

of each critical point; that is to say, if near any critical point a each integral can be put into the form

$$y = (x-a)^k \left\{ \phi_0 + \phi_1 \log(x-a) + \dots + \phi_m [\log(x-a)]^m \right\}$$

where $\phi_0, \phi_1, \dots, \phi_m$ are one-valued functions not infinite at a . These equations are called by Prof. Forsyth "equations of Fuchsian type." The equation of the hypergeometric series is of this type, and is remarkable as being the only one, of order higher than the first, which is completely determined when the positions of the critical points and the indices associated with them are assigned.

An equation of Fuchsian type may have one or more algebraic integrals. If all the integrals are algebraic, the group of the equation must be finite; so here we have a most unexpected concurrence of two apparently disconnected theories. A very interesting problem is that of determining linear equations the groups of which are isomorphic with known finite groups; another is that of finding out whether a given equation has any algebraic integrals.

All the foregoing theory is discussed and illustrated by Prof. Forsyth in a very attractive and lucid manner; thus chapter i. deals with the existence of a synectic integral near an ordinary point and sets of independent integrals; chapter ii. with the expansions near a critical point and with Hamburger's method of grouping them; chapter iii. with regular integrals; chapter iv. with equations of Fuchsian type; and chapter v. with equations of the second and third orders possessing algebraic integrals. Illustrations are supplied by the familiar equations of mathematical physics, by the equation of the elliptic quarter-period, and by that of the hypergeometric series. It is delightful to see how the discussion of these equations is illuminated by the general theory.

After a chapter on equations with only some of their integrals regular, we come to the consideration of integrals with essential singularities. The most familiar example of a function with an essential singularity at a finite place is $\exp(x^{-1})$, which is the integral of $x^2 y' + y = 0$; and it is easy to see that if P is any polynomial in x^{-1} , the expression $\exp P$ has an essential singularity and satisfies a linear equation of the first order.

Suppose now that we find that a given equation has an integral with an essential singularity at the origin; it may be possible to express it in the form $\exp P \cdot x^\rho \psi(x)$, where ρ is constant and $\psi(x)$ holomorphic. Such an integral has been called "normal"; the discussion of these integrals, and others obtained by putting $x^{1/k}$ for x , is given in chapter vii., which contains important results due to Thomé, Hamburger, Poincaré and others. There is also a brief account of "double-loop integrals" after Jordan and Pochhammer, and of Poincaré's theory of asymptotic integrals.

In his paper on the motion of the moon, Hill was led to the solution of a linear equation by a method involving the use of infinite determinants. In chapter viii. Prof. Forsyth discusses this method in some detail, after giving a preliminary account of infinite determinants and their properties. The subject of this chapter is not very attractive in itself, but on account of its practical

importance has naturally attracted a good deal of attention.

Chapter ix. deals with equations with uniform periodic coefficients, and gives an account of this part of the subject which ought to encourage young mathematicians to read the original sources and experiment on their own account. It is, of course, the equations with doubly periodic coefficients that are most interesting. Thanks principally to Hermite, Halphen and Picard, some extremely beautiful results have been already obtained in this field, and there can be no doubt that others are awaiting discovery.

The last chapter of this volume, on equations with algebraic coefficients, must have been very difficult to write, and appeals mainly to the specialist. Its principal topic is Poincaré's celebrated theorem that the integrals of any linear equation with algebraic coefficients can be expressed by means of Fuchsian and Zeta-fuchsian functions. As Prof. Forsyth justly remarks, we cannot hope to make practical use of Poincaré's theorem until the analysis of automorphic functions has reached a higher state of development. To this end the treatise by Klein and Fricke, now in course of publication, will doubtless contribute largely.

In conclusion, it may be well to remark that this volume is in great measure independent of its predecessors, and that a great part of it will be quite intelligible to junior mathematicians provided that they know the elements of the theory of a complex variable. To them, therefore, as well as to their seniors, this book may be heartily commended.

