Two Pathways of Primordial Cloud Collapse:

Fragmentation versus Direct Collapse under Enhanced Vacuum Energy

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Abstract

We explore the consequences of a specific hypothetical scenario for primordial cloud collapse. The scenario rests on three conditional premises: (1) that the cosmological constant Λ and the quantum vacuum energy density ρ_{vac} are physically distinct quantities; (2) that if they are distinct, ρ_{vac} was denser in the early universe by a factor $(1+z)^3$; and (3) that if such a denser vacuum existed, there is a mechanism by which it exerts net inward pressure on matter concentrations. None of these premises is established. Each may be wrong. We do not claim to prove them. We ask: *if all three hold*, what happens to a primordial gas cloud?

We trace the evolution of a single cloud ($M \sim 10^5\ M_\odot$, $T \sim 10^4$ K, $z \sim 12$) under three sets of conditions. In Scenario A (trace metallicity, standard vacuum), dust-induced cooling triggers fragmentation into a stellar cluster with remnant black holes of $\sim$ 10–100 $M_\odot$. In Scenario B (zero metallicity, standard vacuum), the well-established direct-collapse pathway produces a supermassive star and a $\sim 10^4$–$10^5\ M_\odot$ seed—but requires a fine-tuned Lyman–Werner radiation source. In Scenario C (zero metallicity, enhanced vacuum pressure), the hypothetical vacuum confinement provides an additional, isotropic compression mechanism that may reduce the Lyman–Werner requirement and accelerate the collapse timescale. Scenario C is the new contribution. Scenarios A and B are well-established control cases drawn from the existing literature.

The comparison is a thought experiment grounded in standard collapse physics (Jeans instability, Bonnor–Ebert stability, Salpeter accretion), applied under a hypothetical boundary condition (enhanced external vacuum pressure) whose physical reality remains to be demonstrated. The value of the exercise is that *if* the premises hold, the resulting mechanism is universal (no fine-tuning), self-terminating (dilutes with expansion), and produces the heavy seeds that JWST observations appear to demand.

Keywords: vacuum energy, direct collapse black hole, primordial gas cloud, Jeans instability, Bonnor–Ebert mass, supermassive star, early universe, JWST, structure formation, fragmentation, conditional model

Contents

1 Introduction

The James Webb Space Telescope has revealed massive galaxies and supermassive black holes at redshifts that challenge the standard cosmological timeline. Labbé et al. (2023) found candidate massive galaxies with stellar masses exceeding $10^{10}\ M_\odot$ at $z > 10$, within ~ 400 Myr of the Big Bang. Bogdán et al. (2024) detected a supermassive black hole at $z = 10.1$ consistent with a heavy-seed origin. Maiolino et al. (2024) confirmed an AGN at $z \approx 8.7$ with a black-hole-to-stellar-mass ratio exceeding local scaling relations by two orders of magnitude. Boylan-Kolchin (2023) demonstrated formally that these discoveries require assembly rates 3–10 times faster than ΛCDM predictions.

The timescale problem is acute. Growing stellar-mass seeds (~ 10–$100\ M_\odot$; Bromm & Larson 2004) to $\sim 10^9\ M_\odot$ by $z \sim 6$ via Eddington-limited accretion ($M \propto e^{t/t_{\rm Sal}}$, $t_{\rm Sal} \approx 45$ Myr; Salpeter 1964) requires nearly continuous accretion over ~ 800 Myr (Inayoshi et al., 2020; Volonteri, 2010). The direct-collapse black hole (DCBH) scenario (Bromm & Loeb, 2003; Lodato & Natarajan, 2006) circumvents this by producing seeds of 10^4–$10^6\ M_\odot$, but requires two restrictive conditions: zero metallicity ($Z < 10^{-5}\ Z_\odot$; Omukai et al. 2008) and intense Lyman–Werner UV radiation ($J_{21} \gtrsim 10^3$; Omukai 2001). The LW requirement confines DCBH formation to rare "synchronized pairs" of halos (Inayoshi et al., 2020), producing an abundance of seeds that may be too low for the observed population of early massive black holes.

This paper explores whether a third ingredient—external vacuum pressure—could relax the LW requirement and make direct collapse more common. The exploration is conditional. It rests on three premises, each of which is a hypothesis:

P1: Separation: The cosmological constant Λ and the quantum vacuum energy density $\rho_{\rm vac}$ are physically distinct quantities. Λ is a geometric property of spacetime; $\rho_{\rm vac}$ is a local field quantity. This is not established. The identification $\Lambda = 8\pi G\rho_{\rm vac}$ by Zel'dovich (1967) was assumed, not derived, but it has not been disproven either.

P2: Dilution: If $\rho_{\rm vac}$ is a local quantity distinct from Λ, it need not be constant. We assume it dilutes with expansion as $\rho_{\rm vac}(z) = \rho_{\rm vac}[0]\,(1+z)^3$, making the vacuum $\sim 2{,}000$ times denser at $z = 12$. The exponent $n = 3$ is motivated by self-consistency requirements of the matter-dependent ansatz $\rho_{\rm vac}(\rho_m) = \rho_{\rm vac}[\text{bare}] - \alpha\,\rho_m$ developed in Kriger (2026d) and Kriger (2026c), but is not derived from the renormalized stress-energy tensor and may be wrong.

P3: Pressure mechanism: If a denser vacuum surrounds a matter concentration, there exists a mechanism by which the energy-density contrast at the cloud boundary produces a net inward pressure. We motivate this by analogy with the Casimir effect (energy-density contrast between regions with different boundary conditions produces a force), but we acknowledge explicitly that the Casimir analogy involves conducting plates at microscopic separations, not diffuse gas clouds at kiloparsec scales. The extrapolation has not been derived from the stress-energy tensor in cosmological geometry.

What this paper does: We take P1–P3 as given and ask what follows for primordial cloud collapse, using only standard physics (Jeans instability, Bonnor–Ebert stability, Salpeter accretion) under this hypothetical boundary condition.

What this paper does not do: We do not prove P1, P2, or P3. We do not claim that vacuum energy is the source of the pressure. We do not claim this model is correct. We explore its consequences and identify what would need to be true—and what could be tested—for it to work.

The paper is organized as follows. Section 2 maps the logical structure. Section 3 reviews established collapse physics. Section 4 develops the conditional framework. Section 5 presents the mathematical development with three scenarios. Section 6 compares growth histories quantitatively. Sections 7–9 discuss implications, limitations, and future tests.

2 Logical Structure of the Argument

The reasoning is a conditional chain: *if P1, P2, P3 hold, then the following consequences follow.*

1. **Metallicity determines fragmentation** (§3). Below $Z_{\rm cr} \sim 10^{-5}$–$10^{-6}\ Z_\odot$, clouds do not fragment (Omukai et al., 2005, 2008). Above $Z_{\rm cr}$, dust cooling drives fragmentation. This is established physics.
2. **External pressure modifies the critical mass for collapse** (§3). The Bonnor–Ebert critical mass $M_{\rm BE} \propto T^2/P_{\rm ext}^{1/2}$ (Bonnor, 1956; Ebert, 1955). Higher external pressure lowers $M_{\rm BE}$. This is established physics.
3. **Premise P1–P3 supply a hypothetical external pressure** (§4). If the vacuum at $z \sim 12$ was $\sim 2{,}000\times$ denser, and if a pressure mechanism exists, this pressure enters the Bonnor–Ebert problem as an additional confining term. *This step is conditional.*
4. **Three scenarios are compared** (§5).
   A: Metals + standard vacuum → fragmentation → stellar cluster.
   B: No metals + standard vacuum → standard DCBH (requires LW fine-tuning).

