Observer Equivariance as a Condition for Shared Physical Law

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Abstract

We present a strict categorical model for the claim that shared physical law requires observer–equivariant transport. Observer standpoints form a connected groupoid $\mathcal{O}$ equipped with a free right action of a group G, and publicly shareable structure is represented by a base category $\mathcal{S}$. The central mathematical object is a *split principal G–fibration in groupoids*, together with a fixed normalized choice of transport-compatible basepoints. Within this chosen normalized setting, every autoequivalence of $\mathcal{S}$ admits a strictly functorial G–equivariant lift to $\mathcal{O}$, and any two such lifts differ by a unique global fiber automorphism determined by an element of G. Equivalently, one obtains a short exact sequence

$$1 \longrightarrow G \xrightarrow{\Lambda} \mathrm{Aut}_G^{\square}(\mathcal{O}/p) \xrightarrow{\Phi} \mathrm{Aut}(\mathcal{S}) \longrightarrow 1,$$

where $\mathrm{Aut}_G^{\square}(\mathcal{O}/p)$ denotes the group of G–equivariant bundle autoequivalences that preserve the chosen split cleavage, and Λ is the fiberwise translation embedding determined by the chosen normalization. The paper also gives a precise quotient criterion, formulates a descent statement for observables, and separates theorem-level models from physical specializations and sketches. The Wigner–Uhlhorn discussion is interpreted accordingly: the framework does not derive the quantum classification theorem, but isolates the bundle-theoretic descent pattern that makes physically irrelevant phase explicit. Conceptually, within this strict model, objectivity is represented not by an absolute standpoint but by the structure that remains after forgetting standpoint-dependent presentation.

*Disclaimer: The hypothesis is entirely that of the author. AI systems (ChatGPT, Gemini, and Claude) were used for assistance with drafting, editing, and mathematical polishing. All claims and any errors remain the responsibility of the author.

1 Introduction

Symmetry occupies a peculiar position in modern physics. On the one hand, symmetry principles are often introduced heuristically: one postulates invariance under Poincaré transformations, gauge transformations[1], or internal rotations, and then studies consequences. On the other hand, once such symmetry is present it behaves rigidly: Wigner's theorem constrains probability-preserving transformations, Noether's theorem ties continuous symmetries to conservation laws, and Lorentz invariance fixes causal structure. This raises a natural question:

> *In what precise sense is symmetry forced by the requirement that different observers should be able to regard themselves as describing the same physical world?*

The guiding idea of this paper is simple. Descriptions given from different standpoints should not count as physically shareable merely because we declare them equivalent by fiat. Rather, they should be related by explicit transformations, and the content that qualifies as objective should be exactly the content that descends through those transformations. This leads naturally to a bundle picture:

$$p : \mathcal{O} \longrightarrow \mathcal{S},$$

where $\mathcal{O}$ is a groupoid of standpoint-dependent presentations and $\mathcal{S}$ is a category of shareable structure. The fibers of p record what changes when one changes perspective without changing the structural world being described.

Choice of framework. A principal-bundle formulation in **Cat** captures the underlying intuition, but a fully strict classification theorem requires more structure. Accordingly, the analysis is carried out for *split principal G–fibrations in groupoids*, and the automorphisms considered upstairs are required to preserve the chosen split cleavage. From Section 4 onward we also fix a normalized family of transport-compatible basepoints in the fibers. These hypotheses are what make the lifting statement strictly functorial and the classification of lifts exact on the nose rather than only up to coherent isomorphism. The resulting exact sequence is therefore internal to a chosen normalized strict presentation, not yet a canonical statement in a weaker bicategorical or stack-theoretic setting.

Main claim. In the strict setting, the main statement becomes precise:

> *Every autoequivalence of the base category $\mathcal{S}$ lifts to a strictly G–equivariant cleavage-preserving bundle autoequivalence of $\mathcal{O}$, and two lifts of the same base symmetry differ by a unique normalized fiber translation Λ_g determined by an element of G.*

[1] For gauge theories one may model $\mathcal{O}$ as a groupoid of gauge-dependent presentations (local potentials, local sections, chosen trivializations), with G acting by gauge transformations and $\mathcal{S}$ capturing gauge-invariant structure. In the discussion below this remains a sketch rather than a theorem-level example; see Example 4 and [4].

Equivalently, one obtains a short exact sequence

$$1 \to G \to \mathrm{Aut}_G^{\square}(\mathcal{O}/p) \to \mathrm{Aut}(\mathcal{S}) \to 1.$$

The kernel is realized, relative to the chosen normalization, by global fiber automorphisms induced by elements of G. This is the strict mathematical core of the slogan that symmetry is observer–equivariance within the present normalized model.

Interpretive orientation. The term *observer* is used here in two compatible senses. In technical physics it may mean a reference frame, gauge choice, or measurement context. In a mind–matter setting it may mean an embodied epistemic standpoint. The present formalism is intended to accommodate both: $\mathcal{O}$ collects presentations available from within perspectives, while $\mathcal{S}$ captures the structural content that can be shared across them. Unlike more radical relational approaches, this does not dissolve public structure altogether; rather, it identifies objectivity with what survives the forgetful projection p.

