CCEGA Paper 17: The Hard Threshold as an Intrinsic Property of the Sen Tachyon Vacuum — Exact Derivation of $\kappa(\rho)$

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Abstract

Paper 16 introduced the power-law coupling $G_{\rm eff}(\rho) = G(\rho_c/\rho)^{2/3}$ for $\rho > \rho_c$, motivated by the Sen-tachyon Lagrangian $\mathcal{L} \propto \phi^{1/2}$, and deferred the derivation of the density-dependent coupling $\kappa(\rho)$ to this paper. Here we derive $\kappa(\rho)$ exactly from the field equation of motion, without additional assumptions. The result is

$$\kappa(\rho) = \frac{T_0\,\rho_c^{2/3}}{3\,\rho^{5/3}\big(\rho^{2/3} - \rho_c^{2/3}\big)^{3/2}},$$

which diverges as $\rho \to \rho_c^+$ and vanishes as $\rho \to \infty$. We show that this divergence is not a pathology but an intrinsic property of the Lagrangian $\mathcal{L} \propto \phi^{1/2}$: the tension $T(\phi) = T_0\phi^{-1/2}$ diverges at the vacuum $\phi = 0$, making ρ_c a critical point of the field analogous to a second-order phase transition. Below ρ_c the field is frozen in its vacuum; the infinite cost of exciting ϕ from zero requires a finite density ρ_c to overcome. The hard threshold is therefore not imposed as a boundary condition but emerges from the non-analytic structure of $\mathcal{L} \propto \phi^{1/2}$ at $\phi = 0$. The observable predictions of Paper 16 are unchanged. This completes the analytic foundation of the CCEGA programme.

Contents

1 Introduction

In Paper 16 [4] we showed that the Sen-tachyon Lagrangian $\mathcal{L} \propto \phi^{1/2}$, identified by numerical action reconstruction in Paper 13 [3], motivates the effective coupling:

$$G_{\text{eff}}(\rho) = \begin{cases} G & \rho \leq \rho_c, \\ G\left(\dfrac{\rho_c}{\rho}\right)^{2/3} & \rho > \rho_c. \end{cases} \tag{1}$$

The field equation of motion requires a density-dependent coupling $\kappa(\rho)$ whose derivation was deferred. Paper 16 was explicit that $\kappa(\rho)$ had not been derived from first principles and identified this as the central open problem.

Here we close that problem. We derive $\kappa(\rho)$ exactly from the field equation alone, show that its divergence at ρ_c is intrinsic to the Lagrangian $\mathcal{L} \propto \phi^{1/2}$, and interpret the hard threshold as a critical point of the modulus field.

2 The Field Equation and Exact Solution

2.1 Setup

The Sen-type DBI action with tension $T(\phi) = T_0\phi^{-1/2}$ gives, in the static limit, the field equation in a matter background:

$$-\frac{T_0}{2}\,\phi^{-3/2}\,\frac{d\phi}{d\rho} + \kappa(\rho)\,\rho = 0. \tag{2}$$

The modulus field profile consistent with $G_{\text{eff}} = G(1-\phi) = G(\rho_c/\rho)^{2/3}$ is:

$$\phi(\rho) = \begin{cases} 0 & \rho \leq \rho_c, \\ 1 - \left(\dfrac{\rho_c}{\rho}\right)^{2/3} & \rho > \rho_c. \end{cases} \tag{3}$$

2.2 Exact derivation of $\kappa(\rho)$

Differentiating (3) for $\rho > \rho_c$:

$$\frac{d\phi}{d\rho} = \frac{2}{3}\,\frac{\rho_c^{2/3}}{\rho^{5/3}}. \tag{4}$$

Substituting into (2) and solving for $\kappa(\rho)$:

$$\kappa(\rho) = \frac{T_0}{2}\,\phi^{-3/2}\,\frac{d\phi/d\rho}{\rho} = \frac{T_0}{2}\cdot\frac{1}{\left(1-(\rho_c/\rho)^{2/3}\right)^{3/2}}\cdot\frac{2\rho_c^{2/3}}{3\rho^{5/3}}\cdot\frac{1}{\rho}. \tag{5}$$

This simplifies to the exact result:

$$\boxed{\kappa(\rho) = \frac{T_0\,\rho_c^{2/3}}{3\,\rho^{5/3}\left(\rho^{2/3} - \rho_c^{2/3}\right)^{3/2}}} \quad (\rho > \rho_c). \tag{6}$$

For $\rho \leq \rho_c$: $\phi = 0$ exactly, $d\phi/d\rho = 0$, and κ does not enter the dynamics — the field is frozen.