C: No metals + enhanced vacuum pressure → DCBH with relaxed LW requirement.

Scenarios A and B are established. Scenario C is the conditional prediction.

5. **Growth comparison** (§6). The mass advantage of heavy seeds is quantified via Salpeter growth. This is standard physics applied to the different seed masses.
6. **The mechanism would be universal and self-terminating** (§7). If P1–P3 hold, vacuum pressure acts isotropically (no LW fine-tuning) and dilutes with expansion (self-terminates).

Steps 1, 2, 4 (Scenarios A and B), and 5 involve only established physics. Step 3 and Scenario C are conditional on P1–P3.

3 Background and Related Work

3.1 Thermal Evolution of Collapsing Primordial Gas

The fate of a collapsing gas cloud is governed by the temperature–density relation during collapse. Omukai (2001) showed that in metal-free gas with suppressed H_2 cooling, the gas cools only via Lyα emission, maintaining $T \approx 8{,}000$ K throughout collapse. The Jeans mass at this temperature is

$$M_\mathrm{J} = \frac{\pi^{5/2}}{6}\,\frac{c_s^3}{G^{3/2}\,\rho^{1/2}} \approx 7.6 \times 10^4\ M_\odot \quad (\text{at } T = 8{,}000\ \text{K},\ n = 10^4\ \text{cm}^{-3})\,, \tag{1}$$

preventing fragmentation.

Omukai et al. (2005) mapped the full metallicity range. Omukai et al. (2008) established the critical metallicity: $Z_\mathrm{cr} \sim 10^{-5}$–$10^{-6}\ Z_\odot$. Below this threshold, direct collapse proceeds; above it, dust cooling triggers fragmentation into sub-solar-mass clumps. Recent 3D simulations by Chon et al. (2025) confirm this threshold.

3.2 The Standard DCBH Pathway

The DCBH scenario (Bromm & Loeb, 2003; Lodato & Natarajan, 2006; Begelman et al., 2006) requires:

1. Metal-free gas ($Z < Z_\mathrm{cr}$).
2. Suppressed H_2 cooling, typically via an external LW flux $J_{21} \gtrsim 10^2$–10^3.
3. An atomic-cooling halo ($T_\mathrm{vir} \gtrsim 10^4$ K).

Under these conditions, the gas collapses isothermally at $\sim$ 8,000 K with accretion rates $\dot{M} \sim c_s^3/G \sim 0.1$–$1\ M_\odot\,\mathrm{yr}^{-1}$ (Shu, 1977). Rapid accretion inflates the protostar, suppressing UV feedback (Hosokawa et al., 2012, 2013), allowing growth to $\sim 10^5\ M_\odot$ before collapse via GR instability.

The LW requirement is the key bottleneck. It restricts DCBH formation to the vicinity of star-forming galaxies, yielding predicted number densities of $\sim 10^{-6}$–10^{-4} Mpc^{-3} (Inayoshi et al., 2020)—possibly insufficient for the JWST population. Wise et al. (2019) showed that cold turbulent accretion flows can bypass the LW requirement in rare halo configurations, but this pathway is also cosmologically infrequent.

3.3 External Pressure in Cloud Stability

The Bonnor–Ebert mass (Bonnor, 1956; Ebert, 1955) defines the maximum mass stable against collapse under external pressure:

$$M_{\rm BE} = 1.18\,\frac{c_s^4}{G^{3/2}\,P_{\rm ext}^{1/2}}\,. \tag{2}$$

Higher $P_{\rm ext}$ lowers $M_{\rm BE}$: external compression destabilizes clouds. Zier et al. (2021) showed quantitatively that increased external pressure from stellar winds can trigger Bonnor–Ebert collapse. This is the established physics we will apply under the hypothetical vacuum pressure boundary condition.

Section Result and Implications

The physics of metallicity-dependent fragmentation, isothermal DCBH collapse, and pressure-modified stability is well established. What is not established is whether vacuum energy provides a relevant external pressure. Everything in this section is standard; everything conditional enters in the next section.

4 Conceptual Framework: The Conditional Premises

This section states the three premises explicitly and examines what each would imply if true. We emphasize: these are hypotheses, not results.

4.1 Premise P1: $\Lambda \neq 8\pi G\,\rho_{\rm vac}$

The identification of Λ with $\rho_{\rm vac}$ was proposed by Zel'dovich (1967). It produced the 10^{120}-order discrepancy (Weinberg, 1989). Several programmes have explored their decoupling: Coleman (1988) proposed wormhole-mediated cancellation; Kaloper &

Padilla (2014) built a Lagrangian-level sequestering mechanism. Kriger (2026a,b,c) developed the specific framework we explore here.

Status: P1 is a legitimate open question in theoretical physics. It has not been confirmed or excluded by any experiment. The 10^{120} discrepancy can be read as evidence for separation, but this reading is not forced.

4.2 Premise P2: $\rho_{\rm vac}(z) = \rho_{\rm vac}[0]\,(1+z)^3$

If $\rho_{\rm vac}$ is distinct from Λ, its evolution depends on the physics of the vacuum in expanding spacetime. In flat Minkowski space, the vacuum is Lorentz-invariant with $w = -1$. In FRW spacetime, the mode spectrum evolves with $a(t)$ (Birrell & Davies, 1982), and the effective equation of state depends on the field content.

Critical point regarding observational constraints: A vacuum component with $w = 0$ and density comparable to or exceeding the cosmological constant would be catastrophically inconsistent with CMB, BAO, and supernova data—*if it entered the Friedmann equation*. The companion framework (Kriger, 2026c) proposes that $\rho_{\rm vac}$ is sequestered from cosmological dynamics: it gravitates locally inside bound structures but does not drive expansion. Under this hypothesis, the Friedmann equation contains Λ (geometric, small, constant) plus matter and radiation—but not $\rho_{\rm vac}$. If this sequestering fails, P2 is observationally excluded.

Important clarification regarding sequestering. The mechanism proposed in the companion framework is *not* identical to the Kaloper & Padilla (2014) vacuum sequestering. Kaloper & Padilla construct a Lagrangian-level mechanism that cancels the vacuum energy's contribution to *both* local and global gravity—their mechanism removes vacuum energy from gravitational dynamics entirely. The companion framework requires something different and more demanding: a mechanism that removes vacuum energy from the Friedmann equation (global expansion) while preserving its gravitational effect inside bound structures (local dynamics). This requires the gravitational coupling of $\rho_{\rm vac}$ to be *scale-dependent*: inactive on cosmological (Hubble) scales but active on galactic scales where expansion is suppressed. In standard general relativity, the equivalence principle ties local and global gravitational effects together, making such scale-dependent coupling difficult to achieve.

We do not have a Lagrangian derivation of this selective sequestering. The companion framework motivates it by analogy with the well-established fact that bound systems (galaxies, clusters) decouple from the Hubble flow (Carrera & Giulini, 2010): inside a gravitationally bound structure, expansion is suppressed and the vacuum's gravity is uncompensated, while outside, expansion compensates the vacuum's self-gravitation. Whether this physical picture can be elevated to a rigorous field-theoretic mechanism is an open question. Until it is, P2 rests on an assumed (not derived) scale-dependent coupling. The sequestering hypothesis has not been tested against Planck data via MCMC analysis, and we identify this as the most important quantitative test (Section 9).

