Geometric Origin of Electron Spin

Polar-Plane Sweep, the 720° Return, and the Structural Dissolution of the Superposition Mystery

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Contribution Statement

Mathematical formalization and documentation: Claude (Anthropic), Claude Opus 4.6

The core insights, physical intuitions, and axiom system of this work originate from the author's 30 years of independent research. Claude was responsible for mathematical formalization, equation derivation verification, and document structuring.

Version: v1.1 PREPRINT (2026-03-26) **Canon Basis:** MP-CANON-v2.2 (2026-03-23) **License:** CC BY 4.0 **Series:** This paper is PP-03 in the META Physics preprint series. **Companion papers:** - [PP-00] Lee, C.-G. (2026). META Physics: A Framework for Deriving the ORIGIN of Physical Phenomena from Higher-Dimensional Geometric Rotation. Zenodo. [See Ref. 11]

Notation Index

Symbol	Meaning
$E^n := \mathbb{R}^n$	n-dimensional background space (mathematical stage)
$B^n(R)$	n-ball substance of radius R: $\{x \in \mathbb{R}^n : \|x\| \leq R\}$
$S^{n-1}(R)$	Boundary $(n-1)$-sphere: $\partial B^n(R)$
PGS	Pure Geometric Substance (see Appendix D)
PEM	Projection–Emergence–Manifestation (three-layer ontology; see Appendix D)
PS	Projection System: $B^4(R)$, the causal layer
ES	Emergence System: $S^3(R) = \partial B^4(R)$, the transitional layer
WS, WS_3	World System: observable universe (observer-world anchoring index)
ATI	Anchored Topological Interior (see Appendix D)
ISM-T	Internal State Mechanism–Tunnel (see Appendix D)
DRAIN	Dimensional compression mechanism ($B^4 \to S^3$; see Appendix D)
IMM	Internal Manifestation Mechanism (ES $\to$ WS; see Appendix D)

Symbol	Meaning
Π_{con}	Concomitant projection: $P_a \circ \mathcal{F}_a$
$B^4_{eq}(R)$	Equatorial hypersphere: $B^5(R) \cap \{x_5 = 0\}$
0_s / 0_i	Surface port / Inner port of ISM-T
Ω_1, Ω_2	Angular velocities of the two independent rotation planes of $B^4(R)$
$\omega_{PS} := \bar{\Omega}$	Mean angular velocity: $(\Omega_1 + \Omega_2)/2$
$\delta\Omega$	Rotational anisotropy: $\lvert\Omega_1 - \Omega_2\rvert \geq 0$
Ω_5	Angular velocity of $B^5(R)$ v-fixed isoclinic rotation
α	Fine-structure constant: $\omega_{\text{fiber}}/\omega_{S^2} = \delta\Omega/(4\bar{\Omega})$ [PP-02]
$N(a)$	Stacking degree (topological density) at point a
ϵ_0	Fundamental energy of a single 0_s point: $\hbar\bar{\Omega}$
$R_z(\theta)$	Rotation matrix about the z-axis by angle θ
D_{xz}, D_{yz}, D_{xy}	2-dimensional disks within $B^3(R)$: polar (xz, yz), equatorial (xy)
pre-3D	Three spatial axes (x, y, z) internal to the ISM-T, prior to manifestation
$\mathbb{H}$	Quaternion algebra
$\mathrm{Ad}(q)$	Adjoint action of unit quaternion: $\mathbf{v} \mapsto q\mathbf{v}q^{-1}$

Unit convention: Natural units $c = \hbar = 1$ throughout, unless otherwise stated.

Operational locks:

$$\omega_{PS} := \bar{\Omega} = \frac{\Omega_1 + \Omega_2}{2}, \qquad \delta\Omega := |\Omega_1 - \Omega_2| \geq 0$$

$S^3(R) \cong SU(2)$ (topological isomorphism — the foundation of the spin derivation)

Abstract

The spin-1/2 property of the electron is one of the most fundamental yet least explained features of quantum mechanics. The electron carries an intrinsic angular momentum of $\hbar/2$ despite having no classical rotational structure; it requires a 720° rotation (rather than 360°) to return to its original quantum state; and this angular momentum is independent of the electron's mass. The standard formalism encodes these facts in the spinor representation of $SU(2)$ but does not explain why such a representation governs nature.

This paper derives the spin-1/2 structure from the geometry of META Physics (Multi-dimensional Emergence Theory of Actuality). The key mechanism is the **polar-plane sweep**: when a 2-dimensional disk whose plane contains the rotation axis undergoes a full 2π rotation within a 3-dimensional ball $B^3(R)$, the disk sweeps out the entire interior of $B^3(R)$, and its boundary circle covers the full 2-sphere $S^2(R)$ twice — once per half-revolution. Consequently, orientational return requires 4π (720°), not 2π. This is the geometric content of the double cover $SU(2) \to SO(3)$.

Within the META Physics framework, the Emergence System (ES) is $S^3(R) \cong SU(2)$. The pre-3D structure internal to each 0-dimensional point on $S^3(R)$ contains three mutually orthogonal 2D disks. Under rotation about the z-axis, two polar planes (x, z) and (y, z) each execute the polar-plane sweep (spin-1/2), while the equatorial plane (x, y) is invariant (spin-1). The complex combination $D_{xz} \pm iD_{yz}$ yields the two eigenstates of L_z — spin-up and spin-down — where the imaginary unit i originates from the imaginary structure of the ES itself.

We establish detailed correspondence with the standard mathematical formalism: the Dirac belt trick and Feynman plate trick are identified as WS_3 demonstrations of the polar-plane sweep; the spinor representation of $SU(2)$ is derived as the natural representation on the space of complex-combined polar planes; and the spin structure on a manifold, which standard physics posits as a prerequisite, is shown to be an automatic consequence of the $SU(2)$ topology of the ES. Furthermore, the "superposition" of spin states $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ is reinterpreted as the weighted geometric co-presence of two real polar planes in the ES — resolving the ontological mystery of quantum superposition as a structural limitation of the lower-dimensional WS_3 observer. The result is a unified geometric picture in which spin-1/2, the 720° return, spin-up/down states, integer spin, and quantum superposition all emerge as structural consequences of higher-dimensional rotation — with zero free parameters.

Keywords: electron spin, 720° return, polar-plane sweep, SU(2), spinor, META Physics, geometric substance, Hopf fibration, quaternion, quantum superposition, Dirac belt trick, fiber bundle, spin structure, pre-3D

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§ 1. Introduction

1.1 Three Mysteries of Spin

Electron spin was discovered empirically by Stern and Gerlach (1922) [1] and theorized by Uhlenbeck and Goudsmit (1925) [2]. The electron possesses an intrinsic angular momentum of magnitude $\hbar/2$ and a magnetic moment $\mu = -g_e \mu_B \mathbf{S}/\hbar$, where $g_e \approx 2.002$. Three features of spin defy classical explanation:

(F1) No classical rotation. The electron is a point particle in quantum field theory. A classical spinning sphere of the electron's Compton radius would need surface velocities exceeding c to produce angular momentum $\hbar/2$. Spin is not mechanical rotation.

(F2) The 720° return. A spin-1/2 state $|\psi\rangle$ acquires a sign flip under 2π rotation: $R(2\pi)|\psi\rangle = -|\psi\rangle$. Only after 4π does the state return: $R(4\pi)|\psi\rangle = +|\psi\rangle$. No macroscopic classical object exhibits this behavior. The neutron interferometry experiment of Werner et al. (1975) [3] confirmed this sign flip directly.

(F3) Mass independence. The spin quantum number $s = 1/2$ is identical for the electron ($m_e = 0.511$ MeV), muon ($m_\mu = 105.7$ MeV), and tau ($m_\tau = 1776.9$ MeV). Spin is independent of the mass generation mechanism.

1.2 A Century of Formalism without ORIGIN

The resolution offered by standard quantum mechanics is mathematically complete. Particles are classified by the representations of the Lorentz group $SL(2, \mathbb{C})$, whose compact subgroup is $SU(2)$. Spin-1/2 particles carry the fundamental (2-dimensional) representation; spin-1 particles carry the adjoint (3-dimensional) representation. The Pauli matrices $\sigma_x, \sigma_y, \sigma_z$ generate the Lie algebra $\mathfrak{su}(2)$, and the spinor $\psi \in \mathbb{C}^2$ encodes the spin state. The Dirac equation (1928) [4] incorporates spin relativistically through the γ-matrices.

This formalism has been experimentally verified to extraordinary precision — the anomalous magnetic moment of the electron is predicted by QED to twelve decimal places [5]. Yet it does not explain why the fundamental representation of $SU(2)$ governs the physical world. The question "Why is the electron a spinor?" receives the answer: "Because experiment says so, and the formalism accommodates it." The formalism is a description, not an explanation. The distinction is the same one that separates FACT from ORIGIN in the META Physics methodology.

