Heavy Neutrinos at Colliders Worksheet

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I Type I Seesaw Mechanism

Consider a single generation of SM leptons. Now suppose the existence of a right-handed (R.H.) neutrino N_R, which is a singlet under the $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}_Y$ gauge group.

Question: Derive the Dirac neutrino mass $m_D = y_\nu \langle \Phi \rangle$.

Question: Derive the neutrino-Goldstone boson couplings.

Now suppose that N_R is described by the Lagrangian

$$\mathcal{L}_N = \frac{1}{2}\overline{N_L^c} i\gamma^\mu \partial_\mu N_R - \frac{1}{2} m_R^2 \overline{N_L^c} N_R, \tag{1}$$

where $\psi^c = \mathcal{C}\overline{\psi}^T$ denotes the charge conjugate of the spinor field ψ, with $\mathcal{C}$ labeling the charge conjugation operator, and the chiral states satisfy $(\psi^c)_L \equiv (\psi_R)^c = \psi_R^c$.

Question: Supposing $m_N \gg m_D$, show that the two mass eigenstates are

$$m_{\text{light}} \approx m_D^2/m_R \quad \text{and} \quad m_{\text{heavy}} \approx -m_R. \tag{2}$$

II Chiral Couplings to Heavy Neutrinos

Assume that there are three left-handed (L.H.) neutrinos (denoted by $\nu_{aL}, a = 1, 2, 3$) with Dirac masses (m), and n right-handed (R.H.) neutrinos (denoted by $N_{a'R},\ a' = 1, \ldots, n$) with Majorana masses ($m'$). The mixing between chiral states and mass eigenstates can be parameterized by

$$\begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix} = \begin{pmatrix} U_{3\times 3} & V_{3\times n} \\ X_{n\times 3} & Y_{n\times n} \end{pmatrix} \begin{pmatrix} \nu_m \\ N_{m'}^c \end{pmatrix}, \tag{3}$$

In the notation of [1], the flavor state ν_ℓ in the mass basis is explicitly

$$\nu_\ell = \sum_{m=1}^{3} U_{\ell m} \nu_m + \sum_{m'=1}^{n} V_{\ell m'} N_{m'}^c. \tag{4}$$

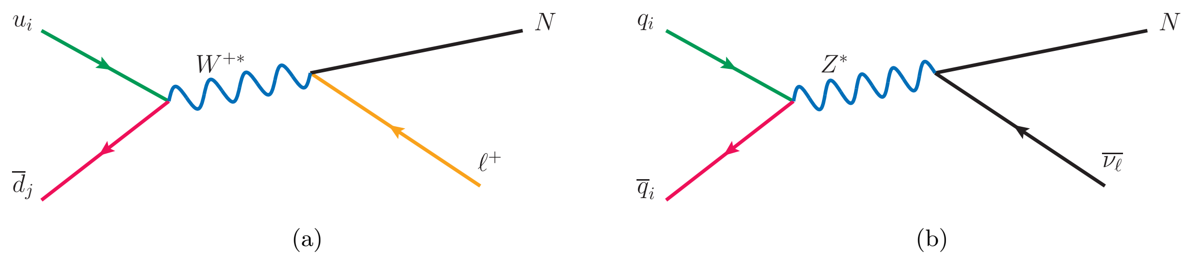


Figure 1: (a) Born-level partonic diagram depicting the production of a heavy neutrino (N) and a charged antilepton (ℓ^+) through the charged-current Drell-Yan process. (b) Same but for a light antineutrino ($\overline{\nu_\ell}$) through the neutral-current Drell-Yan process.

Question: Show that the couplings to electroweak bosons can be written as

$$\begin{aligned}
\mathcal{L}_{\text{Int.}} = \quad & - \frac{g}{\sqrt{2}} W_\mu^+ \sum_{\ell=e}^{\tau} \sum_{m=1}^{3} \overline{\nu_m}\, U_{\ell m}^*\, \gamma^\mu P_L \ell^- - \frac{g}{\sqrt{2}} W_\mu^+ \sum_{\ell=e}^{\tau} \overline{N^c}\, V_{\ell N}^*\, \gamma^\mu P_L \ell^- \\
& - \frac{g}{2\cos\theta_W} Z_\mu \sum_{\ell=e}^{\tau} \sum_{m=1}^{3} \overline{\nu_m}\, U_{\ell m}^*\, \gamma^\mu P_L \nu_\ell - \frac{g}{2\cos\theta_W} Z_\mu \sum_{\ell=e}^{\tau} \overline{N^c}\, V_{\ell N}^*\, \gamma^\mu P_L \nu_\ell \\
& - \frac{g m_N}{2 M_W} h \sum_{\ell=e}^{\tau} \overline{N^c}\, V_{\ell N}^* P_L \nu_\ell + \text{H.c.}
\end{aligned} \tag{5}$$

Note that the above expression is a mixture of mass and interactions states, whereby only one set of neutrinos in each interaction has been decomposed into mass eigenstates.