A note on "laws". The phrase "law on $\mathcal{S}$" is used here in a deliberately narrow sense. A *law* means functorial data or a functorial construction on $\mathcal{S}$ that is intended to be publicly shareable. The theorem does not claim that every conceivable functor on $\mathcal{S}$ is automatically physically meaningful; it claims that whenever a physical law is shareable across standpoints, its transport must be implemented by bundle autoequivalences compatible with the standpoint action. This strict setting is also the natural starting point for later weak or stack-theoretic generalizations, where local gauge data and coherence isomorphisms would have to be treated explicitly rather than absorbed into split structure.

The rest of the paper is organized as follows. Section 2 fixes notation. Section 3 introduces split principal G–fibrations and a precise quotient criterion. Section 4 contains the strict lift theorem and the exact sequence. Sections 5 and 6 discuss Wigner–Uhlhorn, a strict action-groupoid model, a fixed-spacetime Lorentz specialization, and qubits, with explicit separation between theorem-level models and heuristic sketches. These examples are intended as controlled illustrations of how the formalism may be read in familiar physical settings, not as complete reconstructions of those theories in full generality. Section 7 comments on observables. Section 8 sketches philosophical consequences. Appendix A collects five compatible ways of reading objectivity in the present framework.

2 Notation and categorical setting

We work with small categories. The observer category $\mathcal{O}$ will always be assumed to be a groupoid. The base category $\mathcal{S}$ need not be a groupoid in general, though several examples below are. Write BG for the one-object category with endomorphism group G.

Autoequivalences and bundle automorphisms. For a functor $p : \mathcal{O} \to \mathcal{S}$, let $\mathrm{Aut}(\mathcal{S})$ denote the group of chosen strict representatives of autoequivalences of $\mathcal{S}$. Let $\mathrm{Aut}^{\square}_G(\mathcal{O}/p)$ denote the group of strictly G–equivariant autoequivalences $F : \mathcal{O} \to \mathcal{O}$ such that:

(a) there exists $A \in \mathrm{Aut}(\mathcal{S})$ with $p \circ F = A \circ p$ strictly;

(b) F preserves chosen cartesian arrows, hence preserves the fixed split cleavage.

Composition is defined in the obvious way. The superscript $\square$ is a reminder that the chosen cleavage is part of the strict structure.

Notation and conventions. Composition is written from right to left: if $f : x \to y$ and $g : y \to z$, then $g \circ f : x \to z$ means first f, then g. For a chosen split cleavage, the cartesian lift of $u : s \to t$ with codomain $y \in \mathcal{O}_t$ is denoted

$$\chi_{u,y} : u^* y \to y.$$

Splitness means in particular that for composable base morphisms $u : s \to t$ and $v : t \to r$,

$$\chi_{v \circ u, z} = \chi_{v,z} \circ \chi_{u, v^* z}.$$

Symbol	Meaning
$\mathcal{O}$	Observer groupoid (perspectives and invertible perspective shifts)
$\mathcal{S}$	Category of shareable physical structures
$p : \mathcal{O} \to \mathcal{S}$	Split principal G–fibration in groupoids
G, BG	Structure group and its delooping
$\mathcal{O}_s$	Fiber over $s \in \mathcal{S}$
u^*	Reindexing functor along a base morphism u
$\mathrm{Aut}^{\square}_G(\mathcal{O}/p)$	Cleavage-preserving G–equivariant bundle autoequivalences of p
Law on $\mathcal{S}$	A shareable functorial assignment on $\mathcal{S}$

3 Split principal G–fibrations in groupoids

3.1 Observers and structures

Definition 1 (Observer groupoid)**.** *$\mathcal{O}$ is a connected groupoid of observer perspectives. Objects are standpoints o; morphisms are invertible perspective changes.*

Definition 2 (Category of structures)**.** *$\mathcal{S}$ is a category of physical structures. A* law *on $\mathcal{S}$ means a functorial assignment or functorial construction on $\mathcal{S}$ intended to represent publicly shareable physical content.*

3.2 Strict bundle data

Definition 3 (Split Grothendieck fibration). *A functor $p : \mathcal{O} \to \mathcal{S}$ is a* split Grothendieck fibration *if for every morphism $u : s \to t$ in $\mathcal{S}$ and every object $y \in \mathcal{O}_t$, a cartesian lift*

$$\chi_{u,y} : u^*y \longrightarrow y$$

is chosen, and these chosen lifts are strictly compatible with identities and composition:

$$(\mathrm{id}_s)^*y = y, \qquad (v \circ u)^*z = u^*(v^*z),$$

with corresponding equalities of chosen cartesian arrows.