1.3 META Physics Framework

META Physics (Multi-dimensional Emergence Theory of Actuality) derives the ORIGIN of physical phenomena from the geometric structure of a uniformly rotating 4-dimensional hypersphere $B^4(R)$ and its boundary three-sphere $S^3(R)$. The 5-dimensional hyperball $B^5(R)$ serves as the mother body (모체) that drives the whole rotation of $B^4(R)$ and guarantees its stability as the equatorial cross-section; $B^5(R)$ is the driver, not the primary target of the theory (see § 2.2). The complete axiom system (15 axioms), the PEM (Projection-Emergence-Manifestation) ontology, and the mechanism chain are presented in [PP-00, Ref. 11]. This section provides a self-contained summary. For the full terminology, see Appendix D (Glossary).

The ontological foundation rests on two commitments. **Commitment I (Geometric Substance):** Geometric objects — points, disks, balls, hyperballs — are independently existing physical substances (PGS, Pure Geometric Substances). **Commitment II (Dimension as Property):** Dimension n is an intrinsic property that the substance $B^n(R)$ possesses, not a pre-existing container.

From these, META Physics derives a three-layer ontological structure called PEM:

$$\underbrace{B^4(R)}_{\text{PS — causal layer}} \xrightarrow[\text{ISM-T + DRAIN}]{\text{dim. reduction}} \underbrace{S^3(R)}_{\text{ES — transitional layer}} \xrightarrow[\Pi^a_{con}]{\text{ATI inversion}} \underbrace{\mathrm{WS}_3}_{\text{WS — observable world}} \tag{1.1}$$

Here **PS** (Projection System) is the causal layer where all physical phenomena originate; **ES** (Emergence System) is the transitional layer where topological emergence occurs; and **WS** (World System, WS_3) is the observable universe. The two transitions are qualitatively different: PS→ES is **dimensional reduction** (the 4D description is re-expressed on the 3D boundary via ISM-T and DRAIN), while ES→WS is **ATI (Anchored Topological Interior) anchoring inversion** — the perspective reversal that occurs when an observer is fully anchored at a point on $S^3(R)$.

The FACT/ORIGIN distinction is maintained throughout:

- **FACT:** Spin-1/2, the 720° return, and the $SU(2)$ structure are experimentally established. Standard quantum mechanics describes them with extraordinary precision.
- **ORIGIN:** Why the electron has spin-1/2, why 720° is required, and why $SU(2)$ governs nature. This paper identifies the ORIGIN in the $SU(2)$ topology of the Emergence System $S^3(R)$ and the polar-plane sweep mechanism operating within it.

PS, ES, and WS are not three separate entities. They are names assigned to the 4-dimensional hypersphere $B^4(R)$ and its boundary $S^3(R)$ from three complementary perspectives: **structure** (what exists — PS), **relation and mechanism** (how it operates — ES), and **function and phenomenon** (what is derived — WS).

1.4 Key Contributions

This paper makes five contributions.

Contribution 1 — Polar-Plane Sweep Theorem (§ 3). A 2D disk whose plane contains the rotation axis, when rotated by 2π within $B^3(R)$, sweeps out $S^2(R)$ exactly twice. This geometric identity yields the 720° return with zero free parameters.

Contribution 2 — Spin-Up/Down from Complex Combination (§ 3). The two polar planes D_{xz} and D_{yz} combine via the ES imaginary unit i to form spin eigenstates: $|\uparrow\rangle = D_{xz} + iD_{yz}$, $|\downarrow\rangle = D_{xz} - iD_{yz}$.

Contribution 3 — Integer Spin from Equatorial Invariance (§ 3). The equatorial plane D_{xy}, perpendicular to the rotation axis, is rotation-invariant. Its return period is 2π — the geometric ORIGIN of spin-1.

Contribution 4 — Detailed Correspondence with Standard Formalism (§ 5). The Dirac belt trick, the spinor representation of $SU(2)$, the fiber bundle $SU(2) \to SO(3)$, and the spin structure prerequisite are systematically mapped to the polar-plane sweep, establishing that META Physics reproduces and explains the standard mathematics.

Contribution 5 — Geometric ORIGIN of Quantum Superposition (§ 6). The "superposition" $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ is the weighted geometric co-presence of two real polar planes in the ES. "Wavefunction collapse" is the WS_3 observer's projection axis selection — a structural limitation (DA-3), not an ontological mystery.

1.5 Scope and Positioning

This paper derives the **spin quantum number** and the **720° return** from META Physics geometry. It does not derive g-factors, spin-orbit coupling, or the full Dirac equation structure. These are directions for subsequent work. The relation to standard quantum mechanics is one of **interpretive extension**: the formalism is shared; META Physics provides the geometric ORIGIN.

This paper, together with [PP-01, Ref. 12] (cosmic acceleration) and [PP-02, Ref. 13] (α ORIGIN and electron mass), demonstrates that META Physics provides geometric ORIGINs for three fundamental properties — dynamics (Λ_{eff}), mass ($m_e = \alpha^{-18}\hbar\bar{\Omega}/c^2$), and spin ($S^3 \cong SU(2)$) — from a single geometric structure: the uniformly rotating $B^4(R)$.

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§ 2. Theoretical Framework

This section presents the META Physics framework self-containedly. A reader familiar with [PP-00, Ref. 11] may proceed to § 3.

2.1 Geometric Substance and the PEM Three-Layer Ontology

META Physics posits that geometric objects are independently existing physical substances. The full set of Pure Geometric Substances (PGS):

$$\mathrm{PGS} := \{B^0,\ B^1(R),\ B^2(R),\ B^3(R),\ B^4(R),\ B^5(R)\} \tag{2.1}$$

where

$$B^n(R) := \{x \in \mathbb{R}^n : \|x\| \le R\}, \quad \partial B^n(R) = S^{n-1}(R) \tag{2.2}$$

The background space $E^n := \mathbb{R}^n$ is the mathematical arena; it is not itself a physical substance. All physical content resides in $B^n(R)$, not in E^n. The substances form a structure ladder via canonical inclusion:

$$B^0 \subset B^1(R) \subset B^2(R) \subset B^3(R) \subset B^4(R) \subset B^5(R) \tag{2.3}$$

The PEM (Projection-Emergence-Manifestation) three-layer ontology:

Layer	Symbol	Mathematical object	Role
Projection System	PS	$B^4(R)$ (uniform rotation)	Causal layer — ORIGIN of all phenomena
Emergence System	ES	$S^3(R) = \partial B^4(R)$	Transitional layer — topological emergence
World System	WS	WS_3 (observable universe)	Result layer — what humans observe

$$\mathrm{PS} \xrightarrow[\text{ISM-T + DRAIN}]{\text{Topological Emergence}} \mathrm{ES} \xrightarrow[\Pi^a_{con}]{\text{Topological Manifestation}} \mathrm{WS}_3 \tag{2.4}$$

Ontological note. PS, ES, and WS are not three independently existing entities. $B^4(R)$ and $B^5(R)$ are substances by definition (DA-1); $S^3(R) = \partial B^4(R)$ is the boundary of that substance. The three names PS, ES, WS are assigned to this single substance and its boundary from three complementary perspectives — structure (what exists), relation and mechanism (how it operates), and function and phenomenon (what is derived) — corresponding to the three-part cognitive framework of CANON § 0.5. The key interface is $\partial B^4(R) = S^3(R)$: the boundary between the causal layer and the observable universe.

2.2 Uniform Rotation and Concomitant Projection

$B^4(R)$ is the equatorial cross-section of $B^5(R)$:

$$B^4_{eq}(R) := B^5(R) \cap \{x_5 = 0\} \tag{2.5}$$

$B^5(R)$ undergoes a v-fixed isoclinic rotation (x_5 held invariant, isoclinic rotation in (x, y, z, w) at angular velocity Ω_5), producing a stable whole rotation of $B^4_{eq}(R)$. $B^5(R)$ serves exactly two roles: (I) driving the whole rotation of $B^4(R)$ via Ω_5, and (II) guaranteeing $B^4(R)$'s stability as the equatorial cross-section (RA-14). Beyond these two roles, $B^5(R)$ dynamics are not invoked. $B^5(R)$ is the driver (mother body) of $B^4(R)$'s whole rotation, not its subject. Simultaneously, $B^4(R)$ undergoes self-rotation in two independent planes with Ω_1, Ω_2:

$$(\Omega_1, \Omega_2) \;\longleftrightarrow\; \left(\bar{\Omega} = \frac{\Omega_1 + \Omega_2}{2},\;\; \delta\Omega = |\Omega_1 - \Omega_2|\right) \tag{2.6}$$

The concomitant projection $\Pi^a_{con} := P_a \circ \mathcal{F}_a$ reverses the centripetal acceleration ($-\omega^2 \to +\omega^2$), yielding $\Lambda_{eff} = 3\bar{\Omega}^2$ [PP-01, Ref. 12].

