The Compact Fiber as the Canonical Admissibility Structure for Discrete Scalar Motion

A Necessity Result for the Reciprocal System

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April 4, 2026

Abstract

Starting from five postulates of discrete scalar motion (P1–P5), one rotational axiom (A5), and one datum-neutrality principle (P5′), we prove that the canonical compact admissibility structure for the stable local material-closure regime is a degree-2 covered fiber $F \cong \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$ with $|F| = 128$. The construction proceeds through four stages: a canonical primary magnetic 4-cycle from primitive rotational adjacency; a connected electric 8-cycle forming a degree-2 cover, selected uniquely by covering compatibility from a finite enumeration; an independent conjugate magnetic 4-cycle forced by the datum-neutrality principle; and a unique covering channel. We establish that Larson's displacement triple (a, b, c) is faithfully represented by the fiber coordinates, and derive normalized transport and readout classification as structural consequences. All downstream atomic and quantum applications are deferred to companion papers; this paper establishes only the foundational compact structure.

Contents

1 Introduction

1.1 Aim

This paper addresses a foundational question prior to atomic or quantum applications: what compact admissibility structure is forced by discrete scalar motion, three scalar dimensions, vibrational occupation, shared datum, datum neutrality, and rotational closure? We show that the first canonical compact answer is a degree-2 covered fiber with sector form $\mathbb{Z}/4\times\mathbb{Z}/4\times\mathbb{Z}/8$—meaning the 8-state electric sector forms a 2-to-1 cover over the 4-state primary magnetic cycle, while a second independent magnetic 4-cycle completes the restoration structure—and we establish the transport and readout structures that later make atomic theory possible. The aim here is not explanatory range but structural necessity.

In brief: the construction yields two independent 4-state magnetic cycles, one 8-state electric cycle that covers the first magnetic cycle twice (in opposite directions, one per vibrational phase), and a total fiber of $4 \times 4 \times 8 = 128$ admissible states. The present necessity result applies to the regime of stable local material closure—the regime in which persistent matter-like rotational combinations exist. Earlier scalar-motion regimes, including radiation-like or pre-material organization (simple harmonic motion, photons, sub-atomic particles), may be governed by simpler admissibility structures and are not claimed to instantiate the full compact fiber in the same way.

We do not assume or import: the Coulomb potential, Hilbert space, shell structure, the Aufbau principle, Slater screening rules, tensor-product state spaces, or wave-function collapse. The purpose is to derive the first compelled compact structure from the postulates. The paper's only substantive strengthening beyond Larson's base postulates (P1–P5, A5) [1] is a datum-neutrality principle (P5′) that extends the shared datum from a positional reference to a structural one; this principle is needed for the second magnetic sector and is stated explicitly in §2.

The conditional structure is this: with P5′, the full compact fiber $\mathbb{Z}/4\times\mathbb{Z}/4\times\mathbb{Z}/8$ is forced. Without P5′, the theory does not force the second magnetic sector and stops at the smaller structure $\mathbb{Z}/4\times\mathbb{Z}/8$ (capacity 32). All results in this paper that depend on the second magnetic sector are therefore conditional on P5′; results that do not (the primary magnetic cycle, the electric sector, the covering map) hold from P1–P5 and A5 alone. Downstream atomic and quantum consequences are licensed by this paper's foundations but are not developed here; they belong to companion papers.

1.2 Why this paper is needed

The broader research program proposes a compact fiber, transport metric, and downstream effective structures including atomic observables, a spectroscopic bridge to the hydrogen spectrum, and an effective quantum-mechanical representation. But the program's own status audit identifies several central links that remain formalization tasks rather than closed theorems:

- the independence of the second magnetic sector from the first,
- the formal coordinate-identification lemmas connecting Larson's displacement triple (a, b, c) to the fiber coordinates,
- the uniqueness (not merely existence) of the covering channel π.

Downstream claims in companion papers—projection, Born-rule, and Bell-correlation results—depend explicitly on these foundational items. This paper aims to close them.

1.3 Scope boundary

The following topics are excluded from this paper and deferred to companion work:

- the Kustaanheimo–Stiefel coordinate bridge and hydrogen spectrum,
- the quantum-defect law,
- the Born rule and effective Hilbert-space representation,
- Bell correlations and the Tsirelson bound,
- gravitation and the coordinate-time mechanism,
- derived constants and the natural unit system,
- full atomic screening applications (58-pair empirical tests).

2 Primitive Ontology and Admissibility Language

2.1 Primitive objects

Definition 2.1 (Unit displacement)**.** *A* unit displacement *is the smallest departure from the natural datum in one scalar dimension. It is indivisible (P1). A displacement has a* dimension $d \in \{1, 2, 3\}$ *and a* sense $s \in \{+, -\}$*, where* $+$ *denotes outward (away from unity) and* $-$ *denotes inward (toward unity).*

Definition 2.2 (Scalar dimension)**.** *A* scalar dimension *is an independent degree of freedom of motion. It is not a spatial direction; it is an algebraic coordinate carrying a displacement with a sense. There are exactly three scalar dimensions (P3).*

Definition 2.3 (Unity datum)**.** *The* unity datum *is the reference ratio* $s/t = 1/1$ *from which all displacements are measured. All three scalar dimensions share a common datum (P5).*

Definition 2.4 (Admissible state)**.** *An* admissible state *is an assignment of displacement values to the three scalar dimensions that satisfies all constraints derived from P1–P5, P5′, and A5. The set of all admissible states is the* admissible state set*.*

Definition 2.5 (Closure)**.** *A sequence of admissible states* closes *if it returns to its starting configuration after a finite number of steps. A closed sequence is a* cycle*. The* order *of a cycle is its minimal period.*

Definition 2.6 (Restoration)**.** *A* restoration *is a return to the datum configuration after traversal of a complete cycle. Two cycles that require distinct restoration paths are* independently restorable*.*

Definition 2.7 (Equivalence relations)**.** *Two admissible organizations are* equivalent *if they are related by any combination of:*

(i) relabeling*: permutation of dimension indices within a sector,*

(ii) reversal*: replacing* s *by* $-s$ *in a cycle (reversing traversal direction),*

(iii) translation*: shifting the cycle origin,*

(iv) phase-origin shift*: shifting the vibrational phase reference.*

Canonical statements are made up to these equivalences.

2.2 Postulates and axiom

P1 (Discreteness). Motion exists in discrete, indivisible units. The smallest departure from the datum is one unit displacement.

P2 (Two senses). Each scalar dimension admits exactly two senses: inward ($-$) and outward ($+$) relative to the datum.

P3 (Three dimensions). Motion can exist independently in three scalar dimensions.

P4 (Vibrational occupation). One scalar dimension (conventionally dimension 3) is occupied by the primitive vibration: alternating displacement between the two senses within a single dimension.

P5 (Shared datum). All three scalar dimensions share a common unity datum.

P5′ (Datum neutrality). The shared datum is the neutral condition relative to which all admissible structural invariants vanish. Restoration to datum requires annihilation of every invariant the closure structure preserves, not merely return of the visible state. This extends P5 from a positional reference to a structural one and is the paper's only substantive strengthening beyond the base postulates. Without P5′, the primitive theory determines state-return but underdetermines whether restoration excludes retained closure invariants; P5′ resolves this underdetermination. It also preserves fidelity to the displacement algebra (which requires two independently variable magnetic coordinates) and yields useful downstream structure. Whether P5′ can be reduced to P1–P5 is open. See §8.

A5 (Rotational coupling). Rotation—cyclic return to the starting configuration—requires exactly two dimensions.