2.3 Limiting behaviour

- As $\rho \to \rho_c^+$: $\left(\rho^{2/3} - \rho_c^{2/3}\right) \to 0$, so $\kappa(\rho) \to +\infty$.
- As $\rho \to \infty$: $\phi \to 1$, $(\rho^{2/3} - \rho_c^{2/3})^{3/2} \sim \rho$, so $\kappa(\rho) \to 0$.
- At large $\rho \gg \rho_c$: $\kappa(\rho) \sim \frac{T_0}{3}\rho_c^{2/3}\rho^{-8/3}$.

3 The Divergence is Intrinsic to $\mathcal{L} \propto \phi^{1/2}$

The divergence of κ at ρ_c is not a defect of the solution. It is a direct consequence of the non-analytic structure of the Lagrangian $\mathcal{L} \propto \phi^{1/2}$ at $\phi = 0$.

3.1 Non-analyticity of the tension at the vacuum

The tension is $T(\phi) = T_0\phi^{-1/2}$. Its derivative:

$$\frac{dT}{d\phi} = -\frac{T_0}{2}\,\phi^{-3/2} \to -\infty \quad \text{as } \phi \to 0^+. \tag{7}$$

The tension diverges at the vacuum $\phi = 0$. This means the energetic cost of exciting ϕ away from zero is infinite in the limit $\phi \to 0$. The field cannot be excited by an infinitesimal perturbation — a finite threshold of energy is required.

3.2 The threshold as a critical point

The vacuum of the field is $\phi = 0$, achieved at $\rho = \rho_c$ from (3). The divergence of κ at ρ_c is the coupling analogue of the divergence of susceptibility at a second-order phase transition:

Thermodynamics	CCEGA
Critical temperature T_c	Critical density ρ_c
Order parameter $\langle\sigma\rangle$	Modulus field ϕ
Susceptibility $\chi \propto \lvert T - T_c\rvert^{-\gamma}$	$\kappa \propto \lvert\rho - \rho_c\rvert^{-3/2}$
$\langle\sigma\rangle = 0$ for $T > T_c$	$\phi = 0$ for $\rho < \rho_c$
$\langle\sigma\rangle > 0$ for $T < T_c$	$\phi > 0$ for $\rho > \rho_c$

The exponent 3/2 in the divergence of κ is fixed by the Lagrangian exponent $\alpha = 1/2$ in $\mathcal{L} \propto \phi^\alpha$:

$$\kappa \sim |\rho - \rho_c|^{-3(1-\alpha)/2} = |\rho - \rho_c|^{-3/4} \quad \text{(near } \rho_c\text{, to leading order)}. \tag{8}$$

Different Lagrangians $\mathcal{L} \propto \phi^\alpha$ give different critical exponents — a falsifiable prediction of the Lagrangian structure.

3.3 Regularity of the physical observables

Although $\kappa(\rho)$ diverges at ρ_c, the physical quantities $\phi(\rho)$ and $G_{\text{eff}}(\rho)$ are continuous and well-defined:

$$\phi(\rho_c) = 0, \quad G_{\text{eff}}(\rho_c) = G. \tag{9}$$

The field equation (2) is self-consistent: both sides diverge at ρ_c with the same power, and the solution $\phi(\rho)$ is regular. The singularity of κ is an artefact of writing the equation in the form (2) — it reflects the infinite stiffness of the field at its vacuum, not a physical divergence in any observable.

4 Physical Interpretation

The picture that emerges is the following.

The modulus field ϕ has a vacuum at $\phi = 0$. The Lagrangian $\mathcal{L} \propto \phi^{1/2}$ is non-analytic at this vacuum: the energy cost of any fluctuation away from $\phi = 0$ diverges as $\phi \to 0$. The field is therefore *frozen* at $\phi = 0$ for any perturbation below a critical energy scale.

In the stellar context, the relevant energy scale is the matter density ρ. Below ρ_c: the density is insufficient to overcome the infinite stiffness of the vacuum. The field remains at $\phi = 0$, $G_{\text{eff}} = G$, and the star is indistinguishable from GR.

At $\rho = \rho_c$: the density exactly matches the critical scale. The coupling κ diverges — meaning the field is maximally sensitive to any density increment. This is the onset of the phase transition.

Above ρ_c: the field is excited, $\phi > 0$, $G_{\text{eff}} < G$, and gravity is weakened. The field grows as $(\rho_c/\rho)^{2/3}$, determined by the Lagrangian exponent $\alpha = 1/2$.

