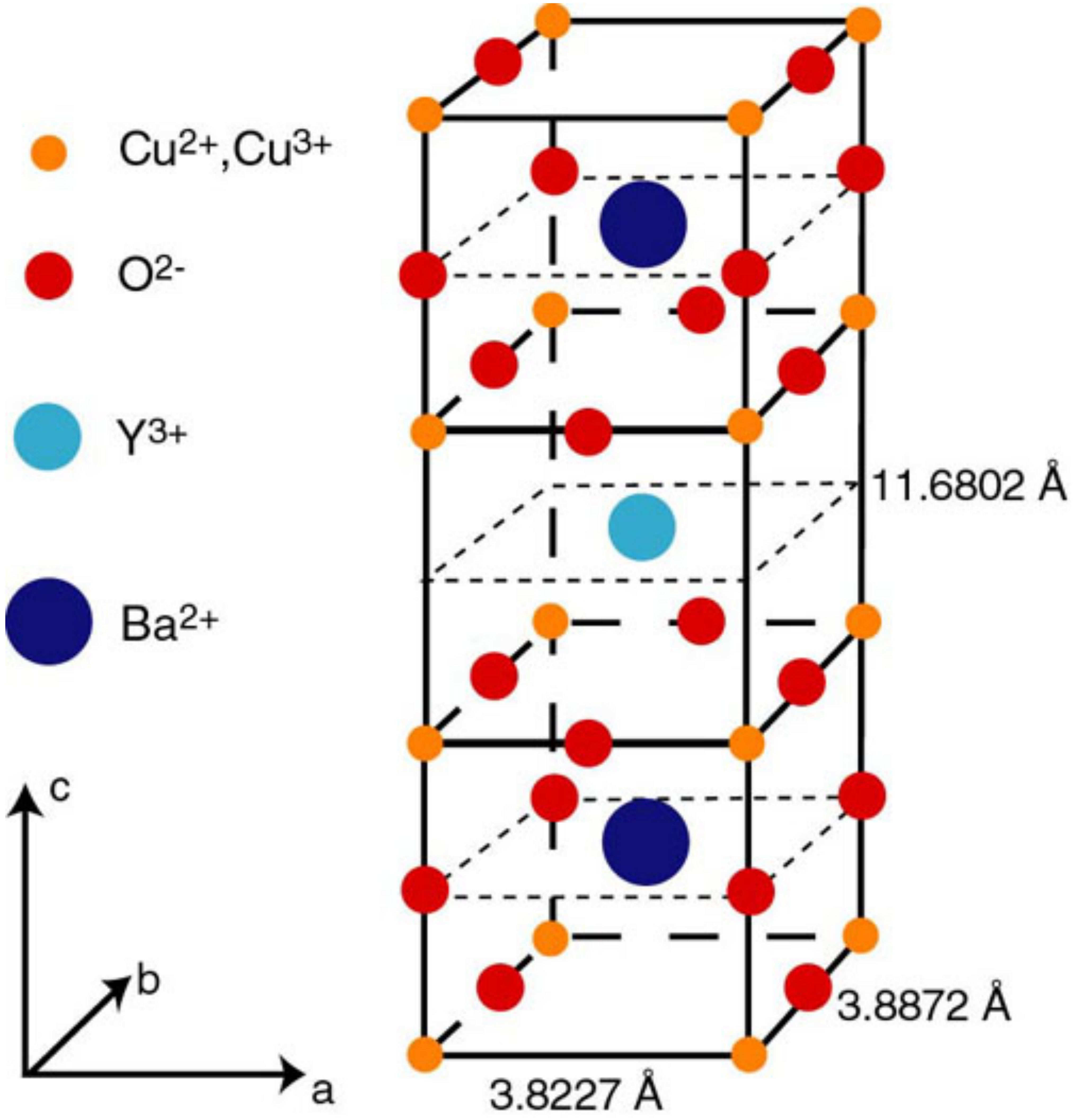
Many-fermion quantum entanglement in the high temperature superconductors and in black holes

Georgia Tech

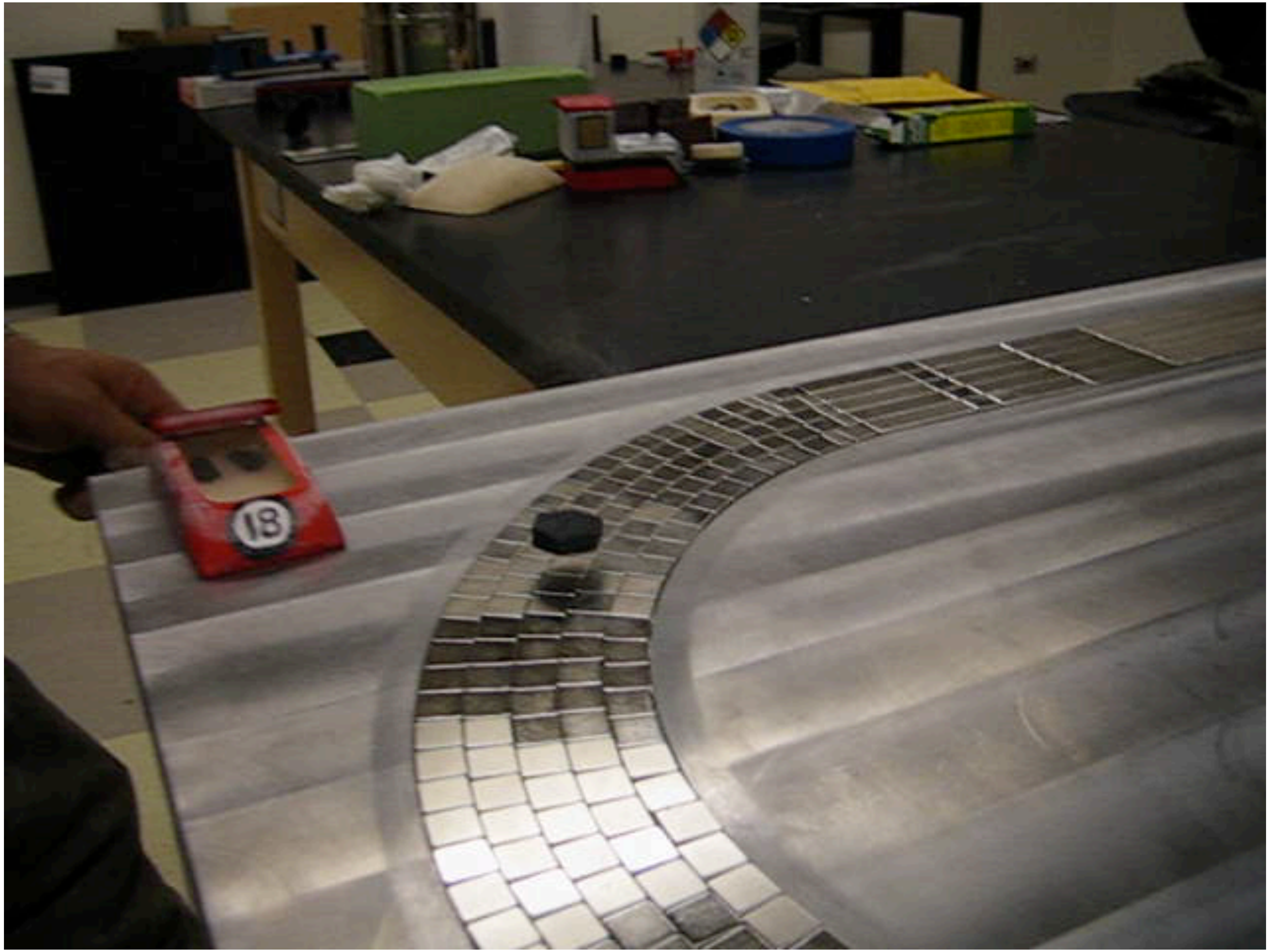
March 2, 2026

Subir Sachdev

Cuprate
high temperature
superconductors



$$YBa_2Cu_3O_{6+x}$$



Nd-Fe-B magnets, YBaCuO superconductor

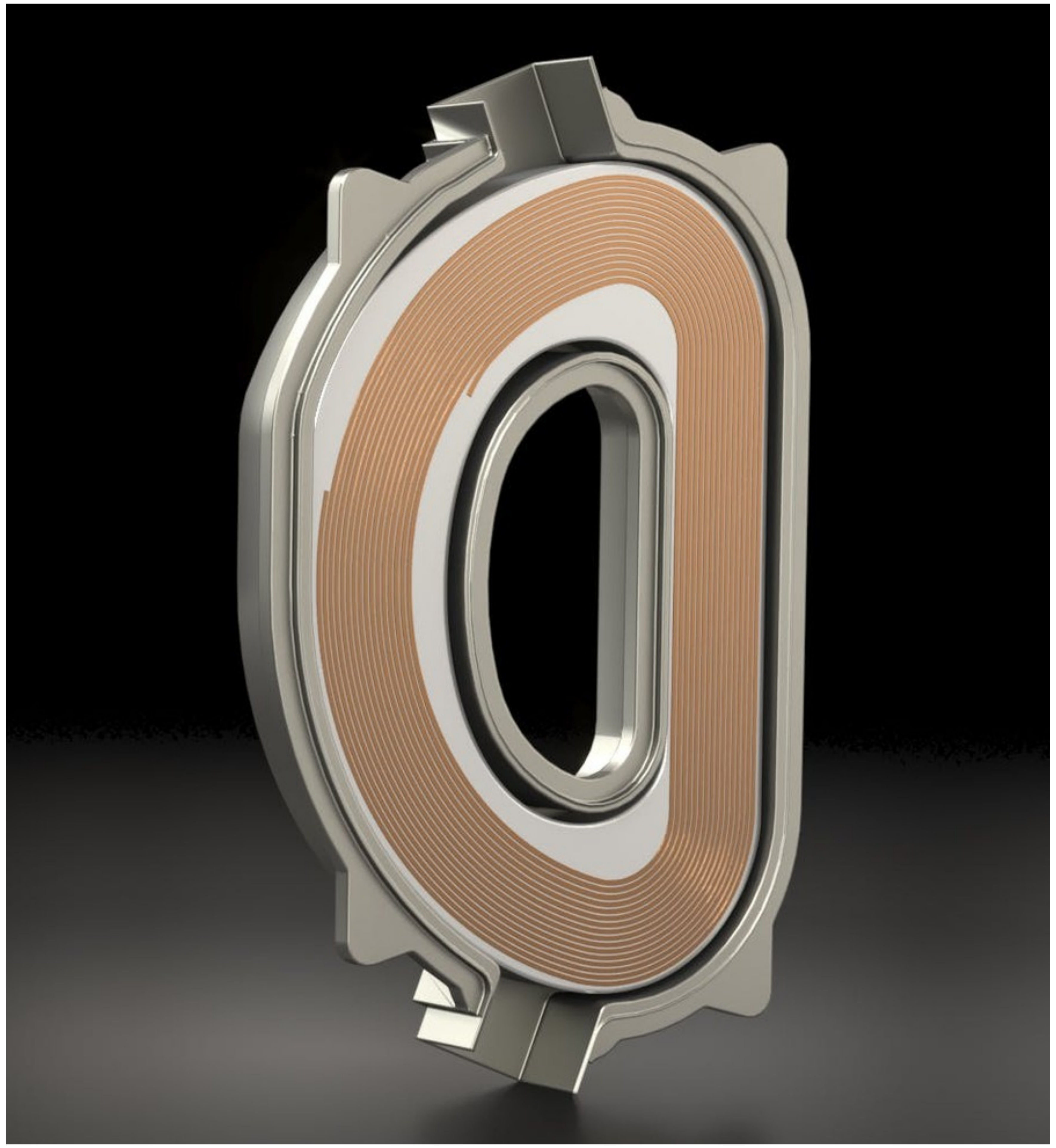
Julian Hetel and Nandini Trivedi, Ohio State University

