Osmotic Dark Energy: Deriving the Evolving Equation of State $w(a)$ from Infodynamic Substrate Thermodynamics

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Abstract

The Dark Energy Spectroscopic Instrument (DESI) 2024 Year 1 data release presents indications that Dark Energy may not be a static cosmological constant ($\boldsymbol{\Lambda}$), but rather a dynamical fluid that evolves over cosmic time. Best-fit phenomenological models using the standard $\boldsymbol{w_0 w_a}$ parameterization suggest Dark Energy transitions from a phantom regime ($\boldsymbol{w < -1}$) in the early universe to a quintessence regime ($\boldsymbol{w > -1}$) today. While standard scalar-field models face mathematical challenges in establishing a mechanism for this phantom crossing, we explore an alternative perspective by modeling the spatial vacuum as a thermodynamic substrate governed by Symbiotic Infodynamic Equilibrium (SIE). We suggest that Dark Energy can be understood not as an intrinsic vacuum energy, but as the macroscopic osmotic pressure required to continuously balance the universal thermodynamic ledger. By constructing the partition function of the discrete substrate, we map this osmotic pressure directly to the free energy shifts driven by the cooling thermal history of the Cosmic Microwave Background ($\boldsymbol{T \propto a^{-1}}$) and the accelerating structural variance of late-stage galaxy clustering ($\boldsymbol{\langle \delta^2 \rangle \propto a^2}$), deriving a physical functional form for the equation of state, $\boldsymbol{w_{SIE}(a)}$. While the underlying mechanics are derived from thermodynamic principles, we calibrate the resulting dimensionless coefficients ($\boldsymbol{\kappa_{osm} \approx 0.311}$, $\boldsymbol{\tau_{land} \approx 0.131}$) against empirical constraints. This semi-empirical equation reproduces the specific phantom-crossing evolution observed by DESI without violating early-universe Big Bang Nucleosynthesis limits. Furthermore, it predicts a steep, non-linear divergence from the CPL parameterization at high redshifts, offering a pathway to transition the analysis of Dark Energy from statistical curve-fitting to thermodynamic hypothesis testing.

1 Introduction: The Evolution of the Cosmological Constant

For over two decades, the standard ΛCDM cosmological model has served as the consensus framework for understanding the macroscopic universe [12, 14]. At its foundation is the Cosmological Constant (Λ), which models Dark Energy as the intrinsic, static zero-point energy of the vacuum, characterized by an unchanging equation of state: $w = -1$.

While this assumption provides a consistent baseline for late-universe acceleration, it has long invited theoretical scrutiny, most notably regarding the Cosmological Constant Problem and the Coincidence Problem [18, 19].

Recently, the 2024 Year 1 data release from the Dark Energy Spectroscopic Instrument (DESI) has prompted the community to re-evaluate the static nature of the vacuum [1]. Utilizing over 6 million extragalactic objects to map baryon acoustic oscillations (BAO), DESI's constraints suggest a dynamic, time-varying equation of state. When combined with cosmic microwave background (CMB) and Type Ia supernovae datasets, the analysis favors an evolving Dark Energy at confidence levels approaching 3.9σ.

Observationally, the data indicates a potential phantom crossing. Parameterized using the conventional Chevallier-Polarski-Linder (CPL) Taylor expansion, $w(a) = w_0 + w_a(1 - a)$ [5, 10], the best-fit trajectories suggest Dark Energy started in the phantom regime ($w < -1$) in the early universe. It then crossed the exact Λ boundary ($w = -1$) around a redshift of $z \approx 0.3$, continuing a gradual ascent into the quintessence regime ($w > -1$) today.

Standard continuous models approach the $w = -1$ boundary with caution. In traditional general relativity, a phantom equation of state implies the energy density of the universe increases as it expands, challenging the Null Energy Condition [4]. Attempts to model a phantom crossing using minimally coupled scalar fields typically require specific mathematical fine-tuning to avoid instabilities [3]. As a result, the CPL parameterization ($w_0 w_a$) remains a useful phenomenological tool—a placeholder pointing toward underlying physics.