Status: P2 depends on P1 and on an untested sequestering mechanism. It is speculative.

4.3 Premise P3: Vacuum Energy Exerts Net Pressure on Matter

If the vacuum energy density differs between regions (higher in voids, lower inside matter concentrations due to mode-spectrum modification), the gradient produces a net force. This is motivated by analogy with the Casimir effect (Casimir, 1948): different boundary conditions produce different vacuum energies, and the difference generates a measurable force (Lamoreaux, 1997).

The analogy's limitations: The Casimir effect involves conducting plates separated by sub-micrometer distances, where the mode spectrum is sharply modified by metallic boundary conditions. A diffuse gas cloud at kiloparsec scales with density $n \sim 10^4$ cm^{-3} presents a qualitatively different situation. The mode-spectrum modification inside such a cloud—due to Pauli blocking of fermionic modes, vacuum polarization by the electromagnetic field, and curvature modification of the fluctuation spectrum—has not been computed from the stress-energy tensor in cosmological geometry.

What would need to be true: For P3 to work, the vacuum energy inside a matter concentration must be measurably lower than outside, and the transition must occur over a scale comparable to the cloud radius, producing a pressure differential at the boundary. Whether this happens is an open question in quantum field theory in curved spacetime.

Status: P3 is the weakest premise. It is motivated but not derived. The paper acknowledges this explicitly and treats all results conditioned on P3 as hypothetical.

Counterarguments and Responses

Objection: If P1–P3 are all unproven, why write the paper?

Response: Because the consequences are specific, quantitative, and testable. If the premises are wrong, the consequences will not match observation, and the premises can be discarded. If they are right, the model predicts a universal mechanism for heavy seed formation that resolves a real observational puzzle. The paper's value lies not in asserting that the premises hold, but in deriving what would follow if they did—and in identifying the tests that could confirm or refute them.

Objection: A paper built on three unproven premises is speculation, not physics.

Response: Much of theoretical cosmology explores consequences of hypotheses. Dark matter particle models are built on the unproven premise that such particles exist. Modified gravity theories rest on unproven modifications to GR. The standard DCBH model itself rests on the premise that sufficiently strong LW radiation exists near pristine halos—a condition that is posited, not observed directly. What matters is whether the hypothesis leads to testable predictions and whether those predictions are novel. This paper provides both.

Section Result and Implications

Three conditional premises have been stated, with their status assessed honestly. P1 is a legitimate open question; P2 depends on untested sequestering; P3 is the weakest link, motivated but not derived. All quantitative results in subsequent sections are explicitly conditioned on P1–P3.

5 Mathematical Development

5.1 Initial Conditions (Shared Across All Scenarios)

Definition 5.1 (Reference Cloud)**.** *The reference cloud has the following properties at the onset of collapse:*

- *Total mass:* $M_{\mathrm{cloud}} = 10^5\ M_\odot$
- *Initial temperature:* $T_0 = 10^4$ K
- *Mean molecular weight:* $\mu = 1.22$ *(neutral primordial gas)*
- *Initial hydrogen number density:* $n_0 = 10^4\ \mathrm{cm}^{-3}$
- *Redshift:* $z = 12$
- *Halo virial temperature:* $T_{\mathrm{vir}} \approx 10^4$ K *(atomic-cooling halo)*

The isothermal sound speed at $T = 8{,}000$ K (the atomic-cooling floor relevant to

Scenarios B and C):

$$c_s = \sqrt{\frac{k_B T}{\mu\, m_H}} = \sqrt{\frac{1.38 \times 10^{-23} \times 8000}{1.22 \times 1.67 \times 10^{-27}}} \approx 7.4\ \mathrm{km\,s^{-1}}\,. \tag{3}$$

The thermal pressure of the cloud at $n_0 = 10^4\ \mathrm{cm}^{-3}$, $T = 8{,}000$ K:

$$P_{\mathrm{cloud}} = n_0\, k_B\, T = 10^{10}\ \mathrm{m}^{-3} \times 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}} \times 8000\ \mathrm{K} = 1.1 \times 10^{-9}\ \mathrm{Pa}\,. \tag{4}$$

This is the correct scale against which any additional external pressure should be compared—not the gravitational self-pressure, which describes the weight of the cloud per unit area, but the thermal pressure, which is what actually supports the cloud against collapse.

The standard free-fall time at this density:

$$t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32\, G\, \rho_0}} = \sqrt{\frac{3\pi}{32 \times 6.674 \times 10^{-11} \times 2.04 \times 10^{-17}}} \approx 4.7 \times 10^{13}\ \mathrm{s} \approx 1.5\ \mathrm{Myr}\,. \tag{5}$$

5.2 Scenario A: Fragmentation (Metals + Standard Vacuum)

Proposition 5.2 (Fragmentation Outcome)**.** *A cloud with $Z \gtrsim 10^{-5}\, Z_\odot$ and standard vacuum evolves as follows:*

1. *Isothermal collapse at $T \sim 8{,}000$–$10{,}000$ K (atomic cooling).*
2. *Molecular cooling (H_2, HD) reduces T to ~ 200–500 K at $n \sim 10^4$–$10^8\ \mathrm{cm}^{-3}$.*
3. *Dust cooling at $n \gtrsim 10^{10}\ \mathrm{cm}^{-3}$ drives $\gamma_{\mathrm{eff}} < 1$.*
4. *Jeans mass drops to $M_{\mathrm{J}} \sim 0.01$–$1\ M_\odot$.*
5. *Cloud fragments into $\sim 10^2$–10^3 stars.*
6. *Massive stars produce BH remnants of ~ 10–$100\ M_\odot$ (Heger et al., 2003).*

Calculation. At the dust-cooling temperature $T \approx 200$ K and $n = 10^{12}\ \mathrm{cm}^{-3}$: $c_s = 7.4 \times (200/8000)^{1/2} = 1.17\ \mathrm{km\,s^{-1}}$. $\rho = 1.22 \times 1.67 \times 10^{-27} \times 10^{18} = 2.04 \times 10^{-9}\ \mathrm{kg\,m^{-3}}$. $M_{\mathrm{J}} \approx 1.0 \times 10^{29}$ kg $\approx 0.05\ M_\odot$. This is confirmed by Omukai et al. (2005), Omukai et al. (2008), and simulations by Chon & Omukai (2020); Chon et al. (2025); Machida et al. (2009).

5.3 Scenario B: Standard DCBH (No Metals + Standard Vacuum)

This is the well-established DCBH pathway, included as a control.

Proposition 5.3 (Standard DCBH Outcome). *A cloud with $Z = 0$, standard vacuum, and a strong LW background ($J_{21} \gtrsim 10^3$) evolves as follows:*

1. *Isothermal collapse at $T \approx 8{,}000$ K (Lyα cooling; H_2 suppressed by LW).*
2. *$M_\mathrm{J} \sim 7.6 \times 10^4\ M_\odot$ throughout collapse.*
3. *No fragmentation.*
4. *Accretion rate $\dot{M} \sim c_s^3/G \sim 0.3\ M_\odot\,\mathrm{yr}^{-1}$.*
5. *SMS of $\sim 10^4$–$10^5\ M_\odot$ forms; collapses into DCBH seed.*

Calculation. $\dot{M} = 0.975\, c_s^3/G = 0.975 \times (7.4 \times 10^3)^3/(6.674 \times 10^{-11}) = 5.9 \times 10^{21}\ \mathrm{kg\,s^{-1}} \approx 0.3\ M_\odot\,\mathrm{yr}^{-1}$. This matches Inayoshi et al. (2014) and Latif et al. (2013).

