On Stereoscopy with a Long Base Line

1906 Proc. Phys. Soc. London 20 607

(http://iopscience.iop.org/1478-7814/20/1/341)

XL. *On Stereoscopy with a Long Base Line.* *By* T. C. PORTER, *M.A., D.Sc., F.R.P.S., F. Phys. Soc. of London, &c.**

THE original communication of the suggestions and extensions of stereoscopic work given in the present paper was made to the Royal Photographic Society on January 29th of the present year; but the lecturer had no opportunity of showing his then audience the results he had obtained, save through a number of small stereoscopes kindly lent to the Society for the occasion. He therefore asked a few members of some of the learned societies to come and see the illustrations for themselves in the author's own laboratory at Eton. A few came, and amongst them the Hon. Sec. of the Physical Society of London, who urged the exhibitor to show the same things to the Physical Society in London, and promised that he would be responsible for the rather considerable preparations involved; moreover, the Council were kind enough to place practically the whole of this evening at the lecturer's disposal. Hence the present paper, which is very largely the same as that read to the Photographic Society; and the illustrations, which contain many shown at that time, and also in the lecture room at Eton. The author would at the very outset express his warm thanks to M. Selb of Brussels, for the magnificent mountain photographs, taken with a long base-line as long ago as 1903, a sufficient proof that the present lecturer is not the first to whom the idea of this application of tele-stereoscopy occurred, though apparently M. Selb did not think of publishing any account of his photographs. So far as the other applications are concerned:—for military and meteorological purposes, the suggestions seem to be really new, though they are of so obvious a character, that it is the more surprising if no one has tried them before †. One more word of preface: the

* Read May 10, 1907.

† Since reading this paper, the author has learnt that Mr. John Tennant took several very successful photographs of distant buildings and of clouds, using a long base-line, some years ago. Prints from the negatives were published in the 'Photogram' at the time.

author has deemed it well to give a brief sketch of the principles of stereoscopy, and, so far as he knows, the attempt to measure the limit of stereoscopic vision for the unassisted eye is new, as well as some other minor points, which seem scarcely worth separate mention.

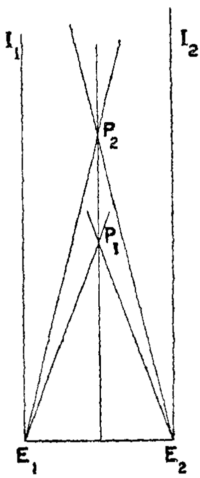
The power we possess of judging of the difference in distance, of two objects in our field of view, depends first, upon various properties of those *objects* which vary with the said distance ; variations to appreciate which we require strictly but one eye, though it may well be that we appreciate them better with two. One of these properties is *the apparent size.* This is one of the most useful properties, if not altogether the most important. The *clearness or haziness of an object* in an atmosphere which is not perfectly transparent is another property of almost universal application—at any rate in our land. A third property, which depends upon the bodies themselves and also on the source of light by which they are illuminated, is *the arrangement of light and shade* they present, and *the shadows they cast.* A fourth, and perhaps it should be termed an accidental property of objects, is their *motion.* As the apparent motion of a body diminishes as the distance increases, its apparent motion gives some clue to its distance, and this is very noticeably the case if it is *we* who are moving, and not the objects which form the landscape. This is well-seen in the cinematograph pictures taken from a train, the more rapid the motion the greater is the distance from which the effect may be observed ; and this suggests a very curious method of exhibiting the true distance between very distant objects, *i. e.* that of exposing a cinematograph film rather slowly, or more correctly with rather long intervals between the different exposures, whilst the whole instrument is moving as fast as is practicable, and then, in showing the result on the screen, practically increasing the speed of the locomotive, by running the pictures through the lantern as fast as possible.

It may be that the well-known rapid side to side motion which birds, and some other animals, make when inspecting the ground, or their food, before pecking, enables them to form a precise idea of the distance of the object at which they are about to aim. The fact that their eyes lie on different sides of their heads necessarily deprives them of certain powers of estimating distance which are possessed by animals like ourselves which have their eyes on the same side of the head.