2.3 The admissibility problem

The central question of this paper:

Given the primitive objects, postulates P1–P5, principle P5′, and axiom A5, what is the minimal compact organization of admissible scalar-motion states, and is it unique up to the equivalences of Definition 2.7?

The remainder of the paper answers this question.

3 Primitive Orientation Set and Admissibility Constraints

3.1 Primitive orientation set

Definition 3.1 (Primitive orientation set)**.** *The* primitive orientation set *is*

$$V = \{(d, s) : d \in \{1, 2, 3\},\ s \in \{+, -\}\}.$$

Each element (d, s) represents a unit displacement in dimension d with sense s. The set V has cardinality $|V| = 6$.

3.2 Constraint C1: Anti-sense exclusion

Proposition 3.2 (Anti-sense exclusion)**.** *No admissible unit state contains both $(d, +)$ and $(d, -)$ in the same scalar dimension simultaneously.*

Proof. A unit displacement is the smallest departure from the datum (P1). The senses $+$ and $-$ are the only two directions of departure (P2). Simultaneous occupation of both senses in the same dimension would constitute zero net displacement—return to the datum—which is not a unit of motion. Since P1 requires discrete indivisible units, no admissible state carries both senses in one dimension. □

3.3 Constraint C2: Rotational two-dimensionality

Proposition 3.3 (Rotational closure requires two dimensions)**.** *Admissible rotational closure requires support on at least two scalar dimensions.*

Proof. By A5, rotation requires two dimensions. A single dimension can oscillate (alternate between $+$ and $-$) but cannot rotate: rotation requires return to the starting configuration via a path that does not retrace itself, which demands a second independent coordinate. □

Remark 3.4 (Minimal rotational closure)**.** *A5 states that rotation requires two dimensions; it does not state that rotation* uses *more than two. Any three-dimensional rotational organization would decompose, under the* $2+1$ *structure of Corollary 3.7, into a two-dimensional rotation plus a separate vibrational component. There is therefore no admissible rotational closure on three dimensions that is not reducible to a two-dimensional rotation coupled to the vibrational dimension. We take the minimal reading throughout: admissible rotation acts on exactly two dimensions.*

3.4 Constraint C3: Vibrational occupation

Proposition 3.5 (Vibrational occupancy)**.** *Under P4, dimension 3 carries alternating two-phase vibrational occupation and is not available as an independent rotational coordinate.*

Proof. P4 assigns dimension 3 to the primitive vibration. The vibration alternates between $(3,+)$ and $(3,-)$. By Proposition 3.2, these cannot coexist; they alternate in sequence. By Proposition 3.3, rotation requires two dimensions; dimension 3 is occupied by the vibration and cannot simultaneously serve as a rotational coordinate. (It may, however, be *visible* to the rotational organization through the shared datum—see Proposition 3.6.) □

3.5 Constraint C4: Shared-datum mediation

Proposition 3.6 (Shared-datum non-isolation)**.** *When all three scalar dimensions share a common datum (P5), the vibrationally occupied dimension is not isolated from the rotational organization: vibrational phase is comparable to rotational state through the shared reference.*

Proof. P5 states that all three dimensions reference the same datum. The vibrational displacement in dimension 3 is measured from this shared datum; so is the rotational displacement in dimensions 1 and 2. Since both are referenced to a common point, a specification of the vibrational phase (which of $(3,+)$ or $(3,-)$) is meaningful relative to the rotational state, and vice versa. This establishes comparability—the two sectors are not informationally isolated from each other. Whether this comparability amounts to a specific structural coupling (such as a covering map) is a stronger claim that must be earned later (§10). □

Corollary 3.7 ($2+1$ decomposition)**.** *The admissibility problem decomposes into a* $2+1$ *organization: two rotational dimensions (1 and 2) and one vibrational dimension (3), with cross-sector comparability through the shared datum.*

Proof. Propositions 3.3 and 3.5 assign dimensions 1 and 2 to rotation and dimension 3 to vibration. Proposition 3.6 establishes that the two sectors are not isolated: vibrational phase and rotational state are comparable through the common datum. The decomposition is $2+1$ with comparability, not $2 \oplus 1$ (direct sum, which would mean isolation). □

4 The Admissible State Set

4.1 Admissible states

The primitive orientation set V has 6 elements. An admissible state is a subset of V satisfying anti-sense exclusion (Proposition 3.2).

Proposition 4.1 (Admissible state count)**.** *The admissible states that select at most one sense per dimension number $3^3 = 27$ (three choices per dimension: $+$, $-$, or unoccupied). Of these, $2^3 = 8$ are fully occupied (one sense chosen in each dimension).*

Proof. Anti-sense exclusion (Proposition 3.2) allows at most one of $(d, +)$ and $(d, -)$ per dimension. For each dimension, the admissible choices are $\{(d, +)\}$, $\{(d, -)\}$, or $\emptyset$. With three independent dimensions, the total count is $3^3 = 27$. The fully occupied states number $2^3 = 8$. $\square$

4.2 Closure classes

Definition 4.2 (Closure class)**.** *A* closure class *on a subset of dimensions is a sequence of admissible states that:*

(i) involves only the specified dimensions,

(ii) visits each admissible sense combination exactly once,

(iii) returns to the starting state after a finite number of steps.

The order *of the closure class is the number of distinct states visited.*

On the two rotational dimensions (1 and 2), each with two senses, there are $2 \times 2 = 4$ fully occupied states: $(+, +)$, $(+, -)$, $(-, -)$, $(-, +)$. Any closure class that visits all four is a 4-cycle.

On the vibrational dimension (3) alone, there are 2 states: $(3, +)$ and $(3, -)$. The vibration alternates between them, forming a 2-cycle.

4.3 Restoration classes

Definition 4.3 (Restoration class)**.** *A closure class is* self-restoring *if its traversal returns to the datum without external input. Two closure classes are* independently restorable *if neither's restoration can be achieved by relabeling, reversal, or translation of the other.*

This definition will be essential in §8, where we must distinguish the second magnetic cycle from a shadow of the first.

4.4 Canonical symmetries

Four types of equivalence are modded out throughout the paper (Definition 2.7):

(i) *Relabeling*: swapping dimension indices (e.g., calling dimension 1 "a" and dimension 2 "b," or vice versa). This is a convention, not structure.

(ii) *Reversal*: traversing a cycle in the opposite direction. The cycle $(+, +) \to (+, -) \to (-, -) \to (-, +)$ and its reverse $(+, +) \to (-, +) \to (-, -) \to (+, -)$ represent the same closure class.

(iii) *Translation*: shifting the origin of a cycle. Starting at $(+, +)$ versus starting at $(+, -)$ does not change the cycle's structure.

(iv) *Phase-origin shift*: in the vibrational dimension, choosing whether the cycle starts at $(3, +)$ or $(3, -)$.

Any claim of "uniqueness" or "canonicality" in this paper means uniqueness up to these four equivalences.

4.5 From admissible subsets to compact closure

The admissible state set catalogues what is *allowed*. It does not yet determine what is *compelled*. The 27 admissible subsets include many that do not close, many that close trivially, and many that are related by the equivalences of Definition 2.7. The task of the next three sections is to classify the closure classes that are canonical under these equivalences and to determine whether the restoration structure requires additional compact sectors beyond the first.