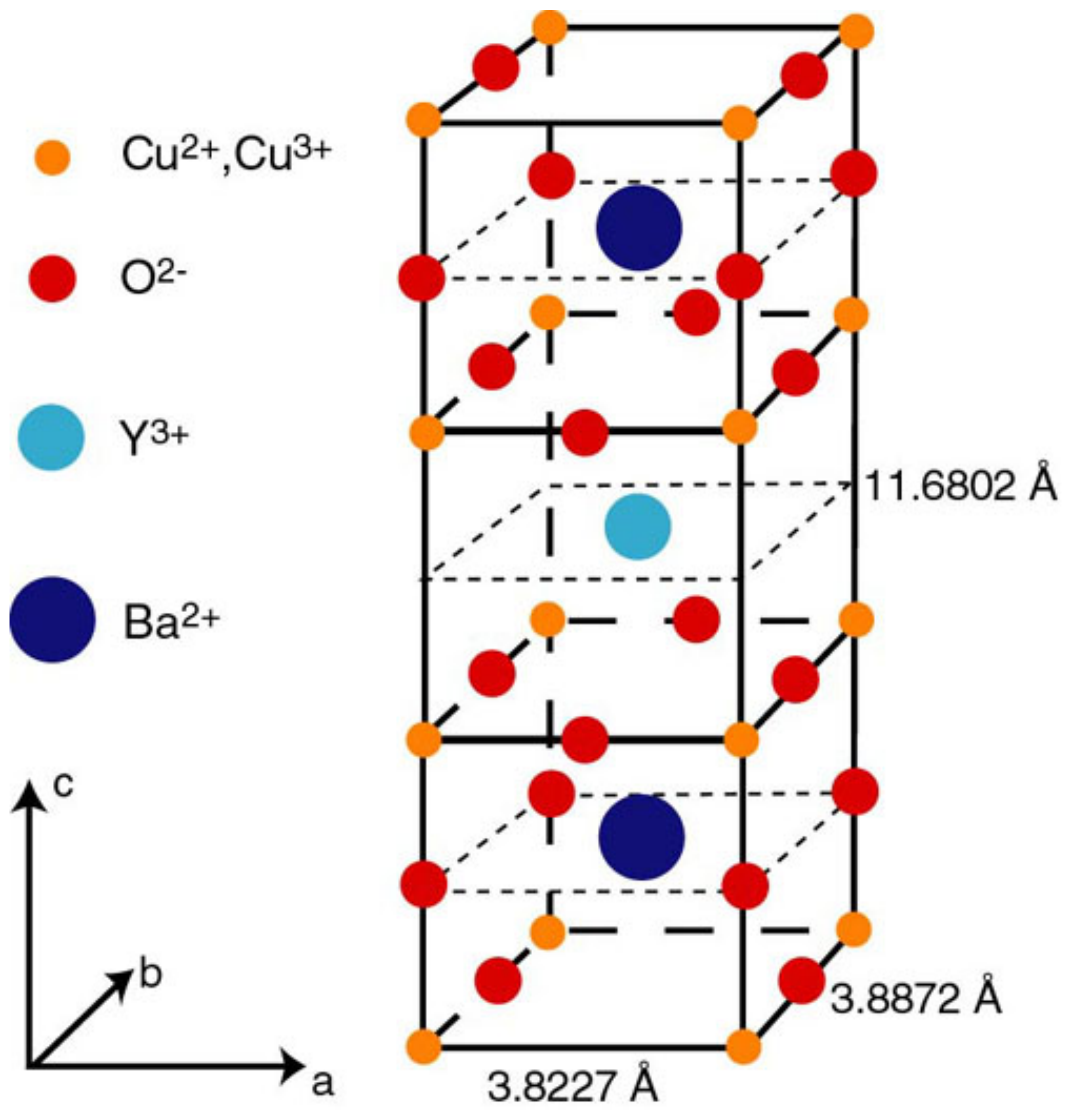
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





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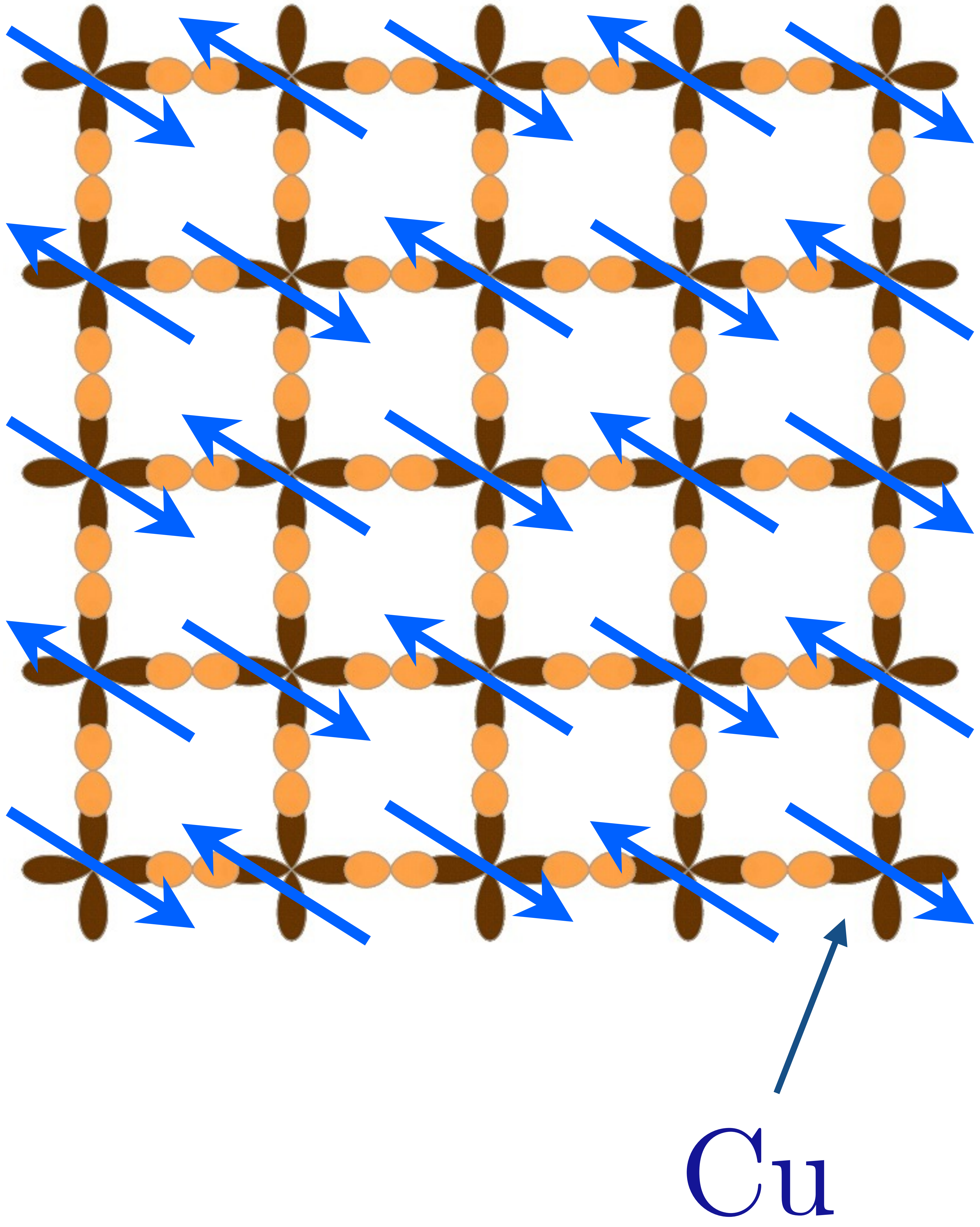
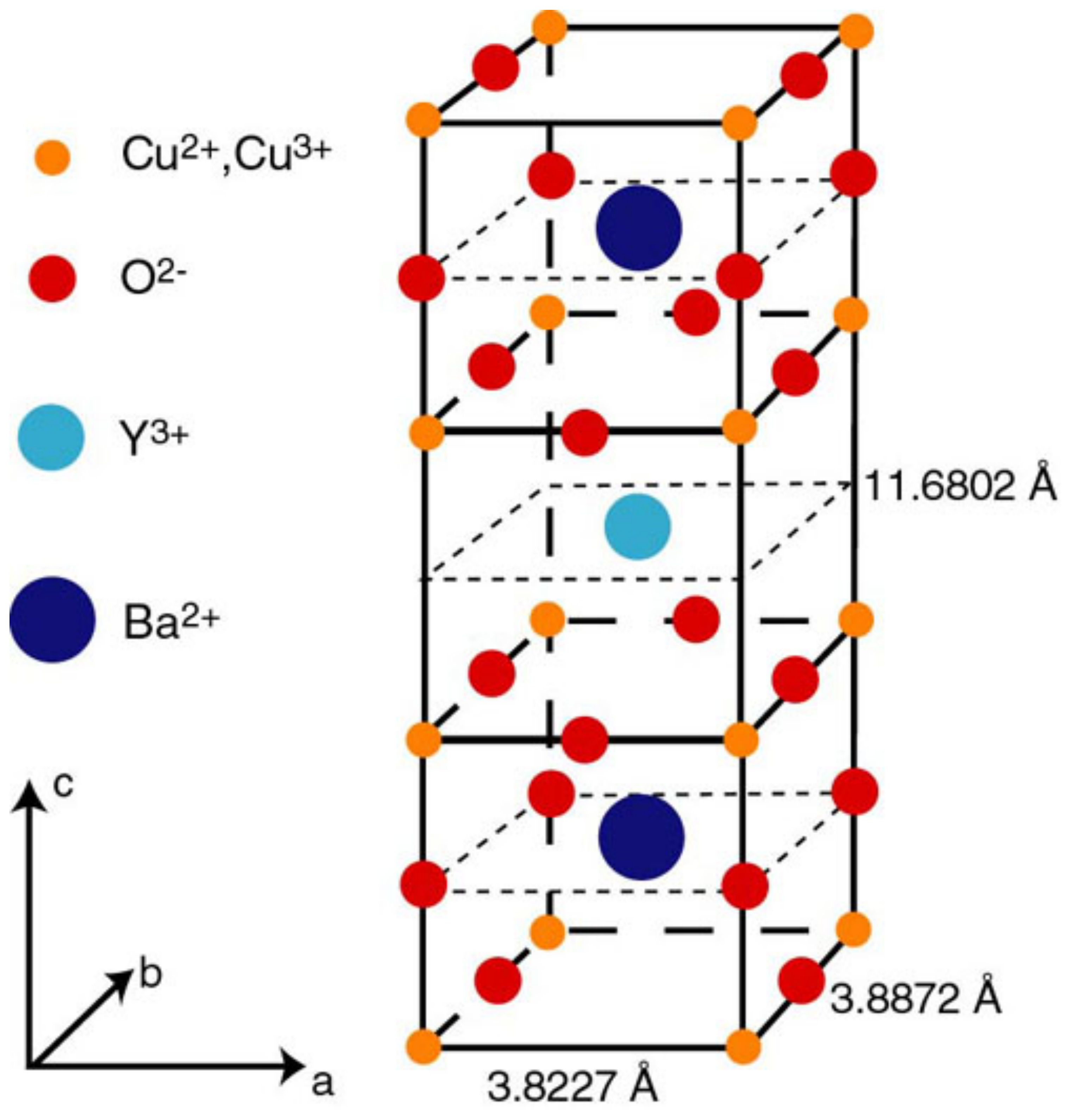


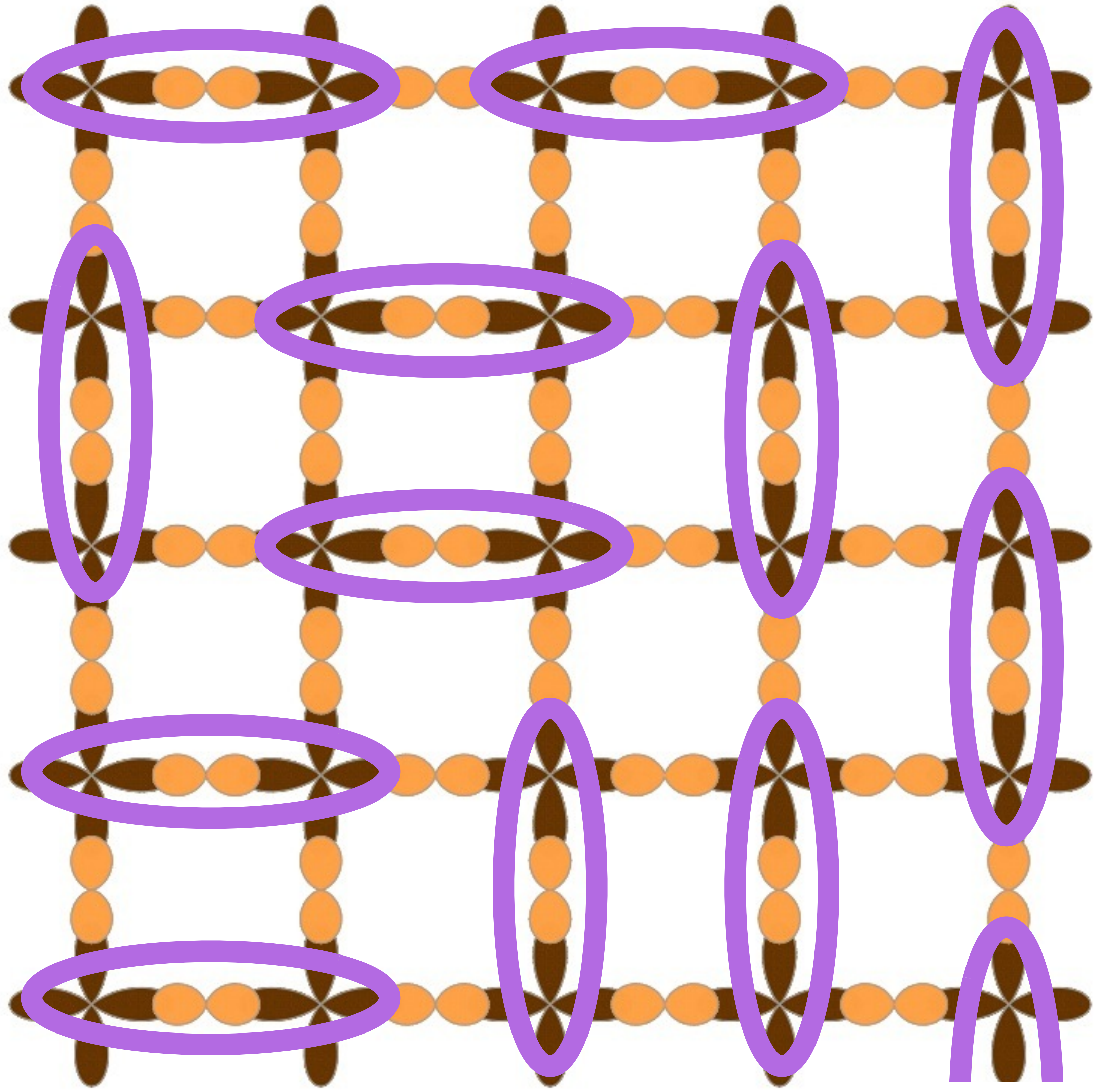
$$\mathcal{H} = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$S = 1/2$ on each site

$$|\uparrow\rangle, \; |\uparrow\rangle$$
$$S_z \, |\uparrow\rangle = (1/2) \, |\uparrow\rangle$$
$$S_z \, |\downarrow\rangle = -(1/2) \, |\downarrow\rangle$$
$$(S_x + i S_y) \, |\downarrow\rangle = |\uparrow\rangle$$
$$(S_x - i S_y) \, |\uparrow\rangle = |\downarrow\rangle$$

