The Compton–de Broglie Tension and a Minimal Lorentz–Hopf Bridge

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Abstract

This paper proposes a minimal geometric bridge between three familiar quantum signatures: Compton-scale rest periodicity, de Broglie longitudinal modulation, and the spinorial double-cover structure associated with spin-$1/2$. I begin with a cylindrical toy model in which a rest-frame phase circulation at the Compton frequency is represented as a transverse angular motion. Under Lorentz transformation, this circulation unfolds into a helical structure whose longitudinal phase gradient reproduces the de Broglie relation.

The toy model is then lifted to a minimal spinorial completion using the standard Hopf-fibration form ($S^3 \to S^2$). In this lifted picture, the same Lorentz-unfolded scalar phase remains the carrier of the de Broglie modulation, while the spinor resolves that phase into half-angle branches and simultaneously introduces the characteristic 4π closure of spin-$1/2$. The purpose is not to claim a full derivation of the electron or of relativistic wave dynamics, but to show that these familiar quantum signatures can be placed within a single compact geometric scaffold. The construction is intentionally modest and kinematic in scope, suggesting that de Broglie modulation and spinorial structure may be understood as different projections of one underlying phase organization.

1 Introduction

Quantum theory assigns two foundational length–phase relations to the electron: the de Broglie wavelength $\lambda_{\mathrm{dB}} = h/p$ associated with motion [1], and the Compton scale $R_c = \hbar/mc$ associated with rest mass [2]. In standard presentations, these are typically treated as separate kinematic facts. What is usually missing, at least at the level of elementary geometric intuition, is a compact geometric picture in which the Compton rest scale and the de Broglie transport scale arise as two aspects of one phase organization rather than as disconnected kinematic inputs.

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As noted by Hestenes [3], the internal circulation and Zitterbewegung of the electron may be deeply linked to its quantum identity. This paper aims to isolate a minimal kinematic framework in which these signatures can be viewed as mutually compatible manifestations of a single internal phase organization. I show that a cylindrical toy model, adapted to the longitudinal–transverse separation of Lorentz boosts, naturally produces a helical phase whose pitch reproduces the de Broglie relation, and that this construction admits a natural spinorial completion via the Hopf fibration [4]. The purpose is not to derive a full relativistic electron theory, but to expose a compact geometric sequence linking these familiar structures.

2 Cylindrical Geometry and de Broglie Emergence

2.1 Lorentz-unfolded phase

Assume a rest-frame internal phase $\psi_0(t_0) = \omega_0 t_0$ with $\omega_0 = mc^2/\hbar$. Under a boost v along the z-axis, the proper time transforms as $t_0 = \gamma(t - vz/c^2)$. The boosted phase field is:

$$\psi(z,t) = \omega_0 \gamma \left(t - \frac{vz}{c^2} \right). \tag{1}$$

Its longitudinal gradient reproduces the de Broglie wavenumber:

$$\frac{\partial \psi}{\partial z} = -\omega_0 \gamma \frac{v}{c^2} = -\frac{\gamma m v}{\hbar} = -\frac{p}{\hbar} \implies k_{\mathrm{dB}} = \frac{p}{\hbar}. \tag{2}$$

2.2 Helical phase organization

To represent transverse circulation, let α denote the transverse cylindrical angle. I define a helical phase field $\Phi(\alpha, z, t) = \psi(z,t) + n\alpha$. At fixed t, the constant-phase condition $d\Phi = 0$ yields:

$$\frac{d\alpha}{dz} = -\frac{1}{n}\frac{\partial \psi}{\partial z} = \frac{p}{n\hbar}. \tag{3}$$

For $n = 1$, the longitudinal advance for one full winding ($\Delta\alpha = 2\pi$) is exactly $\Delta z = h/p = \lambda_{\mathrm{dB}}$. This shows that the de Broglie wavelength can be identified with the helical pitch of a Lorentz-unfolded rest periodicity. However, this $SO(2)$ model remains scalar and cannot yet account for the 4π closure of spin-$1/2$.

3 Minimal Spinorial Completion and Geometric Motivation

The cylindrical construction developed in the previous section provides a natural kinematic origin for the de Broglie modulation, but it remains intrinsically scalar. In particular, it cannot account for the defining property of spin-$1/2$ systems, namely the double-valuedness under 2π rotations.

