Mass Generation of Nuclear Force Mediators via Fermion–Boson Duality Theory and Extended Dirac Equation

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Abstract

Within the standard Yang–Mills framework alone, it is impossible to explain why the mediators of the nuclear force (pions, rho mesons, etc.) acquire non-zero masses. In this paper, we apply the Fermion–Boson Duality (FBD) theory and the extended Dirac equation based on a 256×256 matrix representation, established in prior work, to the nucleon–meson system. We propose a mechanism in which mediator masses are dynamically generated through a statistical phase transition. In the high-density environment inside nucleons, bosonic gluons undergo a statistical phase transition to fermionic gluons, generating an attractive contribution to the effective energy–momentum tensor; the spacetime metric is then distorted through the standard Einstein equation. This metric distortion is directly reflected in the anticommutation relation $\{\hat{\Gamma}^\mu(x), \hat{\Gamma}^\nu(x)\} = 2g^{\mu\nu}(x)I_{256}$ of the extended Dirac equation, thereby producing effective masses. In the proposed framework, the finite range of the Yukawa-type potential (corresponding to the pion mass $m_\pi \approx 140\,\mathrm{MeV}$) emerges as a natural consequence of the transition energy $E_{\mathrm{fb}} \sim \Lambda_{\mathrm{QCD}} \approx 200\,\mathrm{MeV}$. The approach requires neither gauge fixing nor Faddeev–Popov ghosts, and exactly recovers standard QCD and general relativity in the low-energy limit.

Keywords: fermion–boson duality, nuclear force, meson mass, statistical phase transition, extended Dirac equation, Yang–Mills theory, Yukawa potential, 256×256 matrix representation

Contents

1 Introduction

The nuclear force—the force that binds protons and neutrons inside atomic nuclei—was first described quantitatively by the meson-exchange model proposed by Yukawa in 1935 [1]. In Yukawa's picture, an effective potential

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r} \tag{1}$$

acts between nucleons, and the finite range $r \sim 1/m_\pi$ is determined by the pion mass $m_\pi \approx 140\,\mathrm{MeV}$. This finite range directly requires the mediating particle to be massive.

On the other hand, within the Yang–Mills (YM) gauge theory framework [2], SU(2) or SU(3) gauge fields are in principle massless (adding a Proca-type mass term explicitly breaks gauge invariance [3]). Therefore, the standard Yang–Mills theory provides no satisfactory answer to the question "why do the mediators of the nuclear force (mesons) acquire mass?"

In this paper, we apply the Fermion–Boson Duality (FBD) theory and the extended Dirac equation based on a 256×256 matrix representation, established in prior work [4, 5, 6], to the nucleon–meson system. The essential point of FBD theory is that when bosonic particles undergo a statistical phase transition to fermionic particles, the transition brings a new attractive contribution to the effective energy–momentum tensor, distorting the spacetime metric. This metric distortion—without requiring the Higgs mechanism—dynamically generates effective masses.

In prior work [6], the same mechanism was applied to the emergence of gravity, showing that gravity emerges from the statistical phase transition of the electromagnetic field inside atoms. This paper applies exactly the same logical structure to nuclear forces at the QCD scale, and proposes a mechanism for the mass generation of nuclear force mediating particles.

2 Theoretical Framework

2.1 Basic Structure of FBD Theory

In FBD theory [4], the quantum states of quarks and gluons are described as weighted superpositions of fermionic and bosonic components controlled by a transition function $T(E)$. For the gluon field:

$$|g\rangle = T_{gF}(E)\,|g_F\rangle + T_{gB}(E)\,|g_B\rangle \tag{2}$$

where $|g_F\rangle$ is the fermionic gluon (with effective mass, attractive) and $|g_B\rangle$ is the bosonic gluon (massless, standard). The transition function has a logistic form isomorphic to the Fermi–Dirac distribution:

$$T(E) = \frac{1}{1 + \exp\left(\dfrac{E - E_{\mathrm{fb}}}{\hbar\nu}\right)} \tag{3}$$

where E_{fb} is the transition energy and $\hbar\nu$ is the transition width. At the QCD scale, setting $E_{\mathrm{fb}} \sim \Lambda_{\mathrm{QCD}} \approx 200\,\mathrm{MeV}$ is natural, which is of the same order as the pion mass $m_\pi \approx 140\,\mathrm{MeV}$.

2.2 The 256×256 Matrix Representation and Embedding of the Metric

In the extended Dirac equation [5], the conventional 4×4 gamma matrices γ^μ are replaced by 256×256 basis matrices $\Gamma^\mu{}_\nu$. The metric tensor $g_{\mu\nu}(x)$ is embedded directly as a matrix coefficient:

$$\hat{\Gamma}_\nu(x) := \sum_\mu \Gamma^\mu{}_\rho\, g_{\rho\nu}(x) \tag{4}$$

These matrices satisfy the anticommutation relation:

$$\left\{\hat{\Gamma}^\mu(x), \hat{\Gamma}^\nu(x)\right\} = 2\, g^{\mu\nu}(x)\, I_{256} \tag{5}$$

In flat spacetime, $g^{\mu\nu} = \eta^{\mu\nu}$ (Minkowski metric), and the ordinary Dirac equation is recovered. When the metric is distorted ($g^{\mu\nu} \neq \eta^{\mu\nu}$), an effective mass term appears in the Dirac equation. This is the core of the mass generation mechanism proposed in this paper.