Insulating antiferromagnet with one electron per site





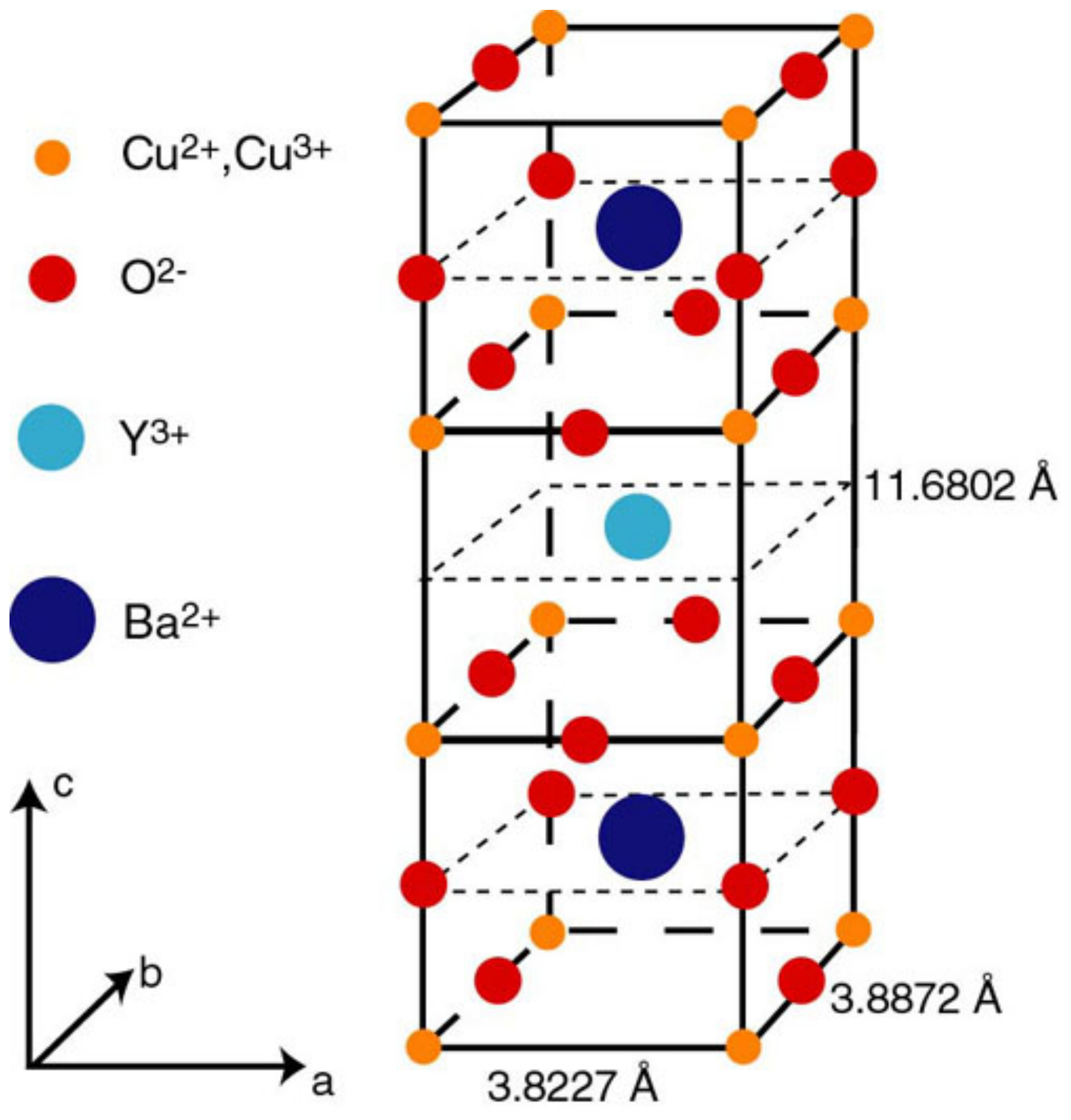
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

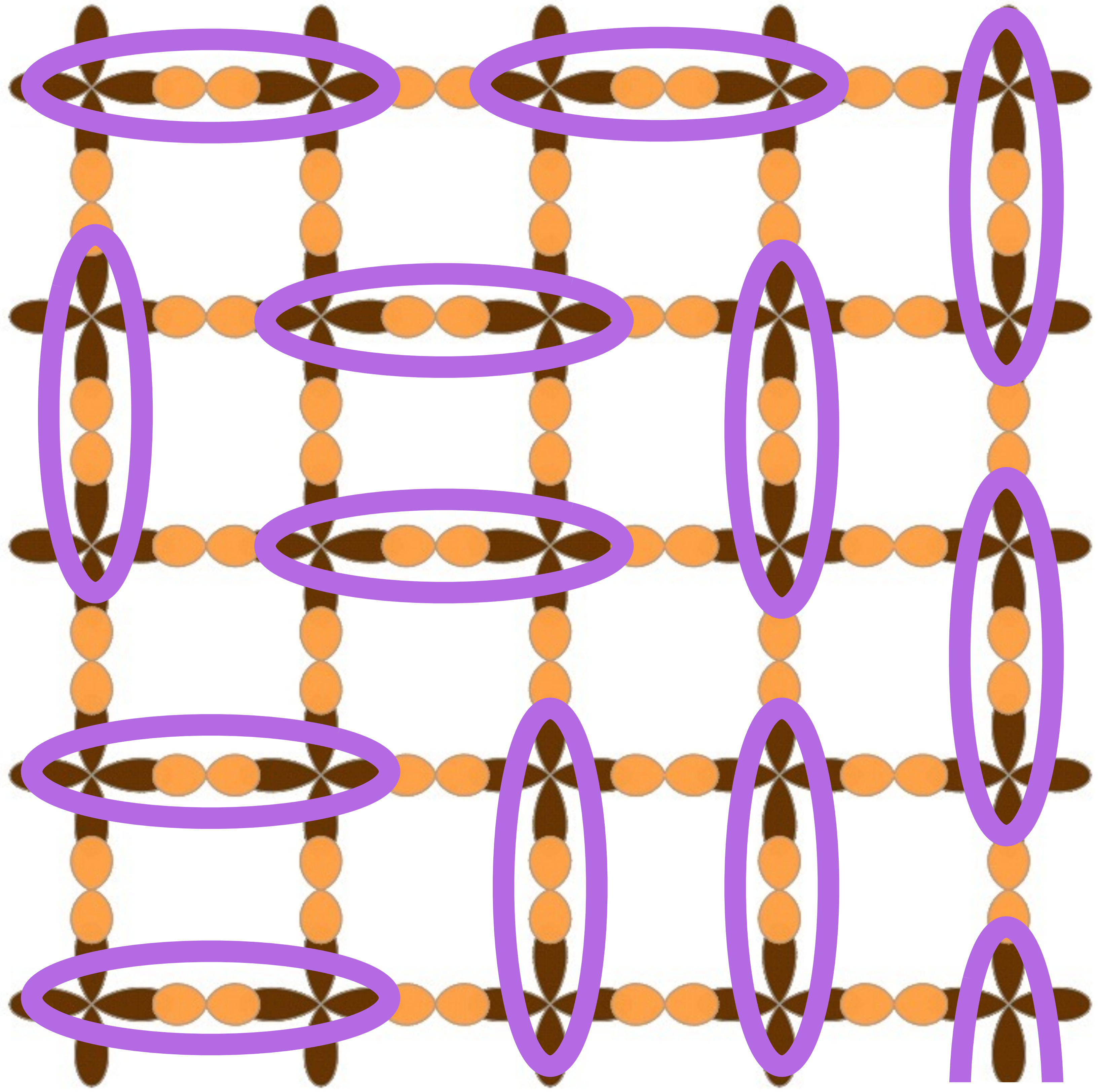
$\mathcal{D} \rightarrow$ dimer covering of lattice

$$\mathrm{YBa_2Cu_3O_{6+x}}$$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P.W. Anderson (1987): The key to high temperature superconductivity is the formation of a "resonating valence bond state".





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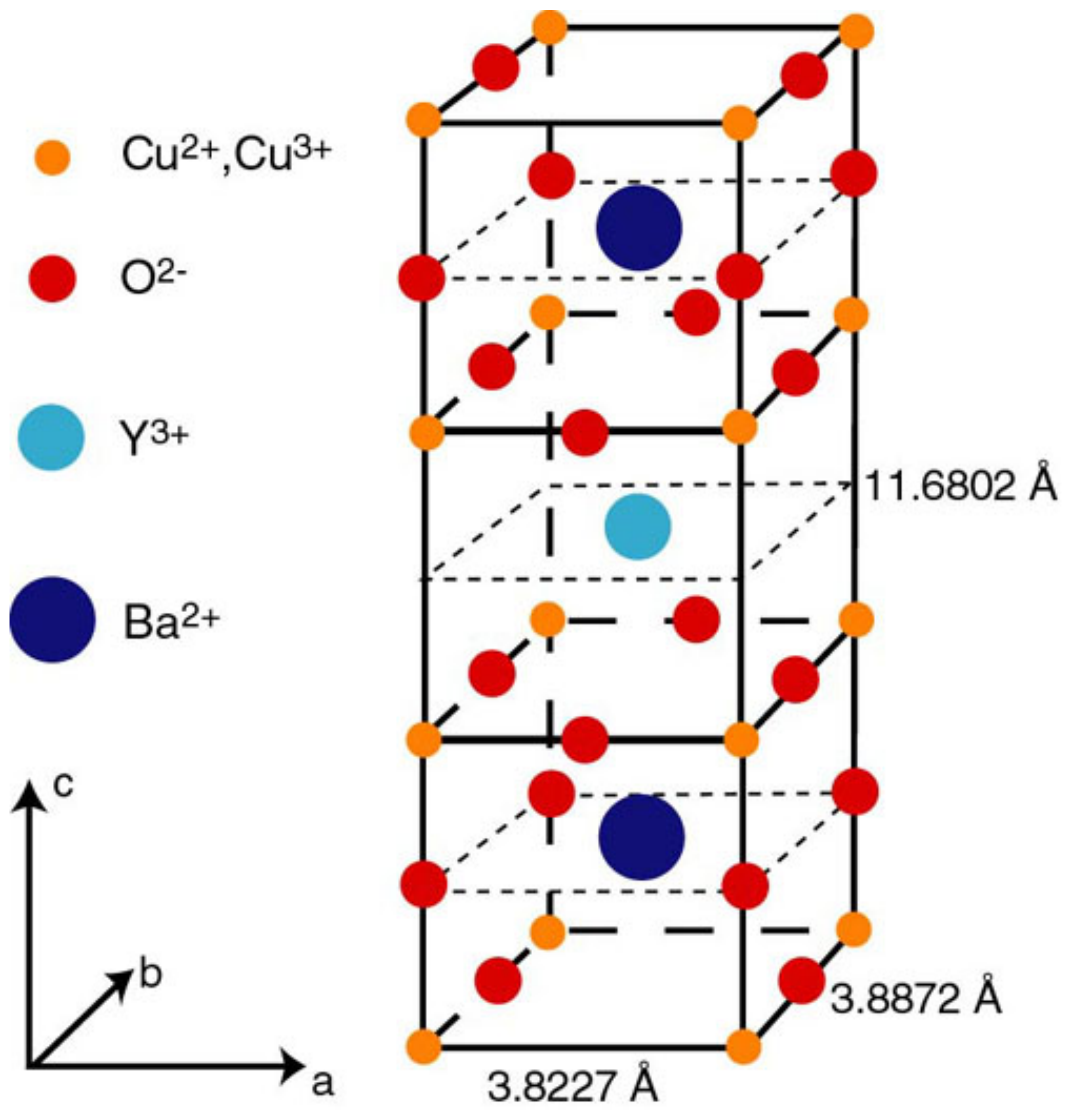