The next means of telling the different distances of objects depends, not on properties of the objects themselves, but on one of the eye itself, namely the effort, made quite instinctively, by which we bring the images of these objects to a sharp focus on the retina, *i. e.* by the effort made in so-called *accommodation*. The power of accommodation is, however, comparatively limited in its scope, far more so than seems to be commonly thought, my experiments on many people putting the limit for ordinary sight at about six feet from the eye. On these five methods, AND ON THE MENTAL DEDUCTIONS FOLLOWING FROM THEM and founded upon previous experience, depend the whole of the powers of appreciating form and distance, so far as each eye is considered *separately*. They probably constitute the most important part of the sum of the powers we have of this kind; and to illustrate how far they are capable of giving us correct ideas of the form and distance of objects, we may present to both eyes precisely the same view, as in the case of a photograph projected on a screen (photograph projected). Our sensation of the forms and distances of the different objects are so nearly those we receive when the *two eyes* regard the *actual view* from which the photograph was taken, that we are driven to the conclusion that any means, apart from motion, of judging of form and distance which are wanting in the viewing of the picture on the screen, must either be quite insignificant in its action, or only act through a range less than that of the observer from the screen; and yet, in thus viewing a photograph, we are completely deprived of the sense of "relief" conferred upon us, under ordinary circumstances by the possession of *two* eyes, which can both be turned so as to converge their lines of sight on each object in turn, thus giving rise to the sensation properly known as stereoscopic relief.

The impressions received by the two eyes when regarding any near object are not the same, because each eye sees from its own position, which differs from that of the other eye, by, let us say, a distance of i inches, i is equal to about 2·5 in most cases. Let E_1 E_2 (fig. 1) be the eyes, distances i inches apart, and let the lines E_1 I_1, E_2 I_2 be their respective lines of sight when looking at a very distant object, so that E_1 I_1, E_2 I_2 are practically parallel. Let P_1 be a near object: then when the observer wishes to examine P_1, he must turn each eye inwards through a certain angle, which we will call θ. This he does by making a definite effort, and it is this effort which gives the sensation of stereoscopic relief.

Fig. 1.



It is true that in order to see P_1 distinctly, at least one eye must focus P_1 by "accommodation" (in people of normal sight, both eyes focus P_1), but this accommodation-effort, which has already been discussed, is altogether different from the effort of convergence, and the stereoscopic relief will be perceived by the convergence effort alone, even if the eyes be unable to focus clearly. This I have proved quite conclusively in more than one way, which I shall describe later. To return to fig. 1. Suppose P_2 be another point, rather further from the eyes, it is clear that the angle of convergence will be smaller, the corresponding effort of convergence less, and that too just in proportion to the distance of P_2 from P_1: thus as the difference in the magnitudes of θ implicitly measures the difference of the distances of P_1 P_2 from the eyes base-line in the figure, so the corresponding efforts of convergence measure the stereoscopic sensation of the difference of distance of P_2 and P_1 from the observer. P_1 P_2 may be any points on a given surface, and the notion of the form and distance of the points which make up the surface, *i. e.* of the surface itself, is got by the variation of θ, as the eyes converge on the different points in succession.

Suppose that two photographs of a view be taken from points the same distance apart as the eyes, and the positive prints from these be mounted side by side and then inspected, each view by its appropriate eye through a suitable instrument, we may arrange (1) that each view shall be in focus; (2) that each shall be viewed at such a distance from each eye, that each object in each photograph shall subtend the same angles at the eyes, as those which it subtended in the actual view from which the photograph is taken, and therefore that it shall look the same size as it did in nature.

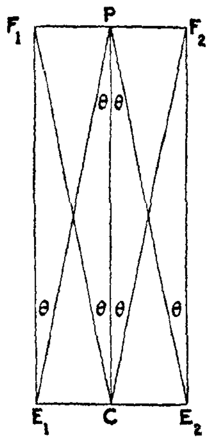
This assumes the use of certain kinds of lenses, both in taking the photographs, and also in viewing them, and in practice these conditions are only approximately realized. That this is so may be proved by taking a photograph of a scene with a fairly long focus lens, and making a positive on glass from it by contact. Now place one eye near the spot where the lens was when the negative was taken, and the positive, as far in front of the eye as the negative was behind the lens when the photograph was taken, and view the landscape through it, moving the eye, if necessary, so as to make the picture of an object just cover the corresponding real object in the view; it will be found that on keeping the eye in this position, and regarding the rest of the view, that the other objects in the transparency do not as a rule "fit" the corresponding objects in the view. This by the way.