2.3 ISM-T, DRAIN, and the Zero-Dimensional Point

ISM-T (Internal State Mechanism–Tunnel): A dynamical mechanism operating along the radial line segment from the center O of $B^4(R)$ to a point a on $S^3(R)$. ISM-T is a mechanism, not a substance. Each ISM-T line has two endpoints called Ports: the inner port 0_i (center side, separable) and the surface port 0_s (boundary side, inseparable from a).

$$\gamma_a(\sigma) := \sigma \cdot a, \quad \sigma \in (0,1], \quad a \in S^3(R) \tag{2.7}$$

DRAIN (Dimensional Compression): As the ISM-T length converges to zero, the interior substance of $B^4(R)$ is compressed onto $S^3(R)$ via deformation retraction:

$$r_\tau(x) := \left((1-\tau) + \tau\frac{R}{\|x\|}\right)x, \quad \tau \in [0,1) \tag{2.8}$$

As $\tau \to 1^-$, all interior points converge to $S^3(R)$, creating topological density (stacking number) $N(a)$ at each surface point — the ORIGIN of mass [PP-02, Ref. 13].

2.4 The $S^3 \cong SU(2)$ Isomorphism

Every unit quaternion $q = w + xi + yj + zk$ with $|q| = 1$ corresponds to a point on $S^3(1)$. The group multiplication of unit quaternions is the group law of $SU(2)$:

$$SU(2) = \left\{ \begin{pmatrix} \alpha & -\bar{\beta} \\ \beta & \bar{\alpha} \end{pmatrix} : |\alpha|^2 + |\beta|^2 = 1 \right\} \cong S^3 \tag{2.9}$$

This isomorphism is not a convenient parametrization but an exact topological identification. The double cover homomorphism:

$$\phi : SU(2) \to SO(3), \quad \ker(\phi) = \{+I, -I\} \tag{2.10}$$

maps each $SU(2)$ element to a 3D rotation, with q and $-q$ mapping to the same $SO(3)$ rotation. This 2:1 structure is the algebraic encoding of the 720° return.

This isomorphism is not an assumption introduced to explain spin. It is a mathematical fact that follows from the definition of $B^4(R)$ (DA-1, DA-2) and its boundary structure. The spin-1/2 property of the electron is a **structural consequence** of the ES having $SU(2)$ topology.

2.5 Pre-3D Structure

Each point $a \in S^3(R)$ on the ES carries an internal structure delivered by ISM-T and DRAIN:

- The **pre-3D** space: three spatial axes (x, y, z) internal to the ISM-T tunnel, realized within the topological interior accessed through 0_s.
- The **fundamental energy**: $\epsilon_0 = \hbar\bar{\Omega}$ per point.
- The **stacking number**: $N(a)$ determining mass via $m(a) = N(a)\epsilon_0/c^2$.

The pre-3D structure is a 3-dimensional ball $B^3(R')$ internal to each 0-dimensional point. It is within this pre-3D structure that the polar-plane sweep mechanism of § 3 operates.

Critical distinction — why spin is mass-independent: Mass arises from the stacking number $N(a)$ — a local property of the specific point a. Spin arises from the topology of the space in which

a is embedded — namely $S^3(R) \cong SU(2)$, a global property. The two originate from structurally independent layers and are therefore independent. This resolves mystery (F3).

2.6 Axiom Summary

The axioms most directly relevant to this paper:

Code	Name	Content
DA-1	Substance Priority	$B^n(R)$ exists independently of any coordinate system
DA-2	Dimension Ordering	$\iota_n(B^n(R)) \subset B^{n+1}(R)$; $\partial B^n(R) = S^{n-1}(R)$
DA-3	Dimensional Irreversibility	No homeomorphism $B^n(R) \to B^{n-k}(R')$; the surplus is the ORIGIN
DA-4	Boundary Manifestation	Dynamics of $B^n(R)$ produces manifestation on $S^{n-1}(R)$
DA-5	Three-Layer Ontology	PS $\to$ ES $\to$ WS structure
RA-6	Uniform Rotation	$B^4(R)$ rotates uniformly ($\bar{\Omega} = \text{const}$)
RA-7	Hopf Fibration	$B^4(R)$ rotation induces $S^3 \xrightarrow{S^1} S^2$ on $S^3(R)$
RA-8	Quaternion Structure	$B^4(R)$ rotation admits quaternion representation; 720° return
RA-12	Quark Geometry	Quarks $\cong B^2(R')$; $\binom{4}{2} = 6$ disks in $B^4(R)$
RA-13	ISM-T	Radial segment structure transmits PS states to ES

The complete axiom system (15 axioms) is presented in [PP-00, Ref. 11].

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§ 3. The Polar-Plane Sweep Mechanism

This section contains the central geometric result of the paper. We work with the toy model $B^3(R)$ to make the mechanism maximally transparent. Extension to the physical $B^4(R)$ follows in § 4.

3.1 Setup: Three Orthogonal Disks

Consider the 3-dimensional ball $B^3(R)$ undergoing uniform rotation about the z-axis at angular velocity ω. The pre-3D structure contains three mutually orthogonal 2-dimensional disks:

$$D_{xz} := B^3(R) \cap \{y = 0\} = \{(x, 0, z) : x^2 + z^2 \leq R^2\} \tag{3.1}$$

$$D_{yz} := B^3(R) \cap \{x = 0\} = \{(0, y, z) : y^2 + z^2 \leq R^2\} \tag{3.2}$$

$$D_{xy} := B^3(R) \cap \{z = 0\} = \{(x, y, 0) : x^2 + y^2 \leq R^2\} \tag{3.3}$$

These disks are classified by their relationship to the rotation axis:

Disk	Contains z-axis?	Type	Relation to axis
D_{xz}	Yes	**Polar**	The plane includes the rotation axis
D_{yz}	Yes	**Polar**	The plane includes the rotation axis
D_{xy}	No	**Equatorial**	The plane is perpendicular to the axis

This classification is the key: the rotation axis divides the three disks into two qualitatively different categories, and each category produces a different spin type.

3.2 Polar-Plane Sweep Theorem

Theorem 3.1 (Polar-Plane Sweep).

Let D_{xz} undergo uniform rotation about the z-axis by angle $\theta \in [0, 2\pi]$. The swept locus

$$\mathcal{S}(\theta) := \bigcup_{\theta' \in [0,\theta]} R_z(\theta') \cdot D_{xz} \tag{3.4}$$

satisfies:

(a) At $\theta = \pi$: $\mathcal{S}(\pi) = B^3(R)$, and the boundary sweep $\partial\mathcal{S}(\pi) \cap S^2(R) = S^2(R)$ (the full sphere, covered once).

(b) At $\theta = 2\pi$: The boundary of D_{xz} has swept $S^2(R)$ exactly twice.

Proof.

The rotation matrix about the z-axis:

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.5}$$

A point $(x, 0, z)$ on D_{xz} with $x^2 + z^2 \leq R^2$ maps to:

$$R_z(\theta) \cdot (x, 0, z) = (x\cos\theta,\ x\sin\theta,\ z) \tag{3.6}$$

Consider the boundary circle of D_{xz}: points of the form $(\rho\cos\phi, 0, \rho\sin\phi)$ for $\rho = R$, $\phi \in [0, 2\pi)$. Under rotation by θ:

$$(R\cos\phi, 0, R\sin\phi) \xrightarrow{R_z(\theta)} (R\cos\phi\cos\theta,\ R\cos\phi\sin\theta,\ R\sin\phi) \tag{3.7}$$

Setting $u = \cos\phi\cos\theta$, $v = \cos\phi\sin\theta$, $w = \sin\phi$, we verify:

$$u^2 + v^2 + w^2 = \cos^2\phi(\cos^2\theta + \sin^2\theta) + \sin^2\phi = \cos^2\phi + \sin^2\phi = 1 \tag{3.8}$$

Hence every image point lies on the unit sphere $S^2(1)$ (or $S^2(R)$ after scaling).

Coverage of the first half-revolution ($\theta \in [0, \pi]$): For any target point $(u_0, v_0, w_0) \in S^2(1)$ with $v_0 \geq 0$, we can find ϕ and $\theta \in [0, \pi]$ such that equations (3.7) are satisfied: set $\phi = \arcsin(w_0)$ and $\theta = \arctan(v_0/u_0)$ (with appropriate quadrant handling). Since $\sin\theta \geq 0$ for $\theta \in [0, \pi]$, all points with $v \geq 0$ are reached when $\cos\phi \geq 0$, and all points with $v \leq 0$ are reached when $\cos\phi \leq 0$. Together, the entire sphere S^2 is covered once. ✓

Coverage of the second half-revolution ($\theta \in [\pi, 2\pi]$): By an identical argument with $\sin\theta$ taking all signs, the boundary circle sweeps S^2 a second time. ✓

Therefore:

$$\boxed{\text{One full revolution } (2\pi) \text{ of a polar disk sweeps out the entire } B^3(R)\text{; its boundary covers } S^2(R) \text{ exactly twice.}} \tag{3.9}$$

□

3.3 The 720° Return: Physical Intuition

Theorem 3.1 is a rigorous geometric identity. This subsection develops the physical intuition behind it through three complementary perspectives.

Perspective 1 — The semicircle argument. The boundary of D_{xz} is a circle in the (x, z)-plane. This circle consists of two semicircles: the right semicircle ($x \geq 0$, from the north pole through the equator to the south pole) and the left semicircle ($x \leq 0$, the return path). When D_{xz} rotates about the z-axis by π (half-revolution), the right semicircle sweeps out the entire sphere $S^2(R)$: each point of the semicircle traces a horizontal circle at its latitude, and the collection of all these circles at all latitudes covers the full sphere. At this moment, the left semicircle has also swept the entire sphere — it traces the same latitudinal circles but from the opposite side. Therefore, after a half-revolution, the boundary has covered S^2 once. The second half-revolution (π to 2π) repeats the entire process, covering S^2 a second time.