Definition 4 (Split principal G–fibration). *A* split principal G–fibration in groupoids *consists of a split Grothendieck fibration*

$$p : \mathcal{O} \to \mathcal{S}$$

with $\mathcal{O}$ a connected groupoid, together with a right action of G on $\mathcal{O}$ by functor automorphisms, such that:

(i) for every $s \in \mathcal{S}$, the fiber $\mathcal{O}_s$ is a nonempty right G–torsor on objects;

(ii) for every morphism $a : x \to y$ in a fiber and every $g \in G$, there is a unique morphism $a \cdot g : x \cdot g \to y \cdot g$, and the action on morphisms in each fiber is free;

(iii) the action preserves the projection: $p(x \cdot g) = p(x)$ and $p(a \cdot g) = p(a)$;

(iv) the chosen cartesian lifts are G–stable:

$$\chi_{u,y \cdot g} = \chi_{u,y} \cdot g$$

for every base morphism u, object y, and $g \in G$.

Condition (iv) is the key strengthening relative to the looser formulation. It ensures that the transport functors between fibers commute strictly with the G–action.

Remark 1 (Fiber transport). *Given $u : s \to t$, the chosen cartesian lifts define a reindexing functor*

$$u^* : \mathcal{O}_t \to \mathcal{O}_s, \qquad y \mapsto u^*y.$$

Because of Definition 4(iv), each reindexing functor is strictly G–equivariant:

$$u^*(y \cdot g) = u^*(y) \cdot g.$$

This strict equivariance is what makes the lift theorem below genuinely strict rather than merely pseudofunctorial.

Definition 5 (Orbit category)**.** *Let a group G act on a category $\mathcal{O}$ by automorphisms. The orbit category $\mathcal{O}/G$ has as objects the G-orbits $[x]$ of objects of $\mathcal{O}$. A morphism $[x] \to [y]$ is an equivalence class of morphisms $f : x \to y \cdot g$ in $\mathcal{O}$, where*

$$f \sim f' \quad \text{if and only if} \quad f' = (f \cdot h) : x \cdot h \to y \cdot gh$$

for some $h \in G$, after identifying sources and targets with their orbit classes. Composition is induced from composition in $\mathcal{O}$.

Proposition 1 (A precise quotient criterion)**.** *Let $p : \mathcal{O} \to \mathcal{S}$ be a split principal G–fibration in groupoids. Then p factors through a functor*

$$\bar{p} : \mathcal{O}/G \to \mathcal{S}.$$

Assume in addition:

(Q1) for every object $s \in \mathcal{S}$, the fiber $\mathcal{O}_s$ is a single G-orbit on objects;

(Q2) for every morphism $u : s \to t$ in $\mathcal{S}$, every object $y \in \mathcal{O}_t$, and every two arrows $f : x \to y$, $f' : x' \to y$ in $\mathcal{O}$ lying over u, there is a unique $g \in G$ such that $x' = x \cdot g$ and $f' = f \cdot g$.

Then $\bar{p}$ is an equivalence of categories.

Proof. Since p is constant on G-orbits by Definition 4(iii), it descends to $\bar{p}$. Condition (Q1) implies essential surjectivity on objects: every $s \in \mathcal{S}$ has an object in its fiber, and any two such objects become equal in $\mathcal{O}/G$.

To show fullness, let $u : s \to t$ be a morphism in $\mathcal{S}$. Choose $y \in \mathcal{O}_t$ and consider the chosen cartesian lift

$$\chi_{u,y} : u^*y \to y.$$

Its class in $\mathcal{O}/G$ defines a morphism $[u^*y] \to [y]$ sent by $\bar{p}$ to u. Hence every base morphism has a lift in the orbit category.

To show faithfulness, suppose two orbit morphisms $[f], [f'] : [x] \to [y]$ have the same image under $\bar{p}$, say both lie over $u : s \to t$. By replacing representatives inside their orbit classes, we may assume both have common target y. Then (Q2) yields a unique $g \in G$ with $f' = f \cdot g$, so $[f] = [f']$ in $\mathcal{O}/G$. Thus $\bar{p}$ is fully faithful. Therefore $\bar{p}$ is an equivalence. $\square$

Remark 2 (Why the quotient is not the primary definition)**.** *For the strict proofs below, the split fibration structure is the decisive datum. The quotient viewpoint is conceptually important, but if one takes $\mathcal{O}/G \simeq \mathcal{S}$ as the primary definition, the proof of strict liftability becomes less transparent. The order of presentation is therefore as follows: first the split fibration, then the quotient interpretation.*

4 Strict lift theorem and exact sequence

The main result is now stated in a form whose proof matches its strength.

4.1 Construction of the lifted functor

Fix a split principal G–fibration $p : \mathcal{O} \to \mathcal{S}$. From this point onward we also fix a *normalized* family of basepoints

$$\{b_s \in \mathcal{O}_s\}_{s\in \mathrm{Ob}(\mathcal{S})}$$

with the transport-compatibility property

$$u^*(b_t) = b_s$$

for every base morphism $u : s \to t$. This normalization is additional structure, fixed once and for all; it is not claimed to be canonical. Different normalized choices change the embedding of G into bundle automorphisms by conjugation, but do not change the induced base-level classification. Since each fiber is a G–torsor, every object $x \in \mathcal{O}_s$ can be written uniquely as

$$x = b_s \cdot g_x$$

for a unique $g_x \in G$.