Remark 4.4 (Faithful representation criterion)**.** *A later section of this paper (§11) will claim that Larson's displacement triple* (a, b, c) *[1] is faithfully represented on the compact fiber. By* faithful representation *we will mean: coordinate assignment preserving sector type (magnetic vs. electric), preserving admissible composition rules, preserving sign and phase structure, and preserving closure and restoration distinctions. This criterion is stated here so that the reader can evaluate the later theorem against it.*

5 Canonical Primary Magnetic Cycle

The two rotational dimensions (1 and 2), each carrying two senses, produce four fully occupied admissible states. This section defines admissible rotational adjacency, proves that the four states form a closure class of order 4, and establishes its cyclicity and canonical form.

5.1 The four rotational states

Label the senses in dimensions 1 and 2 as a pair $(s_1, s_2) \in \{+, -\}^2$. The four fully occupied rotational states are:

$$R = \{(+,+),\ (+,-),\ (-,-),\ (-,+)\}.$$

Lemma 5.1 (Rotational state count)**.** *The set* R *has exactly four elements. No additional admissible fully occupied rotational state exists on two dimensions with two senses per dimension.*

Proof. Each dimension contributes two choices (P2). Anti-sense exclusion (Proposition 3.2) does not reduce this count, since each dimension independently selects one sense. The count is $2 \times 2 = 4$. □

5.2 Admissible rotational adjacency

The closure-class definition (Definition 4.2) requires that a sequence of states close, but it does not yet specify which transitions between states are *primitive*. We need a notion of adjacency.

Definition 5.2 (Primitive rotational step)**.** *A* primitive rotational step *is a transition between two rotational states in* R *that changes the sense in exactly one dimension while preserving the other. The* rotational adjacency graph G_R *has vertex set* R *and edges connecting states that differ in exactly one coordinate.*

Proposition 5.3 (Structure of G_R)**.** *The rotational adjacency graph* G_R *is the 4-cycle graph* C_4*: each vertex has exactly two neighbors, and the graph is connected.*

Proof. Each state in R has two coordinates, each of which can be flipped independently. So each state has exactly two neighbors (one per coordinate flip). Explicitly:

$$\begin{aligned}
(+,+) &\leftrightarrow (+,-) \quad \text{and} \quad (+,+) \leftrightarrow (-,+),\\
(+,-) &\leftrightarrow (-,-) \quad \text{and} \quad (+,-) \leftrightarrow (+,+),\\
(-,-) &\leftrightarrow (-,+) \quad \text{and} \quad (-,-) \leftrightarrow (+,-),\\
(-,+) &\leftrightarrow (+,+) \quad \text{and} \quad (-,+) \leftrightarrow (-,-).
\end{aligned}$$

The resulting graph is $(+,+) - (+,-) - (-,-) - (-,+) - (+,+)$, which is C_4. □

Remark 5.4 (Why simultaneous flips are not primitive)**.** *A transition that flips both coordinates simultaneously (e.g., $(+,+) \to (-,-)$) changes two senses in one step. Since each sense is an independent degree of freedom (P2, P3), a double flip is a composed move: two primitive steps executed simultaneously. It is an edge of G_R^2 (the square of the adjacency graph), not of G_R itself. The distinction matters for cyclicity: the adjacency graph $G_R = C_4$ admits a unique Hamiltonian cycle (up to reversal and origin), whereas the complete graph K_4 would admit three.*

5.3 Cyclic closure

Proposition 5.5 (Existence and uniqueness of the Hamiltonian cycle on G_R)**.** *The adjacency graph $G_R = C_4$ admits exactly one Hamiltonian cycle, up to reversal and translation.*

Proof. C_4 is itself a cycle. Its unique Hamiltonian cycle is the cycle graph itself. Reversal gives the same cycle traversed backward; translation shifts the starting vertex. No other Hamiltonian cycle exists because every edge of C_4 must be used (there are exactly 4 edges and 4 vertices). □

5.4 Canonicality

Theorem 5.6 (Canonical primary magnetic cycle)**.** *Under primitive rotational adjacency (Definition 5.2), the admissible compact closure class on the two rotational dimensions is isomorphic to $\mathbb{Z}/4\mathbb{Z}$, unique up to relabeling, reversal, and translation.*

Proof. By Proposition 5.3, the adjacency graph is C_4. By Proposition 5.5, C_4 has a unique Hamiltonian cycle up to reversal and translation. Relabeling (swapping dimensions 1 and 2) acts on R by $(s_1, s_2) \mapsto (s_2, s_1)$, which is an automorphism of C_4. The quotient by relabeling, reversal, and translation leaves a single equivalence class. The cycle has order 4 and is generated by a single primitive step, so it is isomorphic to $\mathbb{Z}/4\mathbb{Z}$. □

This theorem earns the first magnetic sector. The key structural input is that primitive rotational adjacency forces the adjacency graph to be C_4 rather than K_4, which is what makes the Hamiltonian cycle unique. It is not yet "all magnetism"—the question of whether a second, independent magnetic cycle exists is deferred to §8.

5.5 Realized oriented closure

Theorem 5.6 establishes the *abstract* cycle class: $\mathbb{Z}/4\mathbb{Z}$, unique up to reversal and relabeling. But a realized physical closure is not merely an abstract cycle class. It is a cycle together with a specific generator—the primitive step that is actually executed at each transition. This distinction will be essential in §8.

Definition 5.7 (Realized closure)**.** *A* realized closure *on a state set R is a pair (C, g), where C is an abstract closure class (a cyclic permutation of R) and g is a chosen generator of C: a specific primitive step such that iterating g produces the traversal $r_0 \to g(r_0) \to g^2(r_0) \to \cdots \to g^{N-1}(r_0) \to r_0$.*

Proposition 5.8 (Orientation is intrinsic to realized closure)**.** *On the primary magnetic cycle C_4, the two generators g and g^{-1} determine the same abstract cycle class but distinct realized closures. The distinction is not a labeling convention: it is encoded in the sequence of primitive adjacency steps.*

Proof. The generators g and g^{-1} both generate the same abstract cycle class: the unordered set of states visited is the same, and the set of edges traversed over a full cycle is the same. But a realized closure is the pair (C, g), not the abstract class C alone (Definition 5.7). The pairs (C, g) and (C, g^{-1}) are distinct because $g \neq g^{-1}$ in $\mathbb{Z}/4\mathbb{Z}$ (since g has order 4, not 2).

The realized closure does not identify generators: it records which primitive step is executed at each transition. Since both g and g^{-1} are admissible primitive steps (each changes one sense coordinate per Definition 5.2), the two realized closures are both admissible but structurally distinguishable. Orientation is therefore intrinsic to realized closure, not an external labeling convention. □

Remark 5.9. *This clarifies the role of reversal at two levels. At the* abstract *level, reversal is a legitimate equivalence: g and g^{-1} define the same unoriented cycle, and Theorem 5.6 correctly identifies a unique class up to reversal. At the* realized *level, a physical closure must choose a generator—it must actually traverse the cycle in some direction. The two choices are structurally distinguishable. This does not contradict the canonicality theorem; it refines its meaning. The canonical* class *is unique, but a realized instance of that class carries orientation as intrinsic content.*

6 Connected Compactness and Phase Organization

Before treating the electric sector, we need a principle that governs how non-isolated sectors combine into compact structures. This section provides that bridge.

