . Z.-X. Shen, Science **307**, 901 (2005)

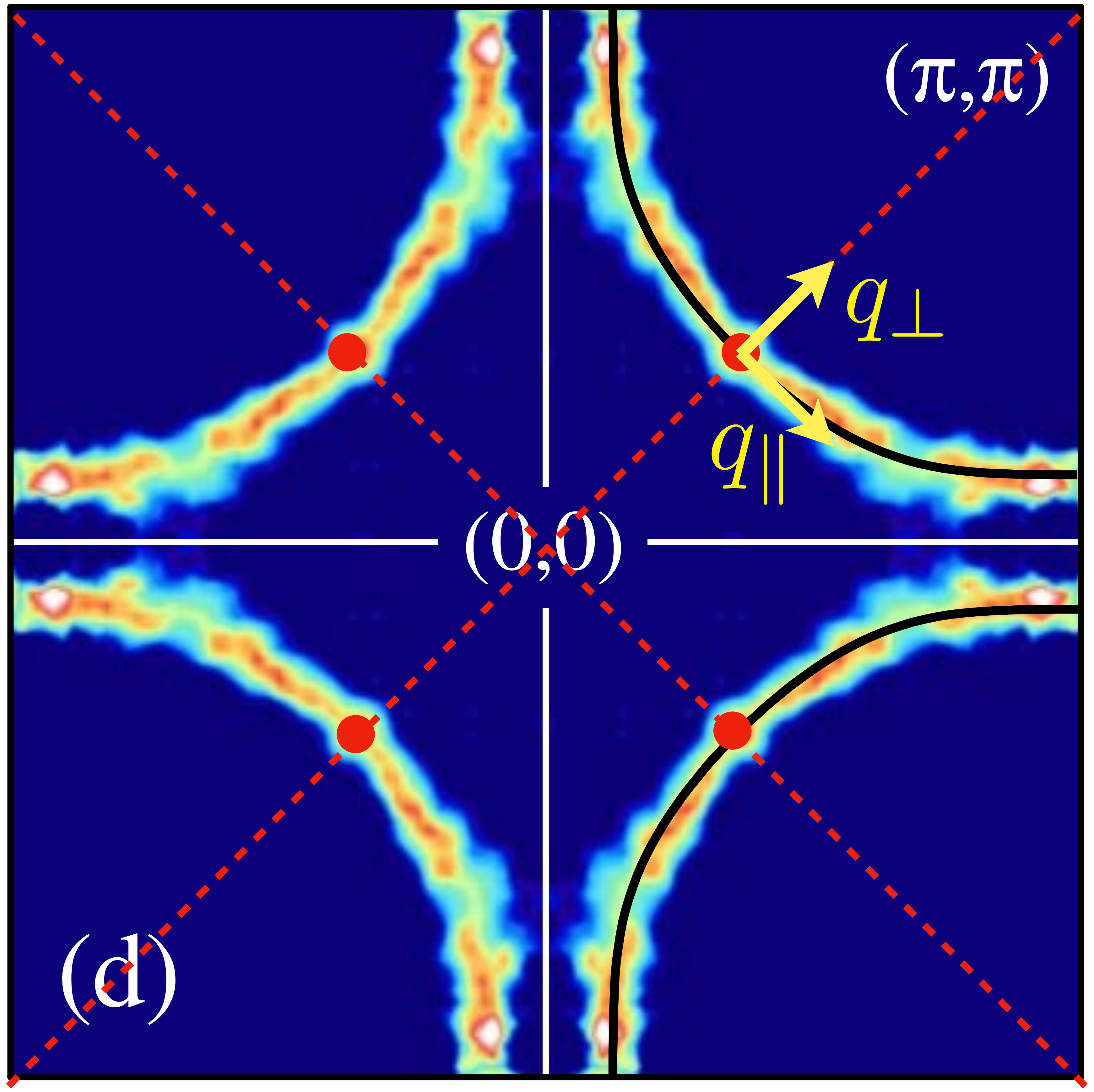
Photoemission observations



Monte Carlo at a temperature $T > T_{KT}$

From FL* to dSC, and the nature of nodal quasiparticles

FL→dSC



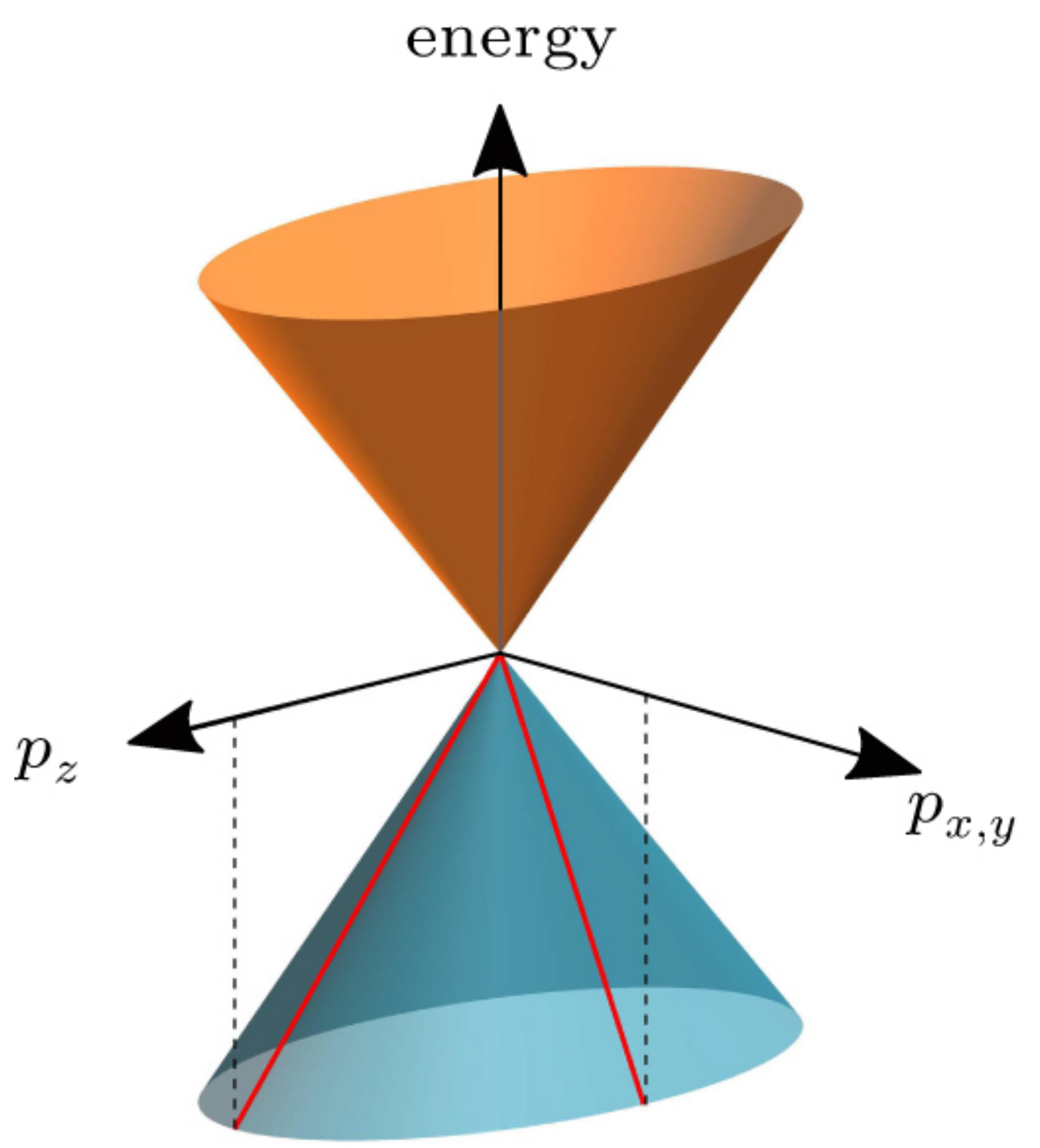
BCS/Bogoliubov quasiparticles in a d-wave superconductor