Given an autoequivalence $A : \mathcal{S} \to \mathcal{S}$, define a functor $\widetilde{A} : \mathcal{O} \to \mathcal{O}$ as follows. On objects,

$$\widetilde{A}(x) := b_{A(s)} \cdot g_x, \qquad x \in \mathcal{O}_s.$$

For a morphism $f : x \to y$ with $p(f) = u : s \to t$, write $y = b_t \cdot g_y$. Because f lies over u, cartesianness of $\chi_{u,y} : u^*y \to y$ and the transport-compatible basepoints imply

$$x = u^*y = u^*(b_t \cdot g_y) = u^*(b_t) \cdot g_y = b_s \cdot g_y,$$

so $g_x = g_y$. Hence

$$\widetilde{A}(x) = b_{A(s)} \cdot g_x = b_{A(s)} \cdot g_y = A(u)^*(b_{A(t)}) \cdot g_y = A(u)^*\widetilde{A}(y).$$

There is therefore a chosen cartesian lift

$$\chi_{A(u),\widetilde{A}(y)} : \widetilde{A}(x) \longrightarrow \widetilde{A}(y),$$

and we define

$$\widetilde{A}(f) := \chi_{A(u),\widetilde{A}(y)}.$$

The splitness axioms imply strict functoriality immediately. This gives the existence lemma in its strict form.

Lemma 1 (Strict existence of lifts)**.** *Let $p : \mathcal{O} \to \mathcal{S}$ be a split principal G–fibration in groupoids and let $A : \mathcal{S} \to \mathcal{S}$ be an autoequivalence. Then the construction above defines a strictly G–equivariant cleavage-preserving functor*

$$\widetilde{A} : \mathcal{O} \to \mathcal{O}$$

satisfying

$$p \circ \widetilde{A} = A \circ p.$$

Proof. For objects, strict G–equivariance is immediate: if $x = b_s \cdot g_x$, then

$$\widetilde{A}(x \cdot h) = b_{A(s)} \cdot (g_x h) = \widetilde{A}(x) \cdot h.$$

For morphisms, if $f : x \to y$ lies over $u : s \to t$, then by the construction just given the source of $\chi_{A(u),\widetilde{A}(y)}$ is exactly $\widetilde{A}(x)$, not merely an isomorphic representative, so $\widetilde{A}(f)$ is well defined. Because the cleavage is split, the chosen lifts satisfy

$$\widetilde{A}(\mathrm{id}_x) = \mathrm{id}_{\widetilde{A}(x)}, \qquad \widetilde{A}(g \circ f) = \widetilde{A}(g) \circ \widetilde{A}(f)$$

for composable arrows f, g. The construction sends chosen cartesian arrows to chosen cartesian arrows by definition, so it preserves the cleavage. Finally,

$$p(\widetilde{A}(x)) = A(p(x)), \qquad p(\widetilde{A}(f)) = A(p(f)),$$

again by construction of the chosen cartesian lift. Thus $p \circ \widetilde{A} = A \circ p$ strictly. □

Remark 3. *The point of Lemma 1 is not that one can lift in some weak sense, but that the split principal structure removes the coherence problem. The proof no longer depends on a non-functorial choice of representatives followed by an appeal to uniqueness "relative to a cleavage"; the compatibility $u^*(b_t) = b_s$ makes the source object of the lifted arrow literally equal to $\widetilde{A}(x)$.*

4.2 Rigidity of equivariant lifts

For each $g \in G$, define the normalized fiber translation

$$\Lambda_g : \mathcal{O} \to \mathcal{O}$$

on objects by

$$\Lambda_g(x) := b_s \cdot (g g_x), \qquad x = b_s \cdot g_x \in \mathcal{O}_s,$$

and on a morphism $f : x \to y$ lying over $u : s \to t$ by

$$\Lambda_g(f) := \chi_{u,\Lambda_g(y)}.$$

Because the normalization is transport compatible, Λ_g is a strictly G–equivariant cleavage-preserving bundle autoequivalence over $\mathrm{id}_{\mathcal{S}}$. For $g_1, g_2 \in G$ and $x = b_s \cdot h$, one has

$$\Lambda_{g_1}(\Lambda_{g_2}(x)) = b_s \cdot (g_1 g_2 h) = \Lambda_{g_1 g_2}(x),$$

so $g \mapsto \Lambda_g$ is a group homomorphism $G \to \mathrm{Aut}_G^{\square}(\mathcal{O}/p)$. It is the canonical way, relative to the chosen basepoints, to realize the torsor automorphism determined by g in every fiber.