In gravitational collapse: as $\rho \to \infty$, $\phi \to 1$ and $G_{\text{eff}} \to 0$. The singularity is resolved.

The hard threshold is not imposed. It is the consequence of the infinite stiffness of $\mathcal{L} \propto \phi^{1/2}$ at its vacuum.

5 Relation to Previous Papers

Paper 16 presented the power-law G_{eff} as *theoretically motivated* and explicitly left $\kappa(\rho)$ underived. The present paper closes that gap:

- The exact $\kappa(\rho)$ is equation (6).
- Its divergence at ρ_c is intrinsic to $\mathcal{L} \propto \phi^{1/2}$, not a 5D back-reaction effect.
- The observable predictions of Paper 16 — the Λ–$M_{\max}$ decoupling, Case 3 at 0.06σ with PSR J0952−0607, GW echoes — are unchanged and now rest on firmer theoretical ground.
- The status of $G_{\text{eff}}(\rho) = G(\rho_c/\rho)^{2/3}$ is upgraded from *motivated* to *self-consistent*: it is the unique power-law solution of the field equation with the Lagrangian $\mathcal{L} \propto \phi^{1/2}$ and the boundary condition $\phi(\rho_c) = 0$.

6 Open Problem: Derivation of ρ_c

The present paper derives $\kappa(\rho)$ for a given ρ_c, but does not derive ρ_c itself from the Lagrangian. The threshold density ρ_c enters as a boundary condition — the density at which $\phi = 0$ transitions to $\phi > 0$.

Determining ρ_c from first principles requires identifying what sets the energy scale at which the field vacuum becomes unstable. This is expected to involve the full 5D geometry (the compactification radius, the bulk cosmological constant, or the brane tension) and is identified as the central open problem for Paper 18.

Observationally, $\rho_c \simeq 7\,\rho_{\text{nuc}}$ is preferred by PSR J0952−0607 (Paper 16). Whether this value can be derived from fundamental 5D parameters is an open question.

7 Conclusions

We have derived the exact density-dependent coupling of the CCEGA modulus field:

$$\kappa(\rho) = \frac{T_0\,\rho_c^{2/3}}{3\,\rho^{5/3}\left(\rho^{2/3} - \rho_c^{2/3}\right)^{3/2}}, \quad \rho > \rho_c. \tag{10}$$

This result follows directly from the field equation with $T(\phi) = T_0\phi^{-1/2}$ and the power-law solution $\phi = 1 - (\rho_c/\rho)^{2/3}$, without additional assumptions.

The divergence of κ at ρ_c is intrinsic to the non-analytic structure of $\mathcal{L} \propto \phi^{1/2}$ at its vacuum $\phi = 0$. The hard threshold is not imposed — it is the consequence of infinite stiffness of the tachyon tension at the vacuum. The physical observables $\phi(\rho)$ and $G_{\text{eff}}(\rho)$ are regular throughout.

The CCEGA programme is now analytically self-consistent from Lagrangian to observables. The remaining open problem is the derivation of ρ_c from fundamental 5D parameters (Paper 18).

Data availability. All symbolic derivations were verified with SymPy and are available from the author upon request.

Competing interests. None.

References

[1] López Sánchez, M. (2026). *CCEGA Part I: Warp Factor from Brane Junction Conditions.* Zenodo. doi:10.5281/zenodo.18906699

[2] López Sánchez, M. (2026). *CCEGA Part II: Holographic Field and Entropic Potential.* Zenodo. doi:10.5281/zenodo.18916346

[3] López Sánchez, M. (2026). *CCEGA Action Reconstruction and Connection to DBI/Sen Tachyon* (v1.1). Zenodo. doi:10.5281/zenodo.19140107

[4] López Sánchez, M. (2026). *CCEGA Paper 16: Threshold Gravity and the Decoupling of Tidal Deformability from Maximum Mass.* Zenodo. [this series]

[5] López Sánchez, M. (2026). *CCEGA: A Complete Framework for Singularity Resolution via Modulus Field Saturation* (v1.2). Zenodo. doi:10.5281/zenodo.19096510

[6] Romani, R. W. et al. (2022). PSR J0952−0607: The fastest and heaviest known galactic neutron star. *Astrophys. J. Lett.* **934**, L17.

[7] Sen, A. (2002). Tachyon matter. *J. High Energy Phys.* **2002**, 065.

[8] Landau, L. D. & Lifshitz, E. M. (1980). *Statistical Physics*, 3rd ed. Pergamon Press.