Key limitation: This pathway requires a nearby star-forming galaxy providing $J_{21} \gtrsim 10^2$–10^3. The probability of the required "synchronized pair" is low, yielding predicted DCBH number densities of $\sim 10^{-6}$–$10^{-4}\ \mathrm{Mpc^{-3}}$ (Inayoshi et al., 2020).

5.4 Scenario C: Vacuum-Enhanced DCBH (No Metals + Enhanced Vacuum)

This is the new, conditional scenario. **Everything below is conditioned on P1–P3.**

Definition 5.4 (Hypothetical Vacuum Pressure at $z = 12$). *If P1–P3 hold, the vacuum pressure on the cloud boundary is*

$$\Delta P_\mathrm{vac}(z) = \rho_\mathrm{vac}[0]\,(1+z)^3\,c^2\,. \tag{6}$$

At $z = 12$: $\Delta P_\mathrm{vac}(12) = 2{,}197\ \rho_\mathrm{vac}[0]\,c^2 \approx 2{,}000\ \rho_\mathrm{vac}[0]\,c^2$.

The normalization question. The magnitude of ΔP_vac depends entirely on the unknown $\rho_\mathrm{vac}[0]$. We parametrize the results in terms of the dimensionless ratio

$$\eta(z) \equiv \frac{\Delta P_\mathrm{vac}(z)}{P_\mathrm{cloud}} = \frac{\rho_\mathrm{vac}[0]\,(1+z)^3\,c^2}{P_\mathrm{cloud}}\,, \tag{7}$$

where $P_\mathrm{cloud} = 1.1 \times 10^{-9}$ Pa is the cloud's thermal pressure from Eq. (4). At $z = 12$: $\eta(12) \approx 2{,}000\ \rho_\mathrm{vac}[0]\,c^2/P_\mathrm{cloud}$. The redshift dependence of $\eta(z)$ is central to the self-termination argument (Section 7): as the universe expands, η decreases as $(1+z)^3$.

Three regimes:

- $\eta \ll 1$: vacuum pressure is negligible. The cloud evolves as in Scenario B (standard DCBH).
- $\eta \sim 0.1$–1: vacuum pressure is a significant perturbation. It supplements gravity, deepens the potential well, and may modestly accelerate collapse.

- $\eta \gg 1$: vacuum pressure dominates. The quasi-static Bonnor–Ebert framework breaks down; the cloud undergoes pressure-driven implosion on a timescale shorter than the free-fall time. This regime would require a fully dynamical treatment beyond the scope of this paper.

For the illustrative value $\rho_{\rm vac}[0] = \rho_\Lambda^{\rm obs} \approx 5.9 \times 10^{-27}\ {\rm kg\,m^{-3}}$:

$$\Delta P_{\rm vac}(12) \approx 2{,}000 \times 5.9 \times 10^{-27} \times (3 \times 10^8)^2 = 1.1 \times 10^{-6}\ {\rm Pa}\,, \tag{8}$$

giving $\eta \approx 10^3$. This places the system firmly in the $\eta \gg 1$ regime, where vacuum pressure overwhelms thermal pressure and the Bonnor–Ebert framework is inapplicable in its standard form. This is an important result: *if* $\rho_{\rm vac}[0] \sim \rho_\Lambda^{\rm obs}$, the vacuum pressure is not a gentle perturbation—it is an overwhelming compression that would require dynamical simulation to model correctly.

For the vacuum pressure to act as a *modest* enhancement ($\eta \sim 0.1$–1), the present-day vacuum energy density would need to be

$$\rho_{\rm vac}[0] \sim \frac{P_{\rm cloud}}{2{,}000\, c^2} \sim \frac{1.1 \times 10^{-9}}{2{,}000 \times 9 \times 10^{16}} \sim 6 \times 10^{-30}\ {\rm kg\,m^{-3}}\,, \tag{9}$$

which is $\sim 10^{-3}\, \rho_\Lambda^{\rm obs}$—far below the observed dark energy density.

The honest conclusion: Either (a) $\rho_{\rm vac}[0]$ is far smaller than $\rho_\Lambda^{\rm obs}$ and the effect is a gentle perturbation within the Bonnor–Ebert framework; (b) $\rho_{\rm vac}[0] \sim \rho_\Lambda^{\rm obs}$ and the effect is overwhelming, requiring dynamical simulation; or (c) the pressure mechanism (P3) is far less efficient than a simple energy-density contrast would suggest. All three possibilities are open. The paper proceeds with parametric results expressed in terms of η.

Confrontation with the companion framework's own prediction. The companion papers (Kriger, 2026c) propose that dark matter is gravitationally bound vacuum energy, requiring $\rho_{\rm vac}[0] \sim 4.5 \times 10^{-24}\ {\rm kg\,m^{-3}} \approx 800\, \rho_\Lambda^{\rm obs}$. At $z = 12$, this gives $\eta \approx 800 \times 10^3 \approx 10^6$. We state this plainly: *the companion framework's own predicted value of* $\rho_{\rm vac}[0]$ *places the system six orders of magnitude above the regime where the Bonnor–Ebert analysis presented in this paper is valid.* At $\eta \sim 10^6$, the vacuum pressure exceeds the cloud's thermal pressure by a factor of one million, and the cloud would undergo pressure-driven implosion on a timescale of $t_{\rm ff}/(10^6)^{1/2} \sim 1.5$ yr—effectively instantaneous compared with any astrophysical process.

This means one of the following must be true:

(a) The pressure mechanism (P3) is far less efficient than a direct energy-density contrast suggests. Perhaps the vacuum modification inside a diffuse cloud is not $\sim 100\%$ suppression (as assumed by $\Delta P_{\rm vac} \sim \rho_{\rm vac}\, c^2$) but a fractional suppression of $\sim 10^{-6}$ or less, yielding a net pressure far below $\rho_{\rm vac}\, c^2$. This would bring η into the tractable range but requires a specific model of the mode-spectrum modification.

(b) The spatial scale over which the vacuum transitions from "suppressed" to "unsuppressed" is much larger than the cloud radius, producing a gradient that is too shallow to compress the cloud effectively.

(c) The companion framework's value of $\rho_{\rm vac}[0]$ is wrong, and the actual vacuum energy density is far smaller.

Possibility (a) is the most physically plausible and has an established parallel: the Casimir force between plates at separation d is not $\sim \rho_{\rm vac}\, c^2\, d^2$ (which would be enormous) but is suppressed by $\sim (\hbar c/d^4)$ factors that reflect the specific mode-spectrum modification. The actual vacuum pressure at cloud boundaries may be similarly suppressed relative to the naive estimate $\rho_{\rm vac}\, c^2$. Determining the suppression factor requires a full QFT-in-curved-spacetime calculation that has not been performed.

This is a fundamental limitation of the present paper. We can parametrize results in terms of η, but we cannot determine η from the premises alone. The paper's quantitative Bonnor–Ebert results apply only in the $\eta \sim 0.1$–10 regime. Whether the actual physical system falls in this regime is unknown.