To build upon this foundation and explore the physical mechanics driving this variance, we consider expanding our functional definition of the vacuum. Transitioning from the traditional, infinitely expanding vacuum of standard cosmology to a closed thermodynamic container represents a distinct ontological shift. We ask the reader to provisionally accept this boundary condition to evaluate the Symbiotic Infodynamic Equilibrium (SIE) framework by its predictive utility: deriving a physical functional form for the DESI phantom crossing, calibrated against current observational limits.

Rather than adding new phenomenological fields, this proposal attempts to operate as a reductionist mechanism. We construct the functional form of the pressure operators from macroscopic boundary constraints, anchoring the final equation to formal statistical mechanics rather than hypothetical non-baryonic particle inventories.

2 The Infodynamic Vacuum and the Cosmic Ledger

Standard continuous cosmology treats space as a smooth, mathematical manifold. The SIE framework builds on this by exploring the implications of treating space as a physical, information-bearing metamaterial operating as a discrete substrate near the Planck scale.

Approached via the Mass-Energy-Information Equivalence Principle [16] and the Second Law of Infodynamics [17], the localized energy density of the vacuum (ρ_{vac}) is viewed not as an arbitrary zero-point field, but as the active thermodynamic maintenance cost required to uphold the structural integrity of the spatial grid against the ambient cosmic noise floor.

Assuming the bulk universe operates as a closed thermodynamic container—bounded by a macroscopic, adiabatic expanding apparent horizon R_H [2, 7]—total entropy and information must be conserved across the macroscopic network. We term this conservation limit the *Cosmic Ledger*. Defining the bulk horizon as adiabatic dictates that while the container's volume evolves over cosmic time, no net information is lost to an outside system, preserving internal conservation.

To prevent infinite energy densities and gravitational collapse, the discrete Substrate structurally throttles its own macroscopic volumetric degrees of freedom. Governed by the Cohen-Kaplan-Nelson (CKN) bound via UV/IR mixing [6], the bulk capacity strictly scales to the 3/4 power of the boundary entropy.

(It is worth noting that unlike standard Holographic Dark Energy models, where the vacuum density continuously evolves with the Hubble parameter as $\rho_{vac} \propto H^2$, we treat ρ_{vac} here as a static geometrical baseline. The CKN bound establishes the initial geometric saturation limit of the discrete network, defining the magnitude of this constant rather than driving its continuous macroscopic evolution).

From this thermodynamic perspective, Dark Energy acts not as a localized repulsive force, but rather as *Osmotic Spatial Decompression*. When mass aggregates into highly organized geometric arrays (such as the Laniakea Supercluster), it generates a localized entropic deficit. To globally balance the thermodynamic ledger and preserve the Second Law of Thermodynamics, the Substrate compensates by introducing new, highly disordered spatial volume into the empty Cosmic Voids [15].

The macroscopic outward acceleration of the universe is therefore the fluid response required to offset the structural compression of matter. Because this osmotic pressure is an active thermodynamic response to the evolving internal state of the universe, it must shift as the universe expands and cools.

2.1 Coarse-Graining the Substrate: Partition Function and Free Energy

To transition from a discrete, infodynamic lattice to the continuous macroscopic fluid mechanics evaluated by observational surveys, we must establish a formal coarse-graining procedure based on standard statistical mechanics. We define the vacuum not as an empty continuum, but as an interconnected network of discrete thermodynamic nodes.

Let the microstate of this Substrate within a comoving spatial cell be described by a localized Hamiltonian H_{sub}. For an ensemble of N discrete nodes, the Hamiltonian incorporates the geometric baseline energy (E_0) and a structural deformation operator ($\hat{O}_{strain}$) driven by the macroscopic clustering of matter.

The partition function for this canonical ensemble is $Z = \mathrm{Tr}\exp(-\beta H_{sub})$, from which the macroscopic Helmholtz free energy is defined by the fundamental relation:

$$F = \langle H_{sub} \rangle - T_{CMB} S_{sub} \tag{1}$$

(In a standard continuous fluid, the entropy S_{sub} would scale extensively with the FLRW scale factor a^3, causing radiation terms to dilute as a^{-4}. However, under the Cohen-Kaplan-Nelson limit, the physical degrees of freedom are holographically bounded. If we assume the substrate operates at geometric saturation, the available information capacity of a comoving cell is invariant under macroscopic expansion. Consequently, the substrate entropy S_{sub} acts as a saturated constant).

