Universal Grid Mechanics (UGM)

An Axiomatic, Admissibility-First Framework for Physical Reality

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J. G. Villarroel H.[1]

[1]Independent Researcher ORCID: 0009-0009-1082-9502

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Abstract

Universal Grid Mechanics (UGM) is an admissibility-first foundational physics framework in which physical existence is restricted to states satisfying continuity, bounded change, and local information consistency under repeated updates. Reality is modelled as a continuous persistent substrate — the grid — that admits deformation, retains history, and resists change in a bounded manner; all evolution preserves a finite admissible domain $\mathcal{D}$. UGM is explicitly pre-phenomenological: no particles, fields, or spacetime geometry are assumed at the axiomatic level.

Starting from five frozen axioms and the minimal local state $X = (S, M)$, where S is a structural deformation state and M is structural memory, this paper derives and consolidates the foundational architecture of UGM. The unique admissibility-minimizing primitive operator L_6 is derived via coordination-normalized spectral anisotropy; its Taylor expansion recovers the isotropic Laplacian. From the UGM gravity Hamiltonian, the Poisson equation and the inverse-square law follow as formal limiting theorems, and the dimensionless Newton constant $G = \sqrt{3}/4\pi^2$ emerges as a geometric consequence of hexagonal coordination without gravitational data. The gravity sector retains a two-regime structure: Newtonian at low memory and MOND-like screened-Poisson behaviour at high memory.

The central new result is a global contraction theorem: admissible evolution is not merely bounded but contractive on $\mathcal{D}$, guaranteeing uniqueness and asymptotic stability of the update dynamics. This upgrades UGM from a collection of admissible states to a dynamically well-posed system, justifies coarse-graining, and strengthens the structural exclusion of General Relativity. The Admissible Path Interpretation replaces globally monotone-memory language with the precisely correct pathwise non-negativity of realized increments, derived from admissibility and branch selection. A causal memory-kernel formulation connects structural memory accumulation to the kinematic discriminator and provides a structural pathway toward the radial acceleration relation, with the characteristic acceleration scale $a^\star \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ treated as an emergent memory–kinematic scale rather than a primitive constant.

Keywords: admissibility-first framework; grid mechanics; primitive operator; structural memory; modified gravity; MOND; radial acceleration relation; pathwise evolution; General Relativity incompatibility.

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1 Introduction

Standard foundational frameworks in physics admit formal infinities, singularities, or divergences as technical features to be regularized or repaired by additional structure. UGM adopts the opposite stance: singularities and infinities are diagnostic markers of ontological inadmissibility, not technical nuisances. Physical existence is restricted, from the outset, to states that are continuous, bounded, and locally consistent under repeated updates.

UGM reverses the usual order of construction. It does not begin with equations and ask which solutions are physical. It begins with a small frozen set of ontological axioms specifying which states and updates are admissible, then allows mathematics only insofar as it describes those admissible structures. The aim is not to overwrite established theories inside their validated domains, but to explain why those domains exist and why breakdowns appear when their assumptions are exceeded.

UGM is explicitly pre-phenomenological. It specifies admissibility constraints prior to the introduction of coordinates, particles, fields, spacetime intervals, or observable inventories. Phenomenological descriptions are treated as projections of admissible structural evolution rather than primitives of the theory.

The radial acceleration relation as a central constraint. The radial acceleration relation (RAR), established by McGaugh, Lelli, and Schombert [13] and extended in later work [14, 18, 19], maps baryonic centripetal acceleration to observed acceleration in rotationally supported galaxies with very low intrinsic scatter across a broad galaxy sample. Any viable foundational framework must reproduce that relation without free per-galaxy tuning.

In UGM, structural memory contributes to the effective gravitational source through a memory-mediated factor $\rho_{\text{eff}} = \rho_\kappa(1 + \chi)$, where χ is a dimensionless structural memory field. This provides a structural pathway toward the RAR through a memory-mediated effective source, with the precise interpolating function $\mu(x)$ and transition scale $g^\dagger$ remaining open. The characteristic acceleration scale $a^\star$ is interpreted as emergent from the interaction between structural memory accumulation and kinematic evolution timescales.

Structure of this paper. Section 2 states the frozen UGM 02 axioms. Sections 3–5 restore the primitive update loop, the coherence–selection mechanism, and the admissibility metric. Section 6 introduces the API. Sections 7–12 cover forward invariance, the primitive operator, the ω_W no-go chain and bifurcation, the primitive write law, coarse reduction, and Route B normalization. Section 13 develops the causal memory kernel and its link to the RAR. Sections 14–15 record the retained gravity, Lorentz, and GR-exclusion results and summarize the empirical programme. A separate accountability package [10] accompanies this paper and contains the derivation-status table and dependency accounting.

2 Frozen Core Axioms and State Structure

The following axioms constitute the frozen UGM 02 canon. They are not modified by any result in this or companion papers; all UGM claims are derived from them alone.

Axiom 0 (Admissibility Is Primitive). Physical existence is restricted to admissible states. Admissibility requires: (i) continuity; (ii) bounded change; (iii) local information consistency under repeated updates.

Axiom 1 (Grid Ontology). Reality is a continuous, persistent substrate. There is no physical void and no background container. The grid admits deformation, retains history, and resists change in a bounded manner.

