Geometric Regularization of the Friedmann Singularity from a Boundary Constraint

Nadav Bashan
Independent Researcher
naduuv@gmail.com
ORCID: 0009-0002-8491-1612

Abstract

The classical Friedmann singular limit is reconsidered in a boundary-based framework. A geometric constraint relating boundary curvature to apparent-horizon scale is combined with a Planck-scale upper cutoff on admissible curvature. This yields a finite minimum apparent-horizon radius together with a finite minimum horizon area, holographic area count, and entropy. The same boundary coefficient is obtained from the black-body normalization on a spherical decoupling surface and from the corresponding Bose–Einstein integral, and is used here as the thermodynamic boundary normalization of a spherical decoupling layer. Since the apparent horizon carries the standard thermodynamic attributes of temperature and entropy, the same boundary normalization is imposed on it as a macroscopic thermodynamic boundary condition. The resulting minimum state is therefore the smallest horizon compatible with that boundary normalization. The analysis remains macroscopic throughout and does not rely on additional dynamical fields, extra dimensions, or a microscopic quantum-gravitational model.

Keywords: cosmological singularity; apparent horizon; cosmological constant; Planck cutoff; horizon entropy; black-body normalization

1 Introduction

Backward extrapolation of the Friedmann equations leads to a singular limit in which the scale factor tends to zero and curvature invariants diverge. This is usually taken to indicate the breakdown of the classical description [1, 2, 3].

Most regularization schemes introduce additional microscopic structure, such as new fields, modified dynamics, or discrete space-time geometry. The present analysis adopts a different starting point. Instead of modifying the bulk dynamics, it imposes a geometric boundary constraint and examines its consequences when combined with a Planck-scale cutoff on admissible curvature.

The horizon scale used throughout is the apparent horizon radius R_A. In the spatially flat limit, $R_A = c/H$, so that the apparent and Hubble horizons coincide. In the present paper the early-universe analysis is formulated directly in terms of R_A, since the apparent horizon is the relevant local thermodynamic boundary [4, 5].

The starting point is

$$\Lambda R_A^2 = \frac{\pi^3}{15}, \tag{1}$$

where Λ is used in the present framework as an effective boundary-curvature parameter and R_A as the horizon scale. The coefficient $\pi^3/15$ is not introduced as a free numerical factor. It is the dimensionless boundary coefficient obtained when the black-body radiative law is written for a spherical decoupling surface and reduced to dimensionless form. Its thermal content enters through the Bose–Einstein integral

$$\int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}, \tag{2}$$

together with the geometric factors of the spherical boundary. No free parameter enters its derivation. In this framework, $\pi^3/15$ is therefore used as the thermodynamic boundary normalization of a spherical black-body decoupling surface.

The apparent horizon is treated on the same footing. Since it carries the standard thermodynamic attributes of temperature and entropy, it is taken here as a macroscopic thermodynamic decoupling surface [4, 5, 6, 7, 8]. The same boundary normalization is therefore imposed on it through Eq. (1).

In the present framework, Eq. (1) is adopted as a boundary postulate and the Planck-scale cutoff introduced below as an independent limiting-curvature assumption. The subsequent minimum radius, area, area count, and entropy follow directly from their combination.

The question addressed here is whether Eq. (1), together with a Planck-scale cutoff, yields a finite minimum horizon radius, and whether that minimum defines the smallest scale at which the horizon remains compatible with a thermodynamic boundary normalization.

2 Boundary Curvature and Horizon Scale

In the present framework, Λ is used as an effective boundary-curvature parameter.

The coefficient $\pi^3/15$ follows from the dimensionless normalization of the Stefan–Boltzmann law for a spherical black-body decoupling surface. For luminosity L, radius R, and effective temperature T,

$$L = 4\pi R^2 \sigma T^4, \tag{3}$$

where

$$\sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}. \tag{4}$$

Writing Eq. (3) in dimensionless form gives

$$\frac{L}{R^2 T^4}\,\frac{\hbar^3 c^2}{k_B^4} = \frac{\pi^3}{15}. \tag{5}$$

Equation (5) supplies the boundary coefficient that is imposed, in the present framework, in the horizon relation (1).