6.1 Connected versus disjoint compact organization

Definition 6.1 (Connected compact closure)**.** *A compact closure class on a state set S is* connected *if it consists of a single cycle visiting every element of S. It is* disjoint *if it decomposes into two or more cycles on proper subsets of S, with no transitions between them.*

Proposition 6.2 (Connected compactness from non-isolation)**.** *Let $S = S_1 \times S_2$ be a state set formed from two sectors, where S_1 carries a compact closure class and S_2 carries a complementary label (such as a phase). If the two sectors are non-isolated—i.e., the label in S_2 is comparable to the state in S_1 through a shared reference (Proposition 3.6)—then the minimal compact closure that organizes the full non-isolated state set S is connected rather than disjoint. (By* minimal full*, we mean: a compact realization that actually traverses states in both values of S_2, rather than merely duplicating a realization of S_1 alone.)*

Proof. A disjoint organization would partition S into subsets $S^{(1)} = S_1 \times \{+\}$ and $S^{(2)} = S_1 \times \{-\}$, with each subset carrying an independent copy of the S_1 closure and no transitions between them. In such an organization, the label in S_2 is never accessed during traversal of either cycle: the two copies are informationally isolated despite sharing a state set. But by hypothesis (non-isolation), the S_2 label is comparable to the S_1 state through the shared datum. An organization that never transitions between phase values fails to realize this comparability in its closure structure. It is therefore not a compact realization of the full non-isolated state set, but merely a duplicated compact realization of S_1 alone. The minimal full compact closure on the non-isolated state set must be connected. □

Remark 6.3 (What this does and does not prove)**.** *Proposition 6.2 establishes that the canonical organization is connected, not that it has a specific cyclic form. The cyclic structure of the connected closure must be proved separately for each case. The proposition rules out the disjoint alternative; it does not by itself determine the specific adjacency or traversal order within the connected cycle.*

7 The Electric Sector

The vibrational dimension (3) alternates between two phases: $(3,+)$ and $(3,-)$. By Proposition 3.6, this phase is comparable to the magnetic state through the shared datum. This section establishes the 8-state electric organization, its connected closure, and the natural degree-2 projection to the magnetic cycle.

7.1 The electric state set

Definition 7.1 (Electric state)**.** *An* electric state *is a pair* (m, ϕ) *where* $m \in \mathbb{Z}/4\mathbb{Z}$ *is a magnetic state and* $\phi \in \{+, -\}$ *is a vibrational phase. The set of electric states is*

$$E = \mathbb{Z}/4\mathbb{Z} \times \{+, -\},$$

with $|E| = 8$*.*

7.2 Connected closure

Theorem 7.2 (Connected electric closure)**.** *The minimal full compact closure class on* E *is connected: it consists of a single cycle of order 8 visiting all electric states.*

Proof. Apply Proposition 6.2 with $S_1 = \mathbb{Z}/4\mathbb{Z}$ (the magnetic cycle, which carries a compact closure by Theorem 5.6) and $S_2 = \{+, -\}$ (the vibrational phase). The two sectors are non-isolated by Proposition 3.6. Therefore the compact closure on $E = S_1 \times S_2$ that realizes the full non-isolated state set is connected rather than disjoint.

A connected compact closure on E must visit all $|E| = 8$ states (Definition 4.2(ii)) in a single cycle (Definition 6.1). Since no state is omitted and no decomposition into subcycles is permitted by connectedness, the closure is a single cycle of order 8. □

7.3 Existence of a connected covering traversal

Proposition 7.3 (Connected 8-state covering traversal exists)**.** *The set* E *admits at least one connected traversal of order 8 that covers the magnetic cycle twice (once per vibrational phase).*

Proof. Consider the sequence

$$(0, +) \to (1, +) \to (2, +) \to (3, +) \to (0, -) \to (1, -) \to (2, -) \to (3, -) \to (0, +).$$

This visits each of the 8 elements of E exactly once and returns to the start, covering the magnetic cycle forward in phase $+$ and then forward in phase $-$. Two of its transitions—$(3, +) \to (0, -)$ and $(3, -) \to (0, +)$—are composed moves (changing both m and ϕ). This establishes existence of a connected covering traversal at the coarse level; the primitive-step decomposition is addressed in §7.6. □

7.4 The covering map

Regardless of which specific 8-cycle is realized, the set E carries a natural projection.

Theorem 7.4 (Degree-2 covering)**.** *The electric state set* E *carries a canonical degree-2 projection to the magnetic cycle:*

$$\pi : E \to \mathbb{Z}/4\mathbb{Z}, \qquad \pi(m, \phi) = m.$$

This map is surjective and 2-to-1, with fiber $\pi^{-1}(m) = \{(m, +), (m, -)\}$ *for each* m*. If one chooses any realized connected 8-cycle identification* $E \cong \mathbb{Z}/8\mathbb{Z}$ *(such as the one exhibited in Proposition 7.3), then* π *is represented by the group homomorphism* $k \mapsto k \bmod 4$*, with kernel* $\{0, 4\} \cong \mathbb{Z}/2\mathbb{Z}$ *(the phase-exchange operation).*

Proof. The map $\pi(m, \phi) = m$ forgets the vibrational phase. Surjectivity: every $m \in \mathbb{Z}/4\mathbb{Z}$ has preimages $(m, +)$ and $(m, -)$. The map is 2-to-1 by construction. Under the exhibited identification $E \cong \mathbb{Z}/8\mathbb{Z}$ (where element k corresponds to ($k \bmod 4$, phase from $\lfloor k/4 \rfloor$)), the map becomes $\pi(k) = k \bmod 4$, which is a group homomorphism with kernel $\{0, 4\} \cong \mathbb{Z}/2\mathbb{Z}$. □

7.5 What has been earned

At this stage, the electric sector provides:

(i) an 8-state state set $E = \mathbb{Z}/4\mathbb{Z} \times \{+, -\}$,

(ii) a connected compact closure of order 8 (Theorem 7.2),

(iii) at least one explicit 8-cycle (Proposition 7.3),

(iv) a canonical degree-2 covering map $\pi : E \to \mathbb{Z}/4\mathbb{Z}$ (Theorem 7.4).

What has *not* yet been earned: the claim that the 8-cycle is *unique* up to the paper's equivalences (this requires formalizing electric adjacency), or that the covering map π is the unique admissible cross-sector projection (this requires a stronger uniqueness argument, addressed in §10). Both of these are addressed in the subsections that follow.

7.6 Primitive electric adjacency

To close the electric-sector canonicality question, we formalize the admissible transitions and enumerate the Hamiltonian cycles.

Definition 7.5 (Primitive electric step)**.** *A* primitive electric step *on $E = \mathbb{Z}/4\mathbb{Z} \times \{+, -\}$ is one of two types:*

(a) Magnetic step*: $(m, \phi) \to (m \pm 1, \phi)$—advance or retreat one magnetic position, phase unchanged. Both forward and backward magnetic steps are primitive because the underlying magnetic adjacency graph $G_R = C_4$ has two adjacency directions at each vertex.*

(b) Phase step*: $(m, \phi) \to (m, -\phi)$—flip the vibrational phase at the same magnetic position.*

No other primitive electric transitions exist: any transition that changes both m and ϕ simultaneously is a composed move.