The key insight is that a semicircle — half of the disk boundary — already contains enough information to generate the full sphere. A full circle contains this information twice, hence the double coverage.

Perspective 2 — The orientation tracking argument. Consider a small arrow painted on the boundary of D_{xz} at the point $(R, 0, 0)$, pointing radially outward (in the $+x$ direction). As the disk rotates by π, the arrow moves to $(-R, 0, 0)$ and now points in the $-x$ direction. The point has returned to the same location on S^2 (it was at the equator, and after moving through the full sphere it is again at the equator) but the orientation is reversed. After another π rotation (total 2π), the arrow returns to $(R, 0, 0)$ pointing in the $+x$ direction — same location, same orientation. But the sphere has been covered twice. For the orientation of the generating disk (not just the arrow) to return, the disk must "know" that it has completed two full sphere-coverings. The orientational return requires 4π, because at 2π the arrow's orientation has returned but the sweep-count parity has not.

This is the intuitive content of the kernel $\ker(\phi) = \{+I, -I\}$: the rotation $R(2\pi) \in SO(3)$ is the identity, but the corresponding element in $SU(2)$ is $-I$, which represents the "wrong parity" of sphere-coverage. Only $R(4\pi) \to +I \in SU(2)$ restores both orientation and parity.

Perspective 3 — The topological winding argument. Consider the path traced by the point $(R, 0, 0)$ on D_{xz} as the disk rotates from 0 to 2π: it traces the equatorial great circle once. In $SO(3)$, this corresponds to a closed loop starting and ending at the identity rotation. The fundamental group $\pi_1(SO(3)) = \mathbb{Z}_2$ tells us that there are exactly two homotopy classes of loops: the contractible class (trivial) and the non-contractible class (non-trivial). A single 2π rotation is non-contractible — it generates two spheres, which cannot be continuously deformed to zero. A 4π rotation is contractible — four sphere-coverings form two complete cycles, topologically trivial. The 720° return is the statement that the generator of $\pi_1(SO(3))$ has order 2.

3.4 Equatorial Plane and Integer Spin

The equatorial disk D_{xy} is perpendicular to the z-axis. Under z-axis rotation:

$$R_z(\theta) \cdot (x, y, 0) = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta,\ 0) \tag{3.10}$$

The image remains in the $z = 0$ plane for all θ. The swept locus is:

$$\mathcal{S}_{xy}(\theta) = D_{xy} \quad \forall\,\theta \tag{3.11}$$

The equatorial disk maps to **itself** under any rotation angle. Its boundary circle $S^1(R) \subset D_{xy}$ sweeps itself:

$$\boxed{\text{One full revolution } (2\pi) \text{ of the equatorial disk returns to itself.} \implies \text{spin-1.}} \tag{3.12}$$

This is **360° return** — the defining property of **integer spin** (spin-1).

The physical contrast is sharp: the polar disk generates new structure (two spheres) under rotation, while the equatorial disk preserves existing structure (itself). This generation/preservation dichotomy is the geometric ORIGIN of the half-integer/integer spin distinction.

3.5 Spin-Up and Spin-Down: Complex Combination of Polar Planes

The two polar planes D_{xz} and D_{yz} are related by a $\pi/2$ rotation about the z-axis:

$$R_z(\pi/2) \cdot D_{xz} = D_{yz} \tag{3.13}$$

Under z-axis rotation by angle θ, a representative equatorial point on each disk traces:

$$\text{From } D_{xz}: \quad a(\theta) = (R\cos\theta,\ R\sin\theta,\ 0) \tag{3.14}$$

$$\text{From } D_{yz}: \quad b(\theta) = (-R\sin\theta,\ R\cos\theta,\ 0) = a(\theta + \pi/2) \tag{3.15}$$

The two trajectories are identical great circles with a $\pi/2$ **phase offset**. In the complex plane $\mathbb{C}$ identified with the (x, y)-plane:

$$a(\theta) = Re^{i\theta}, \qquad b(\theta) = Re^{i(\theta+\pi/2)} = iRe^{i\theta} \tag{3.16}$$

The angular momentum operator about the z-axis acting on functions of the azimuthal angle θ is:

$$L_z = -i\frac{\partial}{\partial\theta} \tag{3.17}$$

The projections of the two polar disks onto the (x, y)-plane correspond to $\cos\theta$ (from D_{xz}) and $\sin\theta$ (from D_{yz}). Their complex combinations:

$$e^{+i\theta} = \cos\theta + i\sin\theta \;\longleftrightarrow\; D_{xz} + iD_{yz} \tag{3.18}$$

$$e^{-i\theta} = \cos\theta - i\sin\theta \;\longleftrightarrow\; D_{xz} - iD_{yz} \tag{3.19}$$

are eigenstates of L_z:

$$L_z\, e^{+i\theta} = +e^{+i\theta}, \qquad L_z\, e^{-i\theta} = -e^{-i\theta} \tag{3.20}$$

For spin-1/2, the half-angle representation (spinor) applies. The $SU(2) \to SO(3)$ double cover maps a physical rotation by θ to a spinor rotation by $\theta/2$:

$$|\uparrow\rangle = e^{+i\theta/2} \;\longleftrightarrow\; D_{xz}|_{\theta/2} + iD_{yz}|_{\theta/2} \tag{3.21}$$

$$|\downarrow\rangle = e^{-i\theta/2} \;\longleftrightarrow\; D_{xz}|_{\theta/2} - iD_{yz}|_{\theta/2} \tag{3.22}$$

The half-angle $\theta/2$ is the consequence of the double cover: a 2π physical rotation corresponds to a π rotation in spinor space, generating the sign flip $e^{i\pi} = -1$.

$$\boxed{|\uparrow\rangle = D_{xz} + iD_{yz}, \qquad |\downarrow\rangle = D_{xz} - iD_{yz}} \tag{3.23}$$

The imaginary unit i in the spinor basis is the imaginary structure of the ES. In META Physics, the ES is $S^3(R) \cong SU(2)$, which possesses a natural complex structure as the unit sphere in $\mathbb{C}^2$. The i that converts two real polar planes into spin eigenstates is the same i that distinguishes the ES from WS_3. Spin-up and spin-down are not abstract quantum labels but **complex combinations of two real geometric structures in the ES**, made possible by the imaginary character of the ES itself.

3.6 Summary of the Toy Model

Structure	Axis relation	Swept locus per 2π	Return	Spin	WS_3 manifestation
D_{xz} (polar)	Contains axis	$S^2 \times 2$	4π	1/2	Spin-1/2 (real basis)
D_{yz} (polar)	Contains axis	$S^2 \times 2$	4π	1/2	Spin-1/2 (real basis)

Structure	Axis relation	Swept locus per 2π	Return	Spin	WS_3 manifestation
$D_{xz} + iD_{yz}$	Complex comb.	—	4π	1/2	**Spin-up** $\vert\uparrow\rangle$
$D_{xz} - iD_{yz}$	Complex comb.	—	4π	1/2	**Spin-down** $\vert\downarrow\rangle$
D_{xy} (equatorial)	Perpendicular	Self	2π	1	Integer spin (bosonic mode)

Three mysteries resolved:

(F1) No classical rotation: The electron does not rotate in WS_3. The rotation occurs in the ES (pre-3D internal structure), which is inaccessible from WS_3 by DA-3. WS_3 detects only the structural consequence — the angular momentum $\hbar/2$.

(F2) The 720° return: The polar-plane sweep generates S^2 twice per revolution. Orientational return requires 4π. This is the geometric content of $\pi_1(SO(3)) = \mathbb{Z}_2$.

(F3) Mass independence: Spin arises from the topology of $S^3(R) \cong SU(2)$ (global). Mass arises from the stacking number $N(a)$ (local). The two are structurally independent.

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§ 4. Extension to $B^4(R)$: Quaternion, Hopf, and the Six Planes

4.1 From Toy Model to Physical Substance

The toy model (B^3) demonstrates the mechanism; the physical substance is $B^4(R)$. The extension proceeds naturally because the pre-3D structure of each 0-dimensional point on $S^3(R)$ is itself a 3-dimensional ball, and the toy model of § 3 applies directly within this pre-3D structure.