3 Planck-Scale Regularization

Equation (1) gives

$$R_A^2 = \frac{\pi^3}{15\Lambda}. \tag{6}$$

By itself, this is a scaling relation: as Λ increases, R_A decreases. In the present framework, Λ is not treated as a dynamical variable. Equation (6) is a kinematic relation between boundary curvature and horizon scale. The Planck cutoff

$$\Lambda_{\max} = \frac{1}{\ell_P^2}$$

represents the maximum admissible value of this curvature parameter, not a quantity treated here as time-dependent. The observed cosmological constant $\Lambda_{\text{obs}} \approx 1.1 \times 10^{-52}\,\text{m}^{-2}$ lies far below this ceiling. To exclude the limit $R_A \to 0$, introduce the independent limiting-curvature assumption

$$\Lambda_{\max} = \frac{1}{\ell_P^2}. \tag{7}$$

Equation (6) then yields the minimum apparent-horizon radius

$$R_{A,\min} = \sqrt{\frac{\pi^3}{15\Lambda_{\max}}} = \sqrt{\frac{\pi^3}{15}}\,\ell_P \approx 1.437\,\ell_P. \tag{8}$$

Within this framework, the singular limit is therefore replaced by a finite initial horizon scale.

4 Minimum Area, Holographic Count, and Entropy

From Eq. (8), the corresponding minimum horizon area is

$$A_{\min} = 4\pi R_{A,\min}^2 = \frac{4\pi^4}{15}\,\ell_P^2. \tag{9}$$

Using the standard Bekenstein–Hawking relation $S = k_B A/(4\ell_P^2)$, define the corresponding dimensionless area count by $N \equiv A/\ell_P^2$. One then obtains

$$N_{\min} = \frac{A_{\min}}{\ell_P^2} = \frac{4\pi^4}{15} \approx 25.98, \tag{10}$$

and

$$\frac{S_{\min}}{k_B} = \frac{N_{\min}}{4} = \frac{\pi^4}{15} \approx 6.49. \tag{11}$$

This equals the Bose–Einstein integral already given in Eq. (2).

The initial state is therefore finite in all three quantities: horizon radius, horizon area, and holographic area count. In this sense, the singular boundary is replaced by a finite initial boundary state.

5 Discussion

The boundary relation $\Lambda R_A^2 = \pi^3/15$, combined with the Planck cutoff $\Lambda_{\max} = 1/\ell_P^2$, yields a finite minimum horizon radius. An algebraically equivalent form of the minimum area count is

$$N_{\min} = \frac{4\pi^4}{15} = 4\pi \left(\frac{\pi^3}{15}\right). \tag{12}$$

This makes explicit that the minimum holographic count combines the full spherical solid angle with the same boundary coefficient that appears in Eq. (1). The minimum state is therefore fixed, in this framework, by the thermodynamic boundary normalization of a spherical black-body decoupling surface distributed over the full horizon area.

This identifies the minimum radius as the smallest scale at which the apparent horizon remains compatible with a thermodynamic boundary normalization. Below $R_{A,\min}$, the horizon falls outside the domain defined here by that normalization together with the limiting curvature. The classical singular limit is therefore replaced by a finite boundary state determined by the boundary postulate and the limiting-curvature assumption.

Within this framework, the Planck cutoff $\Lambda_{\max} = 1/\ell_P^2$ acquires a direct interpretation: it marks the curvature scale at which the thermodynamic boundary normalization reaches its minimum admissible horizon realization. The cutoff is not derived microscopically here; it enters as the limiting curvature compatible with the same boundary structure.