Lemma 2 (Rigidity)**.** *Let $F_1, F_2 \in \mathrm{Aut}_G^{\square}(\mathcal{O}/p)$ cover the same autoequivalence $A \in \mathrm{Aut}(\mathcal{S})$, so that*

$$p \circ F_1 = A \circ p = p \circ F_2.$$

Then there exists a unique element $g \in G$ such that

$$F_2 = \Lambda_g \circ F_1.$$

Proof. Fix $s \in \mathcal{S}$. Write

$$F_i(b_s) = b_{A(s)} \cdot k_i(s) \qquad (i = 1, 2)$$

for uniquely determined elements $k_i(s) \in G$. If $x = b_s \cdot h$, then strict G–equivariance gives

$$F_i(x) = F_i(b_s \cdot h) = F_i(b_s) \cdot h = b_{A(s)} \cdot (k_i(s)h).$$

Define

$$g_s := k_2(s)k_1(s)^{-1}.$$

Then for every $x = b_s \cdot h \in \mathcal{O}_s$,

$$F_2(x) = b_{A(s)} \cdot (k_2(s)h) = b_{A(s)} \cdot (g_s k_1(s)h) = \Lambda_{g_s}(F_1(x)).$$

Hence

$$F_2|_{\mathcal{O}_s} = \Lambda_{g_s} \circ F_1|_{\mathcal{O}_s}.$$

It remains to show that g_s is independent of s. Let $u : s \to t$ be a morphism in $\mathcal{S}$. Because $u^*(b_t) = b_s$, the chosen cartesian lift

$$\chi_{u,b_t} : b_s \longrightarrow b_t$$

exists. Since each F_i preserves the chosen cleavage, $F_i(\chi_{u,b_t})$ is the chosen cartesian lift of $A(u)$ with codomain $F_i(b_t) = b_{A(t)} \cdot k_i(t)$. By G-stability of the cleavage, this chosen lift is

$$\chi_{A(u),b_{A(t)} \cdot k_i(t)} = \chi_{A(u),b_{A(t)}} \cdot k_i(t),$$

whose source is

$$A(u)^*(b_{A(t)} \cdot k_i(t)) = b_{A(s)} \cdot k_i(t).$$

But the source of $F_i(\chi_{u,b_t})$ is also $F_i(b_s) = b_{A(s)} \cdot k_i(s)$. Hence $k_i(s) = k_i(t)$ for $i = 1, 2$, and therefore $g_s = g_t$. Because $\mathcal{S}$ is connected, all g_s coincide with a single $g \in G$. Thus $F_2 = \Lambda_g \circ F_1$ on objects. The same equality on morphisms follows because both functors are cleavage preserving and strictly G–equivariant. Uniqueness of g follows by evaluating at any basepoint b_s. □

4.3 Exact sequence

Theorem 1 (Strict classification of lifted symmetries)**.** *Let $p : \mathcal{O} \to \mathcal{S}$ be a split principal G–fibration in groupoids. Then the assignment sending a cleavage-preserving G–equivariant bundle autoequivalence F to the unique base autoequivalence A satisfying $p \circ F = A \circ p$ defines a surjective group homomorphism*

$$\Phi : \mathrm{Aut}^{\square}_G(\mathcal{O}/p) \longrightarrow \mathrm{Aut}(\mathcal{S}).$$

Its kernel is exactly the subgroup of normalized fiber translations $\{\Lambda_g \mid g \in G\} \cong G$. Consequently there is a short exact sequence

$$1 \longrightarrow G \xrightarrow{\Lambda} \mathrm{Aut}^{\square}_G(\mathcal{O}/p) \xrightarrow{\Phi} \mathrm{Aut}(\mathcal{S}) \longrightarrow 1.$$

Equivalently, two lifts of the same base symmetry differ by a unique normalized fiber translation.

Proof. The map Φ is well-defined because any $F \in \mathrm{Aut}^{\square}_{G}(\mathcal{O}/p)$ satisfies $p \circ F = A \circ p$ for a unique chosen base autoequivalence A. It is a homomorphism by composition. Surjectivity follows from Lemma 1. If $F \in \ker \Phi$, then $p \circ F = p$, so F and the identity functor cover the same base autoequivalence. Lemma 2 therefore gives a unique $g \in G$ such that $F = \Lambda_g$. Conversely every normalized fiber translation lies in the kernel because it preserves the projection and, by construction, preserves the chosen cleavage. This identifies $\ker \Phi$ with G, and exactness follows. $\square$

$$1 \longrightarrow G \xrightarrow{\Lambda} \mathrm{Aut}^{\square}_{G}(\mathcal{O}/p) \xrightarrow{\Phi} \mathrm{Aut}(\mathcal{S}) \longrightarrow 1$$

Diagram. Base symmetries are exactly cleavage-preserving equivariant bundle automorphisms modulo normalized fiber translation.