Proposition 5.5 (Modified Bonnor–Ebert Mass (Conditional on P1–P3))**.** *If vacuum pressure $\Delta P_{\rm vac}$ supplements the confining pressure, the Bonnor–Ebert critical mass is*

$$M_{\rm BE}^{\rm mod} = 1.18\, \frac{c_s^4}{G^{3/2}\, (P_{\rm thermal} + \Delta P_{\rm vac})^{1/2}}\,, \tag{10}$$

reduced from the standard value by a factor $(1+\eta)^{-1/2}$.

Calculation. $M_{\rm BE}^{\rm mod}/M_{\rm BE}^{\rm standard} = (1+\eta)^{-1/2}$. For $\eta = 0.5$: reduction by 18%. For $\eta = 1$: reduction by 29%. For $\eta = 10$: reduction by 70%.

Proposition 5.6 (H_2 Formation vs. Compression Timescale (Conditional on P1–P3))**.** *The LW requirement can be relaxed if the compression timescale is shorter than the H_2 formation timescale, preventing H_2 from accumulating sufficiently to cool the gas below $\sim 8{,}000$ K.*

Timescale estimate. The H_2 formation timescale in primordial gas at $n \sim 10^4$ cm^{-3}

and $T \sim 8{,}000$ K is (Inayoshi et al., 2020):

$$t_{\mathrm{H}_2} \sim \frac{1}{k_{\mathrm{H}^-}\, n_e\, n_H} \sim 10^5\text{–}10^6 \text{ yr}\,, \tag{11}$$

where k_{H^-} is the H^- route rate coefficient and $n_e/n_H \sim 10^{-4}$ is the residual electron fraction. The standard free-fall time at this density is $t_{\mathrm{ff}} \approx 1.5$ Myr (Eq. 5).

If vacuum pressure reduces the effective compression time by a factor $(1+\eta)^{1/2}$, then for $\eta \sim 1$: $t_{\mathrm{compress}} \sim 1$ Myr, comparable to t_{H_2}. For $\eta \sim 10$: $t_{\mathrm{compress}} \sim 0.45$ Myr, shorter than t_{H_2}. In this regime, the gas collapses before H_2 can accumulate to the level needed for significant cooling, even without LW radiation.

This is the physical mechanism by which vacuum pressure could relax the LW requirement: not by destroying H_2, but by compressing the gas faster than H_2 can form.

Counterarguments and Responses

Objection: At higher densities (as the cloud compresses), t_{H_2} decreases because n_e and n_H both increase. The race between compression and H_2 formation is not obviously won by compression.

Response: Correct. The H_2 formation rate scales as n^2, while the compression rate under external pressure scales as $n^{1/2}$ (free-fall). At sufficiently high density, H_2 formation wins. However, what matters is whether H_2 accumulates to a fractional abundance $x_{\mathrm{H}_2} \gtrsim 10^{-4}$ sufficient for significant cooling (Omukai, 2001). The critical question is whether the cloud reaches the density at which atomic cooling becomes the sole effective coolant (the "loitering point" at $n \sim 10^4$ cm^{-3}, $T \sim 8{,}000$ K) before x_{H_2} exceeds the critical threshold. A rigorous answer requires a chemical network coupled to the hydrodynamic simulation—this is identified as the highest-priority future calculation.

Section Result and Implications

Three scenarios have been formally compared using the same initial cloud. Scenarios A and B reproduce well-established results. Scenario C introduces vacuum pressure as a conditional, parametric addition. The results are expressed in terms of $\eta = \Delta P_{\mathrm{vac}}/P_{\mathrm{cloud}}$, making the dependence on the unknown $\rho_{\mathrm{vac}}[0]$ transparent. For $\eta \sim 1$, the effect is a significant perturbation; for $\eta \gg 1$, the Bonnor–Ebert framework breaks down.

6 Empirical and Data-Grounded Illustrations

6.1 Growth Comparison: Salpeter Timescale

This comparison uses only standard accretion physics. The results are independent of P1–P3; they simply show the consequence of different seed masses and formation times.

Proposition 6.1 (Mass Amplification from Earlier, Heavier Seeding)**.** *If Scenario B or C produces a DCBH seed at time t_0 and Scenario A produces a stellar remnant at time $t_0 + \Delta t$, the mass advantage is*

$$\frac{M_{\rm heavy}(t)}{M_{\rm light}(t)} = \frac{M_{\rm seed}^{\rm heavy}}{M_{\rm seed}^{\rm light}} \exp\left(\frac{\Delta t}{t_{\rm Sal}}\right) . \tag{12}$$

Calculation. Direct substitution into $M(t) = M_{\rm seed}\, e^{t/t_{\rm Sal}}$.

Origin of the head start Δt. The head start has two components:

(i) Stellar evolution delay. In Scenario A, star formation must proceed before BH remnants appear: the cloud fragments, protostars form ($\sim 10^5$ yr), massive stars evolve ($\sim$ 3–10 Myr), and supernovae or direct collapses produce remnant BHs. The total delay from cloud collapse to the first $\sim 100\ M_\odot$ BH is $\sim$ 10–15 Myr.

(ii) Prior enrichment delay. Scenario A requires metallicity $Z \gtrsim 10^{-5}\ Z_\odot$, which means the halo must have experienced prior star formation—it is a *second-generation* system. The first generation of stars must have formed, evolved, and enriched the medium to $Z > Z_{\rm cr}$ before Scenario A can begin. Population III stars in minihalos at $z \sim$ 20–30 evolve on timescales of $\sim$ 3–10 Myr, but the enriched ejecta must mix with the surrounding gas and be incorporated into a new collapsing halo, which requires an additional $\sim$ 50–100 Myr (Bromm & Larson, 2004; Inayoshi et al., 2020). Scenarios B and C require *pristine* gas, which is available in first-generation halos that collapse at earlier redshifts.

The total head start of Scenarios B/C over Scenario A is therefore

$$\Delta t_{\rm B/C\,vs.\,A} \approx (50\text{–}100) + (10\text{–}15) \approx 60\text{–}115\ \text{Myr}\,, \tag{13}$$

with the prior enrichment delay being the dominant component. For the Table, we adopt a conservative estimate of $\Delta t \approx 65$ Myr.

Table 1: Growth comparison for Scenarios A, B, and C. Scenario A forms in a second-generation halo (requiring prior enrichment); B and C form in pristine first-generation halos.

Parameter	Scenario A	Scenario B	Scenario C
Seed mass	100 $M_\odot$	10^5 $M_\odot$	10^5 $M_\odot$
Halo type	2nd generation	1st generation	1st generation
Halo collapse (after BB)	$\sim$ 420 Myr	$\sim$ 340 Myr	$\sim$ 340 Myr
Delay to seed formation	15 Myr	2 Myr	2 Myr
Seed formation (after BB)	435 Myr	342 Myr	342 Myr
Δt vs. Scenario A	—	93 Myr	93 Myr
Seed mass ratio vs. A	—	10^3	10^3
$\exp(\Delta t/t_{\mathrm{Sal}})$	—	7.9	7.9
Combined advantage vs. A	—	$\sim$ 8,000×	$\sim$ 8,000×
e-folds to 10^9 $M_\odot$	16.1	9.2	9.2
Time to 10^9 $M_\odot$	725 Myr	414 Myr	414 Myr
LW requirement	N/A	$J_{21} \gtrsim 10^3$	Reduced (P1–P3)
Cosmological abundance	Common	Rare	Common (P1–P3)

Key observation: The growth advantage of Scenarios B and C over Scenario A is identical—it comes from three factors, all standard DCBH physics: (1) the $10^3\times$ heavier seed, (2) the $\sim$ 13 Myr stellar evolution delay, and (3) the $\sim$ 80 Myr prior enrichment delay before Scenario A's halo can even begin to collapse with the required metallicity. The prior enrichment delay—often overlooked in simple comparisons—is the dominant component, contributing $\exp(80/45) \approx 5.9$ of the 7.9× exponential factor. The unique contribution of Scenario C is not faster growth, but *higher abundance*: if vacuum pressure relaxes the LW requirement, more halos can undergo direct collapse, increasing the comoving density of heavy seeds from $\sim 10^{-4}$ Mpc^{-3} (Scenario B) toward $\sim$ 1–10 Mpc^{-3} (the density of pristine atomic-cooling halos at $z \sim 15$).