Axiom 2 (Ontology Precedes Mathematics). Mathematics describes admissible grid states. Formal divergence does not imply physical existence.

Axiom 3 (Inadmissible States Are Forbidden). Singularities, infinities, and breakdowns are inadmissible. No admissible evolution generates divergence from admissible initial conditions.

Axiom 4 (Structural Memory). The grid retains deformation history. Memory is local, constrains future admissibility, and accumulates under admissible updates. Memory cannot diverge.

Operational definitions. The *grid* is the continuous substrate that admits deformation, stores deformation history, and resists change in a bounded manner. No particles, fields, spacetime geometry, or conserved quantities are assumed at the axiomatic level. A state or update is *admissible* if it preserves continuity, bounded change, and local information consistency under repeated updates.

State and domain. The minimal local state is $X = (S, M)$, with S the admissibility/deformation state (finite) and M structural memory (non-negative, finite). The admissible domain is

$$\mathcal{D} = \{(S, M) : S_{\min} \leq S \leq S_{\max},\ 0 \leq M \leq F(S),\ F(S) \leq M_{\max} < \infty\}. \tag{1}$$

All admissible evolution preserves $\mathcal{D}$ (forward invariance). The canonical memory projector is $\Pi_M(z; S) := \mathrm{clip}(z, 0, F(S))$.

3 Primitive Update Loop

All admissible evolution reduces to a closed local update loop:

1. **Geometry stores deformation.** Deformation of the grid is encoded as geometric bookkeeping.
2. **Stored deformation drives motion.** Motion is internal relaxation of stored deformation, not transport through empty space.
3. **Motion updates geometry.** As deformation relaxes, the grid bookkeeping updates.
4. **Geometry redistributes deformation.** Curvature redistributes stored deformation locally and directionally while preserving admissibility.

No additional ontological layer is posited beneath this loop.

A compact structurally admissible update form consistent with this loop is

$$\partial_\tau \Psi = -\nabla \cdot [S(K(\Psi) \circ \nabla\Psi)] + \Lambda(\Psi). \tag{2}$$

This expression is not introduced as a phenomenological field equation. It records an admissible update form whose regime-specific Hamiltonian reductions are developed in the companion gravity paper [9].

4 Coherence–Selection Mechanism

In a locally updated substrate with finite-range interactions and structural memory, initially symmetric excitations coupled to an inhomogeneous environment evolve toward lower-dimensional stable propagation channels. Competing channels are suppressed by faster loss of coherence under repeated interaction [23].

Interpretation. Perfect symmetry is unstable in real environments. The environment does not choose outcomes; it filters them. What persists is the pattern that remains coherent under repeated local disturbances.

Scope. This mechanism identifies stability selection under environmental coupling. It does not postulate measurement, observers, probabilistic collapse, quantum states, or spacetime causality.

The coherence–selection mechanism governs outcome stability under environmental coupling and provides the foundational grounding for the E2 branch-selection problem developed later in this paper (Section 9). A formal derivation connecting coherence selection to the active admissible branch is identified as an open target.

5 Admissibility Metric on Deformation Space

UGM introduces an admissibility metric that does not measure distance in space or time. Instead, it measures the grid's resistance to deformation and classifies proposed updates as admissible, marginal, or forbidden. The term *metric* is used here in the general state-space sense [21, 22] and should not be interpreted as a spacetime interval. States corresponding to infinite curvature, divergence, or breakdown are rejected prior to phenomenological modelling.

6 Admissible Path Interpretation (API)

Earlier versions of this paper used language such as "memory is monotonically non-decreasing" or described UGM evolution as "irreversible" as though these were primitives independent of the axioms. That language overstates what the frozen canon says. Axiom 4 states that memory accumulates under admissible updates and cannot diverge; it does not impose global monotonicity or primitive irreversibility.

The API is a structural clarification that brings the derivations into exact alignment with the frozen axioms. No axiom is changed; all prior results are retained or strengthened.

Definition 6.1 (Admissible path)**.** *An admissible path is a sequence* $X_0 \to X_1 \to \cdots$ *such that each transition is generated by the canonical projected update and each* $X_k \in \mathcal{D}$*.*

Definition 6.2 (Realized update branch)**.** *At each update step, exactly one admissible branch is realized. All statements about evolution apply to the realized branch only.*

Definition 6.3 (Pathwise non-negativity)**.** *Along any realized admissible path, the memory increment satisfies* $\Delta M_{\text{realized}}(\mathbf{r}) := M_{n+1}(\mathbf{r}) - M_n(\mathbf{r}) \geq 0$*. This is a derived property of the write law under admissibility, not a separate axiom.*

Proposition 6.4 (Pathwise consistency)**.** *Given* $\mathcal{D}$ *and* Π_M*, any realized admissible path with non-negative write increment satisfies* $0 \leq M_n(\mathbf{r}) \leq F(S_n(\mathbf{r}))$ *for all* $n \geq 0$*.*

Proof. Induction: $M_0 \in [0, F(S_0)]$ by assumption; $M_{n+1} = \text{clip}(M_n + \Delta M_{\text{realized}}, 0, F(S_{n+1})) \in [0, F(S_{n+1})]$. □

Update ordering and time. Time is not treated as a primitive dimension. It corresponds to the ordered sequence of admissible updates. Directionality arises along realized admissible paths because memory-constrained evolution does not require the same continuation class at each step; this does not amount to a global irreversibility axiom.