(3) We may arrange so that the lines of sight of the two eyes when regarding corresponding points of a very distant object in the two views are parallel. Then, if these three conditions are fulfilled, the efforts of convergence which the two eyes must make to regard the corresponding points of the two views, will be the same as they were in looking at

the corresponding points in the original view, and the stereoscopic effect will be the same. Now in this case there is no *accommodation* necessary to focus different parts of the picture, nevertheless the stereoscopic effect is obtained. This fact therefore constitutes proof positive that variation in the convergence of the eyes is *alone* sufficient to produce stereoscopic effect, apart from "accommodation."

To return to the theory: it can be seen at once from fig. 2 that the total angle ($2\,\theta$) of convergence of the eyes

Fig. 2.



for a point P at any given distance PC is exactly equal to the angle subtended at the eyes $E_1\,E_2$ by a line equal to the distance between them, $F_1\,F_2$, placed at that point, as in the fig., parallel to the line joining the eyes. Now in practice, when P is a certain distance from C, the angle θ, the convergence for one eye, becomes too small to be appreciated, and therefore also the *effort* necessary for the convergence through this small angle is also inappreciable, and the stereoscopic sensation will fail: in other words, stereoscopic sensation will be limited to objects nearer the observer than this distance, which is thus the stereoscopic limit, and it is obviously of interest to find what this distance is for ordinary eyes. The average distance between the eyes may be taken as 2·5 inches, that is $E_1\,E_2 = 2{\cdot}5$ inches, therefore

$E_2C = 1\cdot25$ inches, and the question may be stated thus:—At what distance does the eye fail to appreciate the *effort* made in turning it from F_1 to P?

To find the average value, I drew two black lines on white paper, five-sixteenths of an inch apart, and placing them in bright daylight, measured the distance for each of eleven persons at which the effort to look first at one and then at the other became inappreciable; the mean result was 128 inches, the actual values being those on the accompanying table:—

ft.	in.		ft.	in.		ft.	in.		ft.	in.
5	7	...	9	4	...	11	8	...	14	0
8	1	...	9	6	...	12	0	...	15	8
8	10	...	10	0	...	12	0	...	—	

This shows, by a simple proportion, that the effect to look from F_1 to P becomes inappreciable *for the average person* at about 43 feet distance; this may therefore be taken as the average distance at which stereoscopic relief ceases, though some lose it about 20 feet, and others retain the sensation up to 64 feet.*

It should be particularly noticed that the validity of this proof rests on the assumption, tacitly made, that when θ is very small, *i.e.* near the stereoscopic limit, the effect to turn one eye through a very small angle, θ, is the same, whether the other eye turns with it in the same direction, or in the contrary. This would probably not be true for a large value of θ, but this does not affect the argument. But, whether the proof holds exactly or not, there is no doubt that the limit indicated is not far from its true value: this appears, both by observation of ordinary views, when, however, it is exceedingly difficult to discriminate between the true stereoscopy and the sensations of relief and distance produced by the other causes already enumerated, so that we should naturally infer that the limiting distance for true stereoscopic effect was *greater* than it really is: and a second, and far

* With the "balloon" range-finger, and good eyes, it is certain that the stereoscopic limit is many times greater than that assigned in this paper; but the writer believes that his estimate is not greatly in error for ordinary sight under ordinary conditions.

better proof that the limit established here is fairly correct, is afforded by measurements of the distances to which stereoscopic relief is perceptible in pairs of photographs taken as described later in the present paper, and to compare the results thus obtained with the distances of the limit calculated by the method we are discussing. So far as my work in this direction has gone the values agree as well as I could expect.