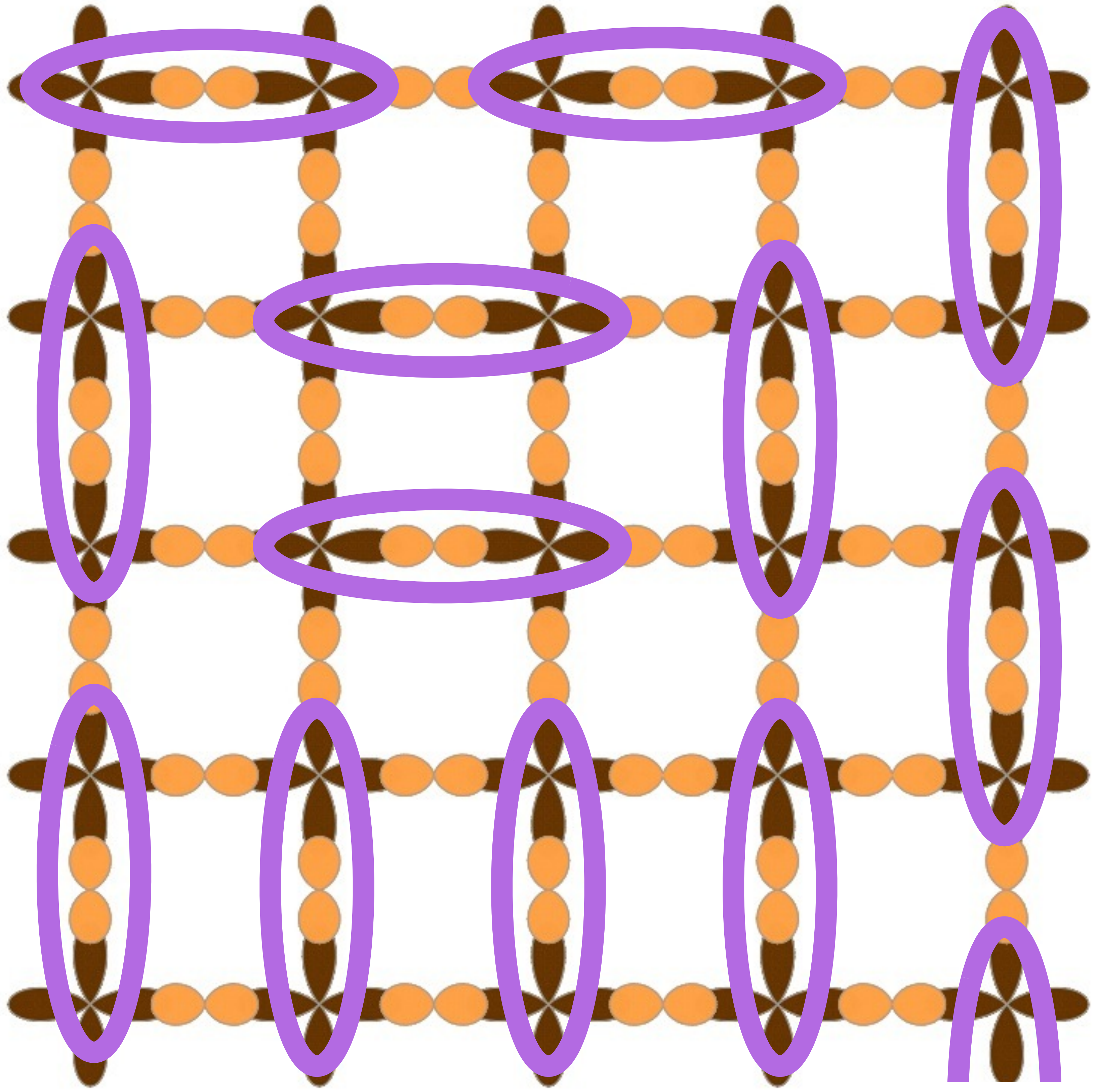

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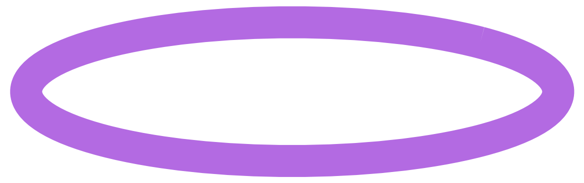 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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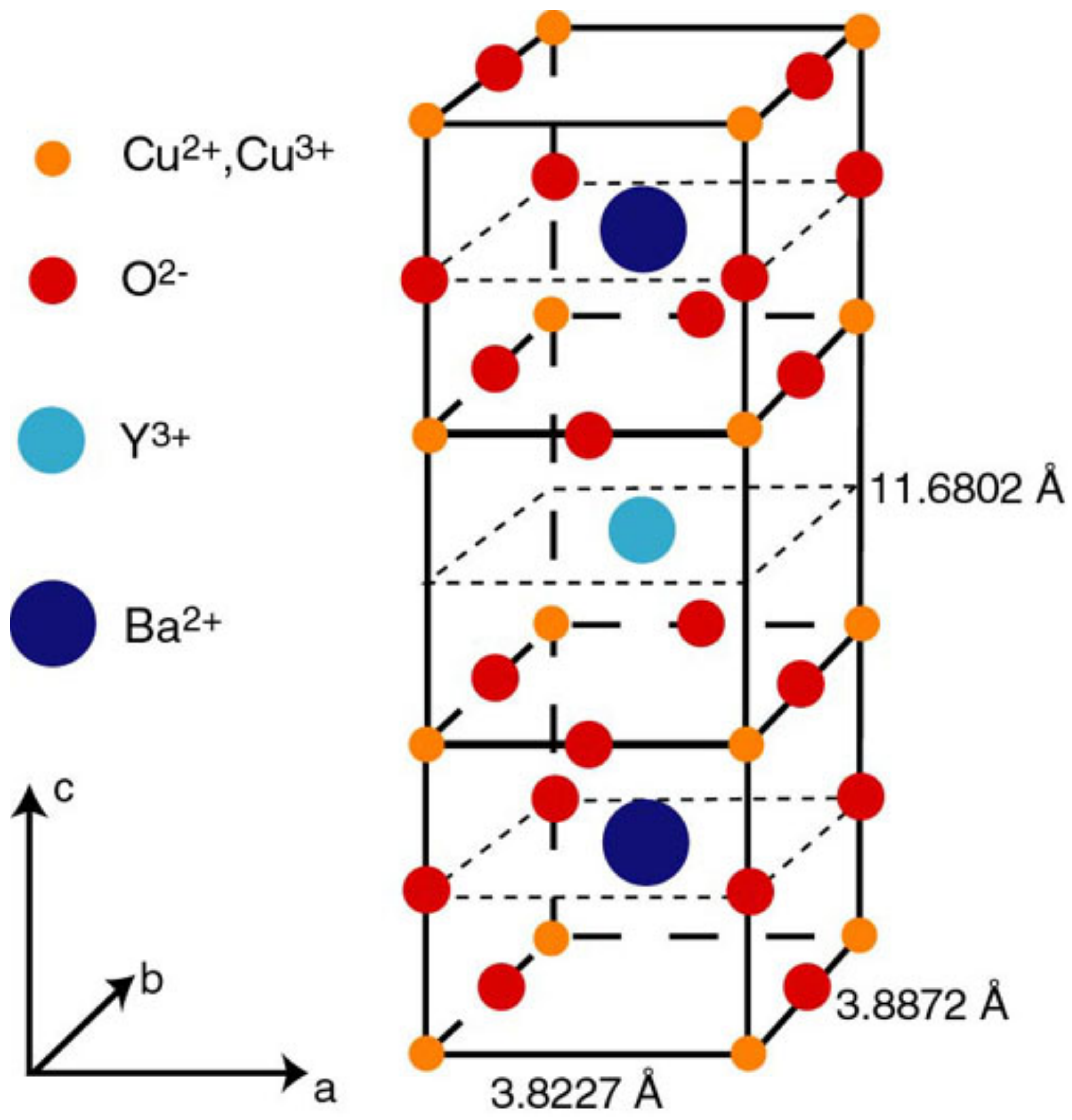

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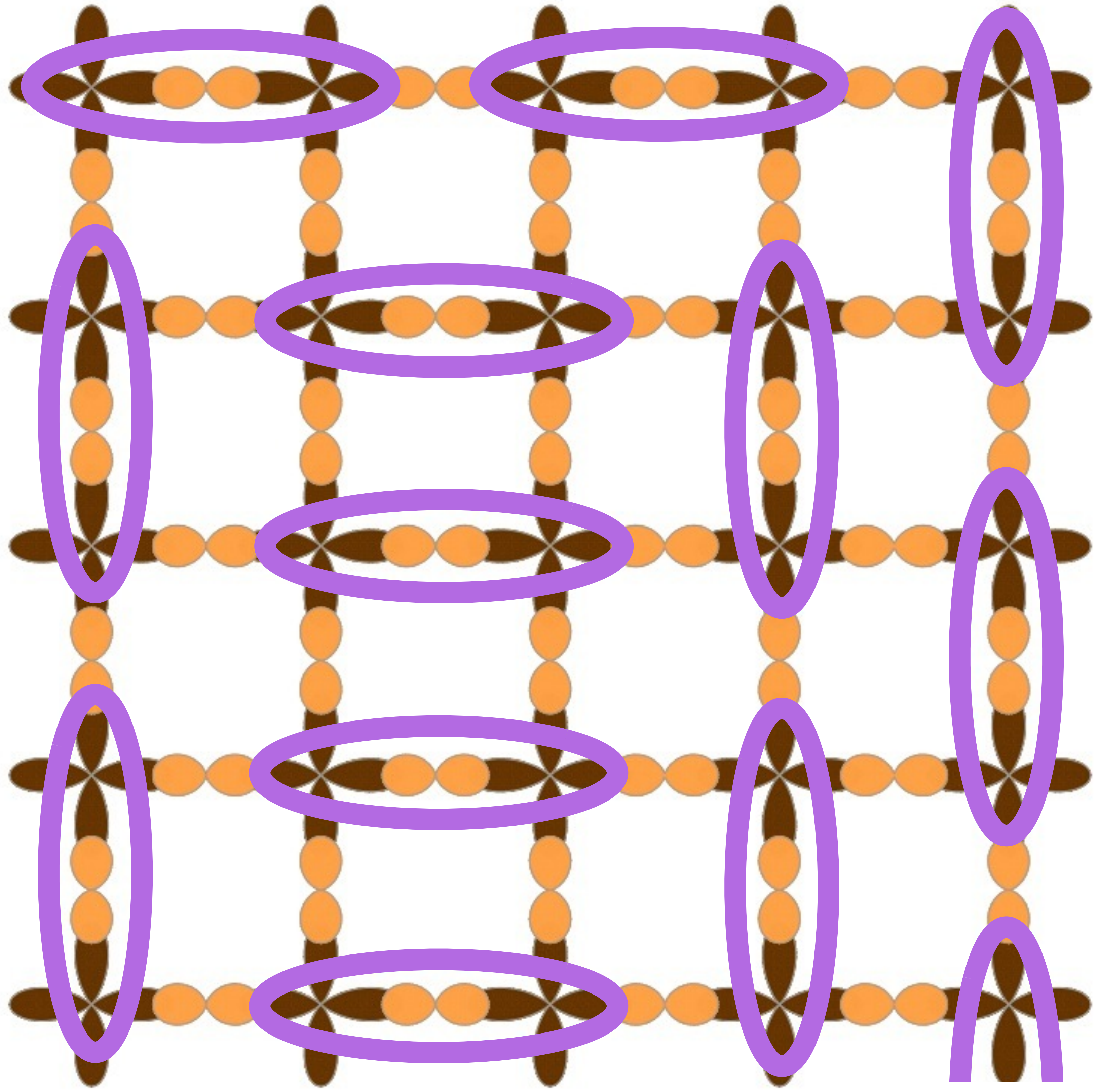

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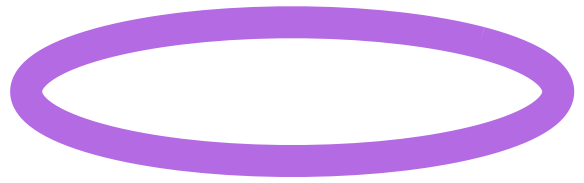