G. B. M.

SCIENTIFIC PSYCHOLOGY.

Grundzüge der physiologischen Psychologie. Von Wilhelm Wundt. Fünfte völlig umgearbeitete Auflage. Erster Band. Pp. xv + 553. (Leipzig: W. Engelmann, 1902.) Price 10s. net.

THIS volume of 553 pages is the first of the three volumes in which the fifth edition of Prof. Wundt's great work is to appear. The rapid increase in size of the work in each of the successive editions is thus maintained in the present one, and, as in the case of the previous editions, has been necessitated by the rapidity of the growth of the youngest of the natural sciences, experimental or, as Prof. Wundt prefers to call it, physiological psychology. And even the increase in bulk of this book does not by any means fully express the rate of growth of the science, a growth towards which this country has contributed so lamentably little. For the book is primarily a record of the work and the views of the author and of his pupils in the great Leipzig school. Nevertheless, Prof. Wundt has found it necessary to rewrite almost the whole of the book, so that, as he tells us, it must be regarded as almost a new one.

The greater part of this first volume is concerned with matters not strictly psychological, but rather with those studies which form an essential part of the equipment of the psychologist, namely, the fine and coarse anatomy, the embryology and the physiology of nervous tissues, both special and comparative. It is, perhaps, open to question whether it is wise to attempt to treat so vast a range of subjects in the scope of a single volume. For

the psychologist may be tempted to content himself with the cursory review that is alone possible in such a work. It should certainly be possible nowadays for the writer on psychology to assume on the part of his readers a competent knowledge of the gross anatomy of the nervous system and of the principles of the conservation of energy. (In the anatomical section occurs an error that is, perhaps, of the nature of a slip. In Fig. 79 and in the accompanying text the uncrossed fibres of the optic nerves are represented as going to the nasal sides of the retinae. Now although v. Kölliker and others still maintain that the decussation of the optic nerve-fibres in the chiasma is complete, and although there is some ground for believing that there occur considerable individual variations in the proportion of crossed and uncrossed fibres, yet all authorities agree that the uncrossed fibres go to the temporal sides of the retinae.) The propriety of including an account of the general physiology of nerves is less open to question, the less so as Prof. Wundt is here on his own ground and can speak with authority. In this section Wundt makes a timely protest against the uncritical acceptance and wholesale application of Hering's doctrine of assimilation and dissimilation now so common among physiologists, and yet he teaches somewhat dogmatically a view that differs but little from the one he rejects. He too groups together under the term "inhibition" (*Hemmung*) all phenomena to which it can in any sense be applied, and assumes that one and all are manifestations of constructive metabolic processes, thus affording one more instance of the fact that the study of logic cannot prevent a man forming illogical conclusions. It cannot be too frequently pointed out that we have no evidence of active inhibitory processes within the nervous system and that all the numerous cases of "inhibition" may, and in the present state of knowledge should, be regarded as cases of interference or prevention only. Wundt goes so far as to assume a differentiation of the bodies of nerve-cells into two parts, the anabolic inhibitory and the katabolic augmentor parts, and applies this hypothesis to the explanation of the valve-like nature of the paths of the spinal cord. But although the hypothesis seems to have been devised in order to explain this phenomenon, it is not by any means clear that it can be made to do so.

The discussions of the functions of the cortex and especially of the "speech-centres" are admirably thorough and suggestive, and here Wundt gives a great development to the conception of a "brain-centre." It is, perhaps, to be regretted that he retains the term "centre," for it properly expresses a crude conception of which the period of usefulness is now at an end.