It is worth noting that both by theory and experiment, the range for stereoscopic relief is *not* the same as the range for "accommodation," *i. e.,* for focussing. In my own case, everything is equally, and well in focus, at distances greater than 5 feet 8 inches, whereas I can perceive stereoscopic effect to a distance of 24 feet: nor is it necessary for the sensation of stereoscopic relief that the objects should be sharply focussed. Even if the images of an object formed by the two eyes seem more or less blurred, so long as the effort of convergence is sensible when the eyes are turned from a more distant object to the nearer, the stereoscopic sensation will be received. This explains why even very short-sighted people possess a tolerably long range for stereoscopic vision, and also why, in cases where one eye is long-sighted, and the other short, the range of stereoscopic vision is still very considerable; but the stereoscopic range is undoubtedly shortened by short-sighted vision, either of one, or of both eyes, for it is obvious that the blurred images of an object formed by each eye are wider than those sharply focussed, and hence the distance at which the images subtend the limiting angle for stereoscopic effect is less with blurred than with sharp images, and therefore the stereoscopic range is reduced.

A corollary of this is also worth noting, namely, that when the focussing has been done for all distances, as by a lens in a pair of stereoscopic photographs, such as have been already described, and in which objects appear of the natural size, the stereoscopic effect reaches its maximum theoretical value, so that a person whose stereoscopic limit is, say, only 20 feet, when looking at a view in nature, may be able to perceive it for 43 feet, the average, or even 64 feet, or more, in the same view, when looked at through the stereoscope.

So far I have stated the fundamental bases of the stereoscopic sensation as simply and shortly as I could, and it is necessary thus to clear the ground before considering the extension of the range of stereoscopic vision, by increasing the distance between the two simultaneous points of view. That it must be so increased is evident from fig. 3, where E_1 and E_2 stand for a pair of eyes 1 inch apart, and E_3 and E_4 form a pair 3 inches apart. The angle θ_1 is the angle which measures the converging effort, and the stereoscopic effect

Fig. 3.

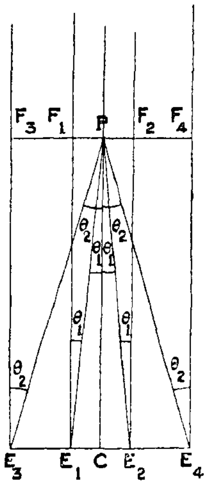
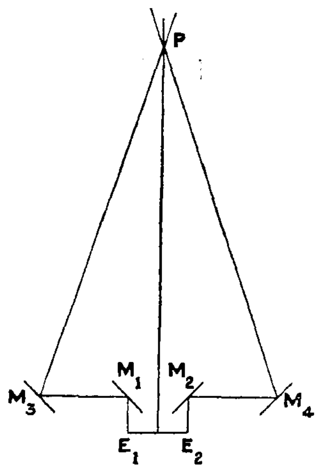


Fig. 4.



for the first pair, when viewing the object P, and the angle θ_2 the similar effort for the second pair, and it is plain that θ_2 is greater than θ_1, and that if P were at such a distance that θ_1 were too small to produce any stereoscopic sensation, θ_2 would still be large enough to produce it, so that the greater the *base-line*, as $E_1 E_2$ may be called, the greater the range.

The exact connexion between the ranges d_1 d_2 with base-lines i_1 i_2 in length being given very simply thus:—

Let θ be the critical value of the angle $F_1 E_1 P$ (fig. 3), so that for any smaller value of θ the stereoscopic sensation fails, then

$$\tan\theta = \frac{E_1C}{CP} = \frac{\frac{1}{2}i}{d_1}, \qquad \text{(i)}$$

and since θ will be the same for the other base-line when stereoscopic sensation vanishes,

$$\tan \theta \text{ also} = \frac{\frac{1}{2}i_2}{d_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(ii)}$$