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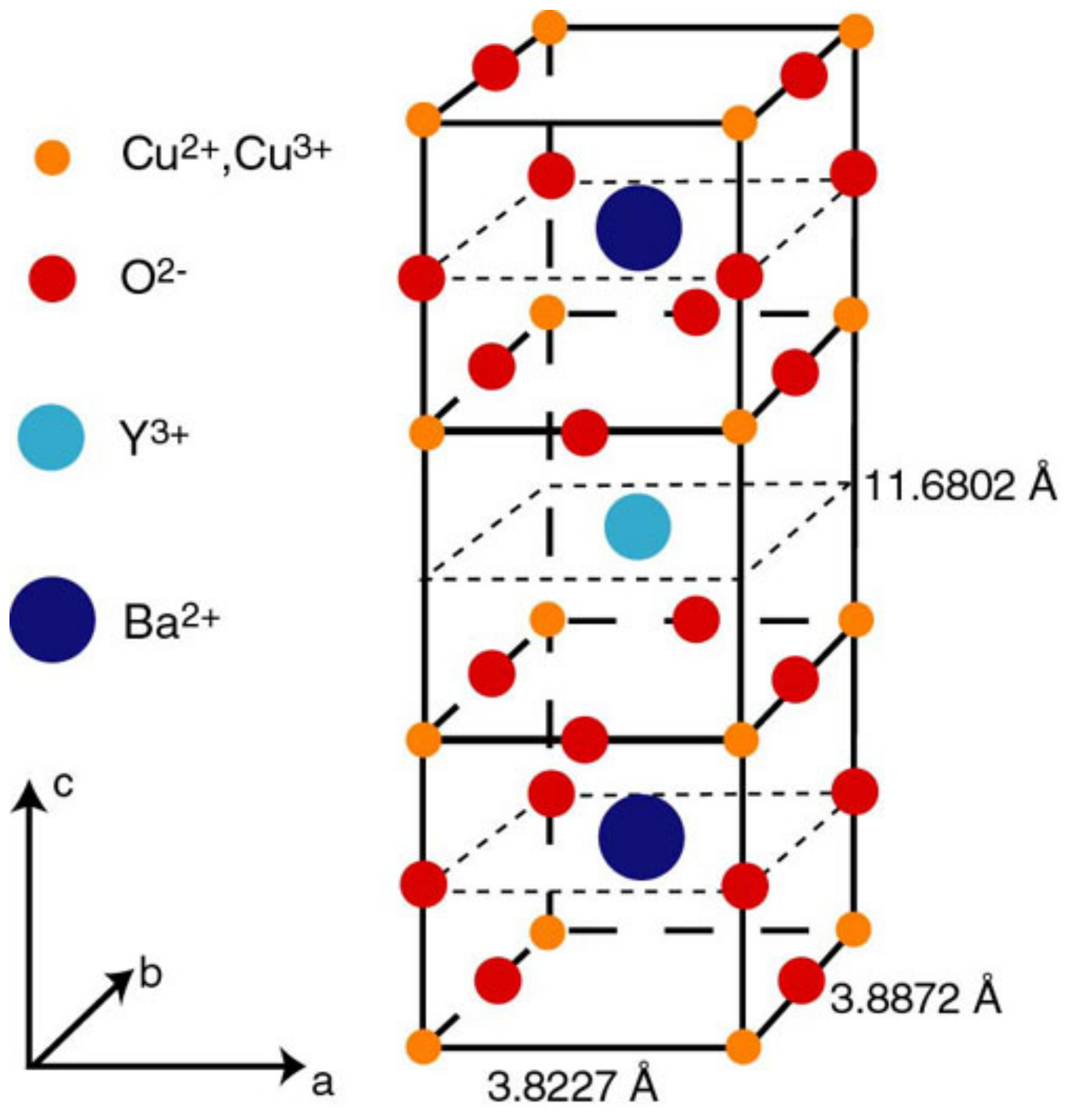

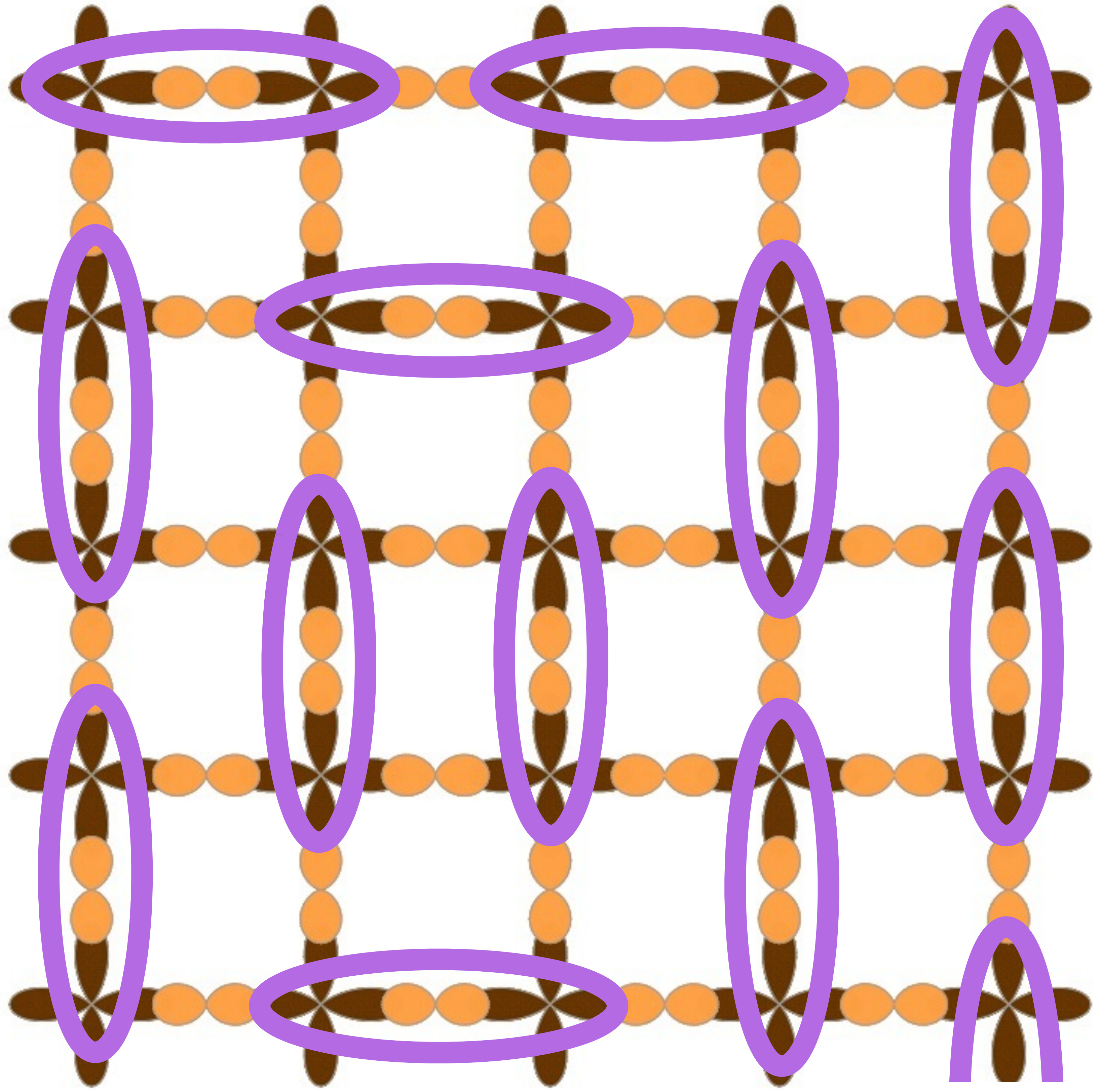

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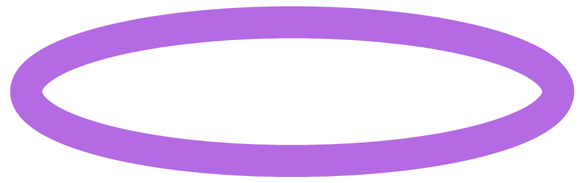 $= \frac{1}{\sqrt{2}} \left(| \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle \right)$

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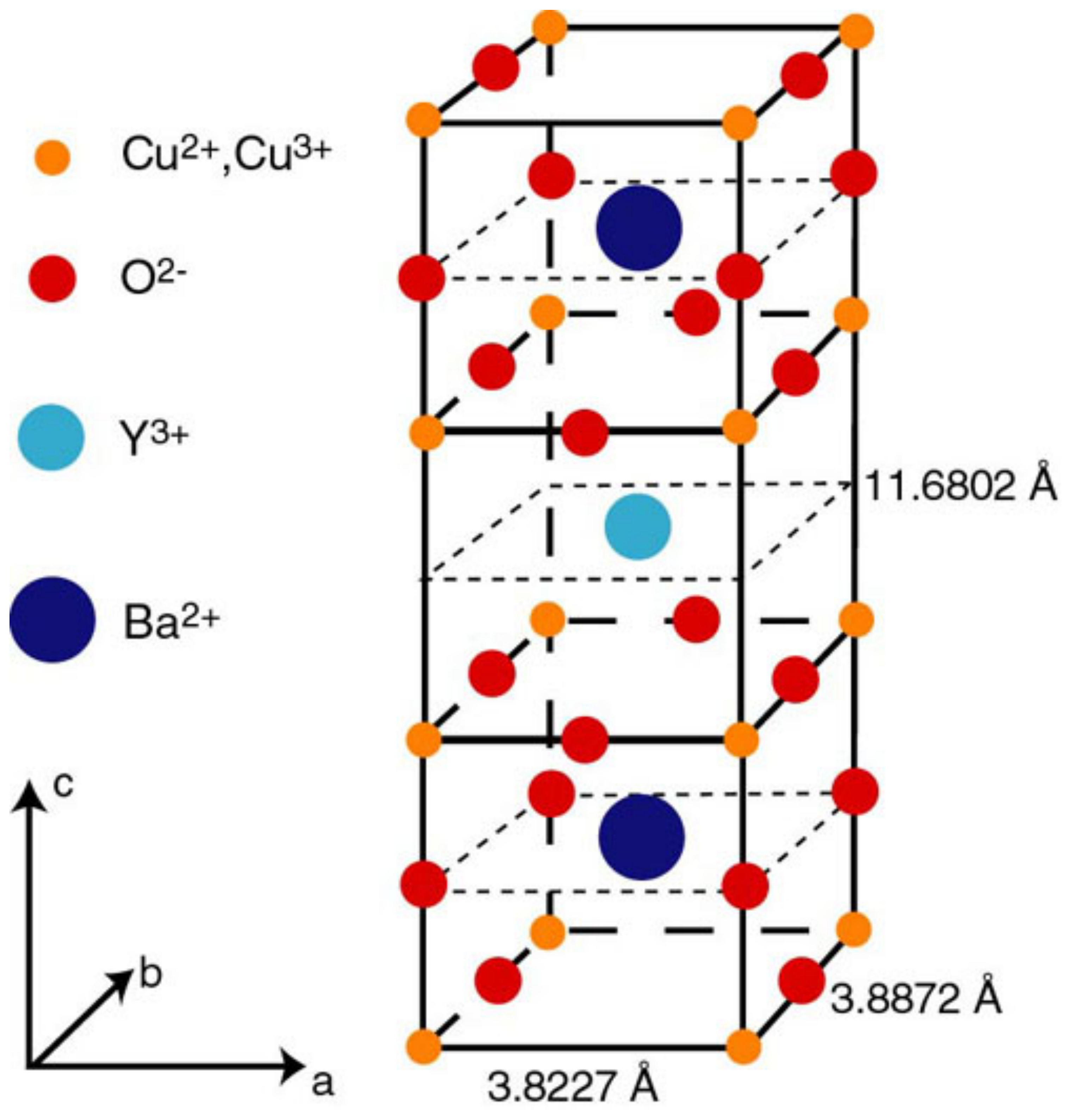

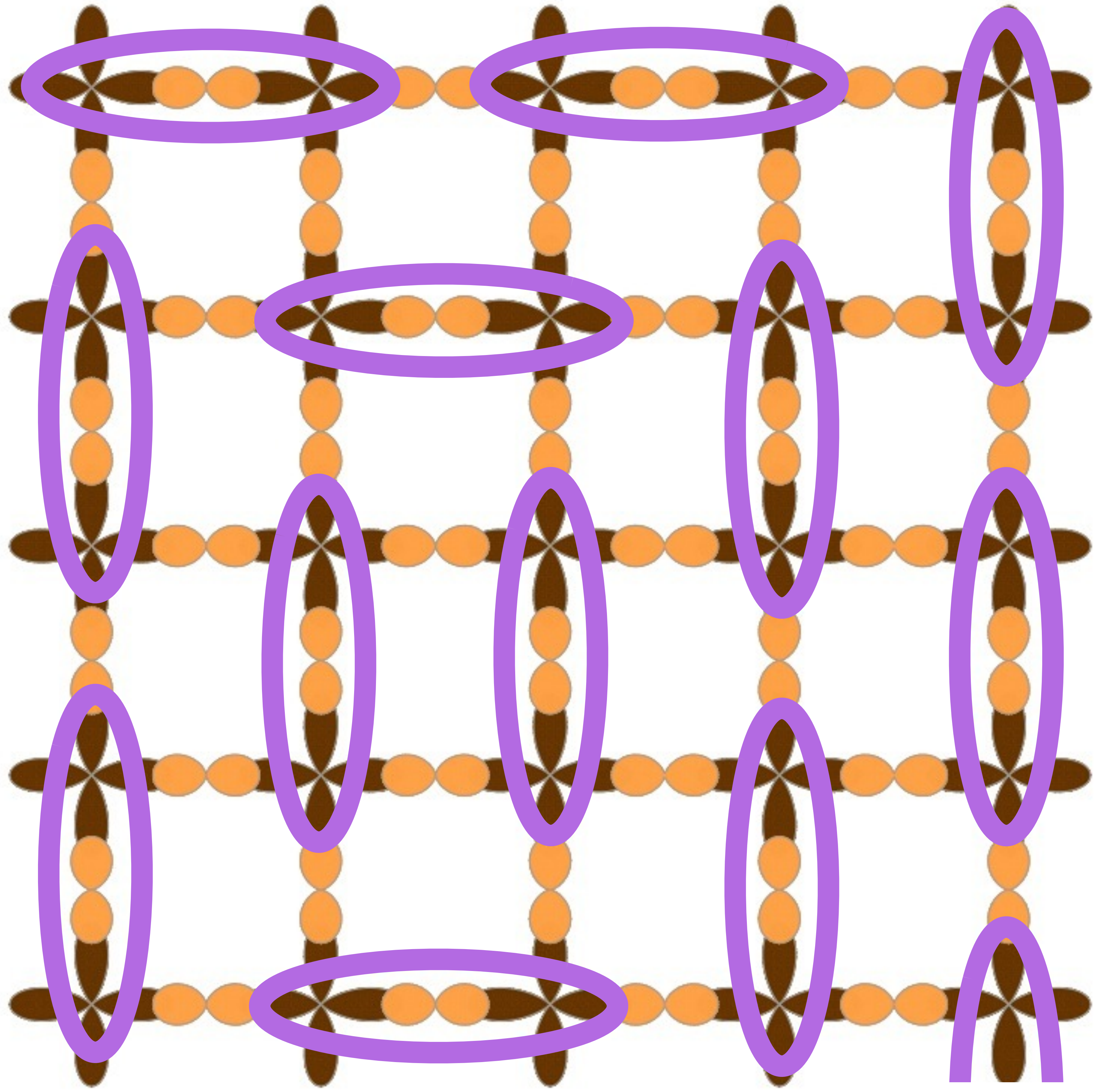

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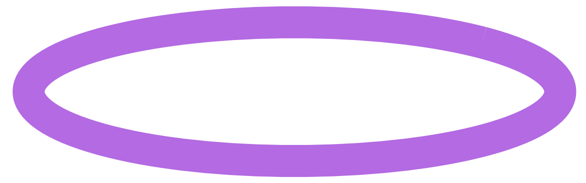