The expectation value of the structural deformation scales directly with the macroscopic variance of galaxy clustering, $\langle \hat{O}_{strain} \rangle \propto \langle \delta^2 \rangle \propto a^2$. Simultaneously, the ambient thermal noise temperature scales inversely with expansion, $T_{CMB} \propto a^{-1}$.

Substituting these operators into Equation 1 yields an explicit scale-dependent function for the free energy:

$$F(a) = E_0 + c_1 a^2 - c_2 a^{-1} \tag{2}$$

where c_1 and c_2 are dimensionful coupling constants governing the structural strain and Landauer thermal erasure, respectively.

In a discrete holographic lattice, the effective macroscopic pressure exerted on the FLRW metric is derived from the volumetric derivative of the free energy, $P = -(\partial F/\partial V)_T$. Because the effective phase-space capacity of the saturated comoving cell (V_{eff}) is invariant due to the CKN bound, the resulting osmotic pressure acts strictly as a volumetric density of the shifting free energy, $P_{osm} \propto -F(a)/V_{eff}$.

Dividing this dynamic osmotic pressure by the static baseline density ($\rho_{vac} = -E_0/V_{eff}$), the macroscopic equation of state emerges directly from the partition function without requiring ad-hoc fluid parameterizations.

3 Deriving the Dynamic Equation of State (w_{SIE})

The total effective equation of state (w) for the cosmos is defined as the ratio of the Substrate's active osmotic pressure (P_{osm}) to its baseline structural density (ρ_{vac}):

$$w(a) = \frac{P_{osm}(a)}{\rho_{vac}} \tag{3}$$

In the standard ΛCDM picture, this ratio is fixed at -1. In the SIE framework, as derived from Equation 2, the required osmotic pressure operates as a dynamic superposition of two distinct, time-varying thermodynamic stressors: the *Landauer*

Thermal Erasure Cost (dominating the early universe) and the *Osmotic Clustering Strain* (dominating the late universe).

Introducing a dynamic pressure against a static baseline density creates a tension with the standard Bianchi identities. In a closed FLRW metric, covariant conservation of energy-momentum ($\nabla_\mu T^{\mu\nu} = 0$) demands the continuity equation: $\dot{\rho}_{vac} + 3H(\rho_{vac} + P_{osm}) = 0$. If the vacuum density (ρ_{vac}) is treated as a static geometric baseline, any dynamic deviation from $w = -1$ violates this conservation law.

To resolve this, we recognize that while the bulk universe is a closed container, the macroscopic FLRW metric (the continuous fluid approximation) acts as an open thermodynamic subsystem [13]. Because the macroscopic metric receives novel entanglement information from the continuous structural collapse of quantum states within the discrete substrate, the standard adiabatic zero is replaced by a non-adiabatic source term (Q_{ent}) representing topological heat injection:

$$\dot{\rho}_{vac} + 3H(\rho_{vac} + P_{osm}) = Q_{ent} \tag{4}$$

(This is conceptually similar to formulations of unimodular gravity, where the cosmological constant arises as an integration constant rather than a strict Lagrangian multiplier, permitting a restricted energy exchange between the macroscopic metric and the microscopic substrate without violating general covariance.)

If we treat the baseline vacuum density as a strict geometric constant ($\dot{\rho}_{vac} = 0$), this modified continuity equation can be solved exactly for the non-adiabatic source term. Substituting our effective equation of state ($P_{osm} = w_{SIE}\rho_{vac}$) into Equation 4 yields:

$$Q_{ent} = 3H\rho_{vac}(1 + w_{SIE}) \tag{5}$$

Anticipating the functional form of w_{SIE} derived in the following subsections (Equation 9), this evaluates directly to:

$$Q_{ent} = 3H\rho_{vac}\left(\kappa_{osm}a^2 - \tau_{land}a^{-1}\right) \tag{6}$$

This explicit formulation demonstrates that the topological heat injection (Q_{ent}) is not an arbitrary free parameter. It is strictly proportional to the Hubble expansion rate (H) and the active thermodynamic stressors on the substrate. This open-system formulation maps the non-adiabatic source term into the observed macroscopic pressure, attempting to bypass the unphysical ghost instabilities required by standard scalar field theories to cross the phantom divide.