$$\therefore \text{ from (i) and (ii) } d_1 : d_2 :: i_1 : i_2,$$

or in ordinary language :—

The range of stereoscopic effect varies directly as the length of the base-line. If then the distance between the eyes could be increased, say ten times, *i. e.* to 25 inches, the stereoscopic effect would be perceptible to a distance of 430 feet for an average person. Now such an increase of distance can be made to all intents and purposes by means of reflectors M_1 M_2 M_3 M_4 placed as in fig. 4, the eyes at E_1 E_2 seeing the object P as reflected in the mirrors M_3 M_4, *i. e.*, as viewed from the ends of a base-line extending from M_3 to M_4 instead of only from E_1 E_2. By suitably inclining the mirrors M_1 M_2, the two images formed by M_3 and M_4 can be brought into apparent coincidence, and the stereoscopic effect witnessed. This experiment was, I believe, first made by Von Helmholtz, and he thus extended the base-line to many inches, and called the instrument the telestereoscope. The larger base-line is also one cause of the increase of range of stereoscopic effect in many modern field-glasses, but it is not the *only* cause, for the magnification of a distant object is equivalent to viewing those parts of it which are visible, at a shorter distance, and this means an increase in the angle θ, but, as will be shown presently, magnification does not act in the same way as actually bringing the object itself nearer, a fact which should be carefully remembered. The stereoscopic effect is also greatly increased in some modern forms of telemeter, and from the same causes. But in all such instruments, the extension of the stereoscopic effect soon reaches a limit. This is due to difficulties of construction felt both in making plane mirrors, and lenses, and also troubles arising from the want of rigidity in bars, and the loss of light in reflexion and by absorption, not only in traversing the lenses, but also any considerable thickness of ordinary air. I cannot find any published account of an extension of base-line beyond that of Helmholtz.

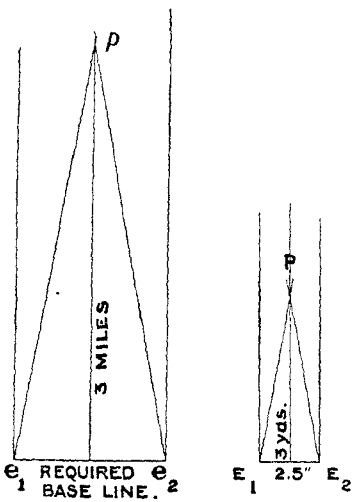
But it is at once evident that no such limitations affect the two photographs, which may be made of a distant object, and afterwards viewed in the stereoscope. However great the base-line, so long as its two ends are about the same distance from the objects to be photographed, the absorption of light caused by the air and the camera may be, and generally will be, the same. Hence, so long as the objects themselves are clear enough and large enough to yield a picture, we have the means of recognizing their forms and relative distances, however far off they may be. I pass at once to the few practical examples, the only stereoscopic photographs of the kind, which I have had time and opportunity for taking, since the thought of the possibility of an enormous practical extension of the field of stereoscopy first occurred to me.

Suppose, then, we have in the distance the confused collection of buildings constituting a fortified city; the nearest points we will say are two miles away, the furthest four. We wish to see at a glance the relative distances between the different objects as plainly as we should see them if an exact small model of the city were made, and viewed by the unassisted eyes, 2·5 inches apart, from a distance, let us say, of three yards. We wish also to see the same parts of the model as we could see of the real city, and moreover the model is to be on such a scale that the parts of the model at a mean distance of three yards shall look the same size as the same parts of the real city at the mean distance of three miles.

To fulfil these conditions, (1) we must use a camera lens and stereoscopic lenses, such that when the photographs are viewed through the stereoscope each object depicted subtends the same angle at the eye as the corresponding part of the original: this can be done without any great difficulty, and since this is not the main question so far as we are concerned, I shall not enter into further details. (2) To get the required stereoscopic effect: if p and P are corresponding points in the real city and in the supposed model respectively. Fig. 5

will show that the base-line for the two photographs must be as many times 2·5 inches as there are three yards in three miles, *i.e.* 122 yards nearly; and if we act accordingly, we shall get the required result, as I think the photograph of Windsor Castle, viewed from a mile's distance, and taken from the ends of a base-line of forty yards, sufficiently shows. This photograph reveals plainly the general arrangement of the Castle buildings round the escarpment of the chalk hill on which it is built. One can see quite plainly the

Fig. 5.



relatively central position of the Round Tower, and also how far behind it the Courtyard side of the Eastern Buildings lies, points which no ordinary photograph or actual view (unless taken from a balloon) could possibly show. The *stereoscopic analysis, as we may call it, in this particular case* is the more surprising because both the points of view, *i.e.* the whole base-line, lies well below the level of all the Castle buildings. Views of this kind may often prove exceedingly valuable in time of war. There are no "rights" for a government to purchase: anyone armed with an ordinary snapshot camera and a stereoscope, such as may be purchased for a few pence, has it in his power to unravel the tangled arrangements of forts and country from a long distance off, for in clear air the two photographs may be taken at five or six miles distance, requiring for a horseman only a few seconds, dismount; and after development, a good rinsing, fixation, and another good rinsing, two prints may be taken without drying the negative, on such a paper as Velox, fixed, and placed in position on a card in the stereoscope, without waiting for them to dry, and the riddle of the strategical position read, as it could be read in no other way.