Two-level write–release interpretation. The gravity-sector effective balance $\partial_\tau \bar{M} = \omega_W \bar{S} - \bar{M}/\tau_M$ applies to a derived coarse/observer field M_Λ, not to primitive M. Fine-grained: $\Delta M_{\text{realized}} \geq 0$ (memory accumulates). Coarse-grained: apparent "release" is admissible redistribution of the memory field's gravitational contribution within the bounded envelope $0 \leq M \leq F(S)$. The projector Π_M enforces boundedness but does not impose directionality of increments prior to projection.

7 Forward Invariance and Global Contraction on $\mathcal{D}$

The canonical projected update is

$$X_{n+1}(\mathbf{r}) = \Pi_{\mathcal{D}}[X_n(\mathbf{r}) + \alpha L_6 X_n(\mathbf{r}) + \beta H_6 X_n(\mathbf{r})]\,, \tag{3}$$

where L_6 is the primitive six-direction operator and $\Pi_{\mathcal{D}} : \mathbb{R}^2 \to \mathcal{D}$.

Theorem 7.1 (Forward invariance of $\mathcal{D}$)**.** *If $X_n(\mathbf{r}) \in \mathcal{D}$ for all $\mathbf{r}$, then $X_{n+1}(\mathbf{r}) \in \mathcal{D}$ for all $\mathbf{r}$.*

Proof. $\Pi_{\mathcal{D}}$ maps every input into $\mathcal{D}$ by definition; hence $X_{n+1}(\mathbf{r}) = \Pi_{\mathcal{D}}[\cdots] \in \mathcal{D}$ regardless of the pre-projection state. □

Corollary 7.2. *If $X_0(\mathbf{r}) \in \mathcal{D}$, then $X_n(\mathbf{r}) \in \mathcal{D}$ for all $n \geq 0$.*

This theorem gives Axiom 3 a rigorous dynamic realization as a provable property of the canonical update rule. Forward invariance via $\Pi_{\mathcal{D}}$ is stronger than intrinsic contraction of the unprojected flow and is sufficient for Axiom 3.

7.1 Global Contraction on $\mathcal{D}$

Forward invariance establishes that admissible solutions remain bounded. The following theorem strengthens this: admissible evolution is contractive, meaning that two nearby admissible trajectories converge, guaranteeing uniqueness and asymptotic stability without importing new axioms.

Definition 7.3 (Admissible distance on $\mathcal{D}$)**.** *For $X = (S_1, M_1)$, $Y = (S_2, M_2) \in \mathcal{D}$, define*

$$d(X, Y) := \|S_1 - S_2\| + \|M_1 - M_2\|, \tag{4}$$

where $\|\cdot\|$ is the L^2-norm over the grid domain. This is finite by Axiom 0 (bounded change) and well-defined on $\mathcal{D}$ by the structural bound $0 \leq M \leq F(S) \leq M_{\max} < \infty$.

Theorem 7.4 (Global contraction on $\mathcal{D}$)**.** *Let $X_n, Y_n \in \mathcal{D}$ be two admissible trajectories under the canonical projected update (3). Assume the admissibility stability condition*

$$\alpha k > \beta C_H, \tag{5}$$

where $k > 0$ is the diffusive-dominance coefficient of L_6 and $C_H \geq 0$ is the bounded operator norm of H_6. Then there exists $\lambda \in (0, 1)$ such that

$$d(X_{n+1}, Y_{n+1}) \leq \lambda\, d(X_n, Y_n) \tag{6}$$

for all $n \geq 0$, with $\lambda = 1 - \alpha k + \beta C_H$.

Proof. Let $T(X) := X + \alpha L_6 X + \beta H_6 X$ denote the pre-projection step.

Step 1 (Pre-projection distance). By linearity of T:

$$d(T(X_n), T(Y_n)) \le d(X_n, Y_n) + \alpha\|L_6(X_n - Y_n)\| + \beta\|H_6(X_n - Y_n)\|.$$

Step 2 (Operator bounds from admissibility). H_6 is bounded under the admissible domain: there exists $C_H \ge 0$ such that $\|H_6\Delta\| \le C_H\|\Delta\|$ for all Δ in the admissible difference space (by Axiom 3, which prevents divergence of H_6 on $\mathcal{D}$). L_6 is the primitive six-direction discrete Laplacian, negative semi-definite with leading continuum form $(\ell^2/4)\nabla^2$. Its diffusive dominance property means that for the difference $\Delta = X_n - Y_n$ restricted to $\mathcal{D}$, there exists $k > 0$ such that

$$\|L_6\Delta\| \le -k\|\Delta\| + \text{boundary terms},$$

where boundary terms vanish by locality and compact support.

Step 3 (Pre-projection contraction). Combining Steps 1 and 2:

$$d(T(X_n), T(Y_n)) \le (1 - \alpha k + \beta C_H)\, d(X_n, Y_n).$$

Step 4 (Admissibility stability condition). For $\lambda := 1 - \alpha k + \beta C_H \in (0, 1)$, condition (5) is required. An update failing (5) generates unbounded difference growth and is excluded by Axiom 0.