III Partonic $q\overline{q'} \to W^{+*} \to N\ell^+$ Scattering

Consider the charged current Drell-Yan process,

$$u(p_u) + \overline{d}(p_d) \to W^{+*} \to \ell^+(p_\ell) + N(p_N), \tag{6}$$

as shown in Fig. 1(a), where N is a heavy neutrino mass eigenstate with with mass m_N.

Question: In the c.m. frame, what are the momentum assignments for the external and internal particles?

Question: In terms of partonic Mandelstam variables $\hat{s}$, $\hat{t}$, $\hat{u}$, and assuming a Breit-Wigner propagator, derive the following matrix amplitudes in the helicity basis:

$$\mathcal{M}(u_L \overline{d}_R \to W^{+*} \to \ell_R^+ N_R) = -i\frac{g^2}{2} V_{ud}^* V_{N\ell} \sin\theta_\ell \, \frac{\hat{s}\,\sqrt{(1-r_N)r_N}}{\hat{s} - M_W^2 + iM_W\Gamma_W} \tag{7}$$

$$\mathcal{M}(u_L \overline{d}_R \to W^{+*} \to \ell_R^+ N_L) = -i\frac{g^2}{2} V_{ud}^* V_{N\ell} (1-\cos\theta_\ell) \, \frac{\hat{s}\,\sqrt{1-r_N}}{\hat{s} - M_W^2 + iM_W\Gamma_W} \tag{8}$$

Here and below, we define the ratio r_i such that $M_i^2 = r_i \cdot \hat{s}$. The angle θ_ℓ is given by

$$\cos\theta_\ell \equiv \frac{\vec{p}_u \cdot \vec{p}_\ell}{|\vec{p}_u|\,|\vec{p}_\ell|}. \tag{9}$$

Question: All other helicity combinations vanish. Why?

Partonic-level cross sections are obtained using the formula

$$d\hat{\sigma}(A+B\to 1+\cdots+n) = \frac{1}{2\hat{s}}\frac{1}{(2s_A+1)(2s_B+1)N_C^A N_C^B}\sum_{\text{d.o.f.}}|\mathcal{M}|^2\cdot \text{dPS}_n, \tag{10}$$

where $s_{A/B}$ and $N_C^{A/B}$ represent the spin and color of initial state parton A/B. dPS_n represents the n-body phase space volume element for total momentum P,

$$dPS_n(P;p_1\dots p_n) = \frac{1}{\Omega}\prod_{k=1}^{n}\frac{d^3p_k}{(2\pi)^3 2E_k}\ (2\pi)^4\delta^4\left(P-p_1-\cdots-p_n\right). \tag{11}$$

Here, Ω is a symmetry factor that accounts for the production of identical particles. It is the product of $(n_l!)$ factors, i.e., $\Omega=\prod_l(n_l!)$, where $n_l\leq n$ is the number indistinguishable final-state particles of species l. Summing over all species recovers the total number of final states: $\sum_l n_l = n$.

Question: For one- and two-body differential phase space volumes, show

$$\begin{aligned}
dPS_1 &= (2\pi)^4\ \frac{d^3p_1}{(2\pi)^3 2E_1}\ \delta^4(P_{\text{Tot.}}-p_1) = 2\pi\ \delta(\hat{s}-m_1^2), && (12)\\
dPS_2(P_{\text{Tot}};p_1,p_2) &= (2\pi)^4\ \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\ \delta^4(P_{\text{Tot.}}-p_1) && (13)\\
&= \frac{d\cos\theta^{p_1}d\phi^{p_1}}{2(4\pi)^2}(1-r_1),\quad \text{assuming}\quad r_2=0. && (14)
\end{aligned}$$

In the above, there is implicit integration over the δ-functions.