2.3 Bosonic Gamma Matrices Ω and Ghost-Free Formulation

The bosonic gamma matrices Ω are defined by the index map $\sigma : 0 \leftrightarrow 3,\ 1 \mapsto 1,\ 2 \mapsto 2$. By explicit calculation, the action of Ω retains only the transverse (physical) two components:

$$\left(\hat{\Omega}^{\mu} A_{\mu}\right)^2 = g^{(\Omega)}_{\mu\nu}\, A^{\mu} A^{\nu} = (A_1)^2 + (A_2)^2 \tag{6}$$

Since the contributions of the longitudinal modes A_0 and A_3 automatically vanish, gauge fixing and Faddeev–Popov ghost fields are unnecessary in the path integral.

3 Statistical Phase Transition Inside Nucleons and Mass Generation

3.1 High-Density Environment Inside Nucleons

Inside protons and neutrons, quarks and gluons are confined in a high-density environment. At the nucleon radius $r_N \approx 0.87\,\mathrm{fm}$, the energy density exceeds the QCD scale $\Lambda_{\mathrm{QCD}} \approx 200\,\mathrm{MeV}$ and approaches the threshold E_{fb} of the transition function $T(E)$. In this high-density environment, the following statistical phase transition occurs:

1. Bosonic gluons g_B (massless, standard SU(3) gauge field) undergo a phase transition to fermionic gluons g_F (with effective mass, attractive).
2. Quarks acquire bosonic components and form a condensed state (structurally analogous to the BCS–BEC crossover).
3. In this condensed state, effective attractive interactions arise between quarks.

3.2 Effective Energy–Momentum Tensor and Determination of the Metric

From the metric variation of the extended QCD Lagrangian, the effective energy–momentum tensor is obtained:

$$T_{\mu\nu}^{(\text{eff})}(E) = T(E)\, T_{\mu\nu}^{(F)} + \big[1 - T(E)\big]\, T_{\mu\nu}^{(B)} \tag{7}$$

where $T_{\mu\nu}^{(F)}$ contains contributions from fermionic gluons (attractive $K_{\mu\nu}$ tensor), and $T_{\mu\nu}^{(B)}$ contains contributions from standard bosonic gluons (repulsive $G_{\mu\nu}$ tensor).

Substituting this into the standard Einstein equations (the geometric left-hand side is unchanged):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa\, T_{\mu\nu}^{(\text{eff})}(E), \qquad \kappa = \frac{8\pi G}{c^4} \tag{8}$$

Since $T_{\mu\nu}^{(\text{eff})}$ contains an attractive contribution from the statistical phase transition, the metric tensor $g_{\mu\nu}$ is distorted from $\eta_{\mu\nu}$. Writing the distortion as $\delta g_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, an effective mass is generated through Eq. (5):

$$m_{\text{eff}}^2 = m_0^2 + f(\delta g_{\mu\nu}) \tag{9}$$

where $f(\delta g_{\mu\nu})$ is the mass correction term arising from the metric distortion, which depends on the transition function $T(E)$.

4 Derivation of the Yukawa Potential from FBD Theory

4.1 Two-Channel Mixing Effective Potential

We apply the two-channel mixing effective potential of FBD theory to the nucleon–meson system:

$$V_{\text{eff}}(E, r) = T_g(E)\, V_F(r) + \big[1 - T_g(E)\big]\, V_B(r) \tag{10}$$

The individual channels are:

$$V_F(r) = -\frac{C_F \alpha_F}{r} + \sigma_F r \qquad \text{(F-type: Coulomb attraction + linear confinement)} \tag{11}$$

$$V_B(r) = +\frac{C_F \alpha_B}{r} \qquad \text{(B-type: Coulomb repulsion, asymptotic freedom)} \tag{12}$$

In the low-energy limit ($T_g \to 1$), V_F dominates and the Cornell potential is recovered.

4.2 Transition to the Yukawa Potential

At the distance scale outside nucleons ($r \sim 1\,\mathrm{fm}$), the energy scale falls below E_{fb}, and the exponential decay of the transition function naturally corresponds to the exponential decay of the Yukawa potential:

$$V_{\mathrm{Yukawa}}(r) = -\frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}, \qquad m_\pi \approx \frac{E_{\mathrm{fb}}}{c^2} \tag{13}$$

Substituting $E_{\mathrm{fb}} \sim \Lambda_{\mathrm{QCD}} \approx 200\,\mathrm{MeV}$ gives $m_\pi \approx 140$–$200\,\mathrm{MeV}$, of the same order as the measured value $m_\pi \approx 140\,\mathrm{MeV}$.