Step 5 (Non-expansiveness of $\Pi_{\mathcal{D}}$). $\Pi_{\mathcal{D}}$ onto the convex domain $\mathcal{D}$ is non-expansive: $d(\Pi_{\mathcal{D}}(A), \Pi_{\mathcal{D}}(B)) \le d(A, B)$ for all $A, B \in \mathbb{R}^2$.

Step 6 (Final bound).

$$d(X_{n+1}, Y_{n+1}) = d(\Pi_{\mathcal{D}}(T(X_n)), \Pi_{\mathcal{D}}(T(Y_n))) \le d(T(X_n), T(Y_n)) \le \lambda\, d(X_n, Y_n). \qquad \square$$

Remark 7.5. *The admissibility stability condition (5) is not an external constraint. Under UGM's axioms, Axiom 0 requires bounded change under repeated updates. An update failing (5) would generate unbounded difference growth between nearby states and is therefore inadmissible. The stability condition is thus derived from admissibility, not added to it.*

Corollary 7.6 (Uniqueness of admissible evolution)**.** *For any $X_0 \in \mathcal{D}$, the admissible trajectory $\{X_n\}$ is unique.*

Proof. If two trajectories $\{X_n\}$ and $\{Y_n\}$ start at $X_0 = Y_0$, then $d(X_0, Y_0) = 0$, and by Theorem 7.4, $d(X_n, Y_n) \le \lambda^n \cdot 0 = 0$ for all $n \ge 0$. $\square$

Corollary 7.7 (Asymptotic stability)**.** *Perturbations of admissible trajectories decay exponentially: $d_n := d(X_n, Y_n) \le \lambda^n d_0$.*

Corollary 7.8 (Attracting structure of $\mathcal{D}$)**.** *The admissible domain $\mathcal{D}$ is not merely forward-invariant but attracting: any two admissible trajectories converge exponentially in the admissible metric. The dynamics on $\mathcal{D}$ possess a unique attractor.*

Proof. By the asymptotic stability corollary, $d_n \to 0$ exponentially. The family of admissible trajectories forms a Cauchy sequence in d, converging to a unique limit in $\mathcal{D}$ by completeness of the bounded domain. $\square$

7.2 Consequences for the Framework

The global contraction theorem has four structural consequences.

(1) **Dynamical well-posedness.** The UGM update rule defines a well-posed dynamical system with a unique trajectory for each admissible initial condition.

(2) **Justification of coarse-graining.** Contraction implies loss of microscopic sensitivity under repeated updates. Two states differing at small scales converge, justifying the coarse-grained observer description $(\bar{S}, \bar{M}) = (C_\Lambda[S], C_\Lambda[M])$ used in the gravity sector.

(3) **Strengthened GR exclusion.** Theorem 7.4 adds to the GR exclusion argument (Theorem 14.3): the contractive irreversible dynamics on $\mathcal{D}$ cannot be mapped to the time-symmetric reversible Cauchy problem of GR, which requires both forward and backward uniqueness without decay.

(4) **Structural basis for the arrow of time.** Contraction is direction-dependent: $d_n \le \lambda^n d_0$ but not $d_n \ge \lambda^{-n} d_0$. The asymmetry is structural, arising from the admissibility-constrained diffusive dominance of L_6 and the memory accumulation in M, without requiring an independent time-asymmetry postulate.

8 The Primitive Operator L_6 and Its Continuum Limit

By minimization of the coordination-normalized spectral anisotropy functional $A[g]$ over equal-radius planar stencils satisfying moment conditions [4, 5], the unique admissibility-minimizing operator is the six-direction hexagonal operator L_6, satisfying $A_{\text{hex}} = w^2/3 < w^2/2 = A_{\text{sq}}$ for all $w > 0$.

Proposition 8.1 (Continuum expansion of L_6)**.** *For sufficiently smooth u:*

$$L_6 u(\mathbf{r}) = \frac{\ell^2}{4} \nabla^2 u(\mathbf{r}) + \frac{\ell^4}{64} \nabla^4 u(\mathbf{r}) + O(\ell^6). \tag{7}$$

Proof. Taylor-expand $u(\mathbf{r} + \mathbf{d}_j)$; odd terms cancel by $\sum_j \mathbf{d}_j = 0$; the second-moment identity $\sum_{j=1}^{6} \mathbf{d}_j \mathbf{d}_j^\top = 3\ell^2 I$ gives $\ell^2/4$; fourth-order symmetry gives $\ell^4/64$. □

The leading term recovers the isotropic Laplacian. This is the rigorous bridge from admissibility to the continuum gravity sector.

9 The ω_W Problem: No-Go Chain and Bifurcation

The gravity sector uses a coarse write–release law at the observer level:

$$\partial_\tau \bar{M} = \omega_W \bar{S} - \frac{\bar{M}}{\tau_M}. \tag{8}$$

Deriving ω_W from L_6 micro-dynamics is the single most consequential open gap in the programme.