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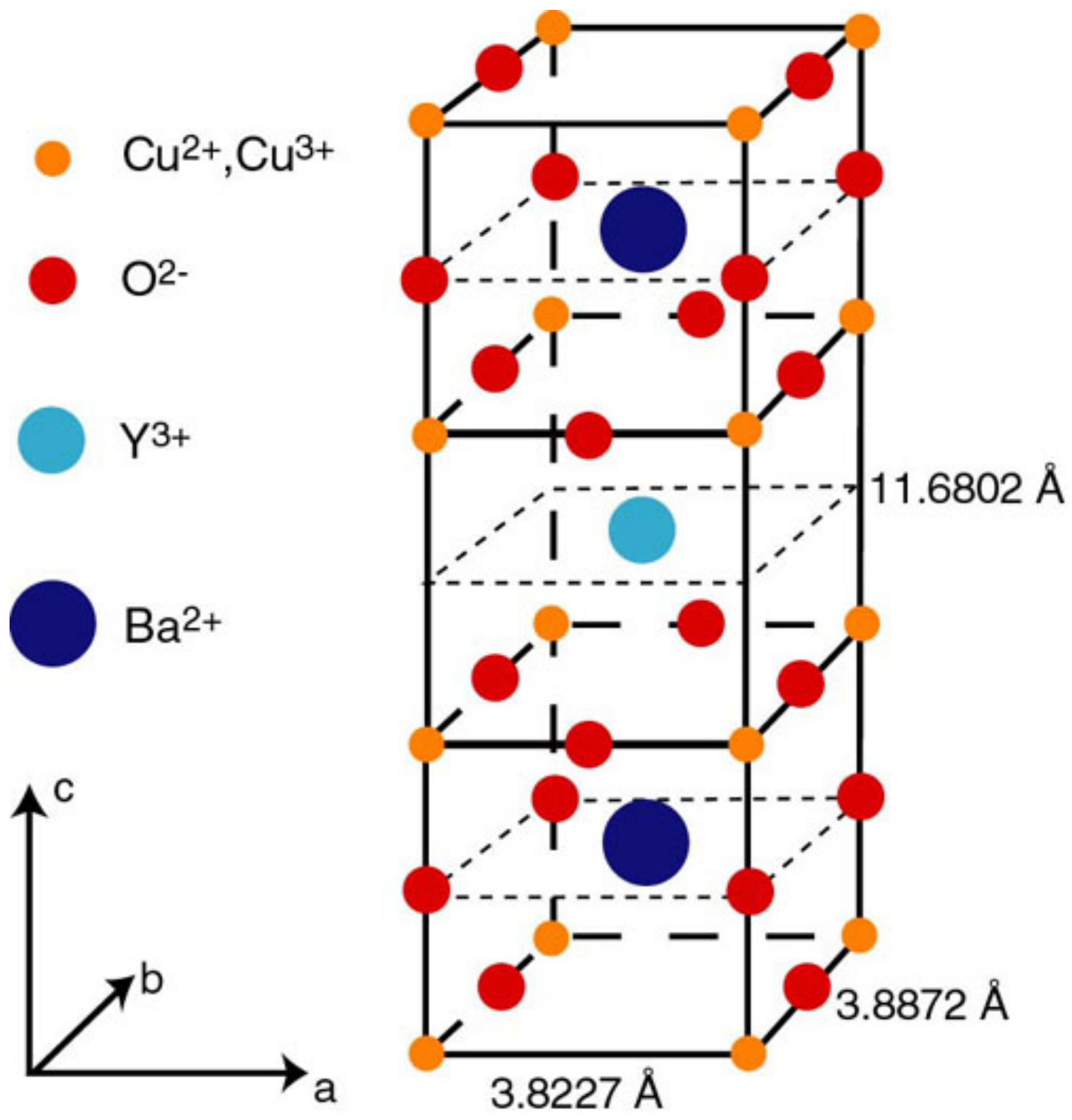
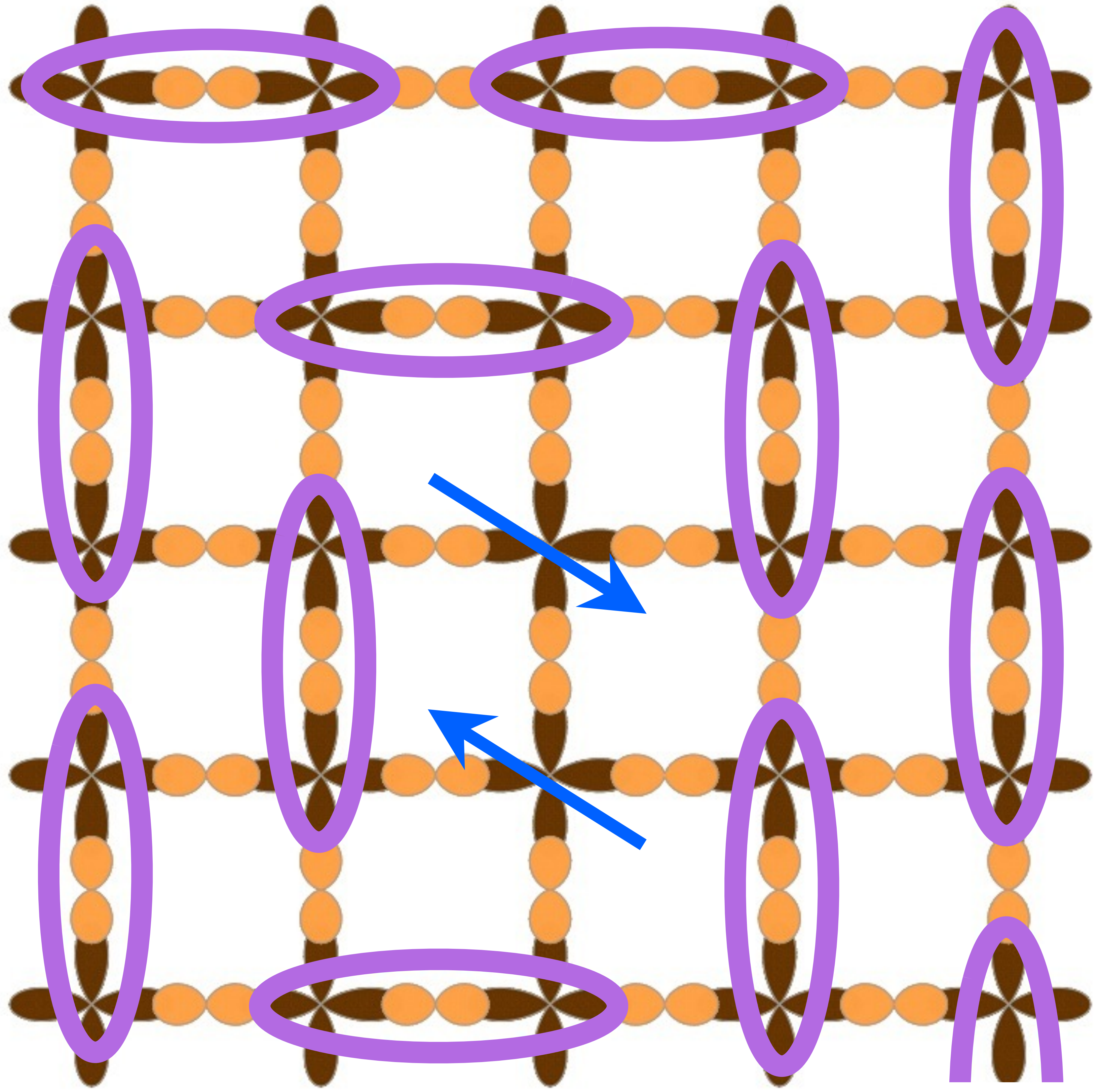
A quantum spin liquid with many-boson (spins on Cu) **entanglement**





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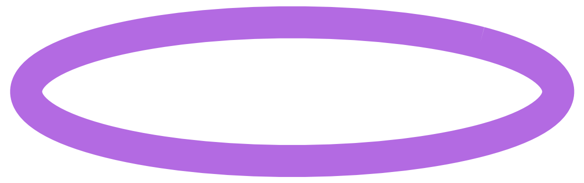
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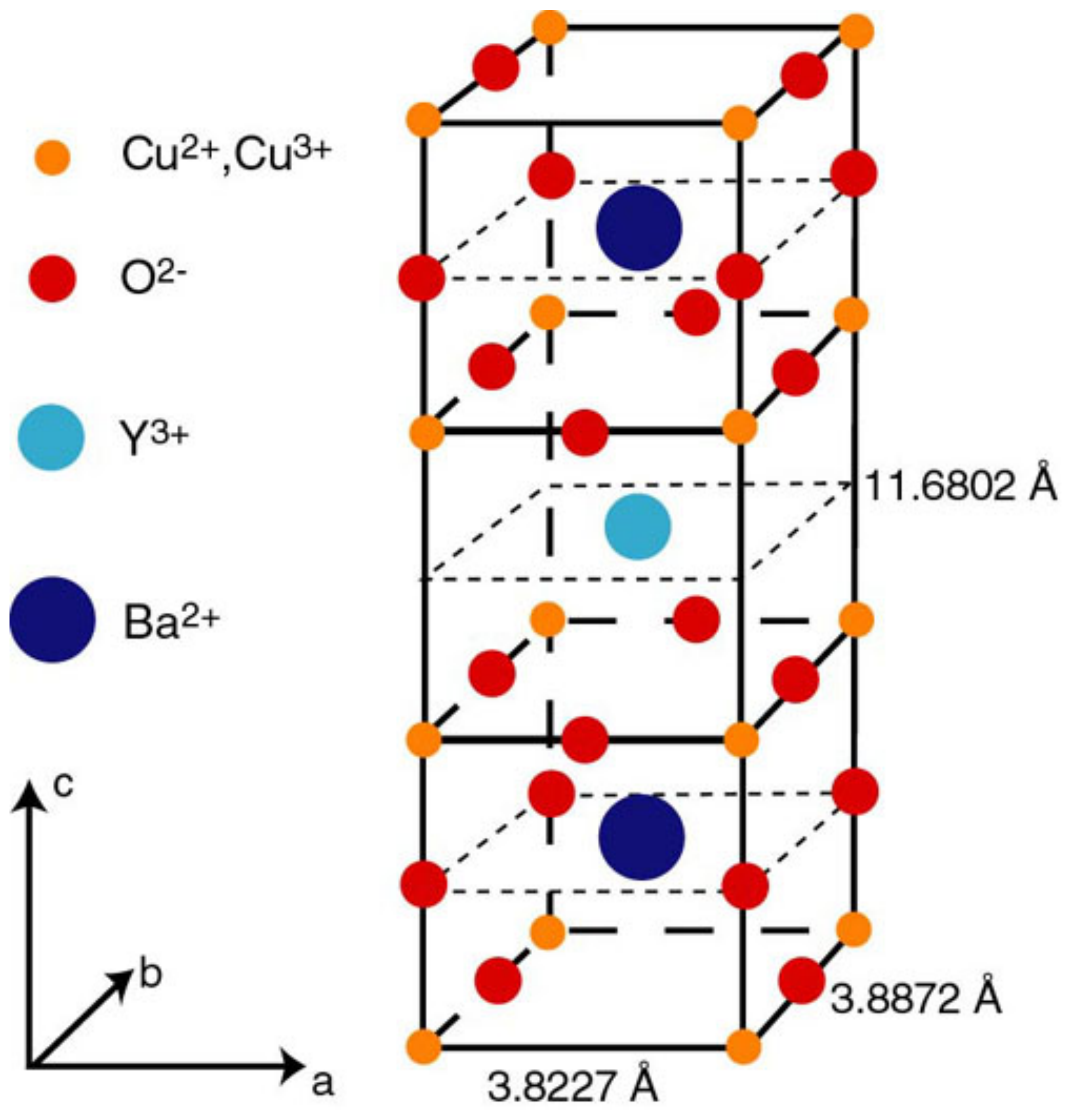
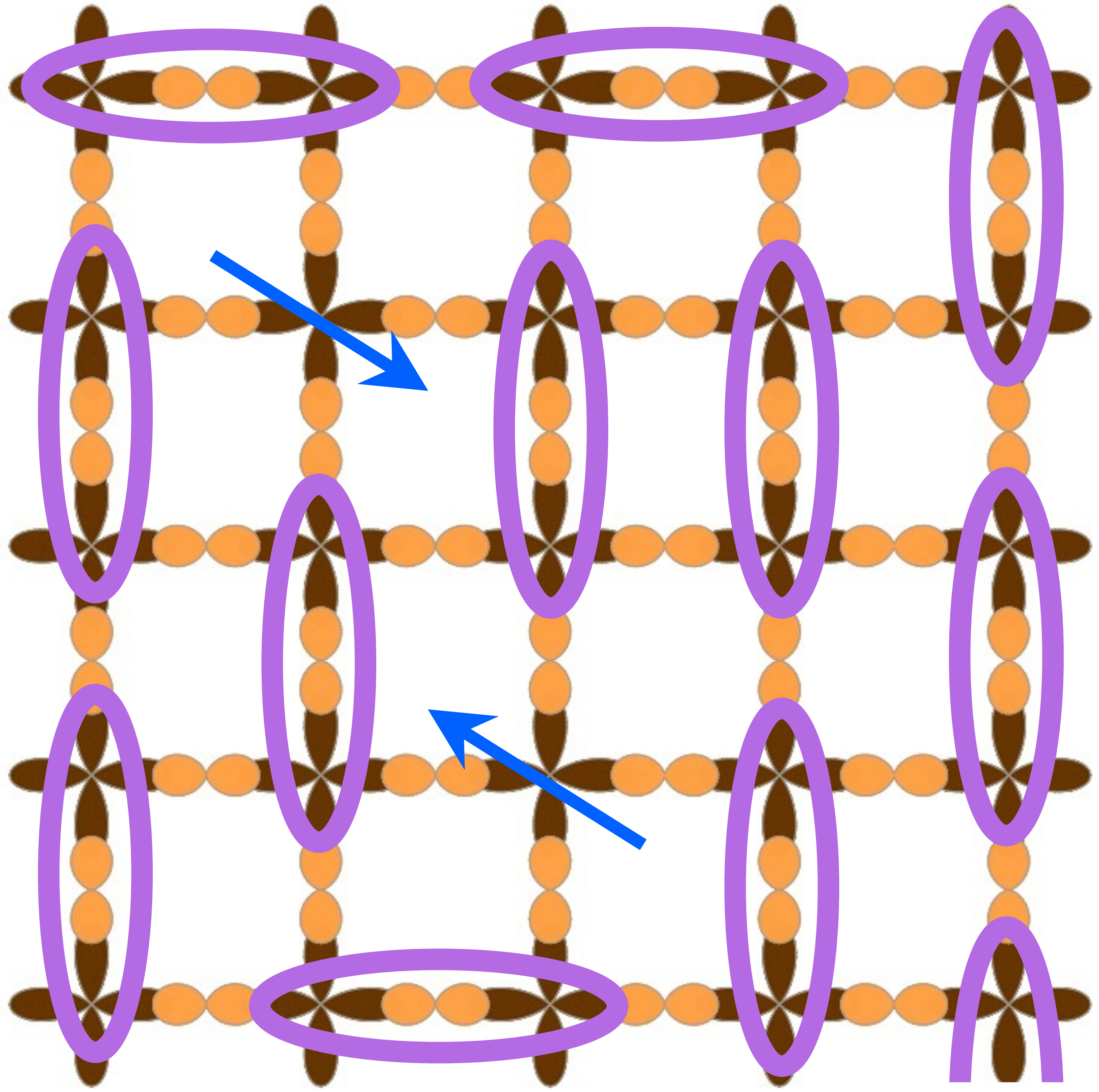
Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.