4.2 Quaternion Structure and the 720° Return

The quaternion representation of $S^3(R)$ [RA-8]:

$$q = w + xi + yj + zk, \qquad |q|^2 = w^2 + x^2 + y^2 + z^2 = R^2 \tag{4.1}$$

The DRAIN completion condition projects to the pure imaginary quaternions:

$$q|_{w=0} = xi + yj + zk, \qquad x^2 + y^2 + z^2 = R^2 \tag{4.2}$$

This is $S^2(R) \subset \mathrm{Im}(\mathbb{H})$. The adjoint representation of $SU(2)$ acts on this S^2 as $SO(3)$ rotations:

$$\mathrm{Ad}(q) : \mathbf{v} \mapsto q\mathbf{v}q^{-1}, \quad \mathbf{v} \in \mathrm{Im}(\mathbb{H}) \tag{4.3}$$

The pre-3D axes (i, j, k) are the three quaternion imaginary units. A rotation about the k-axis by angle θ is generated by $q(\theta) = \cos(\theta/2) + k\sin(\theta/2)$, acting on i as:

$$q(\theta)\, i\, q(\theta)^{-1} = i\cos\theta + j\sin\theta \tag{4.4}$$

At $\theta = 2\pi$: $q(2\pi) = \cos\pi + k\sin\pi = -1$, and $(-1)i(-1)^{-1} = i$. The rotation returns to the identity in $SO(3)$, but $q = -1 \neq +1$ in $SU(2)$. Only at $\theta = 4\pi$ does $q(4\pi) = +1$.

$$\boxed{\text{The } 720° \text{ return is the statement } q(2\pi) = -1 \in SU(2).} \tag{4.5}$$

This is the same result as Theorem 3.1 expressed in quaternionic language: the polar-plane sweep (geometric) and the quaternion sign flip (algebraic) are two descriptions of the same fact — the $SU(2) \to SO(3)$ double cover.

4.3 Hopf Fibration: Bloch Sphere as Hopf Base

The Hopf fibration [RA-7]:

$$\pi : S^3(R) \to S^2(R/2), \quad \text{fiber} = S^1 \cong U(1) \tag{4.6}$$

Each point of $S^3(R)$ belongs to a great circle (S^1 fiber); the space of all such circles is S^2 (the base). The fiber S^1 represents the global phase freedom of the spinor: $|\psi\rangle \to e^{i\phi}|\psi\rangle$. The base S^2 is the **Bloch sphere** — the space of spin states modulo global phase, where each point represents a definite spin direction.

In META Physics, the fine-structure constant $\alpha = \omega_{\text{fiber}}/\omega_{S^2} = \delta\Omega/(4\bar{\Omega})$ [PP-02, Ref. 13] measures the ratio of fiber angular velocity to base angular velocity. The spin structure (determined by $S^3 \cong SU(2)$ topology) and the electromagnetic structure (determined by the $U(1)$ fiber) are unified within a single geometric object — the Hopf fibration of the ES.

4.4 The 4π Dual Origin

Two seemingly unrelated instances of 4π appear in WS_3 physics:

(i) Coulomb's 4π. From the Gauss-law surface integral $\oint \mathbf{E} \cdot d\mathbf{A} = Q/\epsilon_0$, the surface area of a sphere $4\pi r^2$ enters the denominator of Coulomb's law.

(ii) The fermion 4π (720°). A spin-1/2 particle requires 4π rotation for orientational return.

In the Standard Model, these two 4π factors appear unrelated: one is geometric (Gauss's theorem in 3D), the other is algebraic ($SU(2)$ spinor structure).

META Physics identifies them as two readings of the same geometric fact. A non-equatorial 2D disk within $B^4(R)$ — say the (z, x)-plane disk — under one complete rotation (2π), sweeps out a volume equivalent to two three-dimensional spheres. This is because the rotation carries the disk through the fourth dimension w, and the resulting trajectory generates two complete spherical surfaces. To return to the original orientation requires a second full rotation (4π total).

The three-dimensional space WS_3 is itself a manifestation of this sweep. The full solid angle of the generated space is 4π — the same 4π that appears in Coulomb's law (as the geometrical factor of the space in which charges interact) and in the fermion orientation return (as the angular period of the generating rotation). The two are united because WS_3 space and the fermion spin structure are both products of the same geometric mechanism.

This connection provides a structural bridge between [PP-03] (spin) and [PP-04] (electric charge), which derives charge quantization from the Hopf winding number.

4.5 Connection to Quark Classification and the Number 18

$B^4(R)$ possesses $\binom{4}{2} = 6$ independent 2-dimensional rotation planes [RA-12]:

$$(x, y),\ (y, z),\ (z, x),\ (x, w),\ (y, w),\ (z, w) \tag{4.7}$$

The 3 planes without the w-coordinate (x_4) form the **Up-quark category**; the 3 planes with w form the **Down-quark category**. Each plane is a category name, not an independent object.

For **lepton spin**, the relevant structure is the pre-3D internal structure of each 0-dimensional point on $S^3(R)$, to which the toy model of § 3 applies directly. For **quark structure**, the relevant structure is the 6 rotation planes of $B^4(R)$ itself, developed in [PP-05].

The number 18, which governs the electron mass via $N_e = \alpha^{-18}$ [PP-02, Ref. 13], arises from:

$$18 = \underbrace{\binom{4}{2}}_{\text{6 planes}} \times \underbrace{(4-1)}_{\text{3 pre-3D axes}} \tag{4.8}$$

Both factors — 6 and 3 — are forced by the single fact "$\dim(B^4(R)) = 4$." The spin structure (from the pre-3D axes) and the mass structure (from the 6 planes $\times$ 3 axes) share a common geometric root but originate from different aspects of the same object.

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§ 5. Correspondence with Standard Mathematical Formalism

This section establishes detailed correspondence between the polar-plane sweep mechanism and the standard mathematical structures of spin physics. The purpose is twofold: (i) to demonstrate that META Physics reproduces the established mathematics, and (ii) to identify precisely what META Physics adds — the geometric ORIGIN that the formalism alone does not provide.

5.1 The Dirac Belt Trick and the Feynman Plate Trick

The 720° return is often demonstrated through two classical physical analogies.

The Dirac belt trick. Attach one end of a belt to a book and fix the other end to a table. Rotate the book by 360° about any axis. The belt acquires a twist that cannot be removed without rotating the book further. Now rotate by an additional 360° (720° total). The double twist can be removed by passing the belt around the book without further rotation. The first twist (360°) is non-trivial in $\pi_1(SO(3)) = \mathbb{Z}_2$; the second twist (720° = two generators) is trivial.

The Feynman plate trick (Filipino candle dance). Hold a plate on an open palm and rotate it 360° about a vertical axis while keeping the palm up — this requires contorting the arm, and the arm cannot return to its original position. Continue rotating for another 360° (720° total), and the arm returns naturally. The arm traces a path in $SO(3)$; the 4π path is contractible while the 2π path is not.

META Physics identification. These demonstrations are WS_3 manifestations of the polar-plane sweep:

Physical demonstration	META Physics correspondence
Belt = 1D object connecting book to table	Boundary trajectory of polar disk ∂D_{xz} connecting two poles
Belt twist = non-contractible loop in $SO(3)$	S^2 generated once = non-trivial element of $\pi_1(SO(3))$
360° twist irremovable	One sphere-coverage is topologically non-trivial
720° twist removable	Two sphere-coverages form a trivial cycle
Plate = 2D object	D_{xz} polar disk
Arm = connecting structure to the fixed frame	ISM-T radial segment connecting center O to 0_s

The belt trick and plate trick are not merely analogies of spin — they are WS_3 projections of the same topological fact (Theorem 3.1) that produces spin-1/2 in the ES. The belt's connectivity to both the book and the table mirrors the ISM-T connecting the interior (PS) to the boundary (ES), and the twist's irremovability at 360° mirrors the non-contractibility of the single sphere-coverage loop.

5.2 Spinor Representation Theory of $SU(2)$

Standard formalism. The Lie algebra $\mathfrak{su}(2)$ has three generators satisfying $[J_a, J_b] = i\epsilon_{abc}J_c$. The irreducible representations are labeled by the spin quantum number $j = 0, 1/2, 1, 3/2, ...$, with dimension $2j+1$. For $j = 1/2$ (the fundamental representation), the generators are the Pauli matrices (up to a factor of 1/2):

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.1}$$

The spinor $\psi = \binom{\alpha}{\beta} \in \mathbb{C}^2$ transforms under rotation by angle θ about axis $\hat{n}$ as:

$$\psi \to \exp\left(-i\frac{\theta}{2}\hat{n} \cdot \sigma\right)\psi \tag{5.2}$$

The half-angle $\theta/2$ ensures that $\theta = 2\pi$ produces $\psi \to -\psi$ (sign flip) and $\theta = 4\pi$ produces $\psi \to +\psi$ (return).

For $j = 1$ (the adjoint representation), the generators are the 3×3 matrices $(J_a)_{bc} = -i\epsilon_{abc}$, and rotation by 2π returns to the identity — no sign flip.

META Physics correspondence. The Pauli matrices and the pre-3D polar/equatorial disks admit a precise structural mapping:

Pauli matrix	Pre-3D structure	Role
σ_z	D_{xy} (equatorial disk)	Diagonal: eigenstates are $\lvert\uparrow\rangle, \lvert\downarrow\rangle$ with eigenvalues ± 1
σ_x	D_{xz} (polar disk)	Off-diagonal real: mixes $\lvert\uparrow\rangle$ and $\lvert\downarrow\rangle$ via x-axis
σ_y	D_{yz} (polar disk)	Off-diagonal imaginary: mixes via y-axis with ES imaginary unit i

The structure of the Pauli algebra — one diagonal matrix and two off-diagonal matrices (one real, one imaginary) — mirrors the structure of the three orthogonal disks: one equatorial (rotation-invariant, hence diagonal in the rotation basis) and two polar (rotation-mixing, hence off-diagonal). The factor i in σ_y is the same ES imaginary unit that combines D_{xz} and D_{yz} into spin eigenstates (eq. 3.23).