3.1 The Geometric Baseline

Before applying the operators derived from the free energy, we must establish the physical boundary condition of the system. To prevent the loss of information across the Hubble boundary, the spatial Substrate must act macroscopically as an incompressible fluid at the horizon [8]. The baseline restorative pressure pushing against the container mirrors the internal density, establishing a rigid fundamental floor: $w_{baseline} = -1$. Any observed dynamic evolution acts as a dimensionless perturbation of this geometric constraint.

3.2 The Early Universe: Landauer Thermal Erasure

In any information-processing system, maintaining structural memory against ambient thermal noise incurs an energetic cost, defined by Landauer's Principle ($E \geq k_B T \ln 2$) [9].

In the early universe, the ambient temperature of the Cosmic Microwave Background plasma scaled inversely with the scale factor ($T_{CMB} \propto a^{-1}$). This thermal noise floor constantly threatened the coherence of the spatial Substrate. To maintain the baseline grid, the vacuum was required to rapidly transfer and diffuse thermodynamic density, resulting in an expansive outward structural bias.

Mapped from the $T_{CMB} S_{sub}$ term of the partition function, we define this active thermal pressure component ($P_{thermal}$) as proportional to the inverse scale factor:

$$P_{thermal}(a) = -\tau_{land} \left(\frac{1}{a} \right) \tag{7}$$

where τ_{land} is the dimensionless fundamental Landauer coefficient of the Substrate.

Because the saturated entropy S_{sub} prevents standard volumetric dilution, the active thermal pressure scales directly with the local Landauer temperature gradient. This mechanism requires the vacuum to shed energy, drawing the equation of state downward into the phantom regime ($w < -1$).

Regarding the extreme early-universe limit ($a \to 0$), a continuous mathematical manifold implies the equation of state diverges to negative infinity. Because the primordial cosmos transitioned from a discrete Substrate state, the initial scale factor possesses a strict, non-zero geometric minimum ($a_{min} > 0$). This structural boundary caps the a^{-1} scaling term, driving the equation of state negative at low scale factors without diverging to an unphysical mathematical limit. This preserves Big Bang Nucleosynthesis (BBN) constraints, as the total effective energy density of the vacuum (Ω_Λ) is overpowered by the immense energy density of radiation (Ω_r) during this epoch.

3.3 The Late Universe: Osmotic Clustering Strain

As the universe expands and the CMB cools ($T_{CMB} \to 2.7$ K), the thermal noise floor dissipates. The thermodynamic stress on the Substrate shifts from thermal erasure to structural deformation.

Gravity pulls the originally isotropic spatial grid into compressed galactic filaments. The fluid variance of this structural compression is mathematically defined by the density contrast ($\delta = \Delta\rho/\rho$). In the linear growth regime of the matter-dominated era, the macroscopic variance of structural clustering ($\langle \delta^2 \rangle$) scales with the square of the scale factor (a^2) [11].

(We recognize that this a^2 approximation technically breaks down at late times, $z \lesssim 0.6$, due to the suppression of linear growth as vacuum energy begins to dominate the metric. A fully mature iteration of this framework will likely require a dampening factor, $f = d\ln\delta/d\ln a < 1$, to map the late-stage transition. However, as a first-order approximation, the a^2 scaling provides the necessary linear baseline to establish the osmotic coefficient).

To offset the entropic deficit created by these localized galaxy clusters, the Substrate continuously increases the rate of spatial generation in the empty cosmic voids. Mapped from the $\langle \hat{O}_{strain} \rangle$ term of the free energy, we define this structure-driven osmotic component ($P_{cluster}$) as:

$$P_{cluster}(a) = +\kappa_{osm} \left(a^2\right) \tag{8}$$

where κ_{osm} is the dimensionless macroscopic osmotic coefficient. By requiring the vacuum to perform structural work to expand voids, this mechanism lifts the equation of state upward, guiding it out of the phantom regime and into the quintessence regime ($w > -1$).