Proposition 7.6 (Electric adjacency graph)**.** *The electric adjacency graph G_E under primitive electric steps is an 8-vertex graph of degree 3: each vertex (m, ϕ) has two magnetic neighbors and one phase neighbor. The graph is connected.*

Proof. Each state $(m, \phi) \in E$ has magnetic neighbors $(m \pm 1, \phi)$ and phase neighbor $(m, -\phi)$. Total degree: 3. Connectedness follows because any two states can be linked by a sequence of magnetic and phase steps. □

Proposition 7.7 (Enumeration of Hamiltonian cycles)**.** *The graph G_E has exactly 12 oriented Hamiltonian cycles (equivalently, 12 distinct starting-point-normalized traversals). Under the symmetry group (magnetic translation, magnetic reversal, phase-origin shift), these fall into exactly two orbits:*

(i) Zigzag orbit *(4 cycles): alternating phase and magnetic steps. Pattern: phase, magnetic, phase, magnetic, phase, magnetic, phase, magnetic. Representative:*

$$(0, +) \to (0, -) \to (1, -) \to (1, +) \to (2, +) \to (2, -) \to (3, -) \to (3, +) \to (0, +).$$

(ii) Block orbit *(8 cycles): one phase step, three consecutive magnetic steps, one phase step, three consecutive magnetic steps. Representative:*

$$(0, +) \to (0, -) \to (1, -) \to (2, -) \to (3, -) \to (3, +) \to (2, +) \to (1, +) \to (0, +).$$

Proof. Exhaustive enumeration on the 8-vertex graph G_E. Fixing the starting vertex at $(0,+)$ and searching all paths of length 8 using only primitive electric steps that return to $(0,+)$ yields exactly 12 oriented Hamiltonian cycles. Application of the 16-element symmetry group (4 magnetic translations $\times$ 2 magnetic reversals $\times$ 2 phase-origin shifts) partitions these into two orbits of sizes 4 and 8 as stated. $\square$

Theorem 7.8 (Canonical electric cycle)**.** *Among the two Hamiltonian orbit types on G_E, only the block orbit is compatible with the degree-2 covering structure $\pi : E \to \mathbb{Z}/4\mathbb{Z}$. The electric 8-cycle is therefore canonical up to the paper's equivalences.*

Proof. Consider the magnetic projection of each orbit type—that is, the sequence of magnetic states m visited, ignoring phase. The *net magnetic winding* is the sum of magnetic displacements: $+1$ for a forward step $m \to m+1$, -1 for a backward step, 0 for a phase step (which does not change m).

For the block orbit, the projection π traces the magnetic cycle forward in one phase sheet (e.g., $0 \to 1 \to 2 \to 3$ in phase $-$), then backward in the other ($3 \to 2 \to 1 \to 0$ in phase $+$). The net magnetic winding is $+4 + (-4) = 0$: two complete traversals that cancel. This is a degree-2 winding of the magnetic cycle, one traversal per phase sheet.

For the zigzag orbit, the projection visits each magnetic state twice consecutively $(0,0,1,1,2,2,3,3)$ with net magnetic winding $+4 \neq 0$. The zigzag orbit does not traverse the magnetic cycle completely in either phase sheet; it interleaves phase flips with single magnetic steps. It is a connected 8-cycle on E but not a degree-2 covering traversal.

Since the electric sector is defined as a degree-2 cover of the magnetic cycle (Theorem 7.4), only the block orbit is admissible. Within the block orbit, all 8 cycles are equivalent under the symmetry group. $\square$

Remark 7.9 (The exhibited cycle revisited)**.** *The 8-cycle exhibited in Proposition 7.3—*$(0,+) \to (1,+) \to \cdots \to (3,+) \to (0,-) \to \cdots \to (3,-) \to (0,+)$*—contains the composed transition* $(3,+) \to (0,-)$*, which changes both m and ϕ simultaneously. Under the primitive-step definition, this is not a single edge of G_E. The canonical block-orbit cycle corrects this: it uses only primitive steps while achieving the same covering structure. The two are equivalent as coverings but differ in their step decomposition.*

8 Independent Conjugate Magnetic Sector

This is the central structural question of the paper. The primary magnetic cycle (§5) returns the system to its starting *state* after four steps. But does state-return exhaust the restoration requirement, or does the closure carry additional structural content that must also be cancelled? This section argues that a single cycle leaves a nontrivial *restoration residue*, that full datum-restoration requires cancellation of this residue, and that the cancellation requires an independent conjugate sector.

Remark 8.1 (What happens without P5$'$)**.** *Without P5$'$, state-return alone suffices for restoration: a cycle that returns to its starting sense-pair is considered fully restored regardless of traversal history. In that case, the traversal record is present (by Proposition 5.8) but does not block restoration. The second magnetic sector is then not forced: the primary $\mathbb{Z}/4\mathbb{Z}$ plus the electric $\mathbb{Z}/8\mathbb{Z}$ would yield a fiber of the form $\mathbb{Z}/4 \times \mathbb{Z}/8$ with capacity* 32*, not* 128*. The full product $\mathbb{Z}/4 \times \mathbb{Z}/4 \times \mathbb{Z}/8$ requires P5$'$. This is exactly why P5$'$ is stated as an explicit principle: it is the decision point that determines whether the theory has one or two magnetic sectors.*

8.1 State closure versus restoration closure

Definition 8.2 (State closure)**.** *A cycle is* state-closed *if it returns to the same sense-pair after a complete traversal: $g^N = e$ in the state group.*

Definition 8.3 (Restoration closure)**.** *A cycle is* restoration-closed *if it returns to the datum with no residual traversal content. Restoration closure requires state closure but may require more.*

The distinction is this: state closure tracks *where* the system is; restoration closure tracks *how* it got there. A cycle that returns to its starting state may nevertheless carry a record of its oriented traversal—a winding that is invisible to the state but visible to the shared datum.

8.2 Oriented traversal residue

By Proposition 5.8, a realized closure carries orientation as intrinsic content: the generator g (or g^{-1}) is part of the structure, not optional metadata. The traversal record is the natural counting invariant of realized generator applications.

Definition 8.4 (Traversal record)**.** *Let (C, g) be a realized closure (Definition 5.7) on a state set R. The* traversal record *of k complete cycles is $k \cdot [g] \in \mathbb{Z}$, where $[g] = +1$ for the chosen generator and $[g^{-1}] = -1$ for its inverse. The traversal record is additive: concatenating realized closures adds their records.*

Proposition 8.5 (Nontrivial restoration residue of a single cycle)**.** *One complete forward traversal of the primary magnetic cycle is state-closed ($g^4 = e$ in R) but carries traversal record $[g] = +1 \neq 0$. The restoration residue is nontrivial.*

Proof. By construction, $g^4 = e$ restores the state. But the traversal record is additive: one forward traversal contributes $+1$, two contribute $+2$, and so on. After one complete cycle, the record is $+1$, not 0. The record distinguishes a system that has completed one forward traversal from a system that has never been traversed, even though both are in the same state. □

The traversal record is the mathematical object that the previous draft called "rotational handedness." It is now a formal integer-valued invariant rather than an intuition.

8.3 Datum restoration requires zero residue

The key question is whether restoration to the datum requires only state-return or also cancellation of the traversal record. We now apply P5′, the datum-neutrality principle stated in §2: the shared datum is the neutral condition relative to which all admissible structural invariants must vanish, not merely the zero displacement state.

Lemma 8.6 (Residual invariant exclusion)**.** *Under P5′, any admissible invariant of the closure structure that retains a nonzero value after state-return constitutes residual content relative to the datum. Restoration to the datum requires the vanishing of all such invariants.*

Proof. By P5′, the datum is the configuration at which all admissible closure invariants vanish. If an invariant I is admissible (determined by the closure structure and the primitives, not by external convention) and $I \neq 0$ after state-return, then the system is not at the datum. Restoration therefore requires $I = 0$. □

Proposition 8.7 (Full restoration requires residue cancellation)**.** *Restoration to the shared datum requires not only state-return but vanishing of the traversal record.*

Proof. The traversal record $[g] \in \mathbb{Z}$ is an admissible closure invariant: it is determined by the realized closure structure (Definition 8.4) and is intrinsic to the primitive adjacency (Proposition 5.8). It is not an external convention. By Lemma 8.6 (under P5′), any nonzero admissible invariant after state-return constitutes residual content. After one complete primary cycle, the traversal record is $[g] = +1 \neq 0$ (Proposition 8.5). Therefore the system is not fully restored to the datum. Restoration requires $\sum[g_i] = 0$. □

8.4 Reducibility fails to cancel residue

Definition 8.8 (Reducible candidate sector)**.** *A candidate second magnetic sector is* reducible *if it is any of the following:*

(a) Relabeling shadow*: the same cycle with dimension labels permuted.*

(b) Reversal shadow*: the primary cycle traversed in the opposite direction (as a description of the same physical cycle, not as a simultaneously available independent traversal).*

(c) Translation shadow*: the primary cycle with a shifted origin.*

(d) Phase shadow*: a copy distinguished only by vibrational phase (already in E).*

(e) Automorphism image*: the image under any automorphism that does not introduce new traversal content.*

Proposition 8.9 (Reducibility does not cancel the residue)**.** *No reducible operation (a)–(e) annihilates the traversal record of the primary cycle.*

Proof. (a) *Relabeling*: permutes coordinates but does not change the traversal direction or its record. $[g]$ is unchanged.