Question: Show that partonic cross section for Eq. (6) is

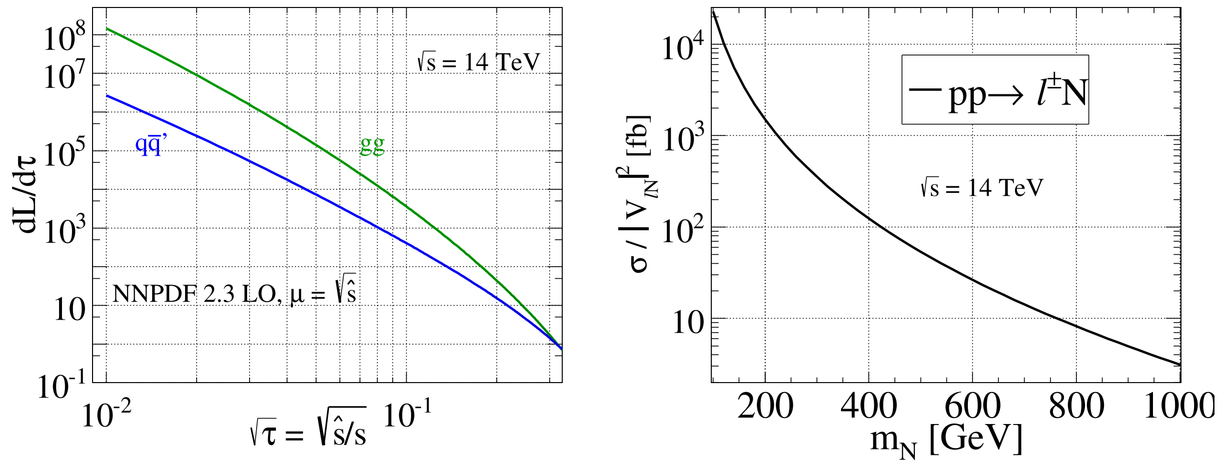
$$\begin{aligned}
\hat{\sigma}_{\text{CCDY}}(u\overline{d}\to W^{+*}\to N\ell^+) &= \hat{\sigma}_{\text{CCDY}}(\overline{u}d\to W^{-*}\to N\ell^-) && (15)\\
&= G_F^2\ \frac{M_W^4\ |V_{ud}|^2|V_{N\ell}|^2}{2^2\cdot 3\ N_C\ \pi}\ \frac{\hat{s}\ (1-r_N)^2(2+r_N)}{[(\hat{s}-M_W^2)^2+(\Gamma_W M_W)^2]}. && (16)
\end{aligned}$$

Now, consider the neutral current DY process as illustrated in Fig. 1(b) and given by,

$$q(p_A)+\overline{q}(p_B)\to Z^*\to N(p_N)+\overline{\nu_\ell}(p_\nu). \tag{17}$$

Question: Show that helicity amplitudes and partonic cross section are given by

$$\begin{aligned}
\mathcal{M}(q_L\overline{q}_R\to Z^*\to\overline{\nu}_{\ell R}N_L) &= -i\frac{g^2 g_L^q}{2\cos\theta_W}V_{N\ell}(1+\cos\theta_\ell)\ \frac{\hat{s}\ \sqrt{(1-r_N)}}{\hat{s}-M_Z^2+iM_Z\Gamma_Z} && (18a)\\
\mathcal{M}(q_L\overline{q}_R\to Z^*\to\overline{\nu}_{\ell R}N_R) &= -i\frac{g^2 g_L^q}{2\cos\theta_W}V_{N\ell}\sin\theta_\ell\ \frac{\hat{s}\ \sqrt{(1-r_N)r_N}}{\hat{s}-M_Z^2+iM_Z\Gamma_Z} && (18b)\\
\mathcal{M}(q_R\overline{q}_L\to Z^*\to\overline{\nu}_{\ell R}N_L) &= \left(-\frac{g_R^q(1-\cos\theta_\ell)}{g_L^q(1+\cos\theta_\ell)}\right)\times\mathcal{M}(q_L\overline{q}_R\to Z^*\to\overline{\nu}_{\ell R}N_L) && (18c)\\
\mathcal{M}(q_R\overline{q}_L\to Z^*\to\overline{\nu}_{\ell R}N_R) &= \left(\frac{g_R^q}{g_L^q}\right)\times\mathcal{M}(q_L\overline{q}_R\to Z^*\to\overline{\nu}_{\ell R}N_R) && (18d)\\
\hat{\sigma}_{\text{NCDY}}(q\overline{q}\to Z^*\to\overline{\nu_\ell}N) &= \hat{\sigma}_{\text{NCDY}}(q\overline{q}\to Z^*\to\nu_\ell N) && (18e)\\
&= G_F^2\frac{|V_{\ell N}|^2 M_Z^4\left(g_L^{q2}+g_R^{q2}\right)}{12\pi N_c}\frac{\hat{s}(1-r_N)^2(2+r_N)}{\left[(\hat{s}-M_Z^2)^2+(M_Z\Gamma_Z)^2\right]}, && (18f)
\end{aligned}$$