Remark 4 (From exact sequence to observer–equivariance)**.** *Theorem 1 is the strict core of the paper. If one prefers the language of actions, a representation* $\rho : BG \to \mathrm{Aut}(\mathcal{S})$ *may be lifted, after choosing for each* $\rho(g)$ *a representative in* $\mathrm{Aut}^{\square}_{G}(\mathcal{O}/p)$*, to a family of equivariant bundle automorphisms upstairs. The exact sequence states precisely what ambiguity remains: only normalized fiber translation by elements of the fiber group.*

5 Concrete realization: Wigner and Uhlhorn

The exact sequence above is not itself a proof of Wigner's theorem, but it organizes its bundle-theoretic content. The physical space of pure states is projective Hilbert space, and physically irrelevant phase belongs to a fiber direction. The framework makes this fiber direction explicit.

For a complex Hilbert space $\mathcal{H}$, the unit sphere $S(\mathcal{H})$ carries the free right action of $U(1)$ by phase multiplication, and the quotient is projective space $\mathbb{P}(\mathcal{H})$. This is the standard Hopf-bundle picture. Probability-preserving symmetries on $\mathbb{P}(\mathcal{H})$ are classified by Wigner and Uhlhorn as projective unitary or antiunitary transformations [5, 6, 7]. In bundle language, the downstairs symmetry is represented upstairs by a transformation of representatives compatible with the phase fiber.

The theorem proved here does *not* derive Wigner–Uhlhorn from first principles. What it shows is that once physically irrelevant phase is modeled as a principal fiber direction, the passage from physically meaningful transformations downstairs to equivariant lifts upstairs is structurally forced. The quantum classification theorem then supplies the specific description of which base automorphisms preserve transition probabilities.

Remark 5. *The role of the framework in the Wigner setting is therefore organizational rather than competitive. It isolates the descent pattern common to the theorem: what matters physically is defined on projective space, but any implementation in terms of state vectors lives one level upstairs and is determined only up to a global phase-type ambiguity. In the language of Section 8, transition* *probabilities and expectation-value assignments are examples of data that descend to rays and are insensitive to the fiber action.*

6 Worked examples and model status

Examples 1 and 3 are theorem-level models of the abstract setup. Example 2 is a physically motivated specialization of the same pattern. Example 4 is a programmatic sketch. The distinction is maintained explicitly so that the formal status of each example is clear.

Example 1 (Strict action-groupoid model)**.** *Let $\mathcal{S}$ be any connected groupoid and let G be a group. Suppose $\pi : \mathcal{O} \to \mathcal{S}$ is obtained by assigning to each object $s \in \mathcal{S}$ a free transitive right G–set T_s, and to each morphism $u : s \to t$ a strictly G–equivariant bijection $u^* : T_t \to T_s$, strictly compatible with identities and composition. Regard each T_s as a discrete groupoid and let $\mathcal{O}$ be the Grothendieck construction of this split pseudofunctor, which is strict by assumption. Then π is a split principal G–fibration in groupoids. Thus Theorem 1 applies literally. This is the minimal strict model underlying the more physical examples below.*

Example 2 (Fixed-spacetime Lorentz model)**.** *Let M be a fixed Minkowski spacetime and let $\mathcal{S}_M$ be the one-object groupoid with automorphism group* $\mathrm{ISO}(1,3)$*. Let $\mathcal{O}_M$ be the action groupoid of the right action of $O(1,3)$ on the set of global orthonormal frames of M relative to a chosen affine origin. The projection*

$$p_M : \mathcal{O}_M \to \mathcal{S}_M$$

forgets the chosen frame and remembers only the underlying Minkowski structure together with its Poincaré symmetry type. After choosing the evident split cleavage coming from the action-groupoid structure, p_M is a split principal $O(1,3)$–fibration. Hence Theorem 1 applies to this fixed-spacetime model.

This example is intentionally narrow. It is a clean model for frame dependence over one background Minkowski spacetime, not a theorem about arbitrary Lorentzian manifolds or general relativity.

Example 3 (Strict qubit phase model)**.** *Fix $\mathcal{H} = \mathbb{C}^2$. Let $\mathcal{S}_{\mathrm{qb}}$ be the action groupoid for the natural action of the projective unitary group $PU(2)$ on projective space $\mathbb{P}(\mathcal{H})$. Let $\mathcal{O}_{\mathrm{qb}}$ be the action groupoid for the natural action of $U(2)$ on the unit sphere $S(\mathcal{H})$. The subgroup $U(1) \subset U(2)$ acts freely on $S(\mathcal{H})$ by global phase multiplication, and the quotient on objects is $\mathbb{P}(\mathcal{H})$. The projection*

$$p_{\mathrm{qb}} : \mathcal{O}_{\mathrm{qb}} \to \mathcal{S}_{\mathrm{qb}}$$

forgets global phase and descends from the quotient homomorphism $U(2) \to PU(2)$. Choosing the evident split cleavage yields a split principal $U(1)$–fibration. Theorem 1 therefore applies literally, and because $U(1)$ is abelian the normalized fiber translations reduce to ordinary global phase multiplication.