Lemma 9.1 (Quadratic onset — uniform background)**.** *Any isotropic-even shell-mismatch write functional $W[S]$ satisfying $W \ge 0$, $W = 0$ at $\Delta_j S = 0$, and even under sign reversal of increments, satisfies $W[S] = O((\delta s)^2)$ about any locally uniform background.*

Theorem 9.2 (No linear isotropic-curvature write response)**.** *For the isotropic-even shell-mismatch functional, a purely isotropic quadratic background produces no first-order memory-write response at the local expansion center. Deviatoric curvature opens the first linear channel.*

The full no-go chain eliminates: uniform backgrounds; affine backgrounds (drift-filtered by $\sum_j \mathbf{d}_j = 0$); and isotropic quadratic backgrounds. Deviatoric quadratic structure opens the first generic linear channel.

Proposition 9.3 (E1/E2 bifurcation theorem)**.** *Within the current canon, any L_6-native write construction falls into exactly one of two classes:*

(E1) Background-assisted effective response, where ω_W^{eff} is branch-dependent through deviatoric background structure.

(E2) Deeper primitive mixed coupling, a bounded local one-cycle write object whose first-order response is not blocked by even-scalar onset arguments.

A universal background-free primitive ω_W requires Route E2.

10 Primitive Write Law and Memory Architecture

Definition 10.1 (Headroom-gated primitive write law)**.** *Define memory headroom $H_M(\mathbf{r}) := F(S(\mathbf{r})) - M(\mathbf{r}) \geq 0$. The minimal canon-safe primitive write law is*

$$W_{\mathrm{prim}}[S, M](\mathbf{r}) := \kappa\, U_{L6}[S](\mathbf{r}) \cdot H_M(\mathbf{r}), \quad \kappa \geq 0, \tag{9}$$

where $U_{L6}[S](\mathbf{r}) \geq 0$ is a local structural-activity scalar built from the one-cycle six-direction neighbourhood of S.

Proposition 10.2 (Non-negative realized increments under headroom gating)**.** *For $U_{L6} \geq 0$, $\kappa \geq 0$, $\varepsilon \geq 0$, the update $M_{n+1} = \Pi_M(M_n + \varepsilon\kappa U_{L6} H_M;\, S_{n+1})$ satisfies $\Delta M_{\mathrm{realized}} \geq 0$ and $M_{n+1} \leq F(S_{n+1})$. Writing shuts off automatically as $M \to F(S)$ without memory erasure.*

Activity scalar candidates. The shell-mean scalar $U_{L6}^{(N1)} = G(L_6 S)$ has quadratic onset at $L_6 S = 0$. The realized one-cycle update scalar $U_{L6}^{(N2)} = G(\Delta_{\mathrm{upd},+}(\mathbf{r}))$, with $\Delta_{\mathrm{upd},+} := \max(S_{n+1} - S_n, 0)$, is strictly preferable: it ties memory accumulation to actual admissible realized structural change.

The bounded normalized realized-update class $G^{(O3)}(z) = \psi(z_+)$ with $0 \leq \psi \leq 1$, $\psi(0) = 0$, $\psi'(0^+) > 0$ is the only admissible candidate class satisfying locality, non-negativity, boundedness, non-negative realized increments, and first-order activation on the realized-update branch.

11 Coarse Reduction: From Primitive Write to ω_W

11.1 Level-1 linear coarse coefficient

The Level-1 envelope-normalized write law uses $\Delta_{\mathrm{env}} := S_{\max} - S_{\min}$:

$$W_{\mathrm{prim}}^{\mathrm{env}}[S, M](\mathbf{r}) = \kappa\, \frac{\Delta_{\mathrm{upd},+}(\mathbf{r})}{\Delta_{\mathrm{env}}}\, H_M(\mathbf{r}). \tag{10}$$

At a local admissible branch $S = S^\star + \delta s$, $M = M^\star + \delta m$, $H^\star := F(S^\star) - M^\star > 0$, $\Delta_{\mathrm{upd},+} = \Delta^\star + \delta\Delta$:

$$\delta W_{\mathrm{prim}}^{\mathrm{env}} = \frac{\kappa}{\Delta_{\mathrm{env}}}\left[H^\star \delta\Delta + \Delta^\star(F'(S^\star)\delta s - \delta m)\right].$$

Proposition 11.1 (Level-1 linear coarse coefficient exists)**.** *If the active realized-update branch has a first-order response $\delta\Delta = \chi^\star \delta s + O(\delta s^2)$, then*

$$\partial_\tau \bar{M} = \omega_W^{\mathrm{env}} \bar{S} - \frac{\bar{M}}{\tau_M^{\mathrm{env}}} + \mathrm{h.o.t.}, \tag{11}$$

with $\omega_W^{\mathrm{env}} = \kappa(H^\star \chi^\star + \Delta^\star F'(S^\star))/(\Delta\tau\, \Delta_{\mathrm{env}})$. E2 survives at Level 1 without any new postulate.

11.2 Saturation postulates P_W and P_Δ

Postulate P_W. Primitive write activity is the normalized fraction of the maximal admissible positive one-cycle realized structural update: $G^\star(z) = \mathrm{clip}(z/\Delta_{\mathrm{upd}}^{\max}, 0, 1)$.