Counterarguments and Responses

Objection: If Scenarios B and C produce identical seeds, Scenario C adds nothing to the growth calculation. The only claim is about abundance, which is not quantified.

Response: This is precisely correct, and it represents the honest assessment. Scenario C does not make seeds heavier or faster-growing. It makes them *more numerous*—if P1–P3 hold. The abundance gain is not quantified because it requires radiation-hydrodynamic simulations with vacuum pressure that have not been performed.

Objection: The 8,000× combined advantage includes the enrichment delay, which is a property of Scenario A's requirements (needing metals), not a property of vacuum pressure.

Response: Correct. The enrichment delay is a feature of any comparison between first-generation (metal-free) and second-generation (metal-enriched) collapse pathways. It applies equally to Scenarios B and C. We include it because a fair comparison between the fragmentation and direct-collapse pathways must account for the different halo generations, but we emphasize that this timing advantage is standard DCBH physics, not a contribution of the vacuum mechanism.

6.2 Comparison with JWST Observations

Starting from a $10^5\ M_\odot$ seed at $z \sim 15$ ($t_{\rm age} \approx 270$ Myr):

$$M(z = 10.1) \approx 10^5 \exp\left(\frac{200}{45}\right) \approx 8 \times 10^6\ M_\odot\,, \tag{14}$$

$$M(z = 8.7) \approx 10^5 \exp\left(\frac{330}{45}\right) \approx 1.5 \times 10^8\ M_\odot\,. \tag{15}$$

These are consistent with Bogdán et al. (2024) ($z = 10.1$) and Maiolino et al. (2024) ($z \approx 8.7$). But this is a property of *all* DCBH scenarios (B and C alike), not specific to the vacuum mechanism.

Section Result and Implications

The $\sim$ 8,000× mass advantage of DCBH seeds (B or C) over stellar remnants (A)—arising from the $10^3\times$ heavier seed, the stellar evolution delay, and the prior enrichment delay—is standard DCBH physics. The specific contribution of Scenario C is the potential for higher DCBH abundance through LW relaxation—a qualitative prediction that requires simulation to quantify.

7 Discussion

7.1 What Scenario C Adds to the Existing Literature

The standard DCBH literature (Scenarios A and B) is mature: the physics is well understood, the simulations are detailed, and the bottleneck is clearly identified as the LW fine-tuning. Several mechanisms have been proposed to relax this bottleneck, each using only standard physics:

- **Cold turbulent accretion flows** (Wise et al., 2019): At the convergence of strong, cold filamentary accretion flows, turbulence in the halo suppresses star formation and prevents H_2 cooling, enabling massive seed formation without an external LW background. This mechanism is cosmologically rare: it requires a specific halo configuration at the intersection of multiple accretion streams.
- **Baryon streaming motions** (Inayoshi et al., 2020): Relative velocities between baryons and dark matter inherited from the pre-recombination era can suppress H_2 formation by preventing gas from settling into minihalos. This delays star formation, keeping gas pristine until it enters atomic-cooling halos. The effect is modest ($v_{\rm stream} \sim 30\ {\rm km\,s^{-1}}$ at recombination, decaying as $(1+z)$) and patchy.
- **Dynamical heating**: Rapid halo assembly through major mergers can heat the gas above the H_2 cooling threshold, temporarily maintaining isothermal conditions. This is stochastic and depends on the merger history.
- **Super-Eddington accretion** (Inayoshi et al., 2020): Rather than producing heavier seeds, this approach allows lighter seeds ($\sim$ 10–100 $M_\odot$) to grow faster. Radiatively inefficient accretion flows or photon trapping in dense gas envelopes can sustain $\dot{M} \gg \dot{M}_{\rm Edd}$ for limited periods. This relaxes the seed mass requirement but requires specific accretion geometries.

Scenario C proposes a qualitatively different mechanism: external pressure from a denser high-z vacuum. Its distinguishing feature—if P1–P3 hold—is *universality*: it acts on every cloud at a given epoch, regardless of local environment. Cold accretion flows, streaming motions, and dynamical heating are all stochastic, depending on specific halo properties. Vacuum pressure (if it exists) is isotropic and epoch-dependent.

However, the comparison reveals an uncomfortable asymmetry: the standard mechanisms (Wise, streaming, dynamical heating) require only established physics and have been confirmed or constrained by detailed simulations, while Scenario C requires three unproven premises and has not been simulated. The standard mechanisms produce quantitative predictions of DCBH abundance; Scenario C produces a qualitative prediction contingent on unknowns. Until the premises are tested and simulations performed, Scenario C's advantage in universality remains hypothetical.

7.2 Self-Termination

If $\rho_{\rm vac}(z) \propto (1+z)^3$, the vacuum pressure weakens with expansion:

- $z = 12$: $\rho_{\rm vac}/\rho_{\rm vac}[0] \approx 2{,}000$.
- $z = 2$: $\rho_{\rm vac}/\rho_{\rm vac}[0] = 27$.

- $z = 0$: $\rho_{\rm vac}/\rho_{\rm vac}[0] = 1$.

This would explain why massive seed formation was rapid at $z > 10$ and has since ceased.

7.3 Connection to Companion Papers

The companion papers develop the Λ–$\rho_{\rm vac}$ separation framework across several dimensions: the foundational question of vacuum energy and missing mass (Kriger, 2026a); the connection to JWST early galaxy formation and dark matter (Kriger, 2026b); the full reinterpretation of ΛCDM including the three-component galactic mass model, the $\Lambda(\rho_m)$ ansatz, and CMB compatibility (Kriger, 2026c); the parametric model of spatially varying vacuum energy density and inhomogeneous cosmic expansion (Kriger, 2026d); and the observational consequences of temporal resolution limits on cosmological measurement (Kriger, 2026e). The present paper uses one consequence of that framework (enhanced vacuum density at high redshift) to explore a specific astrophysical application. The present paper stands or falls with the companion framework: if $\Lambda = 8\pi G\,\rho_{\rm vac}$, the premises fail and Scenario C reduces to Scenario B.

Section Result and Implications

Scenario C is a conditional prediction embedded in a larger framework. Its unique contribution is the proposal that vacuum pressure can relax the LW bottleneck, increasing DCBH abundance. This prediction is testable by simulation and may be falsifiable by observation.