(b) *Reversal as description*: reversing the description of the primary cycle changes $[g]$ to $-[g]$. But this is a re-description of the *same* sector, not a simultaneously available second traversal. The system cannot carry both $[g]$ and $-[g]$ in a single sector—it must be in one orientation or the other. Cancellation requires a second degree of freedom that simultaneously carries $-[g]$ while the primary carries $+[g]$.

(c) *Translation*: shifts origin, does not change traversal record.

(d) *Phase shadow*: the vibrational phase label (already accounted for in E) adds no rotational traversal content.

(e) *Automorphism*: maps the cycle to an equivalent cycle with the same traversal class. $[g]$ is preserved or conjugated, not cancelled.

Since no reducible operation cancels $[g]$, a genuinely independent sector is required. □

8.5 The independence theorem

Theorem 8.10 (Independent conjugate magnetic sector)**.** *Admissible restoration closure under the shared datum requires a second compact cyclic degree of freedom, independent of the primary magnetic cycle, carrying traversal record $-[g]$ to cancel the primary's residue.*

Proof. By Proposition 8.5, a single primary cycle has nontrivial traversal record $[g] = +1$. By Proposition 8.7, datum restoration requires $\sum[g_i] = 0$. By Proposition 8.9, no reducible operation on the primary sector achieves this cancellation. Therefore a second, genuinely independent compact cyclic sector is required, carrying traversal record $-[g]$. □

Corollary 8.11 (Canonical form of the conjugate sector)**.** *The independent conjugate magnetic sector is isomorphic to* $\mathbb{Z}/4\mathbb{Z}$*.*

Proof. The conjugate sector must cancel the primary's traversal record while itself being a compact rotational closure on the same dimension pair (the only available rotational dimensions). By Theorem 5.6, any such closure is canonically $\mathbb{Z}/4\mathbb{Z}$. The two sectors are distinguished by their traversal records: the primary carries $+[g]$, the conjugate carries $-[g]$. $\square$

Remark 8.12 (What the two sectors represent)**.** *The first* $\mathbb{Z}/4\mathbb{Z}$ *closes the state; the second closes the restoration residue. Together they provide full datum restoration. In Larson's notation, these become the two magnetic displacement numbers* a *and* b*. The product* ab *is structurally meaningful because it involves both sectors simultaneously:* a *from the primary traversal record and* b *from the conjugate. A single* $\mathbb{Z}/4$ *plus a label cannot produce an independently variable two-factor product.*

8.6 What has been earned

(i) *Formal restoration residue*: the traversal record $[g] \in \mathbb{Z}$ distinguishes state closure from restoration closure (Definitions 8.2–8.4).

(ii) *Nontriviality*: a single primary 4-cycle is state-closed but carries $[g] = +1 \neq 0$ (Proposition 8.5).

(iii) *Datum restoration requires cancellation*: $\sum[g_i] = 0$ (Proposition 8.7).

(iv) *Reducibility fails*: no shadow or automorphism of the primary sector cancels the residue (Proposition 8.9).

(v) *Independent conjugate sector*: a second $\mathbb{Z}/4\mathbb{Z}$ is required (Theorem 8.10).

The proof-strength caveat is localized at Proposition 8.7: the claim that the shared datum sees the traversal record, not merely the endpoint state. This is the single admissibility principle on which the second sector depends. If the reader accepts it, the rest follows by formal argument. If the reader regards it as an extension beyond P5, the rest of the paper's results are conditional on this principle.

9 Canonical Compact Fiber Theorem

We now assemble the results of §§5–8.

Theorem 9.1 (Canonical compact fiber)**.** *The minimal compact admissibility structure compatible with P1–P5, P5′, and A5 is*

$$F \;\cong\; \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z},$$

with total capacity $|F| = 4 \times 4 \times 8 = 128$*.*

Proof. The three sectors are:

(i) *Primary magnetic*: $\mathbb{Z}/4\mathbb{Z}$ (Theorem 5.6).

(ii) *Conjugate magnetic*: $\mathbb{Z}/4\mathbb{Z}$, independent of the primary (Theorem 8.10, Corollary 8.11).

(iii) *Electric*: $\mathbb{Z}/8\mathbb{Z}$, carrying a degree-2 cover of the primary magnetic cycle (Theorem 7.2, Theorem 7.4).

The three sectors are combined as a direct product because:

- the two magnetic sectors are independently restorable (Theorem 8.10),
- the electric sector is coupled to the primary magnetic sector through the covering map but carries its own cyclic structure (Theorem 7.2),
- no additional compact sector is required beyond these three (the three scalar dimensions are fully accounted for: two rotational, one vibrational, with the vibrational visible through the shared datum).

The product is $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$ with capacity $4 \times 4 \times 8 = 128$. □

Remark 9.2 (What "canonical" means). *The fiber F is canonical up to the equivalences of Definition 2.7: relabeling the dimension indices, reversing cycle directions, translating cycle origins, and shifting vibrational phase origins. Any two realizations of the admissibility structure related by these operations are isomorphic. The claim is that F is the* unique *minimal compact structure, not merely one convenient encoding.*

Remark 9.3 (Conditionality). *The canonical compact fiber theorem is unconditional relative to P1–P5, P5′, and A5. Relative to Larson's base postulates alone (P1–P5, A5), the theorem is conditional on the datum-neutrality principle P5′: without P5′, the second magnetic sector is not forced and the fiber reduces to $\mathbb{Z}/4 \times \mathbb{Z}/8$ (capacity 32). The primary magnetic cycle, electric sector, and covering map are unconditional—they hold from P1–P5 and A5 without P5′.*

Remark 9.4 (Regime of validity). *The compact fiber F is the canonical admissibility structure of the stable local regime: the regime in which persistent three-dimensional rotational closure (matter-like combinations) is available. In Larson's deductive sequence [4], this regime is reached only after simpler structures—the natural progression, simple harmonic motion, and radiation—are already established. The present paper does not claim that radiation or sub-atomic particles must instantiate the full compact structure in the same way as stable atoms. Companion work will address how earlier or lower-dimensional regimes relate to, project into, or fail to realize the stable local compact structure derived here.*

9.1 What has been earned

(i) The compact fiber $F \cong \mathbb{Z}/4 \times \mathbb{Z}/4 \times \mathbb{Z}/8$ with $|F| = 128$.

(ii) The product structure: two independent magnetic sectors and one covered electric sector.

(iii) The covering map $\pi : \mathbb{Z}/8 \to \mathbb{Z}/4$ with degree 2.