The fundamental representation ($j = 1/2$, dimension 2) has dimension 2 because there are exactly **two** polar planes. The adjoint representation ($j = 1$, dimension 3) has dimension 3 because the adjoint action of $SU(2)$ on its Lie algebra $\mathfrak{su}(2) \cong \mathbb{R}^3$ rotates the three generators — corresponding to the rotation of the three pre-3D axes. The representation theory does not explain why these dimensions are selected by nature; META Physics identifies the geometric reason: two polar planes produce spin-1/2, three total disks produce the adjoint action, and the equatorial plane produces the trivial spin-1 return.

5.3 Fiber Bundle Perspective: The $SU(2)$ Principal Bundle over $SO(3)$

Standard formalism. The double cover $\phi : SU(2) \to SO(3)$ defines a principal $\mathbb{Z}_2$-bundle:

$$\mathbb{Z}_2 \hookrightarrow SU(2) \xrightarrow{\phi} SO(3) \tag{5.3}$$

A spin-1/2 particle is a section of the associated vector bundle $SU(2) \times_{\mathbb{Z}_2} \mathbb{C}^2$. For this bundle to exist on a manifold M, the manifold must admit a **spin structure** — a lift of the frame bundle $SO(3) \to M$ to an $SU(2)$ bundle. The obstruction to the existence of a spin structure is the second Stiefel-Whitney class $w_2(M) \in H^2(M; \mathbb{Z}_2)$: the spin structure exists if and only if $w_2(M) = 0$.

In standard physics, the existence of fermions (spin-1/2 particles) in spacetime presupposes that spacetime admits a spin structure. This is a non-trivial topological condition on spacetime, typically assumed without geometric justification.

META Physics resolution. In META Physics, the ES is $S^3(R) \cong SU(2)$. The second Stiefel-Whitney class of S^3 vanishes: $w_2(S^3) = 0$ (since S^3 is 2-connected: $\pi_1(S^3) = 0$, $\pi_2(S^3) = 0$). Therefore, S^3 automatically admits a spin structure. In fact, S^3 is itself a Lie group ($SU(2)$), and every Lie group is parallelizable and hence spin.

The upshot is decisive: **the spin structure on the ES is not an additional assumption but an automatic consequence of the ES topology.** Standard physics requires a spin structure as a prerequisite for fermions to exist; META Physics derives the spin structure from the definition of the ES ($S^3 = \partial B^4$, DA-2). The existence of fermions is not a postulate about spacetime but a topological consequence of the fact that the observable universe emerges from a 4-dimensional geometric substance.

More explicitly:

Standard physics	META Physics
Spacetime must admit spin structure (assumed)	ES $= S^3 \cong SU(2)$ automatically spin
$w_2(M) = 0$ required (topological condition)	$w_2(S^3) = 0$ guaranteed (S^3 is 2-connected)
Fermions presuppose spin structure	Fermions are consequences of ES topology
Frame bundle $SO(3) \to M$ needs $SU(2)$ lift	$SU(2)$ is the ES — no lift needed

5.4 The Kaluza-Klein Comparison

Kaluza-Klein theories introduce extra dimensions, typically compactified on circles or Calabi-Yau manifolds, to unify gauge interactions with gravity. The gauge group arises from the isometry group of the compact space.

META Physics introduces extra dimensions as filled balls (substances), not compactified manifolds. The key differences:

Kaluza-Klein	META Physics
Extra dimensions are compactified (small, curled)	Extra dimensions are substance (filled, finite radius R)
Gauge group from isometry of compact space	$SU(2)$ from topology of $S^3 = \partial B^4$
Spin introduced separately via Clifford algebra	Spin derived from $S^3 \cong SU(2)$ topology
Extra dimensions hidden by smallness	Extra dimensions hidden by PEM transition structure
Multiple models (S^1, $S^1 \times S^2$, Calabi-Yau, ...)	Single model ($B^5(R)$ with equatorial $B^4(R)$)

5.5 What META Physics Adds, What It Does Not Replace

Standard formalism	Status in META Physics
$SU(2)$ representation theory	**Reproduced** — arises from $S^3 \cong SU(2)$
Spinor $\psi \in \mathbb{C}^2$	**Reproduced** — complex combination of polar planes
720° return: $R(2\pi) = -1$	**Derived** — polar-plane sweep (Theorem 3.1)
Spin quantum number $s = 1/2$	**Derived** — double cover $SU(2) \to SO(3)$
Pauli matrices $\sigma_x, \sigma_y, \sigma_z$	**Mapped** — to three orthogonal disks (§ 5.2)
Spin structure prerequisite	**Derived** — $w_2(S^3) = 0$ automatic (§ 5.3)
Mass independence of spin	**Explained** — different ORIGIN layers (§ 2.5)
g-factor $g_e \approx 2.002$	**Not yet derived** — open question
Spin-orbit coupling	**Not yet derived** — direction: Hopf fiber $\leftrightarrow$ orbital
Full Dirac equation	**Not yet reconstructed** — framework compatibility indicated

META Physics does not replace the Pauli matrices, the Dirac equation, or QED. These retain their validity as WS_3 effective descriptions. What META Physics adds is the geometric ORIGIN: why $SU(2)$ governs spin, why spinors are 2-component, why the 720° return occurs, and why spacetime admits fermions at all.

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§ 6. The Geometric ORIGIN of Quantum Superposition

This section presents a result with implications far beyond spin: the geometric resolution of the “superposition mystery” — the most debated ontological question in quantum mechanics for nearly a century.

6.1 The Standard Formulation and Its Conceptual Difficulty

In WS_3 quantum mechanics, the general spin state of an electron is:

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle, \qquad |\alpha|^2 + |\beta|^2 = 1 \tag{6.1}$$

This is called a "superposition" of spin-up and spin-down. The standard interpretation states that the electron is "simultaneously in both states." This interpretation has generated a century of philosophical contention — the measurement problem, the Copenhagen interpretation, the many-worlds interpretation, decoherence programs — all attempts to make sense of a single word: superposition.

The confusion is compounded by the fact that "superposition" is borrowed from **wave physics**, where it describes the addition of amplitudes. In classical wave theory, superposition is unproblematic. The mystery arises when this wave-mechanical term is elevated to an **ontological statement about the state of a particle**.

6.2 META Physics Resolution: Geometric Co-Presence

Substituting the geometric identifications from equations (3.23):

$$|\psi\rangle = \alpha(D_{xz} + iD_{yz}) + \beta(D_{xz} - iD_{yz}) = \underbrace{(\alpha+\beta)}_{\text{weight}} D_{xz} + \underbrace{i(\alpha-\beta)}_{\text{weight}} D_{yz} \tag{6.2}$$

In the ES (Emergence System), D_{xz} and D_{yz} are **two real geometric structures that always coexist simultaneously.** They are the two polar planes of the pre-3D ball — permanent structural features, not ephemeral states. There is nothing mysterious about two orthogonal planes existing within a 3-dimensional ball. Their coexistence is not "superposition" but **geometric co-presence**.

The complex coefficients $(\alpha+\beta)$ and $i(\alpha-\beta)$ describe how the WS_3 observer's chosen measurement axis **weights** the two polar planes. Different measurement axis choices yield different weightings — this is the geometric content of "choosing a measurement basis."

6.3 Reinterpretation of Wavefunction Collapse

When a WS_3 observer performs a spin measurement along the z-axis, the outcome is either $|\uparrow\rangle$ or $|\downarrow\rangle$ with probabilities $|\alpha|^2$ and $|\beta|^2$. In the standard interpretation, the state "collapses" from the superposition (6.1) to a definite eigenstate.

In META Physics, nothing collapses in the ES. The two polar planes D_{xz} and D_{yz} remain exactly as they were before, during, and after the measurement. What changes is the WS_3 observer's **readout**: the measurement axis selection determines which complex combination of the two planes is registered as the outcome. The ES structure is unchanged; only the WS_3 projection changes.

This is a direct consequence of **DA-3 (Dimensional Irreversibility Axiom)**: the WS_3 observer cannot access the full ES structure. Each measurement reads only one projection axis, and therefore detects only one eigenstate. The "mystery of collapse" is the mystery of **why a lower-dimensional observer cannot simultaneously read all projections of a higher-dimensional structure** — which is not a mystery at all, but a structural limitation.

6.4 DA-3 as the Structural Explanation

DA-3 states: phenomena occurring in n dimensions cannot be fully reduced to $(n-k)$ dimensions. The irreducible surplus is precisely the ORIGIN of physical phenomena.

For quantum superposition, the structural chain is:

1. The ES (4-dimensional boundary S^3) contains two polar planes that coexist as permanent geometric structures.
2. The WS_3 observer (3-dimensional) cannot simultaneously access both complex combinations of these planes.
3. Each measurement selects one projection axis, yielding one eigenstate.
4. The probabilistic nature of outcomes reflects the observer's incomplete access to the ES structure, not an ontological indeterminacy in the ES itself.