To satisfy the causality constraints of a fluid medium, the a^2 term cannot grow infinitely. A diverging positive pressure would eventually cause the speed of sound in the spatial fluid to exceed the speed of light. However, because cosmic expansion is driven by the depletion of primordial unstable isotopes, the system possesses a thermodynamic asymptote. As the universe exhausts this finite isotopic inventory, the macroscopic metric arrests into a localized terminal state. The scale factor approaches a geometric capacity (a_{max}). This terminal halting state caps the a^2 term, preventing causality violations in the deep future.

3.4 The Unified $w_{SIE}(a)$ Algorithm

Combining the static geometrical baseline with the two competing thermodynamic operators derived from the partition function, we obtain the Unified SIE Equation of State:

$$w_{SIE}(a) = -1 + \kappa_{osm} a^2 - \tau_{land} a^{-1} \tag{9}$$

This algorithm replaces statistical parameters with causal fluid mechanics, offering a physically grounded alternative to the phenomenological Taylor expansion $w_0 + w_a(1 - a)$.

4 Observational Calibration and the Phantom Crossing

4.1 Quantitative Calibration of Thermodynamic Coefficients

For this framework to be applied as a cosmological tool, we must anchor it to observational data. While the functional form (a^2 and a^{-1}) emerges directly from the thermodynamic properties of the Substrate, the specific dimensionless coefficients (κ_{osm} and τ_{land}) must be semi-empirically calibrated against the DESI 2024 dataset.

We identify two primary mathematical constraints. First, at the physical crossing scale ($a_c \approx 0.75$), the best-fit trajectory crosses the Λ baseline. (We use a_c to denote the physical transition point where $w(a_c) = -1$, distinguishing this from the purely statistical pivot scale of the DESI Fisher matrix). This indicates the dynamic terms must cancel:

$$\kappa_{osm} a_c^2 = \tau_{land} a_c^{-1} \implies \frac{\tau_{land}}{\kappa_{osm}} = a_c^3 \tag{10}$$

Substituting $a_c = 0.75$, the fundamental ratio of thermal erasure to osmotic strain is fixed at $a_c^3 \approx 0.422$. Thus, we establish $\tau_{land} \approx 0.422\kappa_{osm}$.

Second, mapping the DESI best-fit variance (w_0 and w_a) to the present-day epoch ($a = 1$), the net positive drift is observationally constrained to approximately $+0.18$:

$$\kappa_{osm} - \tau_{land} \approx +0.18 \tag{11}$$

Substituting the crossing ratio into the present-day variance ($\kappa_{osm} - 0.422\kappa_{osm} \approx 0.18$), we derive the calibrated numerical values:

$$\kappa_{osm} \approx 0.311 \tag{12}$$

$$\tau_{land} \approx 0.131 \tag{13}$$

These constants represent the dimensionless scaling limits required to balance the Cosmic Ledger over the evolutionary history of the universe. We can test the validity of this calibration against the evolutionary slope of the DESI data.

The standard CPL parameterization defines the evolutionary slope as $dw_{CPL}/da = -w_a$. Differentiating our thermodynamic equation (Eq. 9) with respect to the scale factor yields $dw_{SIE}/da = 2\kappa_{osm}a + \tau_{land}a^{-2}$. Evaluating this derivative at the present epoch ($a = 1$) gives $2(0.311) + 0.131 = 0.753$.

This dictates an effective CPL parameter of $w_a \approx -0.753$. This aligns directly with the DESI 2024 combined-probe central constraint ($w_a \approx -0.75$). The thermodynamic model reproduces the evolutionary slope measured by the instruments.