However hazy the objects may appear, as long as they can be photographed at all, they can be seen by this method with any amount of "relief" desired. It seems to me that for scouts of all kinds the extra kit involved, *i. e.* a single ordinary $3\frac{1}{4} \times 3\frac{1}{4}$ inch camera, with films for lightness, or two plates, if more cannot be carried, would be well worth the extra trouble. It is clear that the presentation of objects by such photographs is quite different from those obtained in any other way, for to revert to the "model city" already mentioned seen from three yards, which expresses well the effect witnessed on looking through the stereoscope at the photographs of the real city three miles off:—it is clear that if an attempt were made to view the real city from three yards distance, only a very small part of it could be seen at all; possibly, at the only points accessible in time of war, a few stones, or a part of a wall, and closed gates. I should like to add that, whilst writing this paper (on the high seas), I showed some of these photographs to Mr. Lloyd Griscom, the Ambassador of the United States at Rome. He informed me that during the late Russo-Japanese war, a man whose name I much regret to say I have forgotten, had made for Admiral Togo some naval photographs, which the ambassador thought must have been taken on much the same principles as these of mine, but he added that no account of these had been published, though he thought they had proved of real value to the Japanese. Mr. Griscom's remark as to the photographs I showed him was that he had never seen anything like them before.

An important point to be noticed is the very great magnitude of the area which may thus be seen at a glance in strong "relief." For, suppose that the stereoscopic view embraces an angle of 25 degrees width of scenery, and suppose that the objects to be examined are very large, and lie at a mean distance of fifty miles, we then have a stereoscopic presentation of a tract of country of more than twenty-one miles from end to end, stretching away to a distance only limited by the curve of the earth, the altitude of the objects, and the clearness of the air; and thus we are directly led to the second great field of stereoscopic work, which this investigation seems to me to open, I mean that of exploration, and in particular the exploration of mountain ranges. Suppose then that the explorer is anxious to study the "lie" and relative heights of peaks standing up in the distance behind the part of the range facing him. Let us take an extreme case, and assume that the highest peaks visible in the distance may be as much as 100 miles off, he wishes to see them as he would see a model of the ranges in which the model of the furthest mountain was at a distance of 30 feet, the model being inspected with the unassisted eyes, say 2·5 inches apart, and the 30 feet lying well within the limit of their stereoscopic vision. We have at once the simple proportion :—

As 30 feet is to 2·5 inches, so must 100 miles be to the base-line required ; and on working this elementary sum out, we find for the necessary base-line a length of $\frac{2}{3}$ of a mile, a perfectly manageable distance in many cases, for note well, the observer does not require a level plane $\frac{2}{3}$ of a mile long and parallel to the range. He only needs two spots, somewhere about the same level above the sea, the higher the better, at $\frac{2}{3}$ of a mile minimum distance. If his base-line is perforce longer, the result will only be to view the "model" mountains nearer ; and if shorter, further off. It will be noticed that the experiment will only fail for any particular mountain, if it should happen to be hidden from one of the points of view : or, if the base-line is not long enough, and the "model" mountains are, some of them at any rate, viewed at a distance beyond the range of stereoscopic effect. It seems therefore worth while to show how we may, by means of what has already been stated, find the smallest base-line which will give stereoscopic effect to any limit, say the 100 miles required in the present example. I purposely give it in a form which all can easily understand.

For the average person with eyes 2·5 inches apart, the stereoscopic sensation fails after 43 feet, thus the "model" mountains must not be viewed at a distance from the furthest of them greater than 43 feet, *i.e.* we have the simple proportion :—As 43 feet is to 2·5 inches, so is 100 miles to the distance required, and this gives the answer as half a mile very nearly.