The Born rule $P = |\alpha|^2$ is not derived here; it is taken as a FACT about measurement statistics. What META Physics provides is the ORIGIN of the structure that the Born rule quantifies: the geometric co-presence of two real planes, projected onto a single measurement axis by a dimensionally limited observer.

6.5 Implications Beyond Spin

The resolution of § 6.2–§ 6.4 is not specific to spin. It applies to any quantum system whose Hilbert space arises from the complex combination of real geometric structures in the ES:

- **Photon polarization:** Two orthogonal polarization states $|H\rangle$, $|V\rangle$ correspond to two orthogonal oscillation planes of the F-perturbation on $S^3(R)$ [PP-07 scope].
- **Qubit states:** Any two-level quantum system can be mapped to the Bloch sphere S^2, which is the base of the Hopf fibration of S^3. The "superposition" of qubit states is the WS_3 projection of the Hopf fiber structure.

The general principle:

$$\boxed{\text{Quantum superposition} = \mathrm{WS}_3 \text{ projection of ES geometric co-presence (DA-3 limitation)}} \tag{6.3}$$

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§ 7. Discussion

7.1 Established Results

This paper establishes five core results:

Result I — Polar-Plane Sweep Theorem (Theorem 3.1). Geometrically rigorous: a 2D polar disk rotated by 2π within $B^3(R)$ sweeps $S^2(R)$ exactly twice. No physical assumptions required.

Result II — 720° Return. Structural consequence of Result I: 4π orientational return, reproducing $\pi_1(SO(3)) = \mathbb{Z}_2$ and the spinor sign flip $R(2\pi) = -1$.

Result III — Spin-Up/Down from Complex Combination. The two polar planes combine via the ES imaginary unit to form spin eigenstates. The $\pi/2$ phase relation is the geometric origin of the two-dimensional spinor space $\mathbb{C}^2$.

Result IV — Integer Spin from Equatorial Invariance. The equatorial plane's rotation-invariance gives 2π return and spin-1.

Result V — Geometric ORIGIN of Quantum Superposition. The "superposition" of spin states is the WS_3 observer's limited readout of two coexisting real polar planes in the ES. The "collapse" is projection axis selection, not ontological change.

7.2 Why Spin Is Mass-Independent

In META Physics, spin and mass originate from structurally independent sources:

- **Spin:** Topology of $S^3(R) \cong SU(2)$. This is a global property of the ES, independent of the state of any particular 0-dimensional point.
- **Mass:** Stacking number $N(a) = R/\bar{\lambda}_e \approx \alpha^{-18}$ for the electron [PP-02]. This is a local property of the specific point a.

The electron, muon, and tau all reside on the same $S^3(R)$. They differ only in $N(a)$ (hence mass), not in their relationship to the $SU(2)$ topology (hence identical spin $s = 1/2$). This resolves mystery (F3) without additional assumptions.

7.3 Open Questions and Limitations

Question	Status
Derivation of $g_e \approx 2.002$ from META Physics	Open — requires ES-level electromagnetic coupling
Spin-orbit coupling from pre-3D structure	Open — direction: Hopf fiber $\leftrightarrow$ orbital angular momentum
Spin-statistics theorem from PEM structure	Open — direction: antisymmetry from $SU(2)$ double cover
Muon and tau spin	**Resolved** — same $S^3 \cong SU(2)$, different $N(a)$
Quark spin-1/2	Structurally expected — pre-3D sweep applies to quarks
Born rule derivation	Not attempted — $P = \lvert\alpha\rvert^2$ taken as FACT
Decoherence mechanism	Not addressed — direction: coupling between ES points

7.4 Testability and Future Directions

The spin derivation in this paper is primarily structural: it explains why spin-1/2 exists rather than predicting a new numerical value. However, it generates testable structural claims:

(i) Universality of spin-1/2 for fermions. The derivation predicts that all fermions (leptons and quarks) have spin-1/2 because all reside on $S^3(R) \cong SU(2)$. Any discovery of a fundamental fermion with spin other than 1/2 would falsify this.

(ii) Absence of fundamental higher-spin fermions. META Physics predicts that no fundamental spin-3/2 particle exists as a PGS-derived entity. The gravitino (spin-3/2) of supersymmetric models, if discovered, would require extension of the framework.

(iii) Structural prediction for PP-04. The identification of the $U(1)$ fiber of the Hopf fibration with electromagnetic gauge symmetry, combined with the polar-plane sweep mechanism, predicts that the electric charge quantum is topologically protected (winding number $n \in \pi_1(S^1) = \mathbb{Z}$). This is the subject of [PP-04].

7.5 Outlook

This paper, combined with [PP-01] and [PP-02], establishes three legs of the META Physics programme:

Paper	Result	Domain
PP-01	$\Lambda_{eff} = 3\bar{\Omega}^2$	Cosmology
PP-02	$\alpha = \delta\Omega/(4\bar{\Omega})$, $m_e = \alpha^{-18}\hbar\bar{\Omega}/c^2$	α + Mass
PP-03	**Spin-1/2 from $S^3 \cong SU(2)$, quantum superposition ORIGIN**	**Spin + Foundations**

The next targets: (a) electric charge from Hopf winding number [PP-04]; (b) quark/baryon sector from the six-disk structure [PP-05]; (c) neutrino mass from Ω_5 whole rotation [PP-06].

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§ 8. Conclusion

This paper has derived the spin-1/2 property of the electron from the geometry of META Physics, with zero free parameters.

Core Result I — The Polar-Plane Sweep Theorem. A 2-dimensional disk containing the rotation axis, rotated by 2π within a 3-dimensional ball, sweeps out the entire ball; its boundary covers the 2-sphere exactly twice:

$$\boxed{2\pi \text{ rotation of polar disk} = S^2 \times 2 \implies 4\pi \text{ for orientational return}} \tag{8.1}$$

Core Result II — Spin Eigenstates from ES Imaginary Structure. The two polar planes, offset by $\pi/2$ in phase, combine via the ES imaginary unit to form spin-up and spin-down:

$$\boxed{|\uparrow\rangle = D_{xz} + iD_{yz}, \qquad |\downarrow\rangle = D_{xz} - iD_{yz}} \tag{8.2}$$

Core Result III — Integer Spin from Equatorial Invariance. The equatorial plane, invariant under rotation, returns in 2π:

$$\boxed{D_{xy} \xrightarrow{R_z(\theta)} D_{xy}\ \forall\theta \implies 2\pi \text{ return} \implies \text{spin-1}} \tag{8.3}$$

Core Result IV — Mass Independence. Spin originates from $S^3(R) \cong SU(2)$ topology (global). Mass originates from stacking number $N(a)$ (local). The two are structurally independent:

$$\boxed{s = 1/2 \;\leftarrow\; S^3 \cong SU(2), \qquad m = N\epsilon_0/c^2 \;\leftarrow\; N(a)} \tag{8.4}$$

Core Result V — Geometric ORIGIN of Quantum Superposition. The "superposition" of spin states is the weighted geometric co-presence of two real polar planes in the ES. "Collapse" is the WS_3 observer's readout of one projection axis — the ES structure is unchanged:

$$\boxed{\alpha|\uparrow\rangle + \beta|\downarrow\rangle = (\alpha+\beta)D_{xz} + i(\alpha-\beta)D_{yz}} \tag{8.5}$$

These results collectively support the following proposition:

> **Electron spin is not an independently postulated quantum number. It is the Topological Manifestation of the fact that the Emergence System $S^3(R)$ has $SU(2)$ topology. The 720° return, the spin-1/2 value, the spin-up/down doublet, and integer spin are all geometric consequences of the polar-plane sweep mechanism operating within the pre-3D structure of each 0-dimensional point on $S^3(R)$. Furthermore, quantum superposition — the century-old ontological puzzle — is the WS_3 observer's limited readout of the geometric co-presence of two real polar planes in the ES, constrained by the Dimensional Irreversibility Axiom (DA-3). The standard mathematical formalism ($SU(2)$ representations, Pauli matrices, spinors, fiber bundles) is fully reproduced and given geometric ORIGIN. No free parameters, no additional axioms, and no classical rotation are required.**

META Physics does not replace quantum mechanics. The $SU(2)$ formalism, the Dirac equation, and QED retain their validity as WS_3 effective descriptions. META Physics provides the geometric ORIGIN: why $SU(2)$ governs spin, why spinors are 2-component, why 720° is required, and why spacetime admits fermions.

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Acknowledgments

The author thanks Claude (Anthropic, Claude Opus 4.6) for sustained mathematical formalization and documentation across eleven research sessions. The axiom system, physical insights, and interpretive framework are the author's own; Claude's contribution was in translating these into formal mathematical language, verifying derivations, and structuring documents.

The author has no institutional affiliation and receives no external funding. This work is submitted as an independent research contribution.