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

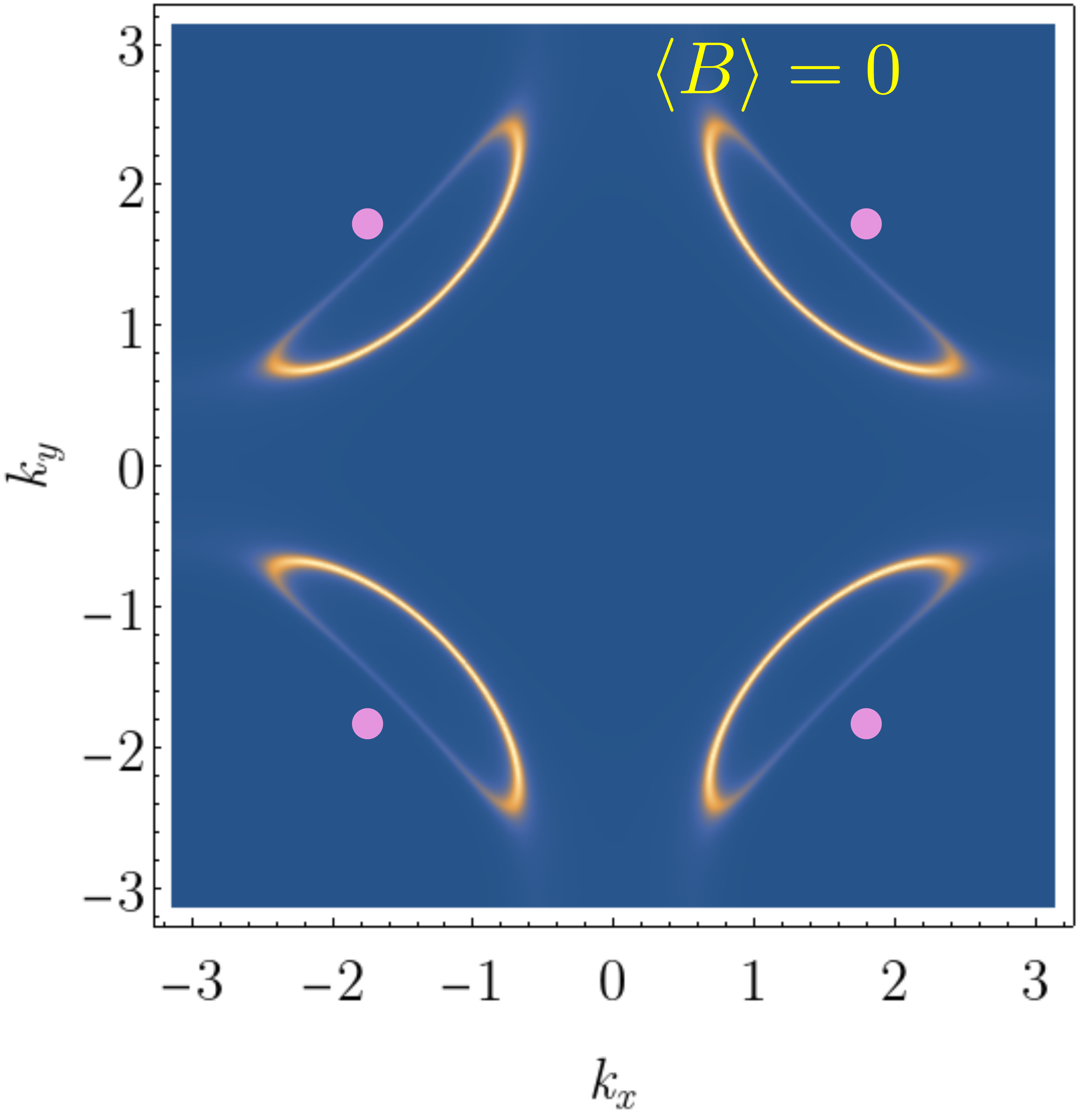
4 nodal points where

$$E_{\boldsymbol{k}_0+\boldsymbol{q}} = \left(v_F^2 q_\perp^2 + v_\Delta^2 q_\parallel^2\right)^{1/2}$$

with $v_F \gg v_\Delta$.