Other approaches to singularity resolution, such as loop quantum cosmology, replace the singular limit through a discrete area spectrum arising from quantum geometry [9]. The present framework differs in that no microscopic structure is assumed; the minimum scale follows from the thermodynamic boundary normalization of the horizon. The two approaches are therefore distinct in method and complementary in scope.

The present approach shares a broader motivation with thermodynamic derivations of gravitational dynamics, in which horizon thermodynamics is taken as a starting point rather than as a consequence. Jacobson [10] derived the Einstein equations from the Clausius relation applied to local Rindler horizons. Padmanabhan [11] developed an emergent gravity program in which spacetime dynamics arises from horizon degrees of freedom. The present analysis does not derive field equations, but operates within the same conceptual direction: the boundary condition constrains the admissible bulk state.

The minimum entropy $S_{\min}/k_B = \pi^4/15$ and the minimum area count $N_{\min} = 4\pi^4/15$ are fixed numerical results of the framework. They are not directly testable by current observations, but they provide definite targets for any future microscopic theory of the initial cosmological state.

The present result is a macroscopic regularization of the admissible thermodynamic boundary state. It is not, by itself, a derivation of a complete nonsingular cosmological dynamics.

6 Conclusion

The boundary postulate

$$\Lambda R_A^2 = \frac{\pi^3}{15} \tag{13}$$

combined with the independent limiting-curvature assumption

$$\Lambda_{\max} = \frac{1}{\ell_P^2} \tag{14}$$

yields the finite minimum radius

$$R_{A,\min} = \sqrt{\frac{\pi^3}{15}}\,\ell_P. \tag{15}$$

The same framework gives

$$A_{\min} = \frac{4\pi^4}{15}\,\ell_P^2, \qquad N_{\min} = \frac{4\pi^4}{15}, \qquad \frac{S_{\min}}{k_B} = \frac{\pi^4}{15}. \tag{16}$$

Within this interpretation, the Planck cutoff marks the curvature scale at which the thermodynamic boundary normalization reaches its minimum admissible realization. The analysis remains macroscopic; the microscopic origin of the boundary relation remains open.

Acknowledgments

The author thanks Dr. Mehmet Keçeci for helpful comments and for independent verification of calculations in earlier work related to the present analysis.

Statements and Declarations

Data Availability: No datasets were generated or analysed during the current study.

Conflict of Interest: The author declares no conflict of interest.

References

[1] R. Penrose, Gravitational collapse and space-time singularities, Phys. Rev. Lett. **14**, 57–59 (1965).

[2] S. W. Hawking and R. Penrose, The singularities of gravitational collapse and cosmology, Proc. R. Soc. Lond. A **314**, 529–548 (1970).

[3] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, 1973).

[4] D. Bak and S.-J. Rey, Cosmic holography, Class. Quant. Grav. **17**, L83–L89 (2000).

[5] R.-G. Cai and S. P. Kim, First law of thermodynamics and Friedmann equations of Friedmann–Robertson–Walker universe, JHEP **02**, 050 (2005).

[6] G. W. Gibbons and S. W. Hawking, Cosmological event horizons, thermodynamics, and particle creation, Phys. Rev. D **15**, 2738–2751 (1977).

[7] J. D. Bekenstein, Black holes and entropy, Phys. Rev. D **7**, 2333–2346 (1973).

[8] S. W. Hawking, Particle creation by black holes, Commun. Math. Phys. **43**, 199–220 (1975).

[9] A. Ashtekar and P. Singh, Loop quantum cosmology: a status report, Class. Quant. Grav. **28**, 213001 (2011).

[10] T. Jacobson, Thermodynamics of spacetime: the Einstein equation of state, Phys. Rev. Lett. **75**, 1260–1263 (1995).

[11] T. Padmanabhan, Thermodynamical aspects of gravity: new insights, Rep. Prog. Phys. **73**, 046901 (2010).