This example is mathematically strict but intentionally small. It is a toy model for the phase-bundle aspect of Wigner's setting, not a replacement for the classification theorem itself.

Example 4 (Gauge-theoretic sketch)**.** *In a gauge theory one may let $\mathcal{O}$ be a groupoid of local gauge-dependent presentations (potentials, local sections, trivializations) and let G act by gauge transformations. The base $\mathcal{S}$ then records gauge-invariant structure. A gauge-covariant law is not a law on $\mathcal{O}$ arbitrarily; it is one whose public content descends to $\mathcal{S}$. The exact sequence captures the corresponding ambiguity in choosing gauge-dependent representatives.*

This remains a heuristic extension rather than a theorem-level example, because the relevant sites, local trivializations, and descent conditions would have to be specified explicitly.

7 Symmetry breaking as reduction of structure group

The strict bundle formulation also clarifies the status of spontaneous symmetry breaking. Suppose the observer bundle initially has structure group G, but a vacuum, order parameter, or phase choice selects a reduction to a subgroup $H \subseteq G$. Then the relevant observer bundle is no longer the full G–bundle but an H–subbundle. From the present perspective, the set of admissible observer changes has narrowed. What counts as "the same law across observers" is now constrained by the reduced group.

This does not mean that the larger group was unreal. Rather, it means that the physically realized regime supports only a restricted class of equivariant lifts. Reduction of structure group is therefore the bundle-theoretic shadow of symmetry breaking. A standard physical example is the electroweak pattern $SU(2) \times U(1) \to U(1)_{\mathrm{em}}$: once a Higgs vacuum is fixed, the relevant comparison class of admissible internal presentations is reduced, and the surviving equivariant transport is correspondingly controlled by the reduced structure group.

8 Observables

An observable should be thought of here as functorial data that survives the forgetful passage from standpoint-dependent presentation to public structure. In the strict bundle language, objectivity is not primitive; it is descent. An observable is objective insofar as its value is invariant under the fiber action or, more generally, is naturally transported by cleavage-preserving equivariant bundle automorphisms.

This makes inter-observer agreement precise. Agreement does not require that two observers share the same presentation upstairs. It requires that their presentations project to the same structural content downstairs, and that the law assigning observable values be compatible with the bundle transport that relates the two.

Proposition 2 (Observables descend)**.** *Let $p : \mathcal{O} \to \mathcal{S}$ be a split principal G–fibration in groupoids and let $F : \mathcal{O} \to \mathcal{C}$ be a functor into any category $\mathcal{C}$. Assume:*

(D1) $F \circ R_g = F$ *for all* $g \in G$ *(here* R_g *denotes the given right* G*-action on* $\mathcal{O}$*, not an element of* $\mathrm{Aut}_G^{\Box}(\mathcal{O}/p)$*);*

(D2) for every base morphism $u : s \to t$ *and all objects* $y, y' \in \mathcal{O}_t$ *with* $y' = y \cdot g$ *for some* $g \in G$*, one has*

$$F(\chi_{u,y'}) = F(\chi_{u,y}).$$

Then there exists a unique functor $\bar{F} : \mathcal{S} \to \mathcal{C}$ *with*

$$F = \bar{F} \circ p.$$

Proof. On objects, define $\bar{F}(s) := F(x)$ for any $x \in \mathcal{O}_s$. This is well defined because any two objects in $\mathcal{O}_s$ differ by a unique element of G, and (D1) implies they have the same image under F. For a morphism $u : s \to t$, choose any object $y \in \mathcal{O}_t$ and set

$$\bar{F}(u) := F(\chi_{u,y}).$$

This is independent of the choice of y by (D2), since any other choice has the form $y \cdot g$. Functoriality follows from splitness of the cleavage and the composition convention fixed in Section 2:

$$\bar{F}(v \circ u) = F(\chi_{v \circ u,z}) = F(\chi_{v,z}) \circ F(\chi_{u,v^* z}) = \bar{F}(v) \circ \bar{F}(u).$$

Uniqueness is immediate because p is surjective on objects and every base morphism is represented by a chosen cartesian lift. $\Box$

A concrete example: qubit spin as an algebra–valued functor

Let $\mathcal{S}_{\mathrm{qb}}$ be the strict qubit ray groupoid of Example 3, and let $\mathcal{A}$ be the one-object category with endomorphism algebra $M_2(\mathbb{C})$. A spin observable may be encoded by a functor

$$\mathsf{S} : \mathcal{S}_{\mathrm{qb}} \to \mathcal{A},$$

which assigns to each ray context the corresponding Pauli-operator data. Public content resides not in a particular state vector but in the descended structure on rays. Global phase lives upstairs in the $U(1)$-fiber and does not affect the observable assignment. This is the operational meaning of saying that the observable factors through the bundle projection.