FL*

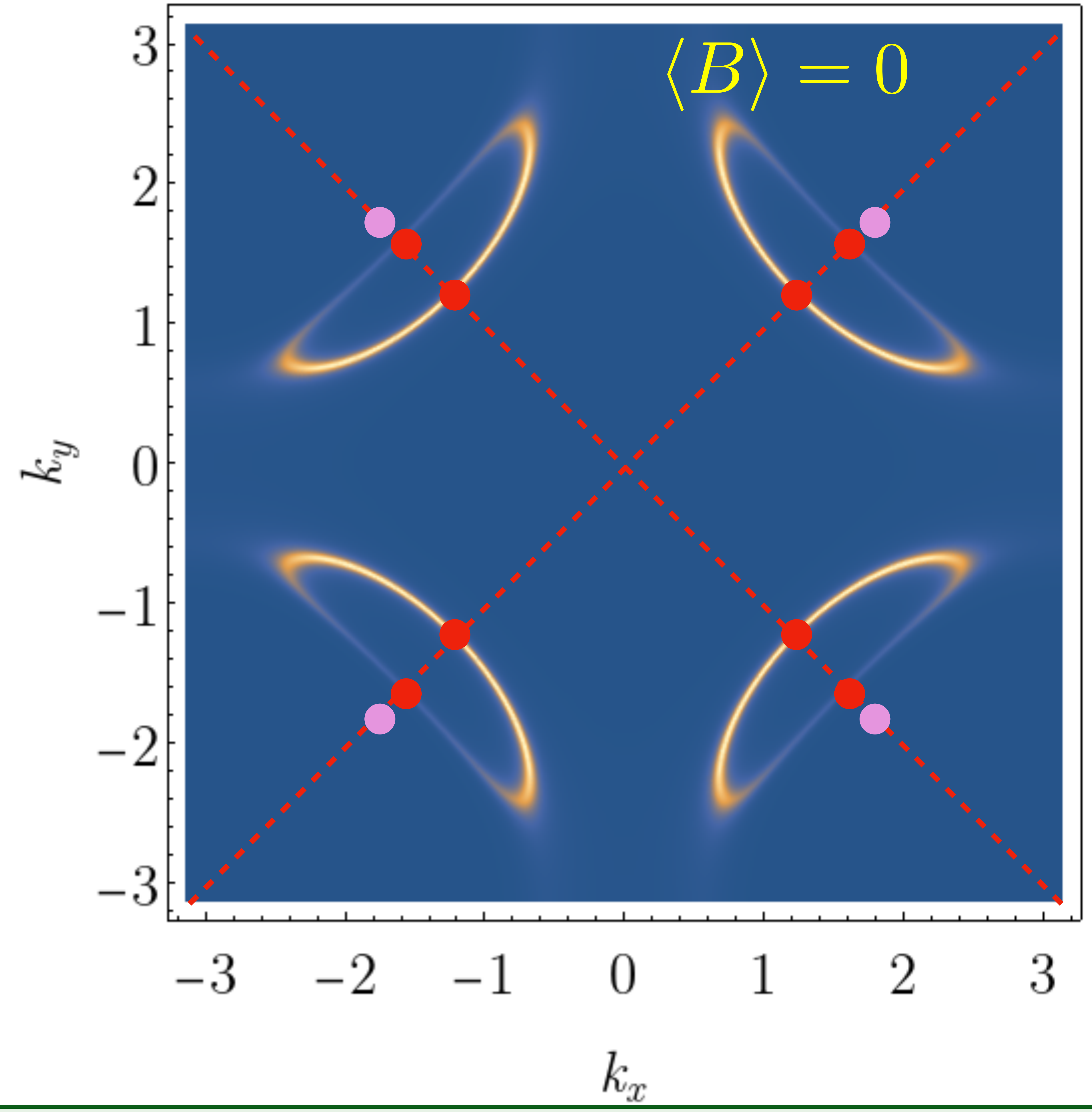


$\mathrm{FL}^* \Rightarrow$ d-SC:

Cooper pairing of the Fermi surface?

FL* has 4 electron-like pockets
and 4 nodal spinons
of the π-flux spin liquid

FL*→d-SC*



FL* $\Rightarrow$ d-SC:

Cooper pairing of the Fermi surface?

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$

$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

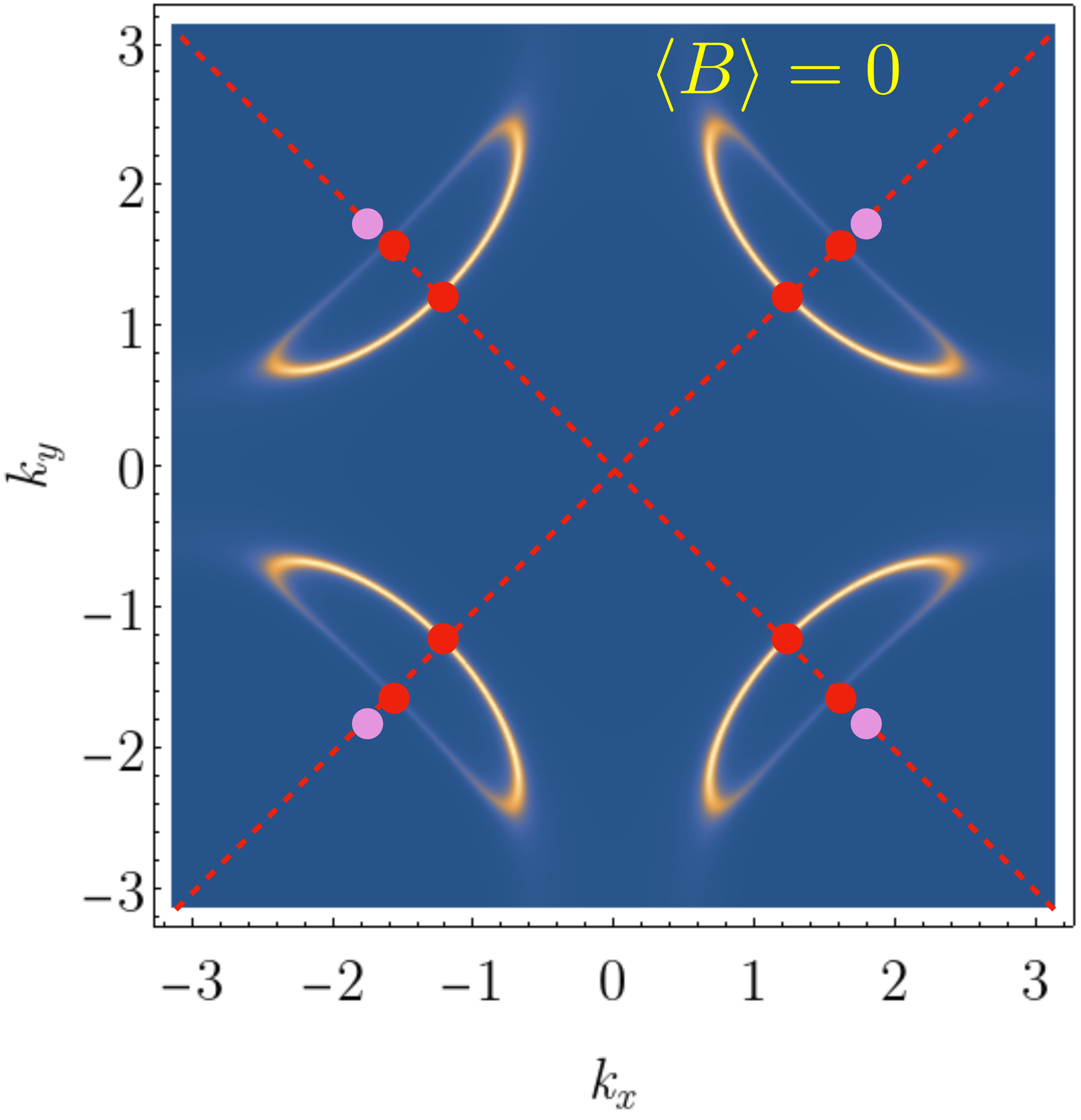
No!