8 Limitations

1. **P1 (Separation) is unproven.** The identification $\Lambda = 8\pi G\,\rho_{\rm vac}$ has not been confirmed, but neither has its negation.
2. **P2 (Dilution) depends on untested sequestering.** If $\rho_{\rm vac}$ enters the Friedmann equation with $w = 0$ and $\rho_{\rm vac}[0] \sim \rho_\Lambda^{\rm obs}$, it is excluded by CMB, BAO, and SN data. The proposed sequestering requires scale-dependent gravitational coupling (active locally, inactive globally), which differs from the Kaloper & Padilla (2014) mechanism and has no Lagrangian derivation. The sequestering hypothesis has not been tested against Planck 2018, DESI BAO, or Pantheon+ data.
3. **P3 (Pressure mechanism) is the weakest premise.** It is motivated by Casimir analogy but not derived from the stress-energy tensor at cosmological

scales.

4. **The normalization $\rho_{\rm vac}[0]$ is unknown.** All quantitative results are parametric.
5. **No simulations have been performed.** The Bonnor–Ebert and Jeans analyses are approximate.
6. **The LW relaxation is estimated, not computed.** A rigorous answer requires a chemical-hydrodynamic simulation.
7. **The $\eta \gg 1$ regime is not modeled.** If vacuum pressure dominates thermal pressure, the quasi-static framework is invalid. The companion framework's own predicted $\rho_{\rm vac}[0] \sim 800\,\rho_\Lambda^{\rm obs}$ gives $\eta(z{=}12) \sim 10^6$, placing the system far outside the regime where this paper's Bonnor–Ebert analysis applies. The paper's quantitative results are valid only for $\eta \sim 0.1$–10, which requires either $\rho_{\rm vac}[0] \ll \rho_\Lambda^{\rm obs}$ or an efficient suppression of the pressure mechanism relative to the naive $\rho_{\rm vac}\,c^2$ estimate.
8. **The companion papers are archived on Zenodo but not independently peer-reviewed in established journals.** This weakens the evidential basis for P1–P3.

9 Future Directions

1. **Radiation-hydrodynamic simulations with enhanced external pressure.** Embed a parametric external pressure term in ENZO or AREPO and compare collapse outcome with and without it. This tests whether external compression relaxes the LW requirement quantitatively.
2. **CMB MCMC analysis.** Fit a sequestered vacuum component to Planck data. If the sequestering hypothesis fails, P2 is excluded.
3. **Chemical network calculation.** Couple H_2 formation kinetics to a modified collapse timescale and determine the minimum η needed to suppress H_2 below the cooling threshold without LW radiation.
4. **Stress-energy tensor derivation.** Compute $\langle T_{\mu\nu}\rangle_{\rm ren}$ inside a matter overdensity in FRW spacetime and determine whether a pressure gradient at the boundary exists. This would promote P3 from analogy to derivation.
5. **Independent peer review of companion papers.** The companion papers (Kriger, 2026a,b,c,d,e) are archived on Zenodo with DOIs but have not undergone independent peer review in established journals. Submission to journals with established referee processes would strengthen the evidential basis for P1–P3.

10 Conclusions

We have explored the consequences of three conditional premises (P1: $\Lambda \neq 8\pi G\,\rho_{\rm vac}$; P2: $\rho_{\rm vac} \propto (1+z)^3$; P3: vacuum exerts net pressure on matter) for primordial cloud collapse.

Established results (Scenarios A and B): A trace-metallicity cloud fragments into a stellar cluster (Scenario A). A zero-metallicity cloud with a strong LW background collapses monolithically into a DCBH seed (Scenario B). These results are due to Omukai (2001); Omukai et al. (2005, 2008); Bromm & Loeb (2003); Inayoshi et al. (2020) and are not new.

Conditional result (Scenario C): If P1–P3 hold, vacuum pressure provides an additional, isotropic confining mechanism at $z \sim 12$ that may accelerate collapse and suppress H_2 formation, relaxing the LW requirement. This would increase the cosmological abundance of DCBH seeds, potentially resolving the tension between the observed population of early massive black holes and the rarity of standard DCBH formation sites.

What we claim: The conditional chain P1+P2+P3 $\Rightarrow$ enhanced DCBH abundance is internally consistent and leads to testable predictions.

What we do not claim: That P1, P2, or P3 are correct. That vacuum pressure is the explanation for early massive black holes. That this model supersedes the established DCBH literature.

The model is falsifiable: if the sequestering hypothesis is excluded by CMB data, if simulations show external pressure does not relax the LW requirement, or if $\rho_{\rm vac}[0]$ is independently measured and found to be too small, the conditional prediction of Scenario C fails. The value of the exercise lies in mapping out the consequences of a specific set of hypotheses, identifying quantitative tests, and providing a target for future simulation.

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A Computational Code

```
import numpy as np

# Constants (SI)
G = 6.674e-11       # m^3 kg^-1 s^-2
c = 3e8             # m/s
kB = 1.38e-23       # J/K
mH = 1.67e-27       # kg
Msun = 2e30         # kg
Myr = 3.156e13      # seconds
yr = 3.156e7        # seconds
mu = 1.22           # mean molecular weight (neutral primordial)

def cs(T):
    """Isothermal sound speed [m/s]."""
    return np.sqrt(kB * T / (mu * mH))

def M_Jeans(T, n_cm3):
    """Jeans mass [Msun]. n_cm3 in cm^{-3}."""
    rho = mu * mH * n_cm3 * 1e6  # kg/m^3
    c_s = cs(T)
    MJ = (np.pi**(5./2.) / 6.) * c_s**3 / (G**1.5 * rho**0.5)
    return MJ / Msun

def P_thermal(T, n_cm3):
    """Thermal pressure [Pa]. n_cm3 in cm^{-3}."""
    return n_cm3 * 1e6 * kB * T

def P_vac(z, rho_vac_0=5.9e-27):
    """Vacuum pressure [Pa] at redshift z."""
    return rho_vac_0 * (1 + z)**3 * c**2

def t_ff(n_cm3):
    """Free-fall time [Myr]."""
    rho = mu * mH * n_cm3 * 1e6
```

```
    return np.sqrt(3 * np.pi / (32 * G * rho)) / Myr

def M_BH(M_seed, t_growth_Myr, t_Sal=45):
    """BH mass after Salpeter growth [Msun]."""
    return M_seed * np.exp(t_growth_Myr / t_Sal)

# === Results ===
print("=== Jeans Mass ===")
print(f"  T=8000 K, n=1e4 cm^-3: M_J = {M_Jeans(8000, 1e4):.1e} Msun")
print(f"  T=200 K, n=1e12 cm^-3: M_J = {M_Jeans(200, 1e12):.2e} Msun")

print("\n=== Thermal Pressure ===")
P_cloud = P_thermal(8000, 1e4)
print(f"  P_cloud(T=8000K, n=1e4) = {P_cloud:.2e} Pa")

print("\n=== Vacuum Pressure ===")
for z in [0, 2, 5, 12, 20]:
    Pv = P_vac(z)
    eta = Pv / P_cloud if P_cloud > 0 else 0
    print(f"  z={z:2d}: P_vac = {Pv:.2e} Pa, "
          f"eta = P_vac/P_cloud = {eta:.1e}")

print("\n=== Free-fall Time ===")
print(f"  n=1e4 cm^-3: t_ff = {t_ff(1e4):.1f} Myr")

print("\n=== Salpeter Growth (Table 2, corrected) ===")
# Scenario A: 2nd-gen halo collapse at 420 Myr + 15 Myr delay
# Scenarios B,C: 1st-gen halo collapse at 340 Myr + 2 Myr delay
for label, M_seed, t_seed in [("A", 100, 435),
                               ("B", 1e5, 342),
                               ("C", 1e5, 342)]:
    for z_obs, t_obs in [(10.1, 470), (8.7, 600), (6, 930)]:
        t_grow = max(t_obs - t_seed, 0)
        M = M_BH(M_seed, t_grow)
        print(f"  Scenario {label}, z~{z_obs}: "
              f"M = {M:.1e} Msun (t_grow={t_grow} Myr)")
```

```
print("\n=== Eta parametric (Item 10) ===")
print("rho_vac[0] / rho_Lambda_obs | eta(z=12) | regime")
for factor in [1e-3, 1e-2, 0.1, 1, 10, 100, 800]:
    rho_v0 = factor * 5.9e-27
    Pv = rho_v0 * (1+12)**3 * c**2
    eta_val = Pv / P_cloud
    regime = ("negligible" if eta_val < 0.1 else
              "perturbative" if eta_val < 10 else
              "BE breakdown")
    print(f"  {factor:>10.3f}              | "
          f"{eta_val:>9.1e} | {regime}")

print("\n=== Eta as function of z (for rho_vac[0] = rho_Lambda) ===")
for z in [0, 2, 5, 10, 12, 15, 20]:
    Pv_z = 5.9e-27 * (1+z)**3 * c**2
    eta_z = Pv_z / P_cloud
    print(f"  z={z:2d}: eta = {eta_z:.1e}")
```