Postulate P_Δ. $\Delta_{\mathrm{upd}}^{\max} = \sup\{\Delta_{\mathrm{upd},+}(\mathbf{r})$ over admissible one-cycle updates$\}$ is attained at primitive admissibility saturation, not merely bounded by Δ_{env}.

Proposition 11.2 ($P_W + P_\Delta$ remove functional-form ambiguity)**.** *Under $P_W + P_\Delta$, the bounded normalized realized-update class has a unique primitive representative up to κ: $W^\star_{\mathrm{prim}} = \kappa\,\mathrm{clip}(\Delta_{\mathrm{upd},+}/\Delta^{\max}_{\mathrm{upd}}, 0, 1$ H_M.*

11.3 Sharp coarse write coefficient

Proposition 11.3 (Conditional existence of sharp ω_W)**.** *Under $P_W + P_\Delta$ and first-order active-branch response $\delta\Delta = \chi^\star \delta s$:*

$$\partial_\tau \bar{M} = \omega_W \bar{S} - \frac{\bar{M}}{\tau_M} + \mathrm{h.o.t.}, \qquad \omega_W = \frac{\kappa(H^\star \chi^\star + \Delta^\star F'(S^\star))}{\Delta\tau\, \Delta^{\max}_{\mathrm{upd}}}. \tag{12}$$

Proposition 11.4 (Final E2 bottleneck)**.** *After $P_W + P_\Delta$ and Proposition 12, the unresolved E2 target is whether there exists a canon-forced admissible branch for which $H^\star\chi^\star + \Delta^\star F'(S^\star)$ is branch-independent, making ω_W a universal primitive scalar. The combination $H^\star\chi^\star + \Delta^\star F'(S^\star)$ contains three branch-dependent quantities: $H^\star = F(S^\star) - M^\star$ is the local memory headroom; $\chi^\star = (\delta\Delta/\delta s)|^\star$ is the first-order linearization coefficient; and $\Delta^\star$ is the background realized-update amplitude. Branch universality requires showing admissibility forces this combination to a branch-independent value.*

12 Route B: Normalization Principle and $A_{\max}$

The spectral papers [4, 5] fix the reduced primitive-shell geometric coefficient $C[\mathrm{hex}] = 1/3$ via exact pairwise ordering, duplication invariance, and the Brillouin-zone trace identity. Route B converts this into $A_{\max} = f_{\mathrm{sat}}(1/3)$.

Proposition 12.1 (B1–B5 do not uniquely determine f_{sat})**.** *Distinct maps $f_1(x) = x$ and $f_2(x) = x/(1+x)$ both satisfy the five admissibility-compatibility conditions B1–B5.*

Postulate P_A. The reduced primitive-shell coefficient is mapped to $A_{\max}$ by the unique zero-based linear saturation map on the reduced geometric interval: $f_{\mathrm{sat}}(0) = 0$, $f_{\mathrm{sat}}(1/3) = A_{\max}$.

Proposition 12.2 (Unique Route B map under P_A)**.** *Under P_A, the unique admissibility-saturation map is $f_{\mathrm{sat}}(x) = 3A_{\max}x$.*

Proof. Linearity with zero baseline gives $f(x) = ax$; the saturation condition $a \cdot (1/3) = A_{\max}$ forces $a = 3A_{\max}$. □

After P_A, the remaining Route B target is the dimensional bridge $\mathcal{B}(\ell, A_{\max}) = 0$, with all dimensional dependence entering only through ℓ. Once proved, ℓ is solved non-circularly and G, c, h in SI close as one block.

13 Causal Memory Kernel and Connection to the RAR

13.1 Memory kernel construction

In the kinematic discriminator programme [8], the structural memory field is driven by velocity-dispersion evolution. Consider the coarse write–release dynamics at the galactic-disk observer level:

$$\frac{d\zeta}{dt} = \frac{\Phi - \zeta}{\tau_0}, \tag{13}$$

where ζ is a dimensionless structural memory variable and Φ is a driver field. This admits the formal solution

$$\zeta(t) = \int_{-\infty}^{t} \frac{1}{\tau_0} e^{-(t-t')/\tau_0}\, \Phi(t')\, dt'. \tag{14}$$

This expresses ζ as a causal, bounded accumulation of past structural activity weighted by a decaying exponential kernel. The kernel satisfies causal support, unit normalization, and finiteness of ζ for bounded Φ. The exponential kernel represents the minimal causal, bounded, single-timescale solution consistent with admissibility constraints. Uniqueness is not claimed; alternative admissible kernels may exist.

13.2 Observational identification

The driver field is observationally defined as

$$\Phi(t) = \frac{d}{dt}[\sigma^2(t)], \tag{15}$$

so ζ represents a temporally filtered record of changes in the velocity-dispersion field. Under the UGM kinematic discriminator framework [8], ζ corresponds to the dimensionless structural memory field χ. The spatial dependence $\chi(\mathbf{r}, t)$ enters through $\sigma^2(\mathbf{r}, t)$; a first-principles derivation of the spatial structure of χ from primitive admissible dynamics remains open.

This paper	**Kinematic formulation**	**Role**
ζ	χ	Structural memory field
Φ	$d[\sigma^2]/dt$	Write-activity driver
τ_0	τ_M	Memory accumulation timescale
$1/(1+\chi)$	$e^{-\zeta}$	Effective response factor

Table 1: Variable identification between the axiomatic and kinematic formulations.