Leads to 8 nodal points of Bogoliubov quasiparticles and 4 nodal spinons of π-flux spin liquid.

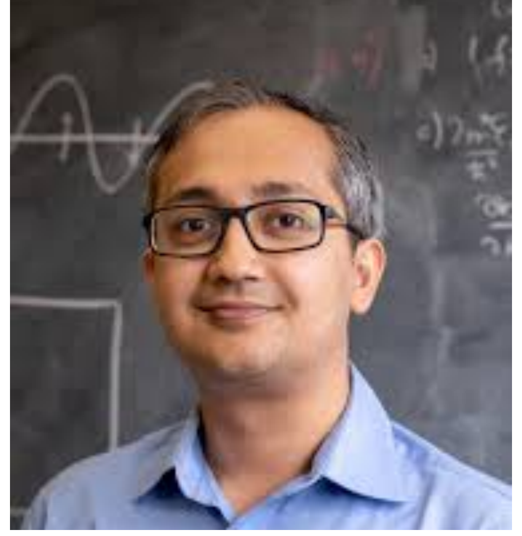
FL* $\Rightarrow$ d-SC*

BCS mechanism applied to FL* pseudogap leads to non-BCS superconductor!

FL*→d-SC



Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)

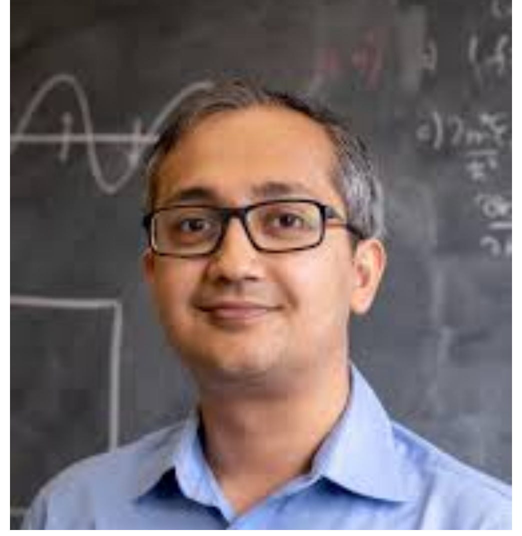


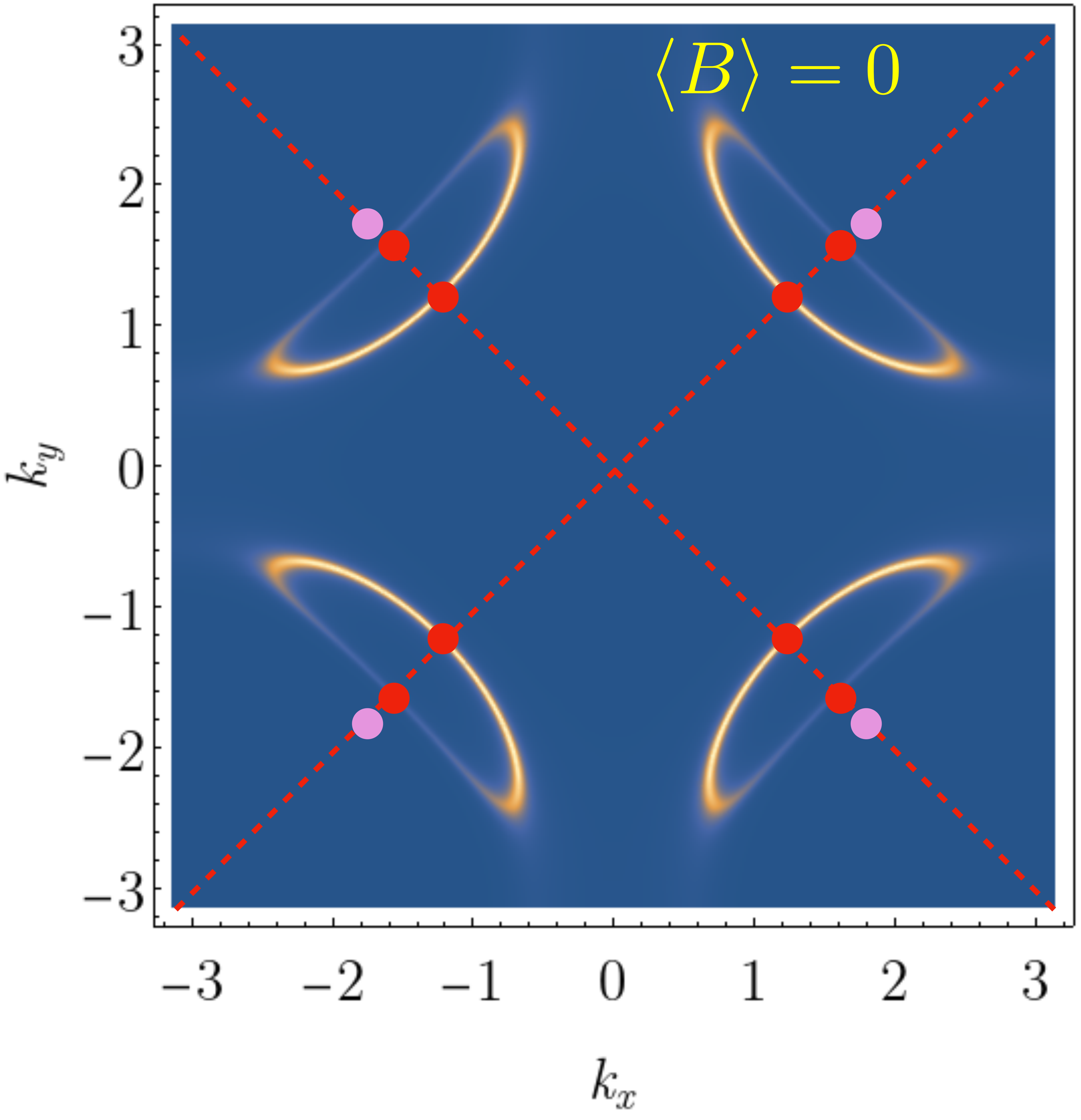
Alternative route to d-wave superconductivity:

Use the pre-existing pairing of the underlying spin liquid and confine the spin liquid!