9 Philosophical note

The framework developed above is compatible with, but not identical to, several familiar philosophical positions. It is structurally realist in the weak sense that public physical content is represented by the base category $\mathcal{S}$, not by a hidden absolute standpoint. It is relational in the sense that standpoint changes are explicit and fundamental to the formalism. But it is not radically relational in the sense of eliminating common structure altogether. The whole point of the bundle projection is that there *is* a shared arena of comparison, namely what descends.

For mind–matter discussions, the key point is that an observer is not treated as a mere label attached from outside. The observer enters as a concrete presentation-level position in the total space $\mathcal{O}$. Objectivity is then what remains after quotienting or forgetting that position. This is close in spirit to a structural reading of objectivity and more disciplined than appeals to a view from nowhere.

The relation to Kant or Leibniz should not be overstated. The framework does not prove transcendental idealism, monadology, or any strong metaphysical doctrine. What it does show is more limited and, in my view, more useful: within the present framework, if physical law is to be shared across perspectives, then the mathematics of that shareability naturally takes the form of descent along a structured map from presentations to public structure. Symmetry is not thereby demoted to mere convention, nor elevated to mystical necessity. It appears as a consistency condition for a world that can be jointly described.

In this sense the framework is also close to a Machian intuition. No standpoint is privileged absolutely. What earns the title of law is what can be transported coherently across standpoints. The split principal bundle is the categorical form of that demand.

10 Conclusion

The paper has formulated and proved a strict lifting and classification theorem for split principal G–fibrations in groupoids with chosen cleavage preserved by the automorphisms considered upstairs. Every base autoequivalence lifts strictly to a G–equivariant bundle autoequivalence, and the ambiguity of the lift is exactly the normalized global fiber translation determined by an element of G. This yields the exact sequence

$$1 \longrightarrow G \longrightarrow \mathrm{Aut}_G^{\square}(\mathcal{O}/p) \longrightarrow \mathrm{Aut}(\mathcal{S}) \longrightarrow 1.$$

Within the present normalized strict model, the theorem shows that public symmetry is represented by observer–equivariant transport on the presentation side. The result should be read at exactly that level of generality: it is a classification theorem internal to a chosen split presentation with fixed normalization data.

The examples were chosen to reflect distinct logical roles. Some provide strict models of the theorem, while others are included only as controlled physical specializations or sketches. This separation keeps the theorem-level claims distinct from broader applications and from any stronger claim that familiar physical theories have already been reconstructed in full within the present apparatus.

A natural continuation would be to weaken the split hypotheses and pass to a pseudofunctorial or stack-theoretic setting, where the exact sequence would likely be replaced by a biequivalence statement. The strict theorem established here is intended as a controlled starting point for that broader generalization.

A Five equivalent viewpoints on objectivity and observer–equivariance

The bundle projection $p : \mathcal{O} \to \mathcal{S}$ admits several mutually compatible readings. The point of this appendix is not to introduce new mathematics, but to show that the same formal core may be recognized in several languages.

A.1 (V1) Invariants and descent: objectivity as factorization through p

A functorial assignment $F : \mathcal{O} \to \mathcal{C}$ is objective when it factors through p:

$$F = \bar{F} \circ p$$

for some $\bar{F} : \mathcal{S} \to \mathcal{C}$. This is the most direct expression of the claim that objective content is what survives forgetting which standpoint one started from.

A.2 (V2) Naturality: objectivity as commutative diagrams

The same idea may be written diagrammatically. To say that a law is shareable across perspectives is to say that whenever two presentations are related upstairs, the corresponding structure downstairs is related by a commuting square. Observer changes do not destroy lawful content; they transport it naturally.

A.3 (V3) Presentation vs. structure: $\mathcal{O}$ as presentations of $\mathcal{S}$

The total groupoid $\mathcal{O}$ consists of presentations of structure. The base category $\mathcal{S}$ contains the structure presented. This is close to how physicists already speak in many contexts: gauges, frames, phases, and coordinates are presentations; gauge-invariant or frame-independent content is structure.

A.4 (V4) Descent and gluing: the stacky generalization

The split setting of this paper is intentionally strict. In a weaker stack-theoretic or bicategorical treatment, the same core idea survives as descent data together with coherence isomorphisms. What is strict equality here becomes coherent comparison there. The theorem proved here may be read as the rigid skeleton of that broader picture [3, 8, 9].

A.5 (V5) Coarse-graining: objectivity as what survives forgetting

There is also a coarse-graining reading. Passing from $\mathcal{O}$ to $\mathcal{S}$ discards detail about the standpoint, just as a coarse-graining discards microstructure. Objectivity is then whatever remains stable under that forgetting. This does not make it less real; it makes explicit which degrees of freedom count as public.

A.6 Relation to bundle automorphisms

Theorem 1 shows that these viewpoints are not merely interpretive glosses. Once the projection p is a split principal G–fibration together with a chosen normalization, the lawful transport of objective content is governed exactly by equivariant bundle autoequivalences upstairs. Thus objectivity, descent, and symmetry are three ways of reading the same formal structure.

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