(a) (b)

Figure 2: (a) Parton luminosities at 14 TeV. (b) Leading order 14 TeV LHC $N\ell^{\pm}$ production cross section, divided by active-heavy mixing $|V_{\ell N}|^2$ a function of m_N.

where the chiral couplings are given by the usual SM vector and axial-vector couplings

$$g_R^f = g_V^f + g_A^f = \frac{1}{2}(T_3^f)_L - Q^f \sin^2\theta_W + \left(\frac{-1}{2}\right)(T_3^f)_L = -Q^f \sin^2\theta_W, \tag{19}$$

$$g_L^f = g_V^f - g_A^f = \frac{1}{2}(T_3^f)_L - Q^f \sin^2\theta_W - \left(\frac{-1}{2}\right)(T_3^f)_L = (T_3^f)_L - Q^f \sin^2\theta_W, \tag{20}$$

with $(T_3^f)_L = \pm 1/2$ and $Q^f = +2/3(-1/3)$ for up-(down-)type quarks.

Question: How do these expressions differ from those for the Drell-Yan process in the Standard Model? How can one recover the Standard Model result?

IV Hadronic Level Production

Hadronic-level cross sections are related to partonic scattering process by the Factorization Theorem. We write the production cross section of a heavy state X as

$$\sigma(pp \to X + \text{anything}) = \sum_{i,j}\int_{\tau_0}^1 d\xi_a \int_{\frac{\tau_0}{\xi_a}}^1 d\xi_b \left[f_{i/p}(\xi_a,\mu^2) f_{j/p}(\xi_b,\mu^2)\hat{\sigma}(ij \to X) + (i \leftrightarrow j)\right] \tag{21}$$

$$= \int_{\tau_0}^1 d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to X). \tag{22}$$

where $\xi_{a,b}$ are the fractions of momenta carried by initial partons (i,j), μ is the parton factorization scale, and $\tau = \hat{s}/s$ with $\sqrt{s}$ ($\sqrt{\hat{s}}$) the proton beam (parton) c.m. energy. For heavy neutrino production, the threshold is $\tau_0 = m_N^2/s$. Parton luminosities are given in terms of the parton distribution functions (PDFs) $f_{i,j/p}$ by the expression

$$\Phi_{ij}(\tau) \equiv \frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_\tau^1 \frac{d\xi}{\xi}\left[f_{i/p}(\xi,\mu^2) f_{j/p}\left(\frac{\tau}{\xi},\mu^2\right) + (i \leftrightarrow j)\right], \tag{23}$$

and are a measure of the incoming flux of particles. Consider only light quarks (u, d, c, s).
Question: Show that at 14 TeV, the DY, gq, and gg luminosities are given by Fig. 2(a).

Question: Stipulating all input parameters, e.g., $\sqrt{s}$, PDF set, CKM values, G_F, μ, etc., show that Eq. (22) is given by Fig. 2(b).

Question: Compare your results with MadGraph and Ref. [2].

V Heavy Neutrino Decay

For m_N of a few hundred GeV or more can decay through on-shell SM gauge and Higgs bosons.
Question: Show that the partial widths of the lightest heavy neutrino are

$$\begin{aligned}
\Gamma(N \to \ell^\pm W_0^\mp) &\equiv \Gamma_0 = && \frac{g^2}{64\pi M_W^2}|V_{\ell N}|^2 m_N^3 (1-r_W)^2 \\
\Gamma(N \to \ell^\pm W_T^\mp) &\equiv \Gamma_T = && \frac{g^2}{32\pi}|V_{\ell N}|^2 m_N (1-r_W)^2 \\
\Gamma(N \to \ell^\pm W^\mp) &\equiv \Gamma_W = && \Gamma_0 + \Gamma_T = \frac{g^2 m_N^3}{64\pi M_W^2}|V_{\ell N}|^2 (1-r_W)^2(1+2r_W) \\
\Gamma(N \to \nu_\ell Z) &\equiv \Gamma_Z = && \frac{g^2 m_N^3}{64\pi M_W^2}|V_{\ell N}|^2 (1-r_Z)^2(1+2r_Z) \\
\Gamma(N \to \nu_\ell H) &\equiv \Gamma_H = && \frac{g^2 m_N^3}{64\pi M_W^2}|V_{\ell N}|^2 (1-r_H)^2
\end{aligned} \tag{24}$$

where $W_{0\ (T)}$ is the longitudinally (transversely) polarized W, and $r_i = M_i^2/m_N^2$. The total width is given by

$$\Gamma_{\mathrm{N}} = \sum_{\ell=e}^{\tau} \left(2(\Gamma_0 + \Gamma_T) + \Gamma_Z + \Gamma_H\right). \tag{25}$$