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)



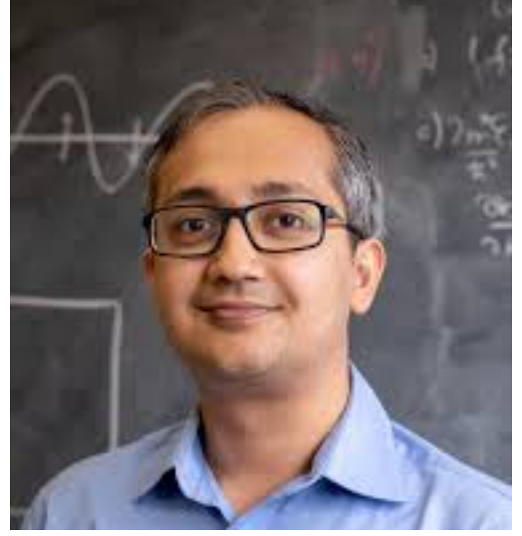



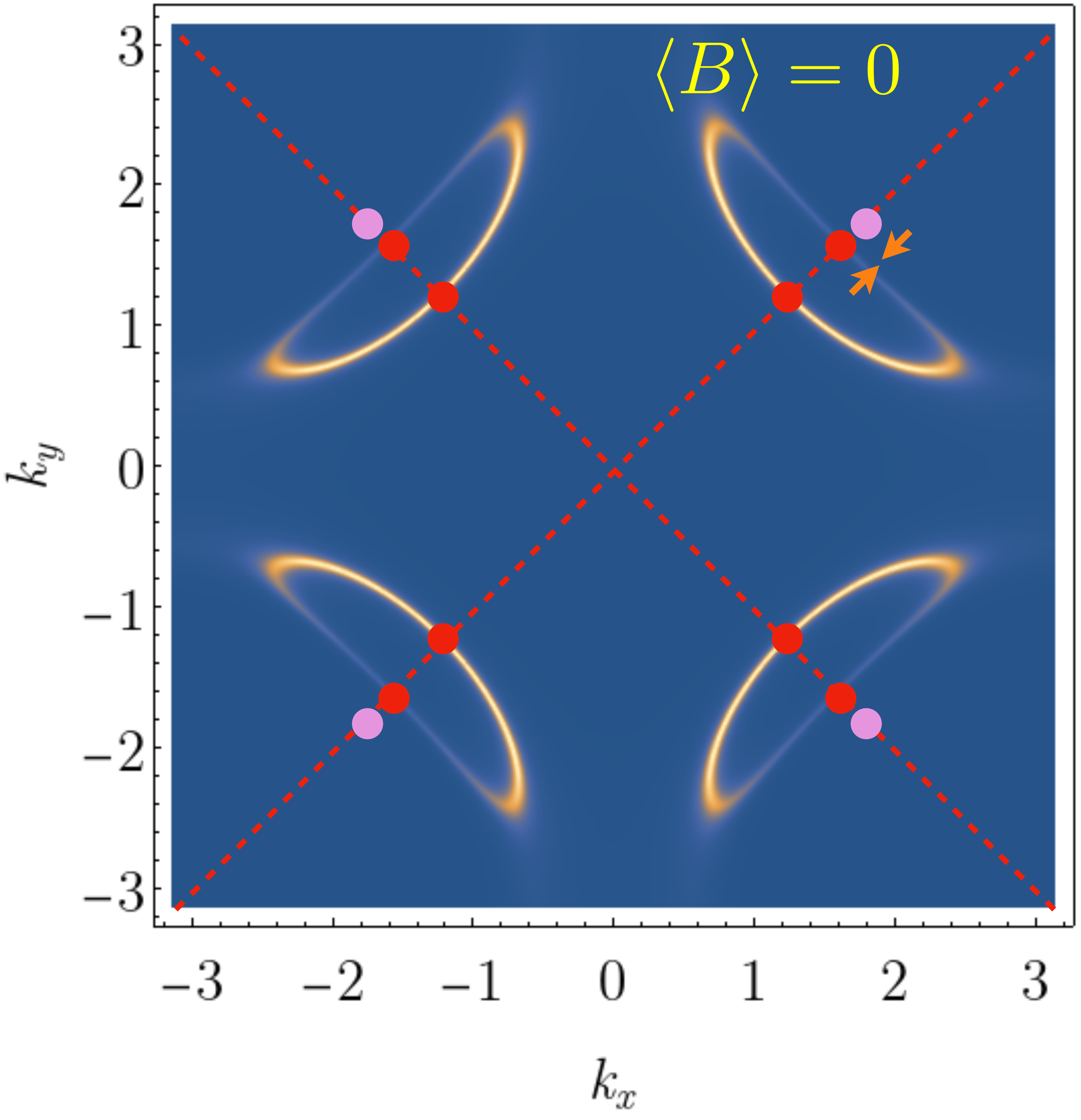
Alternative route to d-wave superconductivity:

Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$. This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)