In treating of the fundamental constituents of psychical processes, Wundt distinguishes two fundamental kinds of psychical element, the sensations and the feelings (*Empfindungen und Gefühle*), the former including all those that have an objective reference and that are determined directly or indirectly by stimulation of sensory nerve-endings both within and on the surface of the body, the latter being the purely subjective elements. Compounded of sensations is the presentation (*Vorstellung*) and of feelings the emotion (*Gemüthsbewegung*). Wundt thus sets aside the old distinction of sensation and idea as that which is excited from without and from

within respectively, asserting that the distinction is purely logical and not at all psychological. Though we may admit that Wundt's use of the terms is a convenient one, yet it is impossible to follow him in denying the psychological character of the distinction usually made by English authors, or to admit his claim that the occurrence of hallucinations, which are purely pathological states, necessitates this denial. If the distinction were not psychologically valid, if we did not immediately recognise in the presentation the peculiar quality of reality that distinguishes it from the representation, the term hallucination would have no meaning.

Perhaps the most interesting part of the volume is the discussion of the "law of specific nervous energies." This principle Wundt would replace by one which he declares to be directly opposed to it, and which he describes as "the principle of the adaptation of the sensory functions to the stimulus and of the sensory apparatus to the functions." This is based upon and assumes the truth of the following principles: that of the original similarity of function of all nerve-elements, which Wundt establishes by tracing in a most interesting manner the differentiation of the various senses from the general sensibility of the amoeba upwards; the principle of the adaptation of nerve-elements through use or habituation; and the possibility, which we seem compelled to assume in some cases, that nerve-elements may come gradually to discharge the functions of others when those others are in any way rendered incapable of functioning. Now, admitting that the "law of specific nervous energies," as set up by Johannes Müller and by Helmholtz, is not in any sense an explanatory principle, but merely a *résumé* of a large group of facts, and admitting that it demands genetic treatment such as Wundt supplies, yet it is not possible to admit that even the most complete account of the evolution of the specific differentiations of sense can abolish the truths of which this "law" is the summary expression; to account for the origin of a thing or belief is not necessarily to explain it away. The fact remains that any specialised nerve of sense, when subjected to stimuli whether normal or abnormal, leads only to the kind of affection of consciousness peculiar to that sense. Wundt's account of the adaptations of the senses to stimuli is admirable and no doubt true so far as it goes, but it is far from being a complete explanation of the genesis of the specific functions.

Reducing the problem to its simplest terms, suppose a primitive sense-organ to be affected in the same way by two classes of stimuli, say two rates of vibration of the circumjacent medium—and then suppose that either rate of vibration comes in the course of evolution to determine a differentiation of one part of the nerves of the organ, so that one set of nerves comes to respond in one way to the one vibration-rate only and the other set in another way to the other (or that all the nerves come to respond in two distinct ways), and suppose the difference of response to consist in a difference in rate of vibration of the substance of the nerves, or in a difference of propagated chemical changes. Up to this point we may accept Wundt's account of the differentiation-process as adequate. But when we inquire—How comes it that the soul reacts to these two vibration-rates (or two kinds of chemical change) with two different

qualities of sensation? then we find ourselves still completely in the dark. Wundt himself seems to have felt this inadequacy and to have introduced in consequence towards the close of his exposition a new factor, the "entgegen kommende Triebe des empfindenden Wesen." This introduction of the "feeling being" amounts, it would seem, to nothing more than an admission of our ignorance. And indeed we have here reached the very kernel of the problem of life, of that mystery of the relations of soul and body which has vexed the thinkers of all ages, of that "master knot of human fate" of which the Persian poet wrote eight hundred years ago

"There was the door to which I found no key,
There was the veil through which I might not see."

And these words remain equally true to-day, in spite of the splendid labours of Johannes Müller, of Fechner, of Wundt and of many others.

It is interesting to note that Wundt assumes the principle of the inheritance of acquired characters as absolutely necessary to the explanation of the evolution of the nervous system, and that in this he is in agreement with most of the psychologists who have considered the problem. For the principle of natural selection, which is so satisfactory when we are dealing with the neck of the giraffe or the protective colouring of a butterfly, seems hopelessly inadequate when we have to account for those million-fold coordinated details of nervous disposition which together determine in large part, if not wholly, the tendencies and character of a human being.