The factor of two in front of $\Gamma_{0,T}$ is from the sum over positively and negatively charged leptons. Also of interest is the branching ratio (BR) of heavy neutrinos into charged leptons:

$$\mathrm{BR}\left(\mathrm{N} \to \ell^\pm \mathrm{W}^\mp\right) = \frac{\sum_{\ell=\mathrm{e}}^{\tau}\left(\Gamma_0 + \Gamma_{\mathrm{T}}\right)}{\Gamma_{\mathrm{Tot}}} \tag{26}$$

Figure 3(a) shows the total decay width (solid) and the partial decay widths to positively charged lepton (dashed) normalized to the sum over the mixing matrices. Although the width appears to be large at high neutrino mass, for small mixing angles the width is still narrow.

Question: Why does the N width grow dramatically with mass? Show this by estimating the matrix element, phase space, and $d\Gamma$ dependence on m_N.

Figure 3(b) shows the total BR of the heavy neutrino into positively charged leptons (solid) and individually the BR into longitudinally (dashed) and transversely (dotted) polarize W's as a function of neutrino mass. The BR's into negatively charged leptons are the same.

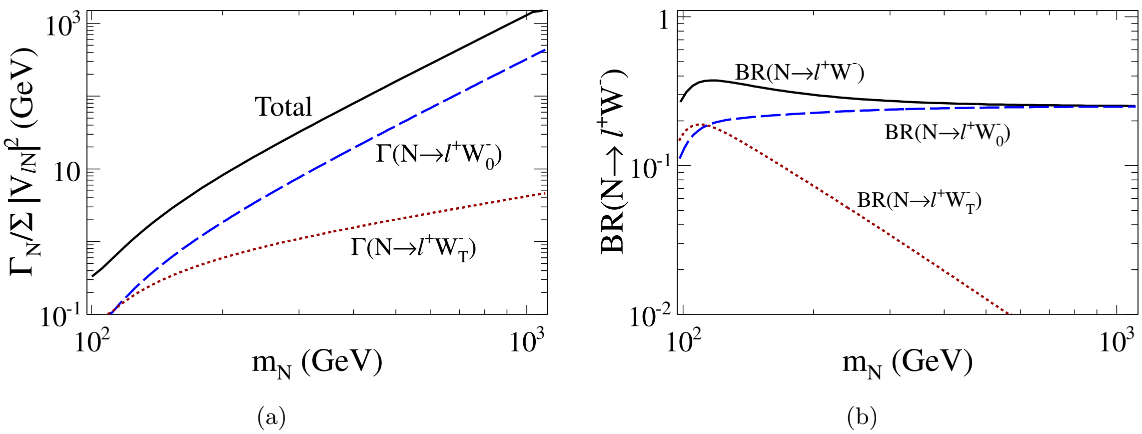


Figure 3: As a function of heavy neutrino mass, (a) the total N width and the $N \to \ell^+ W^-_\lambda$ partial widths, and (b) the combined $N \to \ell^+ W^-$ and individual $N \to \ell^+ W^-_\lambda$ branching ratios for longitudinal ($\lambda = 0$) and transverse ($\lambda = T$) W polarizations.

Question: What is the origin of the bump at low m_N?

Question: Explain the asymptotic behavior at large m_N.

In the narrow width approximation (NWA), a resonant production and decay cross section can be estimated as

$$\sigma(pp \ \to \ A \ \to C \ B) \approx \sigma(pp \ \to \ A) \times \text{BR}(A \ \to \ B \ C). \tag{27}$$

Note that this ignores spin correlation between A and its decay products.

Question: Assuming the NWA, plot the total cross section as a function of m_N for $pp \to \ell^\pm N \to \ell^\pm \ell^\pm q \overline{q}'$. It may be useful to first draw the corresponding Feynman diagrams.

References

[1] A. Atre, T. Han, S. Pascoli and B. Zhang, JHEP **0905**, 030 (2009) [arXiv:0901.3589 [hep-ph]].

[2] R. E. Ruiz, arXiv:1509.06375 [hep-ph].