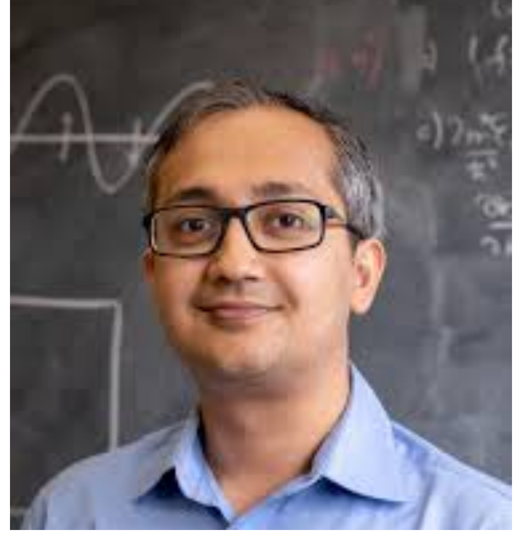


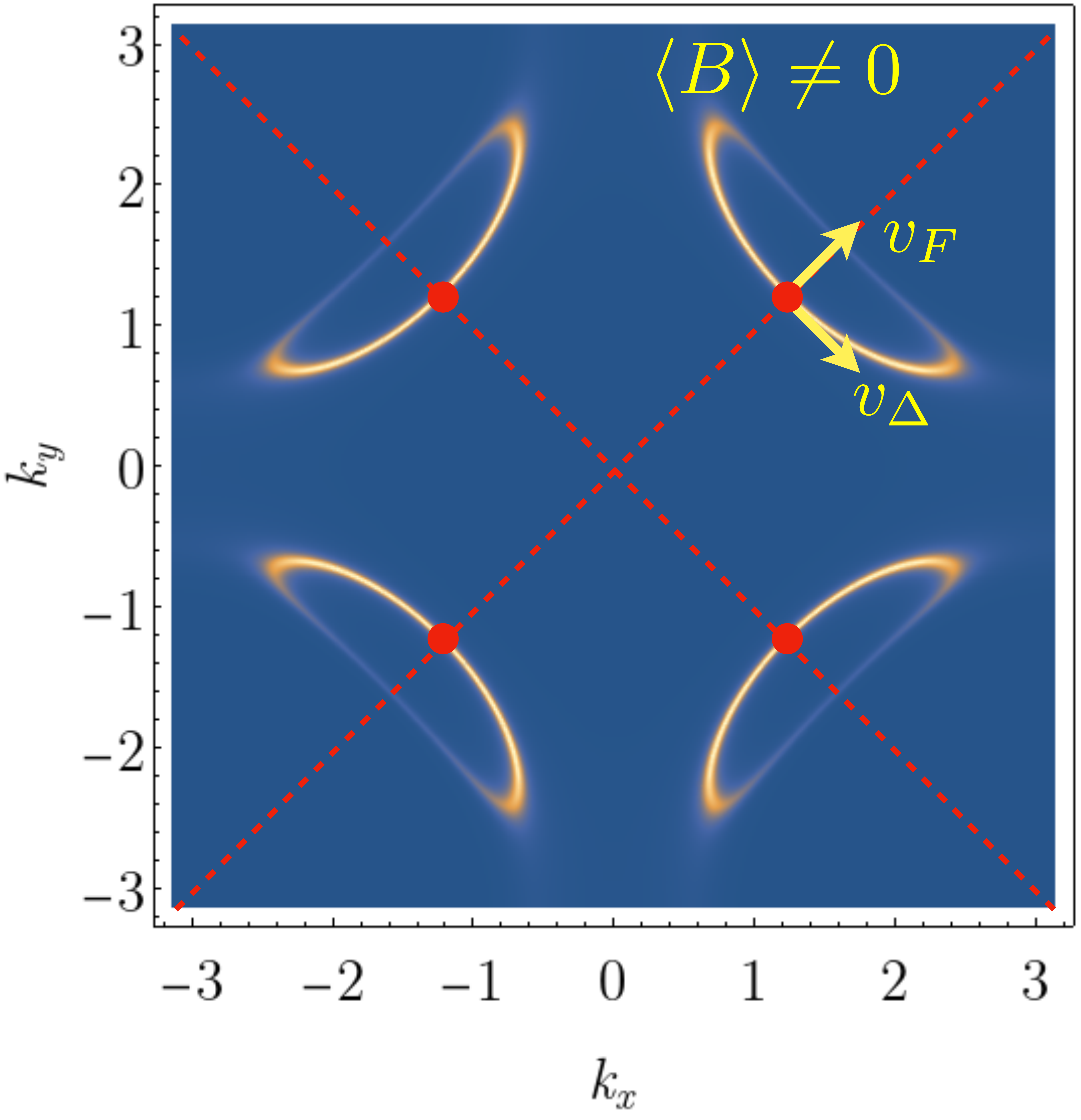
Alternative route to d-wave superconductivity:

Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$.
This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)





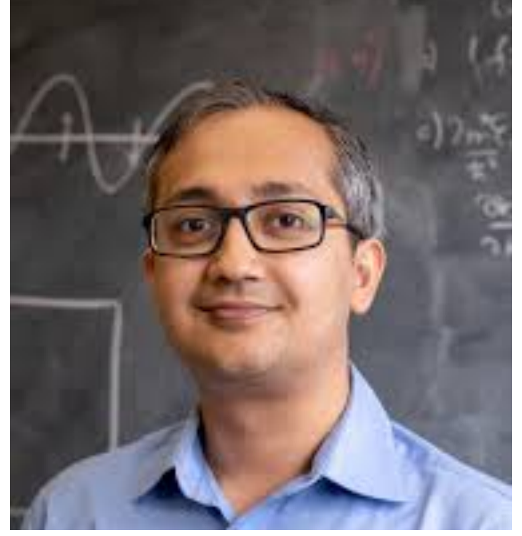
Alternative route to d-wave superconductivity:

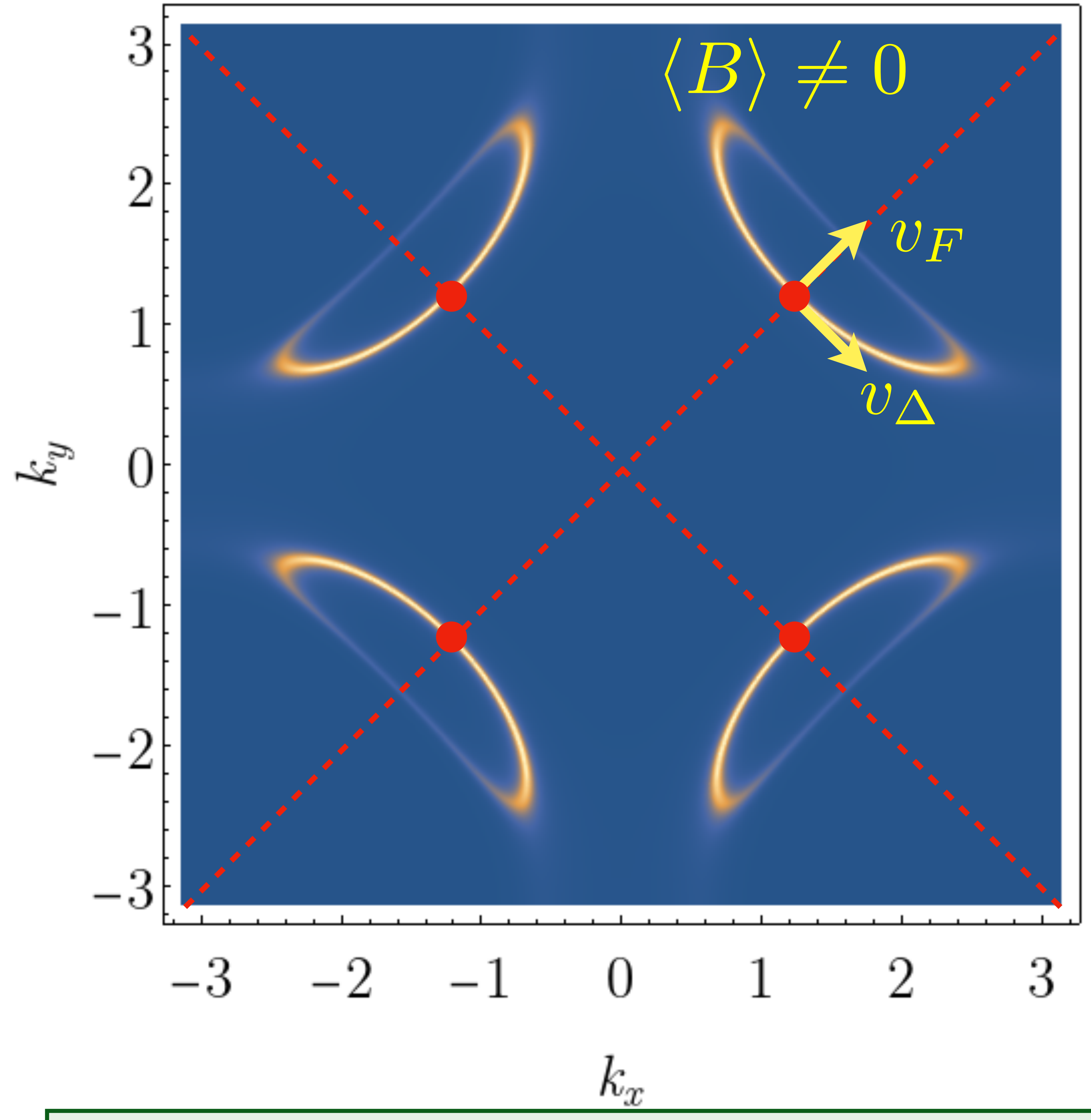
Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$.
This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