4.2 The Three Eras of Cosmic Evolution

Applying these calibrated coefficients, the equation for $w_{SIE}(a)$ maps three broad eras of cosmic evolution:

1. *The Phantom Past ($a \ll 1$):* In the early universe, the a^2 structural clustering term is negligible, while the a^{-1} Landauer thermal term dominates. The equation dictates that Dark Energy must start in the phantom regime ($w \ll -1$). This mimics an open-system phantom regime, reflecting a hot system venting internal pressure to maintain equilibrium.
2. *The Crossing ($w = -1$):* As the universe evolves, the cooling thermal noise and the accelerating structural clustering intersect. At the crossing scale ($a_c \approx 0.75$, corresponding to $z \approx 0.3$), the competing thermodynamic terms balance ($\kappa_{osm}a_c^2 = \tau_{land}a_c^{-1}$). The equation of state crosses the Cosmological Constant baseline, operating as a non-equilibrium phase transition without requiring specific mathematical fine-tuning.
3. *The Quintessence Present and Future ($a \geq 1$):* In the present cosmological epoch, the CMB rests at 2.7 K, rendering the thermal erasure term negligible. However, the a^2 structural variance of modern superclusters strains the Substrate. This term dominates the equation, yielding a net positive drift and locking the present-day value above the baseline ($w_{today} > -1$).

5 Conclusion and High-Redshift Prediction

The dynamic equation of state indicated by the DESI collaboration may not be an instrumental artifact, but rather a reflection of the underlying thermodynamic nature of the universe.

The standard Cosmological Constant provides utility, but its static nature might be extended by transitioning from an open geometry to a closed thermodynamic container. Modeled as a physical infodynamic system, Dark Energy emerges as Osmotic Spatial Decompression.

The Symbiotic Infodynamic Equilibrium (SIE) equation of state (w_{SIE}) indicates that the observed phantom crossing is an outcome of the thermodynamic intersection between the cooling CMB plasma and the accelerating variance of cosmic structural clustering.

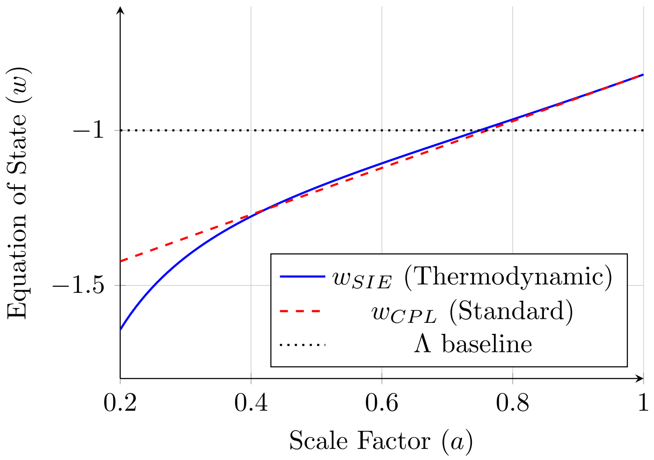

Fig. 1 Theoretical divergence of the Symbiotic Infodynamic Equilibrium equation of state (w_{SIE}) versus the standard Chevallier-Polarski-Linder (w_{CPL}) parameterization. Both models intersect the phantom divide ($w = -1$) at the physical crossing scale $a_c \approx 0.75$, but w_{SIE} predicts a steeper, non-linear descent into the phantom regime at high redshifts ($a < 0.5$).

Because the SIE model relies on an inverse-scale thermal term (a^{-1}) rather than a standard linear Taylor expansion, it creates a testable divergence from phenomenological models at high redshifts (Figure 1). We predict that upcoming Lyman-α forest data ($z > 2$) will reveal a steeper, non-linear plunge into the phantom regime ($w \leq -1.50$) than the standard CPL parameterization permits.

(We acknowledge that while this background formulation navigates the standard ghost instabilities associated with the phantom crossing via the open-system continuity equation, a complete cosmological perturbation analysis will be required to verify that the effective speed of sound, c_s^2, remains positive and stable across the $w = -1$ transition).

Future theoretical work will need to investigate whether the empirical ratio of these constants ($\kappa_{osm}/\tau_{land} \approx 2.37$) derives from a geometric limit of the infodynamic lattice, such as the ratio of discrete vacuum nodes to macroscopic particle horizons. We offer this thermodynamic algorithm, alongside the calibrated derivations, to the data analysts within the DESI and upcoming Euclid collaborations to provide a definitive, testable coordinate to evaluate the thermodynamic nature of Dark Energy.

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