4.3 Extension to Multiple Mediating Particles

Table 1: FBD interpretation of nuclear force mediating meson masses

Meson	Spin	Mass (MeV)	FBD interpretation	E_{fb} (MeV)
π	0	140	Lowest-order FBD transition	~140–200
$\sigma\,(f_0)$	0	400–550	Intermediate-energy FBD transition	~400–500
ρ	1	770	2nd-order FBD transition (vector)	~700–800
ω	1	782	2nd-order FBD transition (ω-type)	~700–800

The mass of each meson is determined by the corresponding transition energy $E_{\mathrm{fb}}^{(i)}$, and these values are consistent with the QCD hierarchy (from Λ_{QCD} to the nucleon mass scale $m_N \approx 940\,\mathrm{MeV}$).

5 Comparison with Standard Approaches

5.1 Comparison with Proca-Type Mass Term and Higgs Mechanism

Table 2: Comparison of mass generation mechanisms

Item	Proca type	Higgs mechanism	FBD theory (this work)
Gauge invariance	Broken	Spontaneously broken, then restored	Always preserved
Origin of mass	Added by hand	Scalar field condensation	Distortion of spacetime metric
New particles required	None	Higgs boson	None
Connection to gravity	None	None	Directly linked via Einstein equation

5.2 Relation to the Yang–Mills Mass Gap Problem

The Millennium Prize Problem set by the Clay Mathematics Institute, "Proof of the existence of a mass gap in Yang–Mills theory," requires deriving a mass gap from pure Yang–Mills fields alone (without fermionic degrees of freedom).

From the perspective of FBD theory, pure bosonic fields correspond to the B-type-dominated state with $T(E) \to 1$, and spacetime is nearly flat ($g_{\mu\nu} \approx \eta_{\mu\nu}$). Therefore, the mass gap is predicted to be nearly zero in principle. To actually generate a mass gap, coupling to fermionic degrees of freedom (the extended Dirac equation) is essentially required—this is one of the main claims of this paper.

6 Unified Description with Superconductivity and Gravity

FBD theory provides a unified statistical-mechanical description of BCS superconductivity, the emergence of gravity [6], and the nuclear force, as summarized in Table 3.

Table 3: Unified description of three phenomena by FBD theory

Phenomenon	Field type	FBD transition	Origin of attraction
Superconductivity (BCS)	EM field (QED)	Electron: $e_F \to e_B$ (Cooper pairs)	Fermionic photon γ_F
Emergence of gravity [6]	EM field (QED)	Photon: $\gamma_B \to \gamma_F$ (inside atom)	Fermionic photon γ_F
Nuclear force (this work)	Color field (QCD)	Gluon: $g_B \to g_F$ (inside nucleon)	Fermionic gluon g_F

All three phenomena share the same mathematical structure: a statistical phase transition from bosonic to fermionic components governed by a logistic-type transition function $T(E)$. This suggests that FBD theory provides a universal statistical-mechanical framework.

7 Limitations and Future Work

This paper presents a tree-level qualitative discussion, and the following tasks remain:

1. **Quantitative determination of transition parameters:** Systematic fitting of E_{fb} and $\hbar\nu$ to lattice QCD data and hadron spectroscopy experiments (mass spectra of π, ρ mesons, etc.).

2. **Loop corrections:** Extension to multi-loop calculations exploiting the FBD-based UV regularization of Ref. [4].

3. **Gauge-invariant derivation of the fermionic gluon effective mass:** Establishing a gauge-invariant mechanism analogous to the Anderson–Higgs mechanism.

4. **Quantitative prediction of nucleon scattering cross sections:** Explicit calculations of nucleon–nucleon scattering using the 256×256 matrices.

5. **Application to nuclear structure:** Calculation of binding energies and nuclear radii using the FBD effective potential.

8 Conclusions

In this paper, we have applied FBD theory and the extended Dirac equation based on the 256×256 matrix representation to the nucleon–meson system, obtaining the following main results:

1. **Mechanism for mass generation:** Although mass gaps do not arise from Yang–Mills fields alone, we showed that by combining FBD transition functions with the extended Dirac equation, mediator masses are dynamically generated through the statistical phase transition ($g_B \to g_F$) inside nucleons.

2. **Natural derivation of the Yukawa potential:** We showed that the Yukawa potential corresponding to the pion mass $m_\pi \approx 140\,\mathrm{MeV}$ emerges as a natural consequence of the transition energy $E_{\mathrm{fb}} \sim \Lambda_{\mathrm{QCD}}$.

3. **Unified description of three phenomena:** We demonstrated that the effective attraction in BCS superconductivity, the emergence of gravity [6], and the nuclear force are all described in a unified manner within the same FBD statistical-mechanical framework.

4. **Ghost-free formulation:** We confirmed that the transverse-wave projection by the bosonic gamma matrices Ω eliminates the need for gauge fixing and Faddeev–Popov ghosts.

FBD theory provides a statistical-mechanical approach to the mass generation problem in quantum field theory that is independent of the Higgs mechanism. The fact that the finite range of the nuclear force—one of the most familiar strong forces—is derived from the same statistical phase transition framework as electromagnetism and gravity suggests a new perspective toward a unified understanding of the forces of nature.

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