In the last section Wundt deals with Weber's law and maintains his well-known psychological interpretation of it, in opposition to the now very generally accepted physiological interpretation. The attention of English readers may be called to the novel and ingenious explanation suggested by Heymans in the *Zeitschrift für Psychologie*, Bd. 26.

W. McD.

THE MODERN DYNAMO.

The Generators of Electricity at the Paris Exhibition of 1900. By C. F. Guilbert. Pp. iv + 766. (Paris: C. Naud, 1902.) Price 30 fr.

THERE were probably few who went to the Paris Exhibition two years ago who did not pay a visit to the Palais d'Électricité; and no one who did so can have failed to have been impressed by the enormous size of the electric generators exhibited there. We even know of feminine sightseers, on pleasure bent, sparing a few hours from the fascinating display of M. Worth to look at, and possibly learn a little about, the "purrin' dynamos." The massive grandeur of these magnificent machines, examples of the best design and workmanship of all nations; the complicated nature of their parts working in perfect harmony and smoothness, and obedient to the control of one or two men; their spotless cleanliness and the impression of reserved power which they conveyed; all these must have moved even the most matter-of-fact observer into sympathy with the ideas which inspired Mr. Kipling to write "M'Andrew's Hymn." Such a collection merited the permanent record which it has obtained in the pages of M. Guilbert's book. Something of the spirit of the machines which he describes seems to have entered into the author, for his

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book, like the dynamos, is very large. There are nearly 800 pages, with, to use the author's own words, "615 engravings and plans, of which 118 plates." M. Guilbert has adopted a somewhat novel plan with the laudable desire of making his book attractive to foreign readers. The title pages and preface are in the language of the country in which the copy is to be sold; the chapter and section headings, the descriptions of the illustrations and the tables, are given in French, German and English. We rather doubt the wisdom of this innovation, since it increases the size of a volume already bulky, and still the most important part, the text, remains only in French. The result of the translation, too, is apt at times to be rather humorous, as, for example, when the author translates *résumé* (which the mere Englishman is content to use in the original French) into a non-existent English equivalent.

Criticism of a book of this kind is almost out of the question. M. Guilbert begins by describing the system of classification which he has adopted, and then, taking each division in turn, gives a more or less detailed description of the principal exhibits which come within it. Photographs of the generators and clearly executed diagrams of the whole machine or of important details greatly help out the letterpress. The book is therefore, in a way, like a descriptive catalogue, but it is one which gives a large amount of very valuable information, and M. Guilbert deserves great credit for the painstaking way in which he has collected and the clear manner in which he has arranged the data supplied by the manufacturers. It may be objected that the work is two years out of date and that the machines of 1900 are almost ready for the scrap heap in 1902. But rapid as the advance of electrical engineering is, there are few engineers who will not benefit to-day by the careful study of what was best two years ago, especially as it is the best, not of one country only, but of all countries; there will be many also interested in the design and improvement of electric generators who will desire to possess this book, even though it should become in the course of a few years of historical interest only.

As we turn over the pages of M. Guilbert's book, we find difficulty in selecting any particular machine for special notice. As the most noticeable feature in dynamo development in recent years has been the steadily increasing size of the unit, we may perhaps be pardoned if we pick out one of the largest machines exhibited at Paris. The Allgemeine Elektrizitäts Gesellschaft exhibited a three-phase alternator of 4000 h.p. The output of this machine was 3000 kilovolt-amperes with a power factor of 0.9, making 2700 kilowatts. This alternator is one of a set of twenty-two, eight of which are already installed at the Berlin Electricity Works, the remaining thirteen being under construction. To bring this machine to Paris and to erect it in the German annexe, where there was no travelling crane, was a work of no small difficulty. The total weight was 160 tons, the armature frame weighing 80 and the field magnet 70 tons, the remaining 10 tons being due to the bedplate. The armature and field magnet were brought to the exhibition in quarters, each quarter being carried mounted between two railway trucks in the position most suited