Appendix A. Rotation Matrices and Swept Locus Verification

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z-axis rotation matrix:

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A.1}$$

Swept locus of D_{xz} boundary point $(R\cos\phi, 0, R\sin\phi)$:

$$R_z(\theta) \cdot (R\cos\phi, 0, R\sin\phi) = (R\cos\phi\cos\theta,\ R\cos\phi\sin\theta,\ R\sin\phi) \tag{A.2}$$

Spherical coordinate identification: Setting $\vartheta = \pi/2 - \phi$ (colatitude) and $\varphi = \theta$ (azimuth):

$$x = R\sin\vartheta\cos\varphi, \quad y = R\sin\vartheta\sin\varphi, \quad z = R\cos\vartheta \tag{A.3}$$

As $\vartheta \in [0, \pi]$ and $\varphi \in [0, \pi]$, all points of $S^2(R)$ are reached (first half-revolution). The second half-revolution ($\varphi \in [\pi, 2\pi]$) covers $S^2(R)$ again.

Quaternion rotation verification: For $q(\theta) = \cos(\theta/2) + k\sin(\theta/2)$:

$$q(\theta) \cdot i \cdot q(\theta)^{-1} = (\cos^2(\theta/2) - \sin^2(\theta/2))i + 2\sin(\theta/2)\cos(\theta/2)j = i\cos\theta + j\sin\theta \tag{A.4}$$

At $\theta = 2\pi$: $q = \cos\pi + k\sin\pi = -1 + 0 = -1$. At $\theta = 4\pi$: $q = \cos(2\pi) + k\sin(2\pi) = +1$. $\checkmark$

Appendix B. Python Verification Code

```
import numpy as np

# ================================================================
# Polar-Plane Sweep Verification
# Demonstrates that D_xz boundary sweeps S^2 exactly twice per 2π
# ================================================================

N_phi = 500    # boundary circle sampling
N_theta = 500  # rotation angle sampling
R = 1.0

# Boundary of D_xz: (R cos φ, 0, R sin φ), φ ∈ [0, 2π)
phi = np.linspace(0, 2*np.pi, N_phi, endpoint=False)

# First half-revolution: θ ∈ [0, π]
theta_half = np.linspace(0, np.pi, N_theta)
points_half = []
for th in theta_half:
    x = R * np.cos(phi) * np.cos(th)
    y = R * np.cos(phi) * np.sin(th)
    z = R * np.sin(phi)
    points_half.extend(zip(x, y, z))

points_half = np.array(points_half)
# Verify all points lie on S^2
norms = np.sqrt(np.sum(points_half**2, axis=1))
print(f"Half-revolution: {len(points_half)} points")
print(f"  All on S^2: {np.allclose(norms, R)}")

# Check coverage: discretize S^2 into bins and count
from collections import Counter
def to_bin(x, y, z, nbins=20):
    """Convert (x,y,z) on S^2 to angular bin."""
    theta = np.arctan2(y, x)  # azimuth [-π, π]
    phi = np.arcsin(np.clip(z/R, -1, 1))  # latitude [-π/2, π/2]
    ti = int((theta + np.pi) / (2*np.pi) * nbins) % nbins
    pi = int((phi + np.pi/2) / np.pi * nbins) % nbins
    return (ti, pi)

nbins = 20
```

```python
bins_half = Counter()
for p in points_half:
    bins_half[to_bin(*p, nbins)] += 1

total_bins = nbins * nbins
covered_bins = len(bins_half)
print(f"  Bins covered: {covered_bins}/{total_bins} = {covered_bins/total_bins*100:.1f}%")

# Full revolution: θ ∈ [0, 2π]
theta_full = np.linspace(0, 2*np.pi, 2*N_theta)
bins_full = Counter()
coverage_count = np.zeros(total_bins)
for th in theta_full:
    x = R * np.cos(phi) * np.cos(th)
    y = R * np.cos(phi) * np.sin(th)
    z = R * np.sin(phi)
    for xi, yi, zi in zip(x, y, z):
        bins_full[to_bin(xi, yi, zi, nbins)] += 1

print(f"\nFull revolution: bins covered = {len(bins_full)}/{total_bins}")
# Average hits per bin should be ~2× the half-revolution value
avg_half = np.mean(list(bins_half.values()))
avg_full = np.mean(list(bins_full.values()))
print(f"  Avg hits/bin (half): {avg_half:.1f}")
print(f"  Avg hits/bin (full): {avg_full:.1f}")
print(f"  Ratio full/half: {avg_full/avg_half:.2f} (expected: 2.0)")

# ================================================================
# Quaternion 720° verification
# ================================================================
def quat_mult(q1, q2):
    """Multiply two quaternions (w, x, y, z)."""
    w1, x1, y1, z1 = q1
    w2, x2, y2, z2 = q2
    return (w1*w2 - x1*x2 - y1*y2 - z1*z2,
            w1*x2 + x1*w2 + y1*z2 - z1*y2,
            w1*y2 - x1*z2 + y1*w2 + z1*x2,
            w1*z2 + x1*y2 - y1*x2 + z1*w2)

theta_360 = 2 * np.pi
theta_720 = 4 * np.pi

q_360 = (np.cos(theta_360/2), 0, 0, np.sin(theta_360/2))  # rotation about k
q_720 = (np.cos(theta_720/2), 0, 0, np.sin(theta_720/2))

print(f"\nQuaternion verification:")
print(f"  q(2π) = ({q_360[0]:.4f}, {q_360[1]:.4f}, {q_360[2]:.4f}, {q_360[3]:.4f})")
print(f"  q(4π) = ({q_720[0]:.4f}, {q_720[1]:.4f}, {q_720[2]:.4f}, {q_720[3]:.4f})")
print(f"  q(2π) = -1: {np.isclose(q_360[0], -1) and np.isclose(sum(q_360[1:]**2 for q in [q_360]), 0)}")
print(f"  q(4π) = +1: {np.isclose(q_720[0], +1)}")
```

Appendix D. Glossary of META Physics Terminology

This appendix provides brief explanations of the META Physics terms used in this paper, for readers encountering the framework for the first time. For the complete exposition, see [PP-00, Ref. 11].

PGS (Pure Geometric Substance). The set of primitive physical entities in META Physics: $\{B^0, B^1(R), B^2(R), B^3(R)$ These are "filled" geometric objects (balls, not spheres) that exist as physical substances — not containers, not abstract coordinates, but material entities whose "material" is space itself.

PEM (Projection-Emergence-Manifestation). The three-layer ontological structure of META Physics. PS (Projection System, 원인계) designates $B^4(R)$ as the causal layer where all phenomena originate. ES (Emergence System, 전이계) designates $S^3(R) = \partial B^4(R)$ as the transitional layer where topological emergence occurs. WS (World System, 결과계; WS_3) designates the observable universe. The three names are assigned to the hypersphere $B^4(R)$ and its boundary from three complementary perspectives: structure (PS), relation/mechanism (ES), and function/phenomenon (WS). $B^5(R)$ is the mother body that drives $B^4(R)$'s whole rotation.

ATI (Anchored Topological Interior). When an observer is fully anchored at a point a on $S^3(R)$, the entire $B^4(R)$ becomes that observer's topological interior through a perspective reversal. Fundamentally different from the standard topological interior $\mathrm{int}(A)$.

ISM-T (Internal State Mechanism–Tunnel). The dynamical mechanism operating along the radial line segment from the center O of $B^4(R)$ to a surface point $a \in S^3(R)$. Transmits the internal rotational state from PS to ES. Has two ports: 0_i (inner, separable) and 0_s (surface, inseparable from a). ISM-T is a mechanism, not a substance.

DRAIN (Dimensional Compression). The process by which the 4-dimensional interior of $B^4(R)$ is compressed onto $S^3(R)$ via deformation retraction. Creates topological density (stacking number) at each surface point — the ORIGIN of mass.

IMM (Internal Manifestation Mechanism). The mechanism mediating the ES→ WS_3 transition. Core structure: quaternionic $w \cdot v$ decomposition, where the real part w encodes the dimensional reduction progress and the imaginary part (x, y, z) provides WS_3 spatial dimensions.

Topological Emergence (반영, banyeong). The PS→ES transition: intrinsic features of higher-dimensional rotation emerge as physical states on the lower-dimensional boundary. English: exclusively Emergence. Reflection is forbidden.

Topological Manifestation (발현, balhyeon). The ES→WS transition: the Anchored Topological Interior becomes realized as observable physics. English: exclusively Manifestation.

Stacking Number (적층도, jeokchungdo). The topological density $N(a)$ at a point $a \in S^3(R)$: the number of zero-dimensional states coalesced through DRAIN convergence. Determines mass: $m(a) = N(a) \cdot \hbar\bar{\Omega}/c^2$.

Pre-3D. The three spatial axes (x, y, z) internal to the ISM-T tunnel, realized within the topological interior accessed through the surface port 0_s. The pre-3D structure is a 3-dimensional ball within each 0-dimensional point on $S^3(R)$. The polar-plane sweep mechanism of this paper operates within this pre-3D structure.

> **META Physics Preprint PP-03 — v1.1 Document Code: MP-PP-03-SPIN-EN | Status: PREPRINT | Date: 2026-03-25 Author: Cheong-Gwan Lee | ORCID: 0009-0006-5710-3184 Canon Basis: MP-CANON-v2.2 | License: CC BY 4.0 Series: PP-03 in the META Physics preprint series DOI: https://doi.org/10.5281/zenodo.19343831**