This length of base-line, then, would be sufficient to bring all the mountains visible into stereoscopic relief, but it would, no doubt, in all cases be advisable to take a base-line considerably larger than the minimum.

There is one point in particular which should be carefully remembered in connexion with viewing these stereoscopic photographs. Hitherto I have supposed in every case that the objects "looked as big in the stereoscope as they did in nature." I put the words in inverted commas because this is the simplest way of saying that when viewed in the stereoscope, the objects in the photographs subtend the same angles at the observer's eye as their originals did at the observer's eye in the natural scene; and the ordinary expression "look as big as" is a convenient abbreviation, when its exact meaning is understood. Generally, however, the objects as seen in the stereoscope do not look the same size as they do in nature: they are magnified, *i. e.*, subtend greater angles, and at first sight it would perhaps seem as if magnification must increase the stereoscopic effect in exactly the same way as taking a correspondingly longer base-line. This is, however, not the case, for a moment's reflection will show that, however much we may magnify two views A and B, taken 2·5 inches apart, say, we can never make them the same as two views—C and D, say, taken, let us suppose 2·5 yards apart, for the views themselves are different in the two cases: even if A and C are taken from the same spot, and are therefore the same, B and D cannot be the same; and the stereoscopic result of magnifying A and B thirty-six times (the number of inches in a yard) will not be the same as that of C and D. The stereoscopic effect of what objects there are on the photographs A and B is undoubtedly multiplied thirty-six times, when A and B are so magnified, and so far as these photographed objects go, it is equivalent to viewing them at one-thirty-sixth the distance, *i. e.*, the stereoscopic effect for the photographed objects is thirty-six times as great; but there are actually objects on the photographs C and D, taken together, which are not to be found on the other pair, whilst some of the objects which appear in A and B will not be found on C and D. The greater base-line used for C and D makes us see, as it were, further "round" the objects than we did in A and B, and supplies us with fresh information for the stereoscopic estimation of the form and distance of each object. One curious effect of the magnification of any pair of stereoscopic photographs must be to increase unduly the apparent depth of the objects themselves, and also the apparent distance between them:—in short to produce what may be called "stereoscopic distortion."

The last, and perhaps the most interesting application of this extension of the base-line from the ends of which the two necessary photographs are taken, is to the sky, i. e., *to the analysis of the distances at which the different layers of cloud lie.*

If we take the greatest vertical height at which any cloud is seen to be as great as 20 miles—and this is undoubtedly of rare occurrence—the smallest base-line necessary will be for the average person about one-tenth of a mile; but in practice, it will be better to use about twice this distance, or say 350 yards. This is for clouds overhead; but in most cases, since the average height of clouds is not more than 7 or 8 miles above sea-level, if indeed that—a base-line of 150 yards, or even less, will be found to work well. If, however, the clouds to be analysed lie near the horizon, the base-line will probably have to be greater. Let us take the *extreme* case of clouds which are 20 miles above sea-level, and viewed when they appear to be resting on the observer's horizon, which we will further suppose is the sea, while the height of the observer above sea-level is one-fifth of a mile. Such a cloud would be, if visible, 310 miles from the observer, as the crow flies, and the base-line must be at least a mile and a half, no doubt two miles would be better, but then such a case is not likely to occur, and I only quote it to give some idea of the superior limit to the base-line necessary for our purpose. Under ordinary circumstances with clouds anywhere up to 3 miles in vertical height, and the height of the observer above sea-level less than 400 feet, a base-line of rather over a quarter of a mile will be sufficient to exhibit even the furthest clouds (which may be taken as about 50 miles off) in relief. The stereoscopic cloud photographs shown were taken as snap-shots from ships going 15 knots, with an interval of about a minute between the exposures of the two negatives which together constitute one of the pairs of photographs; and this implies a base-line of approximately 450 yards. Some of them exhibit the curve of the sky, *i. e.* the curve of the earth, very beautifully. Subsequent experience has shown the writer that in many cases the base-line was unnecessarily long, with the result that the resulting photographs are more difficult to combine subjectively in the stereoscope than need have been the case.