13.3 Effective source and acceleration scaling

The accumulated structural memory contributes multiplicatively to the instantaneous structural density:

$$\rho_{\text{eff}} = \rho_\kappa(1 + \chi). \tag{16}$$

The corresponding acceleration scaling takes the form

$$a(\mathbf{r}, t) = \frac{a_N(\mathbf{r})}{1 + \chi(t)}, \tag{17}$$

representing memory-mediated screening of the Newtonian field. In the quasi-steady limit, the memory factor produces a continuous transition between Newtonian behaviour ($\chi \to 0$) and a memory-dominated regime ($\chi \gg 1$), reproducing the qualitative structure of the RAR without introducing an independent acceleration constant. The precise MOND interpolating function $\mu(x)$ is not derived here; the present result establishes the structural origin of the transition rather than its exact functional form.

13.4 The timescale τ_0 and the characteristic acceleration scale

The memory timescale τ_0 is not treated as a free fitting parameter. It is constrained by the requirement that the quasi-steady scaling reproduce the observed characteristic acceleration $a^\star \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$. As a dimensional estimate under the assumption that the relevant structural velocity scale is $v \sim \ell/\tau_0$, so that $a \sim v/\tau_0$:

$$a^\star \sim \frac{\ell}{\tau_0^2}. \tag{18}$$

The canonical dimensional statement is Appendix A, which isolates the bridge through the explicit observational quantities $\{\lambda, M_0\}$.

The value $\tau_0 \approx 3\,\mathrm{Gyr}$ is consistent with this constraint and with independent determinations from the MMGS framework linking velocity-dispersion evolution to galactic structural timescales [6, 8]. τ_0 is an empirically anchored structural parameter, not a free fit. A first-principles derivation of $a^\star$ from ω_W, τ_M, Γ_U, ℓ, c remains a landmark target.

The characteristic acceleration scale is therefore interpreted as an emergent quantity arising from the interaction between structural memory accumulation and kinematic evolution timescales, rather than as a fundamental constant of nature.

14 Retained Results: Gravity Hamiltonian, Lorentz, GR Exclusion

14.1 Gravity Hamiltonian and branch structure

The UGM gravity-sector Hamiltonian $\mathcal{H}_{\mathrm{UGM}} = \mathcal{H}_{\mathrm{grad}} + \mathcal{H}_{\mathrm{mem}} + \mathcal{H}_{\mathrm{src}}$ is constructed from admissible fields alone [9]. Stationarity recovers the scalar gravity closure $\Gamma_U \nabla^2(\bar{S} + \bar{M}) = \rho_\kappa + \bar{M}$. Here $\Gamma_U = \pi\sqrt{3}/3$ is the hexagonal geometric constant derived in [9].

Theorem 14.1 (Newtonian branch)**.** *In the limit $\bar{M} \ll \bar{S}$ with $\partial_\tau \bar{S} = 0$, $\Gamma_U \nabla^2 \bar{S} = \rho_\kappa$ with*

$$G = \frac{1}{4\pi\Gamma_U} = \frac{\sqrt{3}}{4\pi^2} \approx 0.0439. \tag{19}$$

The inverse-square law follows as the spherically symmetric Green's function. G is not a free parameter.

Theorem 14.2 (MOND-like branch)**.** *In the quasi-steady memory-dominated limit $\partial_\tau \bar{M} \approx 0$, $\bar{M} \gg \bar{S}$: $\Gamma_U \nabla^2 \bar{M} = \rho_\kappa + \bar{M}$ (screened Poisson). A single-argument effective modification $a_{\mathrm{eff}} = \mu(a_N/g^\dagger)a_N$ is structurally indicated by the Hamiltonian equations of motion. The transition scale*

$$g^\dagger = \frac{\omega_W \tau_M - 1}{\tau_M}\,\bar{S}_0 \tag{20}$$

is structurally identified; its numerical value awaits derivation of ω_W.

Falsifiable post-Newtonian correction: $\delta F \sim \ell^2 GM/r^4$.

14.2 Lorentz package

From the hexagonal Gram matrix, irreversible count N, propagation ceiling $c = \ell/\Delta\tau$, and saturation condition, one obtains the invariant $\mathcal{Q} := c^2t^2 - |\mathbf{R}|^2$, yielding $\tau = t\sqrt{1 - v^2/c^2}$, $L = L_0\sqrt{1 - v^2/c^2}$, $E = \gamma E_0$, $\mathbf{p} = \gamma m_0 \mathbf{v}$, $E^2 - p^2c^2 = E_0^2$. No spacetime metric is imported.

14.3 Structural exclusion of General Relativity

Theorem 14.3 (GR is not a limit of UGM)**.** *No sequence of admissible UGM states converges to a time-symmetric Lorentzian manifold while remaining within the admissible domain $\mathcal{D}$.*

Proof sketch. (1) By the state definition and Axiom 4, M is a non-trivial state variable within $\mathcal{D}$ whenever the grid has retained deformation history. (2) GR has no primitive memory field; its Cauchy state is encoded in $g_{\mu\nu}$ and first derivatives. Memory-dependent admissibility (M constrains future evolution through $F(S)$) has no GR counterpart. (3) For a GR limit, $M \to 0$ is required. This would require either a trivially static grid with no gravitational dynamics, or driving M to the degenerate boundary $\partial\mathcal{D}$, excluded by forward invariance (Theorem 7.1).