10 Covering Channel Uniqueness

The covering map $\pi : E \to \mathbb{Z}/4\mathbb{Z}$ was established as a natural degree-2 projection in Theorem 7.4. This section addresses the stronger question: is π the *unique* admissible cross-sector channel?

10.1 Admissible cross-sector maps

Definition 10.1 (Admissible cross-sector projection). *An* admissible cross-sector projection *is a surjective map $\varphi : E \to \mathbb{Z}/4\mathbb{Z}$ that:*

(a) respects the shared datum (P5): states related by vibrational phase project to the same magnetic state,

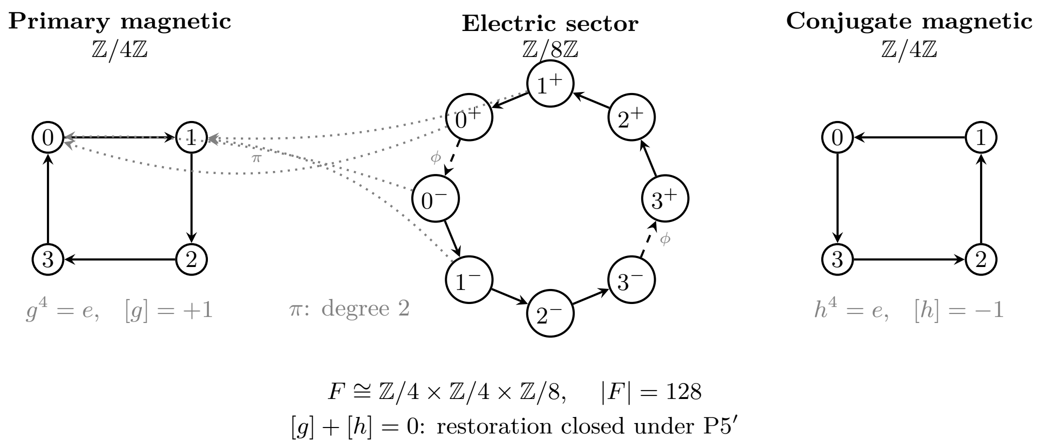

Figure 1: The compact fiber F. *Left*: primary magnetic 4-cycle (generator g, traversal record $[g] = +1$). States labeled 0–3 correspond to the four sense-pairs of dimensions 1 and 2. *Center*: electric 8-cycle in the canonical block-orbit traversal (Theorem 7.8): phase step (dashed, labeled ϕ), three forward magnetic steps in phase $-$, phase step, three backward magnetic steps in phase $+$. Dotted arrows show the degree-2 covering π: each magnetic state has two preimages. *Right*: conjugate magnetic 4-cycle (generator $h = g^{-1}$, opposite orientation, $[h] = -1$). Together, $[g] + [h] = 0$: full datum restoration under P5′.

(b) preserves cyclic order: if $e_1 \to e_2$ is an admissible transition in E, then either $\varphi(e_1) = \varphi(e_2)$ (same magnetic state, phase transition) or $\varphi(e_1) \to \varphi(e_2)$ is an admissible transition in $\mathbb{Z}/4\mathbb{Z}$,

(c) is uniform: each magnetic state has the same number of preimages.

Theorem 10.2 (Uniqueness of the admissible covering channel)**.** *The only admissible cross-sector projection $\varphi : E \to \mathbb{Z}/4\mathbb{Z}$ satisfying (a)–(c) is $\pi(m, \phi) = m$, up to relabeling and phase-origin shift.*

Proof. Condition (a) requires that $(m, +)$ and $(m, -)$ map to the same element. This forces $\varphi(m, +) = \varphi(m, -)$ for each m, making φ a function of m alone on the fibers. Condition (c) requires uniformity: each target has the same number of preimages. Since $|E| = 8$ and $|\mathbb{Z}/4| = 4$, each target has exactly 2 preimages. Since the fibers are already phase pairs $\{(m, +), (m, -)\}$, the 2 preimages of $\varphi(m, \cdot)$ must be exactly the phase pair at m. Therefore φ maps each magnetic state to itself: $\varphi(m, \phi) = m$. Condition (b) is then automatically satisfied. Up to relabeling (permuting the target indices) and phase-origin shift, this is exactly π. □

10.2 What has been earned

The covering channel π is the unique admissible cross-sector projection from E to $\mathbb{Z}/4\mathbb{Z}$, under the three conditions of Definition 10.1. This promotes π from “a natural candidate” to “the only admissible choice.”

11 Faithful Representation of the Displacement Algebra

Larson’s displacement triple (a, b, c) [1, 2] specifies an atom’s rotational structure: a and b are the magnetic displacement numbers, and c is the electric displacement number. This section establishes that the fiber F provides a canonical coordinate system for the displacement algebra, satisfying all four criteria of Remark 4.4: sector type, admissible composition, sign/phase structure, and closure/restoration distinctions.

11.1 Coordinate assignment (sector type)

Theorem 11.1 (Canonical displacement coordinates)**.** *The two magnetic coordinates of F canonically represent Larson's a and b, and the electric coordinate canonically represents c:*

(i) $a \in \{1, 2, 3, 4\}$ *is the position in the primary magnetic cycle* $\mathbb{Z}/4\mathbb{Z}$*.*

(ii) $b \in \{1, 2, 3, 4\}$ *is the position in the conjugate magnetic cycle* $\mathbb{Z}/4\mathbb{Z}$*.*

(iii) $c \in \{1, \ldots, 8\}$ *(or the negative-displacement convention* $c \in \{-7, \ldots, -1, 1, \ldots, 8\}$*) is the position in the electric cycle* $\mathbb{Z}/8\mathbb{Z}$*.*

The assignment preserves sector type: magnetic coordinates map to magnetic sectors, the electric coordinate maps to the electric sector, and no cross-assignment is admissible.

Proof. The fiber $F = \mathbb{Z}/4 \times \mathbb{Z}/4 \times \mathbb{Z}/8$ has exactly three cyclic coordinates, one per sector. The two magnetic sectors are independent (Theorem 8.10), carry the same canonical form $\mathbb{Z}/4\mathbb{Z}$ (Corollary 8.11), and are distinguished by their traversal records ($[g] = +1$ vs. $[h] = -1$). Larson's a and b are likewise two independent magnetic displacement numbers with identical internal structure. The electric sector is a single cycle of order 8 with a degree-2 covering of the primary magnetic cycle (Theorem 7.4). Larson's c is a single electric displacement with twice the period of the magnetic displacements. No cross-assignment (e.g., mapping a to the electric sector) is consistent with these structural constraints: the periods, independence relations, and covering structure uniquely determine which fiber coordinate represents which displacement number. □

11.2 Admissible composition

Proposition 11.2 (Composition preserved)**.** *Displacements combine by addition within each cyclic sector, modulo the sector capacity. This reproduces the additive composition rules of Larson's displacement algebra.*

Proof. The group operation in $\mathbb{Z}/N\mathbb{Z}$ is addition modulo N. Two displacements a_1, a_2 in the primary magnetic sector combine as $a_1 + a_2 \pmod 4$; likewise for b in the conjugate sector and c in the electric sector. This is precisely Larson's rule: displacements within a sector add, and the sum wraps around at the sector capacity. The cross-sector product ab is well-defined as the product of two independent cyclic coordinates in $\mathbb{Z}/4 \times \mathbb{Z}/4$; its structural role in cross-sector relations is inherited from the covering map π, which connects the electric sector to the primary magnetic sector while the conjugate sector provides the independent factor. □

11.3 Sign and phase structure

Proposition 11.3 (Sign structure preserved)**.** *The first half of each cycle (*$1, \ldots, N/2$*) corresponds to positive displacement; the second half (*$N/2 + 1, \ldots, N$*) corresponds to negative displacement. This reproduces Larson's sign convention.*