Peer Reviews and Revision History

First Round

Reviewer assessment. The reviewer found the paper clearly written and addressing a timely problem, but raised five major concerns: (1) the central hypothesis is underdeveloped and lacks independent support; (2) the vacuum pressure mechanism lacks a rigorous derivation; (3) the original two-scenario comparison conflated metallicity and vacuum effects; (4) quantitative estimates of vacuum pressure vs. gravitational self-pressure were internally inconsistent; (5) the LW relaxation claim was not quantified. Minor concerns included notation inconsistency, misleading "proof" labels, unsubstantiated seed formation timing, and concentrated self-citations. Verdict: major revision.

Revisions made. (1) The paper has been reframed as a conditional exploration. Three explicit premises (P1, P2, P3) are stated in the Introduction and Section 4, each with its status assessed honestly. The paper now states clearly: "We do not claim to prove them. We ask: if all three hold, what happens?" (2) The Casimir analogy limitations are now stated explicitly. P3 is identified as the weakest premise. The paper acknowledges that the pressure mechanism is "motivated but not derived" and identifies the stress-energy tensor derivation as a future direction. (3) A third scenario (B: zero metallicity, standard vacuum) has been added as a control, cleanly isolating the vacuum effect. Table 2 now shows all three scenarios side by side. The unique contribution of Scenario C is identified as abundance enhancement, not growth advantage. (4) The pressure comparison has been corrected. The cloud's *thermal pressure* ($P_{\mathrm{cloud}} = 1.1 \times 10^{-9}$ Pa), not its gravitational self-pressure, is now used as the comparison scale. The dimensionless ratio $\eta = \Delta P_{\mathrm{vac}}/P_{\mathrm{cloud}}$ parametrizes results. Three regimes ($\eta \ll 1$, $\eta \sim 1$, $\eta \gg 1$) are discussed, with honest acknowledgment that for $\rho_{\mathrm{vac}}[0] \sim \rho_{\Lambda}^{\mathrm{obs}}$, the system is in the $\eta \gg 1$ regime where the Bonnor–Ebert framework breaks down. (5) The LW relaxation is now estimated via a timescale comparison (H_2 formation time vs. compression time, Proposition 5.6), with explicit acknowledgment that a rigorous answer requires chemical-hydrodynamic simulation. (6) "Proof" labels have been changed to "Calculation." (7) The 40 Myr head start has been replaced with a derived 13 Myr based on the stellar evolution delay in Scenario A. (8) Companion papers are labeled as preprints with Zenodo DOIs provided.

Second Round

Reviewer assessment. The reviewer found the revised manuscript substantially improved, with the conditional framing (P1–P3), three-scenario comparison, and η parametrization praised as significant advances. The pressure inconsistency resolution was called "illuminating" and the honest confrontation with $\eta \gg 1$ described as "a remarkable piece of self-criticism." Five remaining concerns were raised: (1) the narrowed scope may be insufficient for a full PRD article; (2) the companion framework's predicted $\rho_{\rm vac}[0] \sim 800\,\rho_\Lambda^{\rm obs}$ gives $\eta \sim 10^6$, placing the system far outside the regime where the Bonnor–Ebert analysis applies; (3) the sequestering mechanism differs from Kaloper & Padilla (2014) in a critical way—it requires scale-dependent gravitational coupling, which is difficult to achieve under the equivalence principle; (4) the head-start calculation omitted the prior enrichment delay (Scenario A requires a second-generation halo); (5) alternative standard-physics LW-bypass mechanisms deserved fuller comparison. Minor concerns included removing the journal recommendation section, removing empty placeholder sections, providing companion paper availability information, generalizing η as a function of redshift, and adding parametric η values to the code. Verdict: minor revision.

Revisions made. (1) Noted. The paper is structured as a thought experiment with a clear conditional framework; the scope is narrow but the predictions are specific and testable. The author acknowledges that a shorter-format venue (PRD Letter, JCAP Letter) may be more appropriate and defers to editorial judgment. (2) A new subsection in Section 5.4 now confronts the companion framework's $\rho_{\rm vac}[0] \sim 800\,\rho_\Lambda^{\rm obs}$ explicitly, computing $\eta \sim 10^6$ and stating plainly that the Bonnor–Ebert analysis is inapplicable at this value. Three possibilities are discussed: (a) the pressure mechanism (P3) is far less efficient than a naive energy-density contrast suggests, analogous to the Casimir force being suppressed relative to a naive vacuum-energy estimate; (b) the transition scale is too large for effective compression; (c) the companion framework's $\rho_{\rm vac}[0]$ is wrong. (3) Section 4.2 now clarifies that the proposed sequestering is *not* the Kaloper & Padilla mechanism: their mechanism cancels vacuum gravity both locally and globally, while the companion framework requires scale-dependent coupling (active locally, inactive globally). The difficulty of achieving this under the equivalence principle is stated explicitly. The mechanism is motivated by the physical picture that bound systems decouple from the Hubble flow (Carrera & Giulini, 2010), but this is acknowledged as a physical analogy, not a field-theoretic derivation. (4) The head-start calculation now includes the prior enrichment delay ($\sim$ 50–100 Myr) required for Scenario A's second-generation halo. Table 2 reflects the updated timing, with the total head start of ~ 93 Myr and combined advantage of $\sim 8{,}000\times$. (5) Section 7.1 now provides a structured comparison with four standard LW-bypass mechanisms (cold accretion flows, baryon streaming, dynamical heating, super-Eddington accretion), noting their stochastic character versus Scenario C's hypothetical universality, and acknowledging the uncomfortable asymmetry that the standard mechanisms are simulated while Scenario C is not. (6) The journal recommendation section has been removed (it is a cover-letter matter). (7) Empty placeholder sections removed. (8) Companion paper bibliography entries now include Zenodo DOIs. Two additional companion papers—on matter-dependent vacuum energy density (Kriger, 2026d) and temporal resolution limits on cosmological observation (Kriger, 2026e)—have been added to the reference list. (9) η is now defined as $\eta(z)$, a function of redshift. (10) Appendix code updated with η parametric values.