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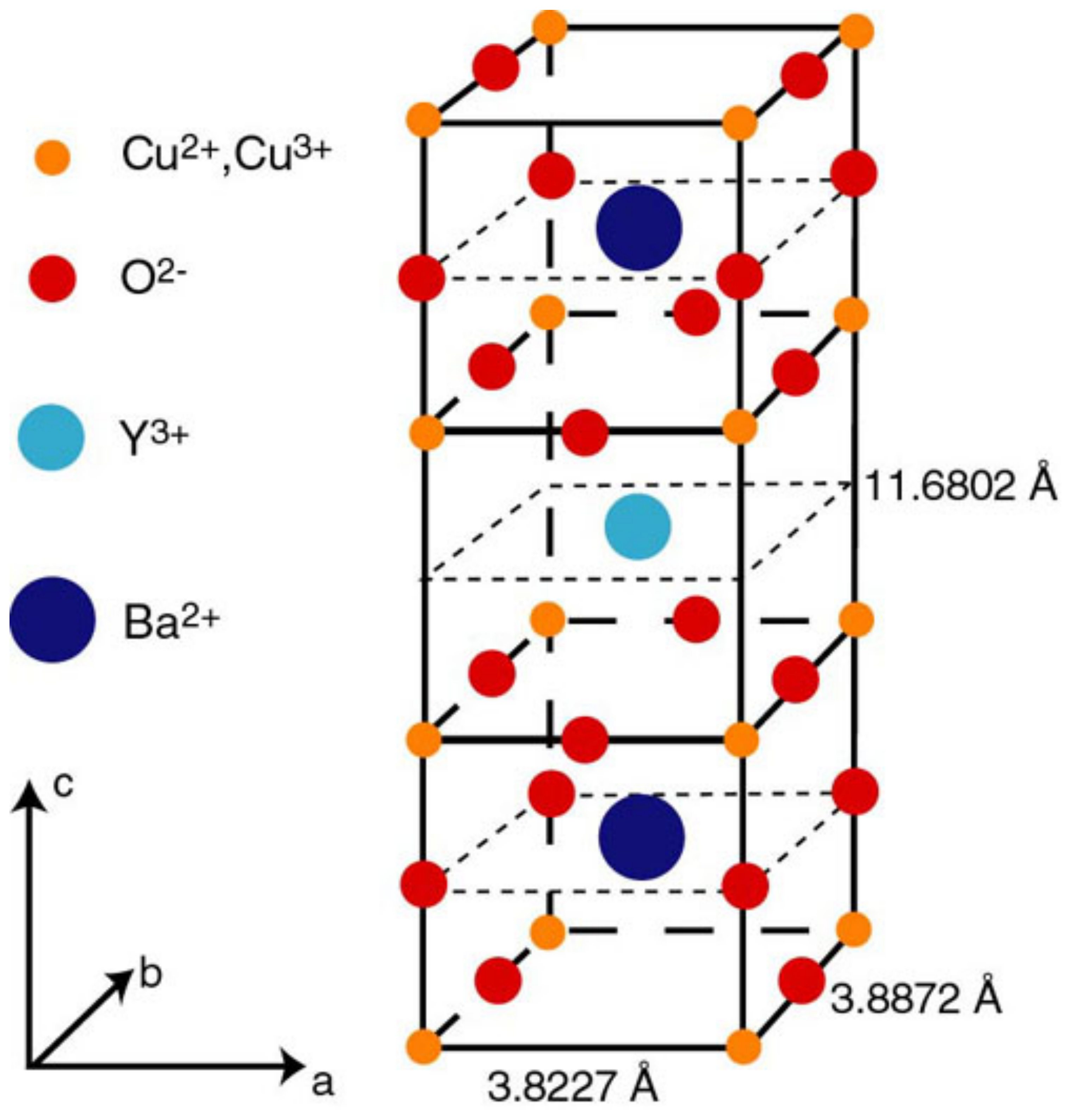

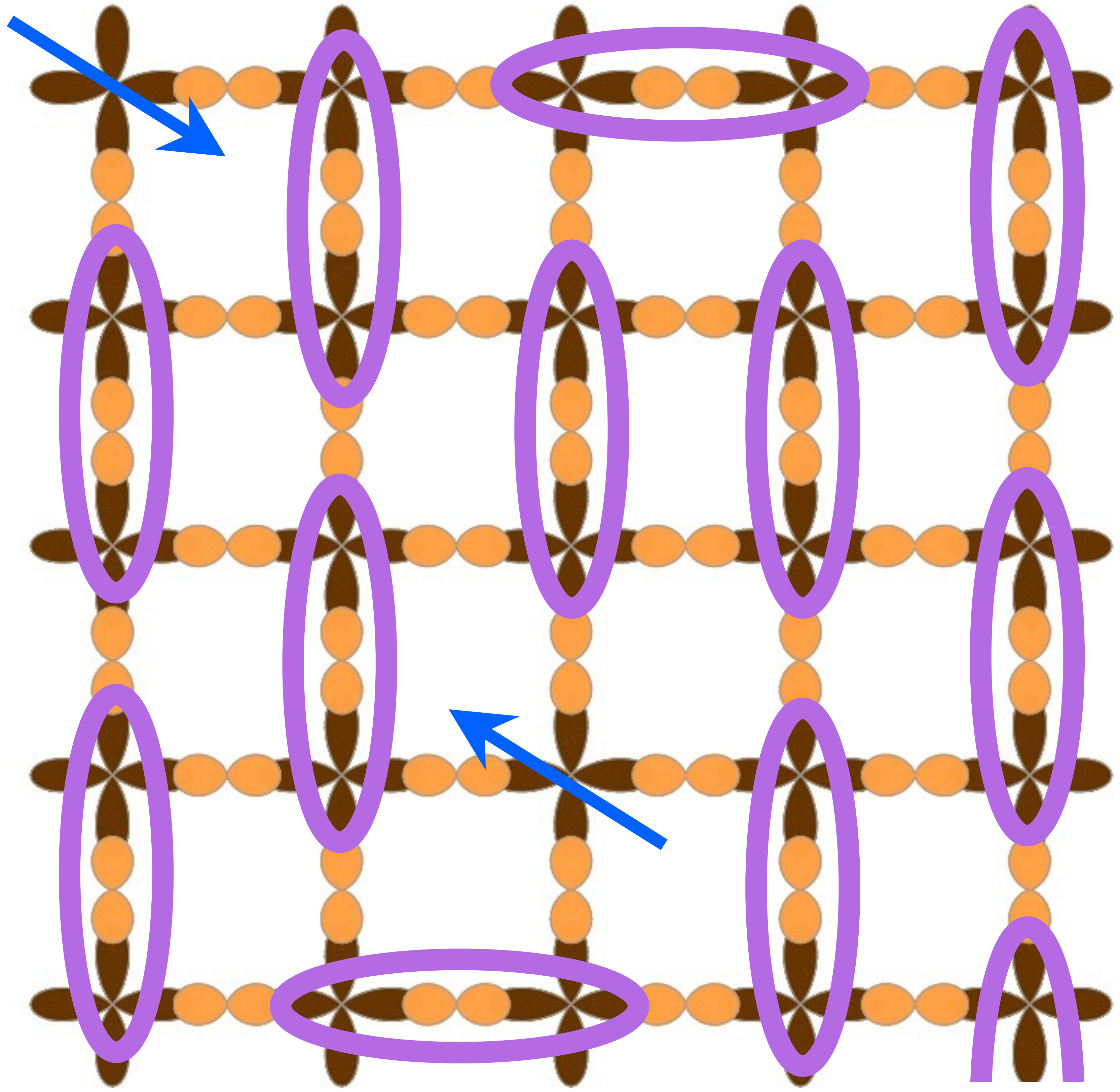

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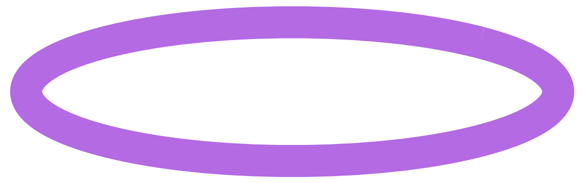 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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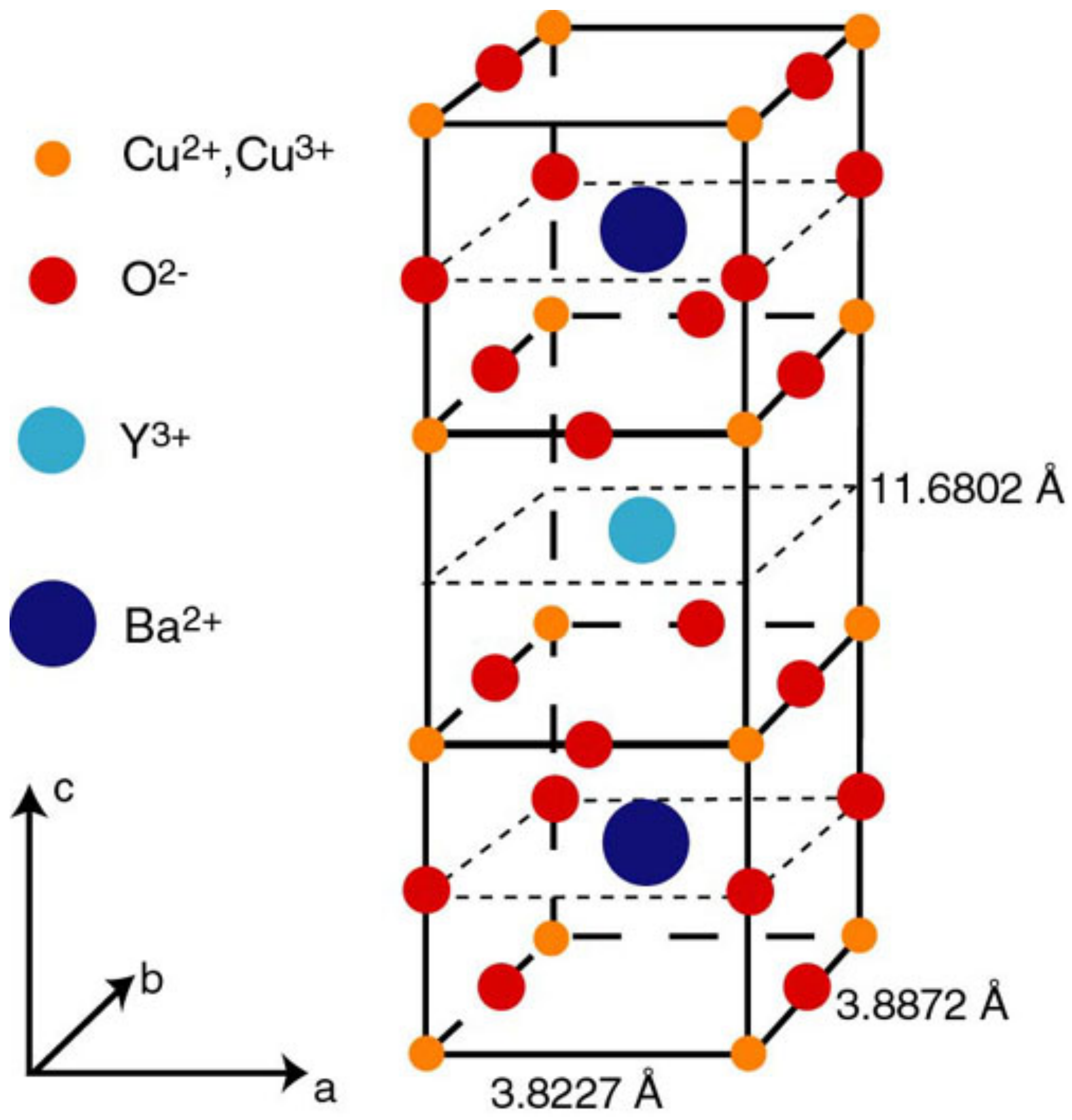