There are two considerations which affect this third great field for experiment:—

(1) The clouds are moving objects; hence if they are moving fast in a line at right angles to the direction in which the camera is pointed, the camera itself need not be moved, save perhaps to give it a slight rotation about its axis. The two views can be taken from the same spot at an interval of time long or short at the operator's will, and depending also on the rate of the clouds' motions, and also on the rate at which the clouds may be changing their forms;—for

(2) The clouds are continually changing, and hence theoretically two SIMULTANEOUS photographs should be taken at the proper distance apart. Such an arrangement is absolutely necessary in all cases where the height, distance, and velocity of the clouds are to be measured from the photographs; but for a picture giving a general view of the cloud layers, this is scarcely necessary, as the photographs exhibited sufficiently prove.

Lastly, not with clouds only, but with any distant moving object, a stereoscopic presentation of that object may be obtained by taking two successive photographs of it, with a suitable time-interval between the two. The principle involved is plain. A change in the position of an object, whilst the observer, or rather his camera, remains at rest, is precisely equivalent to a change in the position of the camera in the opposite direction, whilst the object remains at rest. The results of the former case are, however, often very curious, for although the moving object is presented in relief by the photographs, yet naturally its surroundings are not so; in fact these last may be different in the two views, in which case the stereoscopically presented object stands forth on a ground which changes slowly from that on one picture to that on the other, through retinal rivalry, caused by the gross dissimilarity of the two views. This retinal rivalry lies indeed at the root of all stereoscopic vision, for it is this retinal rivalry, caused by the dissimilarity of the two stereoscopic views, which prompts the eyes to change their angle of convergence, so as to bring corresponding points of images of the two pictures to "corresponding points" of the two retinas.

Here this paper must close: in the first part of it I have restated as simply as I could the fundamental truths on which stereoscopy rests. This led at once to the deduction of a working estimate of the average distance at which true stereoscopic sensation fails. In the last part I have pointed out three great fields of survey work:—Military, Geographical, and Meteorological, in which the great extension of the base-line advocated in this paper may, I hope and believe, prove a valuable, if not an indispensable implement in further research.

Upton Park, Bucks,
May 10th, 1907.

Discussion.

Mr. J. Tennant expressed his interest in the paper, and said he had shown pictures of clouds, taken by the same method as that employed by the author, in a paper published in 1897. He was not certain that the feeling of convergence was the only thing which determined stereoscopic vision. The estimate of 43 feet as the limit of stereoscopic vision seemed to him too small, as he had observed stereoscopic effects at distances up to 125 yards. In 1896 he devised a stereoscopic camera and used it to measure the heights and depths of clouds. In some of his cloud experiments he had used a base-line $\frac{3}{4}$ of a mile long. The method was applicable to many subjects and was a most valuable one.

Mr. C. W. S. Crawley did not quite understand the author's stereoscopic limit which he put at 43 feet. The only definite limit to the angle θ was the smallest difference of angle of convergence, say θ_0, by which we could judge difference of distance. In ordinary use, many things prevented our employment of stereoscopic power to the full, but he thought 43 feet was much below the limit in any case. Von Helmholtz, from a rough test, gave us θ_0 as about 1 minute, which would mean that we could say by pure stereoscopic power that an object at 200 metres was nearer than one at infinity.

As a matter of fact, von Helmholtz's figure was much too large; for English eyes at any rate. The speaker had repeated the rough experiments in a way which, while it allowed the stereoscopic power fuller scope, did not allow any help from other sources. He had tested all sorts and ages of people and found the usual value of θ_0 about 10 secs.; giving a limiting distance of 1200 metres. Several were, however, much better than this and had θ_0 about 2 secs. of arc, or a limiting distance of 6000 metres. Personally, the speakers varied from 3 seconds when in good health to 8 or 10 secs. when not; and he found a stereoscopic test most useful in hepatometry. In answer to a question, the speaker said these values were fully borne out by others on the Forbes range-finder, where judgment of distance by any other means was absolutely impossible.

He quite agreed with the author that wide-base stereoscopic photography might be much more widely used in surveying and in cloud-work, and hoped that the paper would greatly promote such use. He thought the author would be interested in some of the work of Dr. Pulfrich of Jena, who had gone a long way in the former branch.