This raises a structural question: what is the minimal geometric extension of a scalar phase circle that can simultaneously support (i) an internal $U(1)$ phase and (ii) a double-cover structure compatible with spinorial behavior?

A natural answer is provided by the Hopf fibration,

$$S^1 \hookrightarrow S^3 \to S^2, \tag{4}$$

which realizes a nontrivial $U(1)$ bundle over the two-sphere. As emphasized in geometric analyses [4,5], this structure is the simplest compact configuration in which a global phase degree of freedom is intrinsically linked to a spinorial $SU(2)$ geometry. In this sense, the Hopf fibration may be viewed as a minimal spinorial completion of the scalar phase circle.

In the present context, this lift is not introduced as a dynamical hypothesis, but as a natural geometric completion once one requires compatibility between internal periodicity and spin-1/2 structure. It is worth noting that structures of this type are not purely formal. Exact localized solutions of the Dirac equation exhibiting Hopfion-like topology have been constructed by Bialynicki-Birula [6]. These solutions demonstrate that relativistic electron wavefunctions can possess nontrivial topological organization, with velocity fields forming linked flow lines closely related to the Hopf fibration.

The role of the present construction is different. Rather than deriving such solutions, it identifies a minimal geometric pathway by which a scalar rest-frame periodicity, when combined with Lorentz kinematics, naturally admits a spinorial lift of Hopf type. In this sense, the geometric framework developed here and the known Dirac hopfion solutions may be viewed as complementary: the former addresses structural origin, while the latter establishes dynamical realization.

4 Lorentz-Unfolded Spinor and Half-Phase Branches

I define a rest-frame spinor ansatz of standard Hopf form:

$$z_{\mathrm{rest}}(t_0) = \begin{pmatrix} \cos\frac{\vartheta}{2}\, e^{\frac{i}{2}(\omega_0 t_0 - \varphi)} \\ \sin\frac{\vartheta}{2}\, e^{\frac{i}{2}(\omega_0 t_0 + \varphi)} \end{pmatrix}. \tag{5}$$

Under boost, each spinor component carries the Lorentz-unfolded phase only through a half-angle factor $e^{i\psi/2}$, giving a longitudinal gradient:

$$\frac{\partial}{\partial z}\left(\frac{\psi}{2}\right) = -\frac{p}{2\hbar}. \tag{6}$$

The spinor therefore resolves the underlying modulation into two half-angle branches. This should not be read as a modification of the de Broglie relation: the full de Broglie modulation remains encoded in the common scalar phase ψ, while the spinorial representation introduces a double-cover description whose single-valued bilinear observables (e.g., $z^\dagger \boldsymbol{\sigma} z$) are insensitive to the overall sign of z. This separation between underlying phase and observable quantities is consistent with the standard role of phase in quantum mechanics. Meanwhile, the spinorial carrier exhibits the familiar double-cover behavior:

$$\varphi \mapsto \varphi + 2\pi \implies z \mapsto -z; \quad \varphi \mapsto \varphi + 4\pi \implies z \mapsto z. \tag{7}$$

This is the standard sign ambiguity of a spinorial representation, rather than by itself a full derivation of physical spin dynamics.

5 Conclusion

The central suggestion of this paper is deliberately limited but structurally specific: the Compton-scale rest periodicity, the de Broglie longitudinal modulation, and the spinorial double-cover behavior associated with spin-$1/2$ need not be treated as entirely independent ingredients. At least at the level of a minimal kinematic construction, they can be placed within a single geometric sequence.

The value of the present construction is not dynamical completeness but geometric compression: it places three familiar quantum signatures—Compton rest periodicity, de Broglie modulation, and spinorial double-valuedness—within a single minimal kinematic scaffold. Whether such a scaffold can be embedded into a fully dynamical account of the electron, including charge and field structure, remains an open question for future work. In the present paper, the Hopf lift is introduced not as a demonstrated ontological necessity, but as a natural minimal geometric completion once one asks how a scalar internal phase might be upgraded to accommodate known spinorial behavior.

References

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