A Note on the Graphs of the Bessel Functions of Integral Grder

1916 Proc. Phys. Soc. London 29 211

(http://iopscience.iop.org/1478-7814/29/1/319)

XVII. *A Note on the Graphs of the Bessel Functions of Integral Order.* By B. HAGUE, *B.Sc., D.I.C.*

THE differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$ is of very frequent occurrence in many branches of mathematical physics and it is shown in the standard treatises * that the solution of the equation is expressible in terms of special functions, known as Bessel functions. The solutions of the equation met with in the commonest practical problems are composed of functions of the first kind, defined by the infinite series,

$$J_n(x)=\sum_{r=0}^{r=\infty}\frac{(-1)^r}{\Gamma(n+r+1)\,.\,\Gamma(r+1)}\cdot\left(\frac{x}{2}\right)^{n+2r},$$

where Γ is the Eulerian gamma function, and n is generally a positive integer.

In a recent investigation the writer had occasion to plot the graph of the function for certain integral values of n, and the resulting curves seemed so interesting that a complete series of graphs was prepared for all integral values of n from zero to 12 inclusive, using the tables recently published by the British Association Committee.

In the hope that the curves may be of interest to others engaged in practical work the author submits Figs. 1, 2 and 3 for the consideration of the Society. In Fig. 1, $J_n(x)$ for $n=0$, 1, 2, and 3 is shown ; in Fig. 2 the series is extended up to $n=7$; and in Fig. 3 is completed to $n=12$. On the latter diagram a few points are shown of $J_{18}(x)$ in order to emphasise the striking and well-known tendency of $J_n(x)$ towards zero as n becomes large.

Apart from the purpose for which the graphs were constructed, they have also proved a useful means of verifying by graphical means the commoner properties of the functions and some of the simpler solutions of Bessel's equation.

The mathematical theory of the J functions has been investigated with the greatest thoroughness, and tables of the values of the functions have been published from time to time. However, with the exception of J_0 and J_1—the graphs of which are

* Forsyth, "Differential Equations," and Gray and Mathews, "A Treatise on Bessel Functions."