Resolves difficulty in early theories of d-SC from spin liquids
(Zhang, Gros,Rice, Shiba (1988);
Kotliar, Liu (1988)),
in which spinons turned into Bogoliubov quasiparticles with $v_F \approx v_\Delta$.

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)





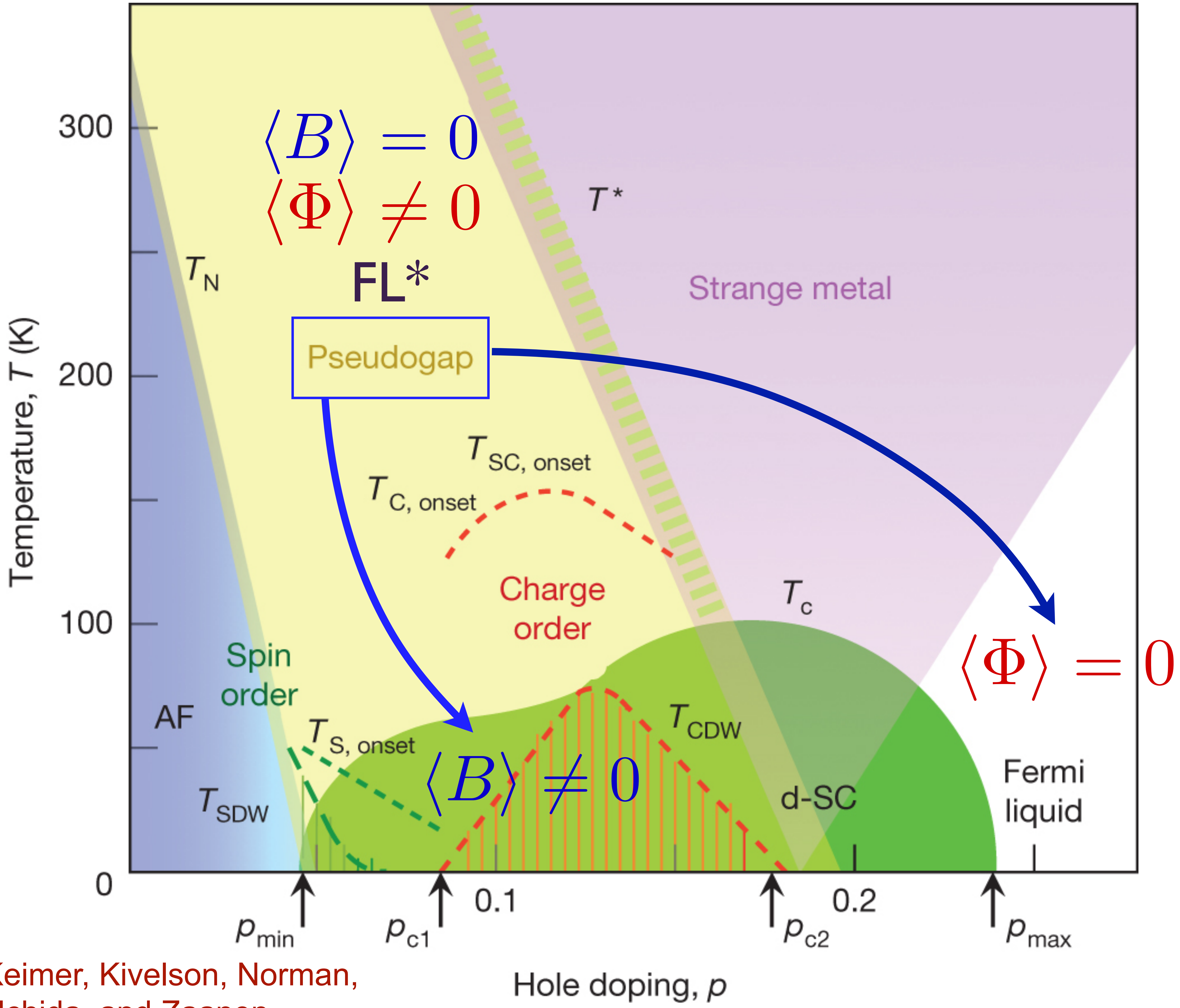
Alternative route to d-wave superconductivity:

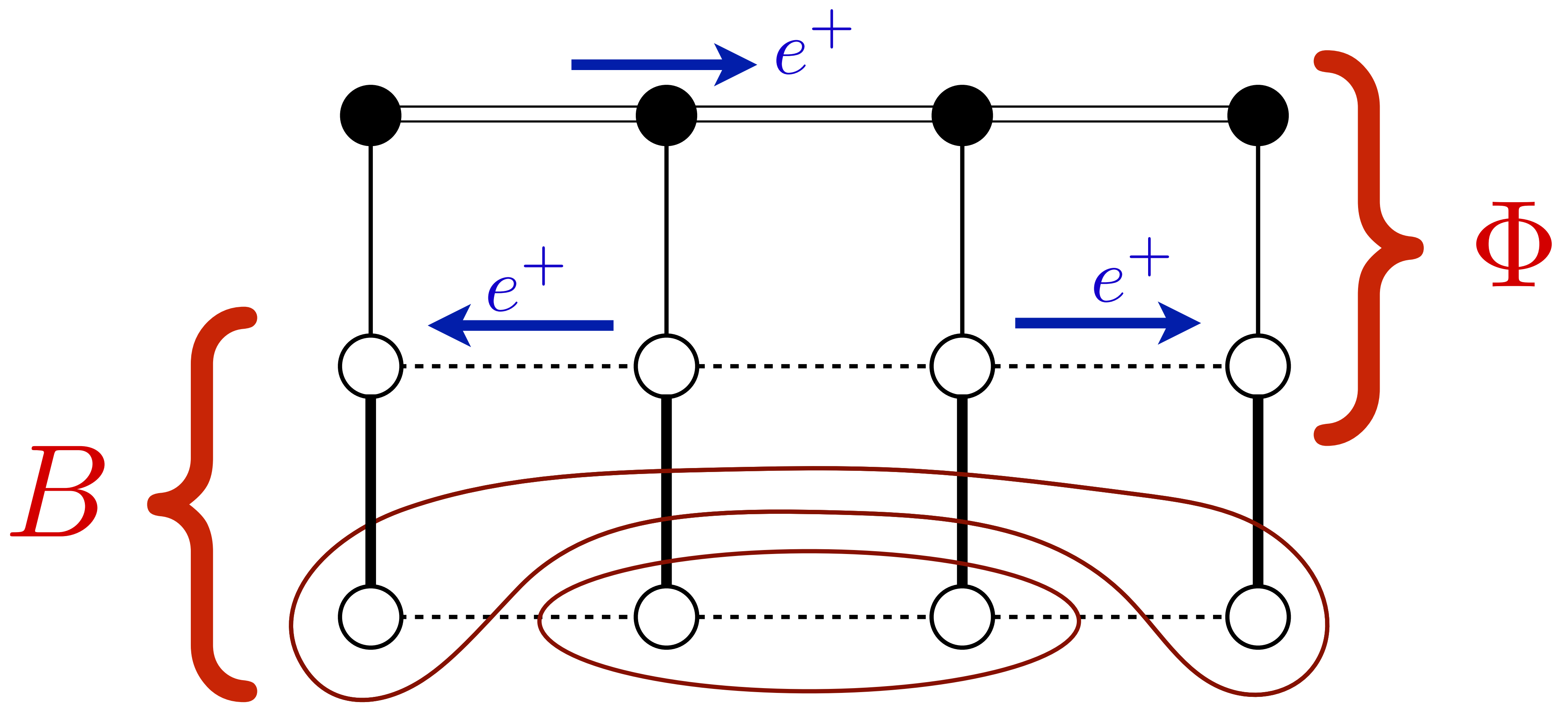
Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$. This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

Resolves difficulty in early theories of d-SC from spin liquids (Zhang, Gros,Rice, Shiba (1988); Kotliar, Liu (1988)), in which spinons turned into Bogoliubov quasiparticles with $v_F \approx v_\Delta$.

Non-BCS mechanism applied to pseudogap leads to BCS superconductor!

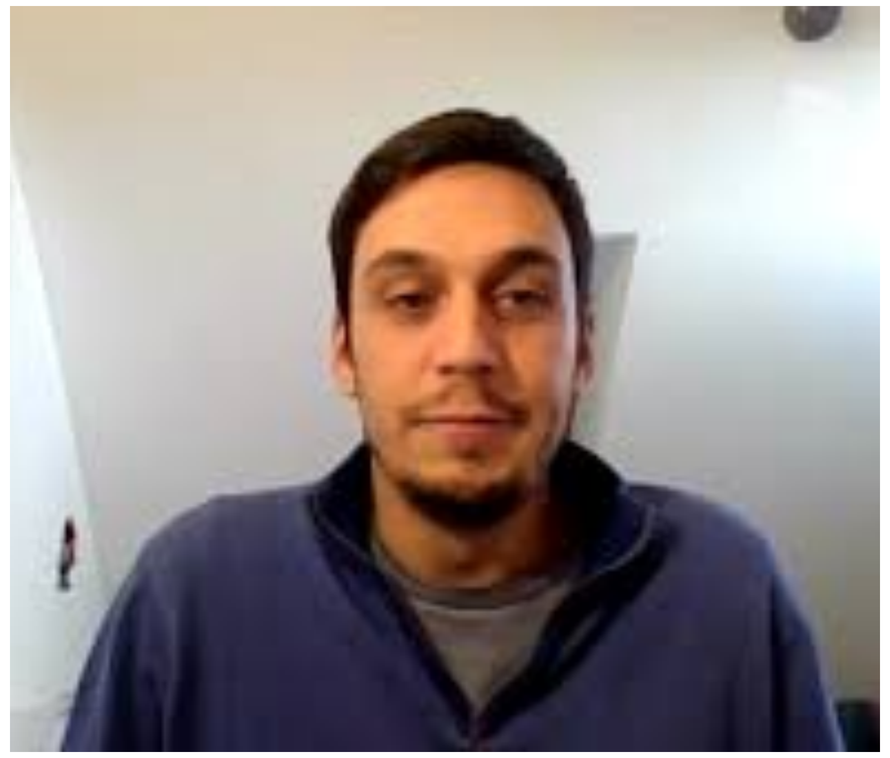
From the FL* pseudogap to d-SC and FL





Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

Maine Christos
Caltech



Pietro Bonetti
Stuttgart

Alexander
Nikolaenko

Aavishkar Patel
ICTS, Bengaluru

The Institute of Mathematical Sciences, Chennai

Harshit Pandey

Ravi Shanker

Sayantan Sharma

- *Lectures on insulating and conducting quantum spin liquids*, S. Sachdev, arXiv:2512.23962
- *Fractionalized Fermi liquids and the cuprate phase diagram*, P. M. Bonetti, M. Christos, A. Nikolaenko, A.A. Patel, and S. Sachdev, Reports on Progress in Physics **89** 044501 (2026).
- *Thermal SU(2) lattice gauge theory for intertwined orders and hole pockets in the cuprates*, H. Pandey, M. Christos, P. M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, and S. Sachdev, PNAS in press, arXiv:2507.0533