(4) Independently, UGM update ordering admits only one realized branch per step; GR's bidirectional Cauchy structure has no admissible UGM correspondent. Both routes independently exclude GR. □

Corollary 14.4. *The Hawking–Penrose singularity theorems [20] have no purchase in UGM: their assumption of geodesic completeness on a smooth causal manifold fails, and singularities are inaccessible by Axiom 3.*

15 Empirical Programme

Pre-registered; archival analysis pending. The planetary atmospheric programme [7] defines operators, observables, and falsification thresholds in advance of archival Cassini/JunoCam analysis.

Pre-registered. The spiral-disk programme [6] declares six Rubin/LSST-facing predictions in advance of survey testing. The MMGS/kinematic programme anchors $\tau_0 \approx 3\,\text{Gyr}$ empirically [8].

Partial — galactic disk / RAR programme. The current Gaia-based kinematic discriminator [8] returns a non-detection of non-quasi-steady behaviour at present resolution ($D(R) \approx 10^{-2}$, detection floor $D \approx 0.03$–0.09 at 95% power), and the NGC 3198 outer-regression test is a null result at current sensitivity. These outcomes are consistent with the quasi-steady branch assumption and define the current empirical detection floor rather than a completed external confirmation. The admissibility condition $\text{Var}[X] < \infty$ is identified as a structural pathway toward the empirically observed scatter suppression in the RAR sample [13]. The memory-mediated effective source $\rho_{\text{eff}} = \rho_\kappa(1 + \chi)$ provides the structural mechanism. The exact values of $g^\dagger$ and $\mu(x)$ are not yet derived from first principles.

The framework is falsifiable through deviations from predicted memory-mediated scaling relations in galaxy kinematics, breakdown of the quasi-steady/active branch structure under improved time-domain surveys, or failure of the fixed-parameter programme across galaxy classes.

16 Conclusion

V02.10 consolidates the full foundational architecture of UGM and closes one previously open gap: the global contraction theorem. The frozen axioms remain unchanged. The primitive update loop, coherence–selection mechanism, and admissibility metric are stated explicitly so that the document functions both as a foundational paper and as the current submission draft.

The API replaces globally monotone-memory language with pathwise non-negativity of realized increments, a weaker and more accurate statement derived from admissibility and branch selection. The global contraction theorem (Theorem 7.4) establishes that admissible evolution is not merely bounded but contractive on $\mathcal{D}$: two admissible trajectories converge exponentially, guaranteeing uniqueness, asymptotic stability, and an attracting structure on $\mathcal{D}$. This upgrades the framework from a collection of admissible states to a dynamically well-posed system, justifies coarse-graining, and strengthens the GR exclusion argument.

The primitive write programme is reduced to two explicit closure bottlenecks: E2 branch universality and the dimensional bridge $\mathcal{B}(\ell, A_{\max}) = 0$. The causal memory kernel connects structural memory accumulation to galactic kinematics and provides a structural path toward the RAR without a new primitive acceleration constant.

Two parallel open bottlenecks remain:

- **E2 branch universality:** is there a canon-forced admissible branch making ω_W background-independent?
- **Dimensional bridge:** does $\mathcal{B}(\ell, A_{\max}) = 0$ hold with dimensional dependence entering only through ℓ?

The complete derivation chain stands:

$$\text{Admissibility} \to L_6 \to \frac{\ell^2}{4}\nabla^2 \to \Gamma_U \nabla^2 \Phi = \rho_\kappa \to G = \frac{\sqrt{3}}{4\pi^2} \to F \propto r^{-2},$$

with the Lorentz package and the memory-mediated gravity branch structure carried in parallel. Every closed arrow is presented as a derivation, not an assertion.

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A Provisional Observational Normalization of G in SI Units

The present framework derives the dimensionless gravitational coupling $G_{\text{UGM}} = \sqrt{3}/4\pi^2$ from admissibility and grid geometry alone. The conversion to SI units requires a dimensional bridge between structural units and physical units. That bridge is not yet derived from the frozen axioms and is therefore isolated explicitly.

Define the observational bridge

$$a^\star = \lambda \frac{\ell}{\tau_0^2}, \tag{21}$$

where $a^\star$ is the characteristic acceleration scale inferred from the RAR, τ_0 is the empirically constrained memory timescale, ℓ is the structural length scale, and λ is a dimensionless bridge factor. Then

$$\ell = \frac{a^\star \tau_0^2}{\lambda}. \tag{22}$$

The SI gravitational constant takes the form

$$G_{\text{SI}} = G_{\text{UGM}} \frac{\ell^3}{M_0 \tau_0^2}, \tag{23}$$

where M_0 is the SI mass corresponding to one unit of structural mass.

This appendix does not claim a derivation of G in SI units from the frozen axioms alone. It isolates the remaining dimensional normalization into explicit observational bridge quantities $\{\lambda, M_0\}$. Their determination from the combined Gaia–Rubin kinematic programme is a defined next step in the empirical closure programme.

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