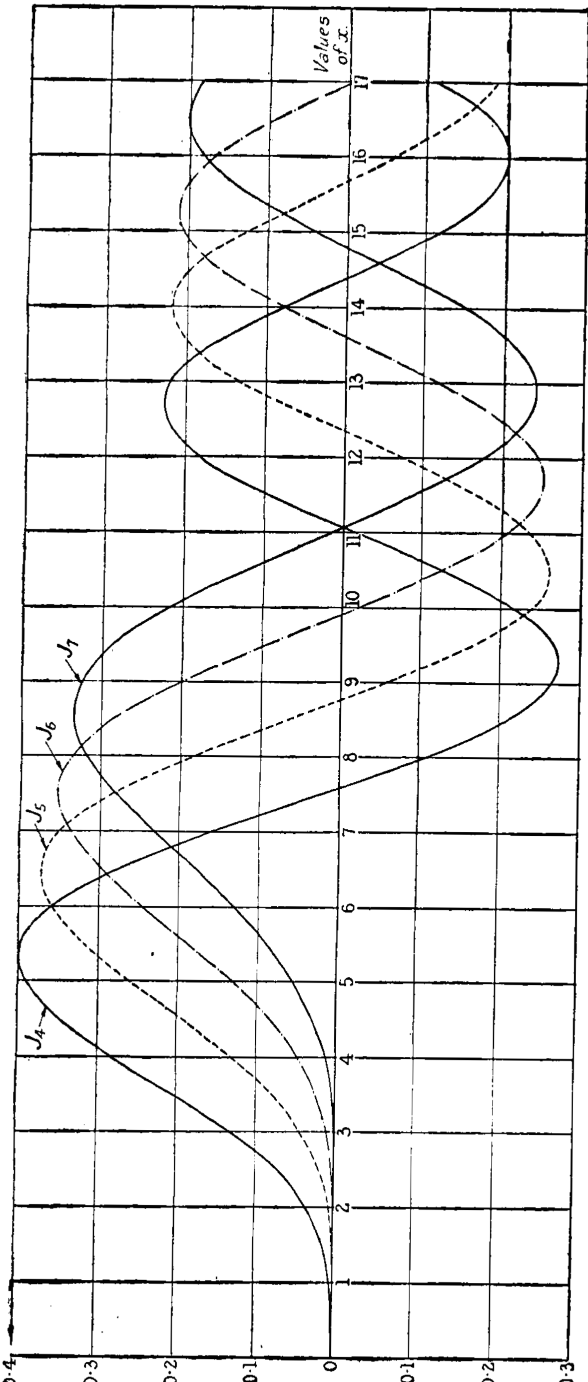

Fig. 2.—Bessel Functions, $J_n(x)$, of Integral Order. $n=4, 5, 6, 7$.

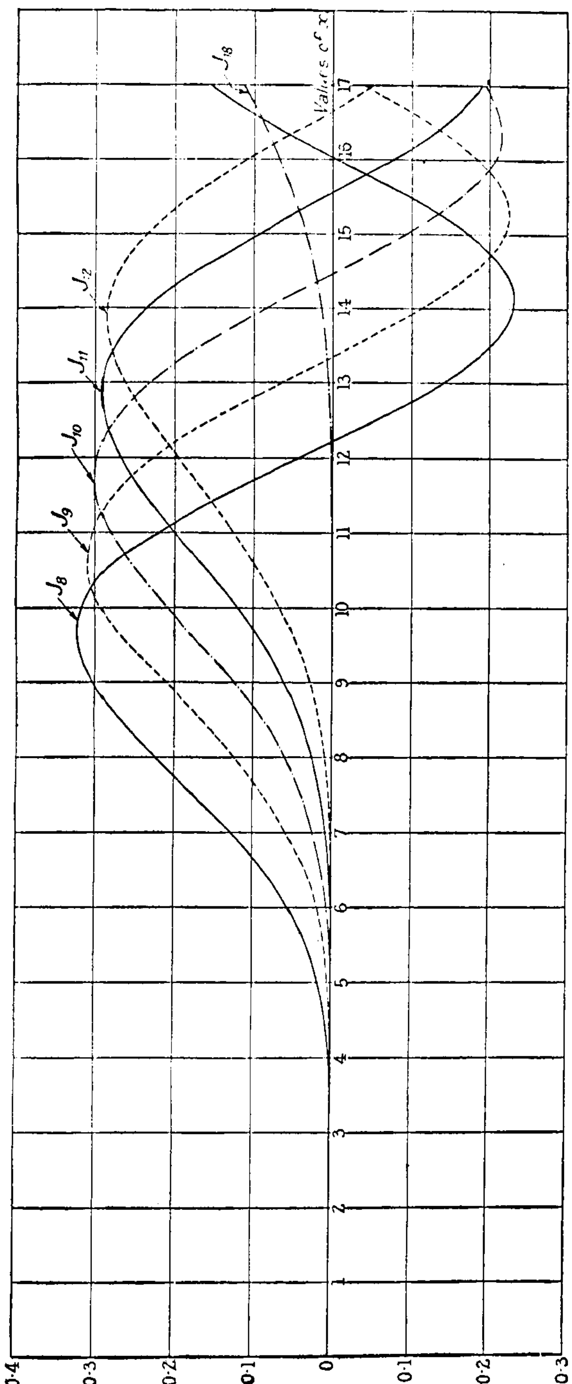

FIG. 3.—BESSEL FUNCTIONS, $J_n(x)$, OF INTEGRAL ORDER. $n=8, 9, 10, 11, 12$ & 18.

given in Gray and Mathew's treatise and of these and a few other orders in Jahnke and Emdes' "Functionentafeln"—the writer does not recollect seeing the march of the functions illustrated for integral values of the parameter n.

In concluding this short note the author wishes to thank Dr. A. R. Forsyth, F.R.S., for the interest which he has shown in the work, which was carried out at the City and Guilds (Engineering) College.