Proof. Each cycle has two senses (P2), and the half-cycle boundary is the midpoint of the cyclic group. In $\mathbb{Z}/N\mathbb{Z}$, the elements $1, \ldots, N/2$ are reached by forward steps from the datum; the elements $N/2+1, \ldots, N-1$ are equivalently reached by $1, \ldots, N/2-1$ backward steps. Forward corresponds to positive displacement (outward from datum); backward corresponds to negative displacement (inward, or equivalently, measured from the opposite end of the cycle). This reproduces Larson's convention that negative electric displacement counts from the noble-gas end. For example, $c = -3$ in Larson's notation corresponds to position $8-3 = 5$ in $\mathbb{Z}/8\mathbb{Z}$: three steps before cycle completion, measured backward. □

Proposition 11.4 (Phase structure preserved)**.** *The two vibrational phases* $(+,-)$ *in the electric sector correspond to the two sheets of the degree-2 covering. Phase exchange is the kernel element of* π*.*

Proof. The covering map $\pi : E \to \mathbb{Z}/4\mathbb{Z}$ has kernel $\{0,4\} \cong \mathbb{Z}/2\mathbb{Z}$ (Theorem 7.4). The kernel element maps $(m,+) \mapsto (m,-)$: it exchanges vibrational phase while preserving magnetic position. This is the phase-exchange operation. The two sheets of the cover—the $+$ sheet and the $-$ sheet—are the two cosets of the kernel, corresponding to the two vibrational phases. □

11.4 Closure and restoration distinctions

Proposition 11.5 (Closure/restoration preserved)**.** *A displacement that reaches the cycle capacity returns to the datum (closure). Full restoration requires vanishing of all admissible invariants including the traversal record (P5′). These distinctions are preserved by the fiber representation.*

Proof. In $\mathbb{Z}/N\mathbb{Z}$, the element $N \equiv 0$ is the datum. A displacement sequence that sums to 0 (mod N) is state-closed. This reproduces Larson's shell-closure rule: an atom whose displacement reaches the sector capacity has completed a full shell. Restoration closure additionally requires vanishing traversal record (Proposition 8.7), which is tracked by the realized-closure structure (Definition 5.7). The fiber coordinates carry both the displacement value (state) and the generator choice (orientation), so both closure and restoration are represented. □

11.5 What has been earned

The displacement triple (a,b,c) is not fitted onto the fiber post hoc. It is the natural coordinate system of F, satisfying all four faithful-representation criteria of Remark 4.4:

(i) sector type preserved (Theorem 11.1),

(ii) admissible composition preserved (Proposition 11.2),

(iii) sign/phase structure preserved (Propositions 11.3 and 11.4),

(iv) closure/restoration distinctions preserved (Proposition 11.5).

12 Normalized Transport and Readout Classification

The compact fiber determines the state space. This section derives the first structural consequences: the transport metric (how observables weight each position in a cycle) and the readout classification (how many primitive types of observable projection exist).

12.1 Normalized transport

Theorem 12.1 (Normalized transport on compact cyclic sectors)**.** *On a cyclic sector* $\mathbb{Z}/N\mathbb{Z}$*, translation invariance and full-cycle normalization uniquely determine the per-step transport weight as* $1/N$*.*

Proof. Let $w(k)$ be the weight assigned to position $k \in \mathbb{Z}/N\mathbb{Z}$. Translation invariance requires $w(k) = w(k')$ for all k, k' (the weight depends only on the step, not the position). Full-cycle normalization requires $\sum_{k=0}^{N-1} w(k) = 1$. Together: $N \cdot w = 1$, hence $w = 1/N$. □

12.2 Readout classification

An atomic observable is a projection that reads a displacement value from the fiber. Different types of projection access different parts of the fiber structure.

Theorem 12.2 (Readout classification)**.** *The admissible first-level readout classes on F are exactly three:*

(i) Self-sector readout*: the projection accesses positions within a single cyclic sector. The observable weight is the complementary fraction: $(N-e)/N$ for e occupied positions in a sector of capacity N.*

(ii) Cross-sector readout*: the projection accesses positions in a different sector from the one being displaced. The observable weight is e'/N' where e' is the displacement in the cross sector.*

(iii) Cover-to-base readout*: the projection uses the covering map π to translate between the electric and magnetic sectors. The weight includes a resolution correction of $1/(N_m \cdot N_e)$ from the covering degree and sector capacities.*

Proof. The fiber $F = \mathbb{Z}/4 \times \mathbb{Z}/4 \times \mathbb{Z}/8$ has three sectors. A first-level readout projects onto one sector coordinate. The three types correspond to:

(i) same sector as the displacement being measured (self),

(ii) a different magnetic sector (cross),

(iii) translation between the electric and magnetic sectors via π (cover-to-base).

That these are the only three types follows from the structure of F: the only sectors are the two magnetic $\mathbb{Z}/4$'s and the electric $\mathbb{Z}/8$. A readout either stays in the same sector (type i), crosses between the two magnetic sectors (type ii), or crosses between the electric and a magnetic sector via π (type iii). There is no fourth option because F has exactly three sectors and the covering map is the only inter-type connection. □

Corollary 12.3 (No fourth readout class)**.** *There is no additional primitive first-level readout class compatible with the admissibility structure of F.*

Proof. The proof of Theorem 12.2 enumerates all sector combinations. The three sectors and one covering map exhaust the inter-sector relations. Any proposed fourth class would require either a fourth sector (excluded by the compact fiber theorem) or a second inter-sector map (excluded by the covering uniqueness theorem). □

12.3 What has been earned

The transport metric ($1/N$ per step) and the three-class readout structure are structural consequences of the compact fiber, not additional postulates. They are the starting point for atomic observable calculations in the companion paper.

13 Conclusion

This paper establishes that the canonical compact admissibility structure for discrete scalar motion, under postulates P1–P5, axiom A5, and the datum-neutrality principle P5′, is the fiber

$$F \;\cong\; \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}, \qquad |F| = 128.$$

The construction proceeds through ten theorems and supporting propositions: primitive admissibility constraints yield a $2+1$ decomposition; primitive rotational adjacency forces a canonical primary magnetic 4-cycle; connected compactness and covering compatibility select a unique electric 8-cycle from a finite enumeration; datum neutrality (P5′) forces an independent conjugate magnetic 4-cycle via traversal-residue cancellation; and the covering channel and displacement representation are proved unique under explicitly stated admissibility conditions.

The conditional structure is explicit. Results 1–6 (the primary magnetic cycle, electric sector, and covering map) hold unconditionally from P1–P5 and A5. Results 7–11 (the second magnetic sector, the full fiber, faithful representation, transport, and readout classification) additionally require P5′. Without P5′, the theory does not force the second magnetic sector and the admissibility structure reduces to $\mathbb{Z}/4 \times \mathbb{Z}/8$ with capacity 32.

The single open foundational question is whether P5′ can be derived from P1–P5 or is an irreducible additional principle. Either resolution would sharpen the program's foundational status.

The compact fiber, covering map, transport metric, readout classification, and faithful displacement representation established here provide the structural foundations for companion papers on atomic observables and effective quantum structure. The result is best understood as a foundational local admissibility theorem for later atomic and effective-quantum development, rather than as a claim that all scalar-motion regimes are governed by the same full compact structure. Companion work will address how earlier or lower-dimensional regimes—including radiation and sub-atomic particles—relate to, project into, or differ from the stable local compact structure derived here.

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