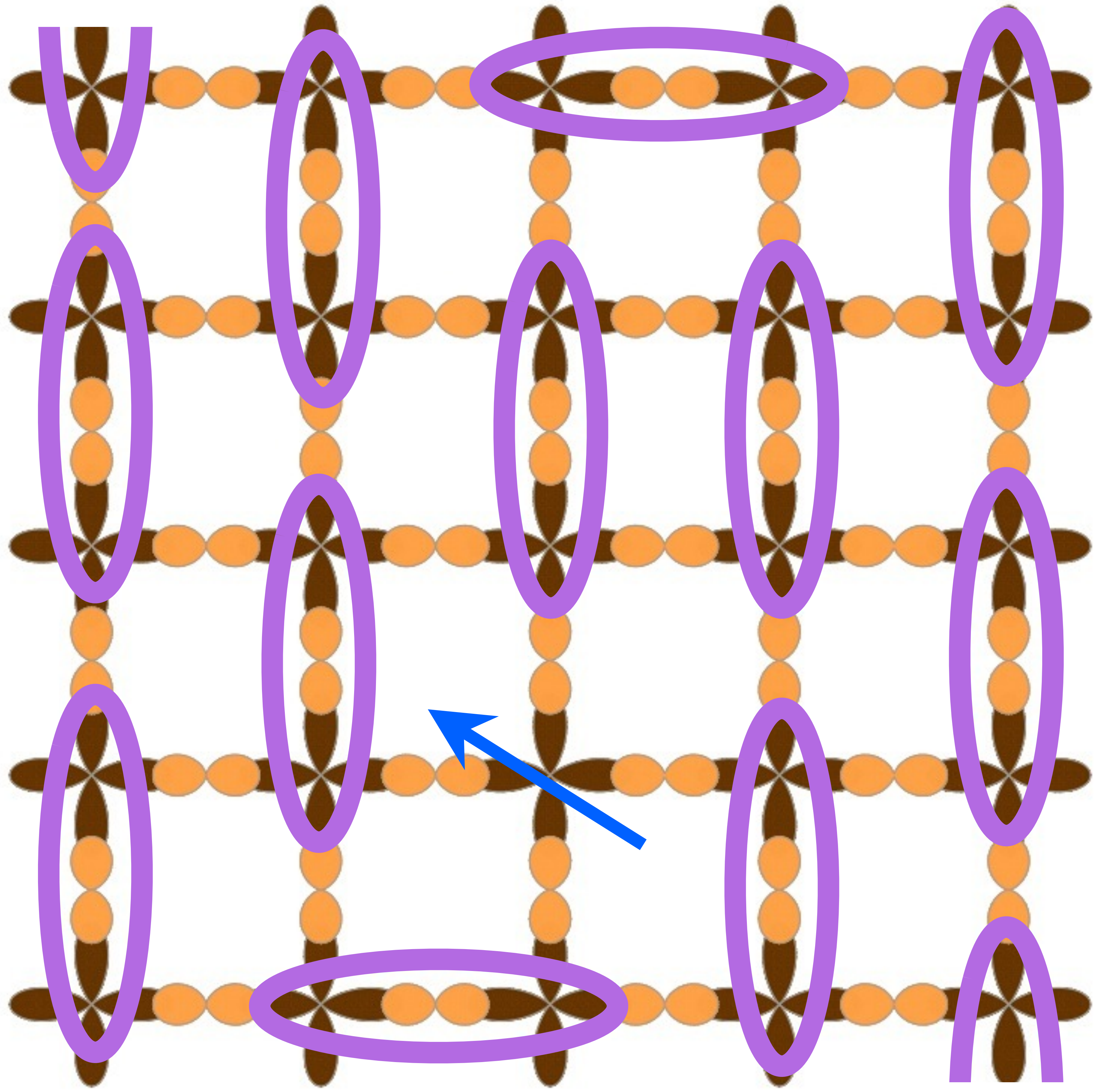

Spin S=1/2,
charge
neutral
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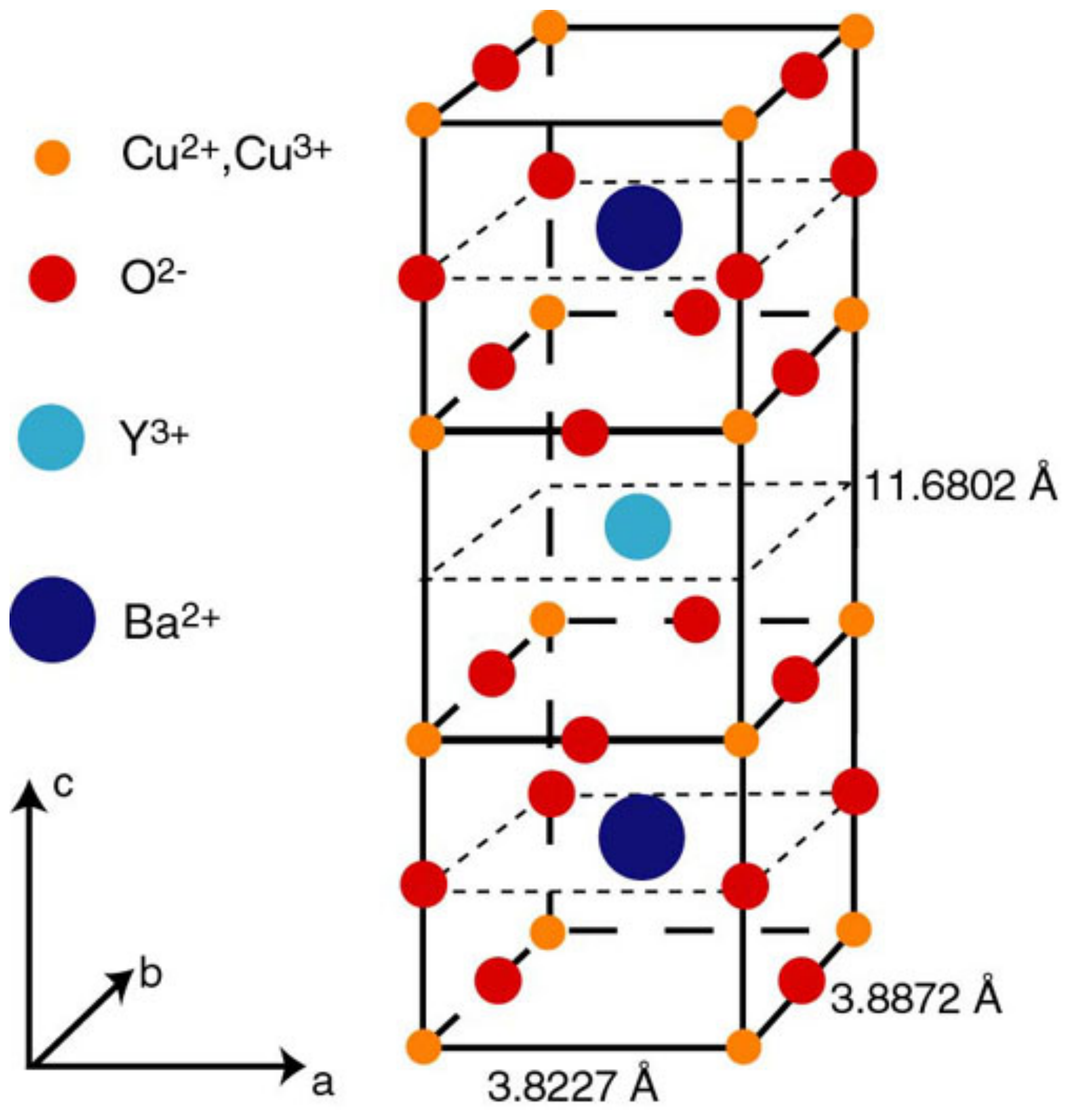

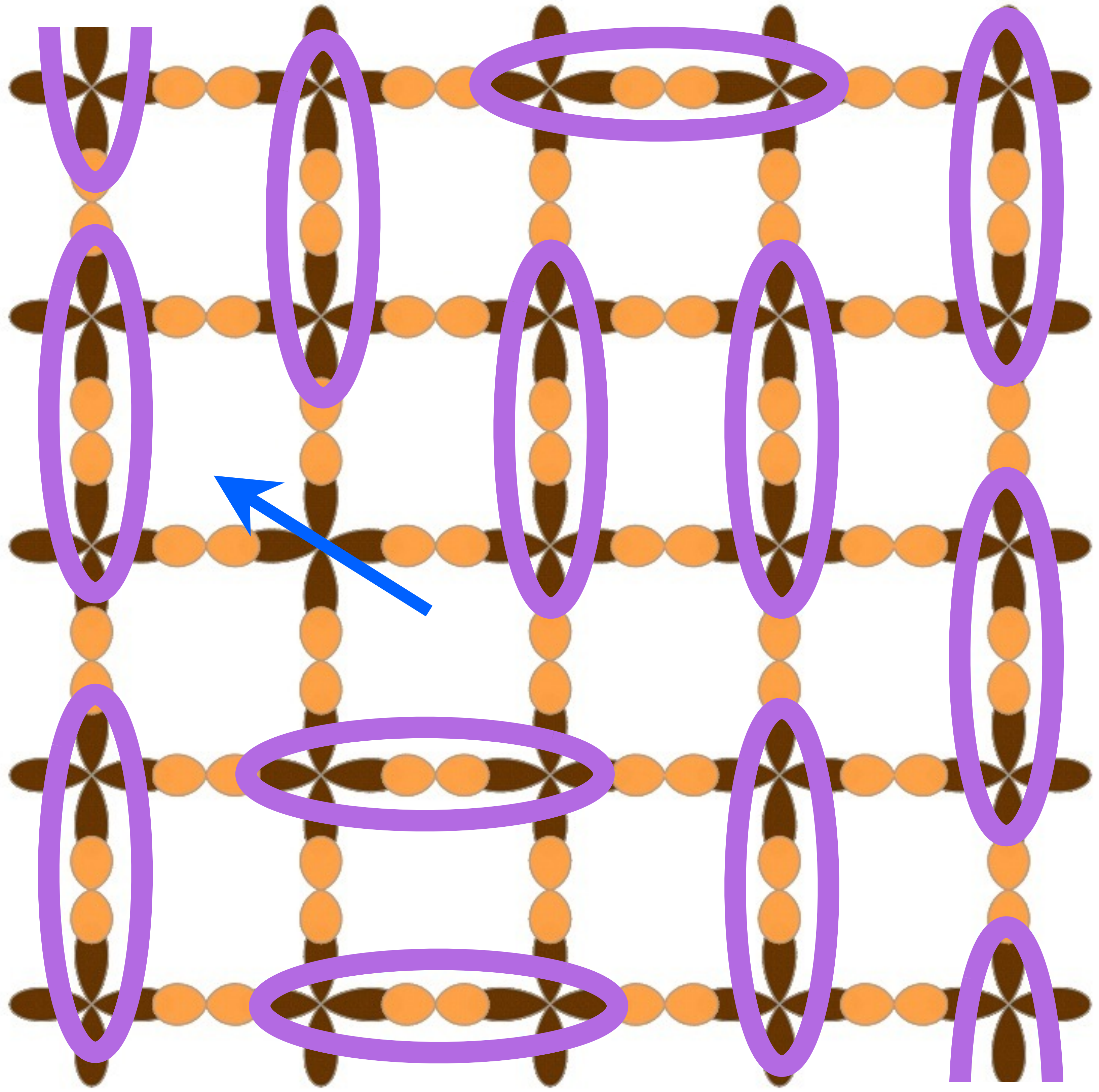

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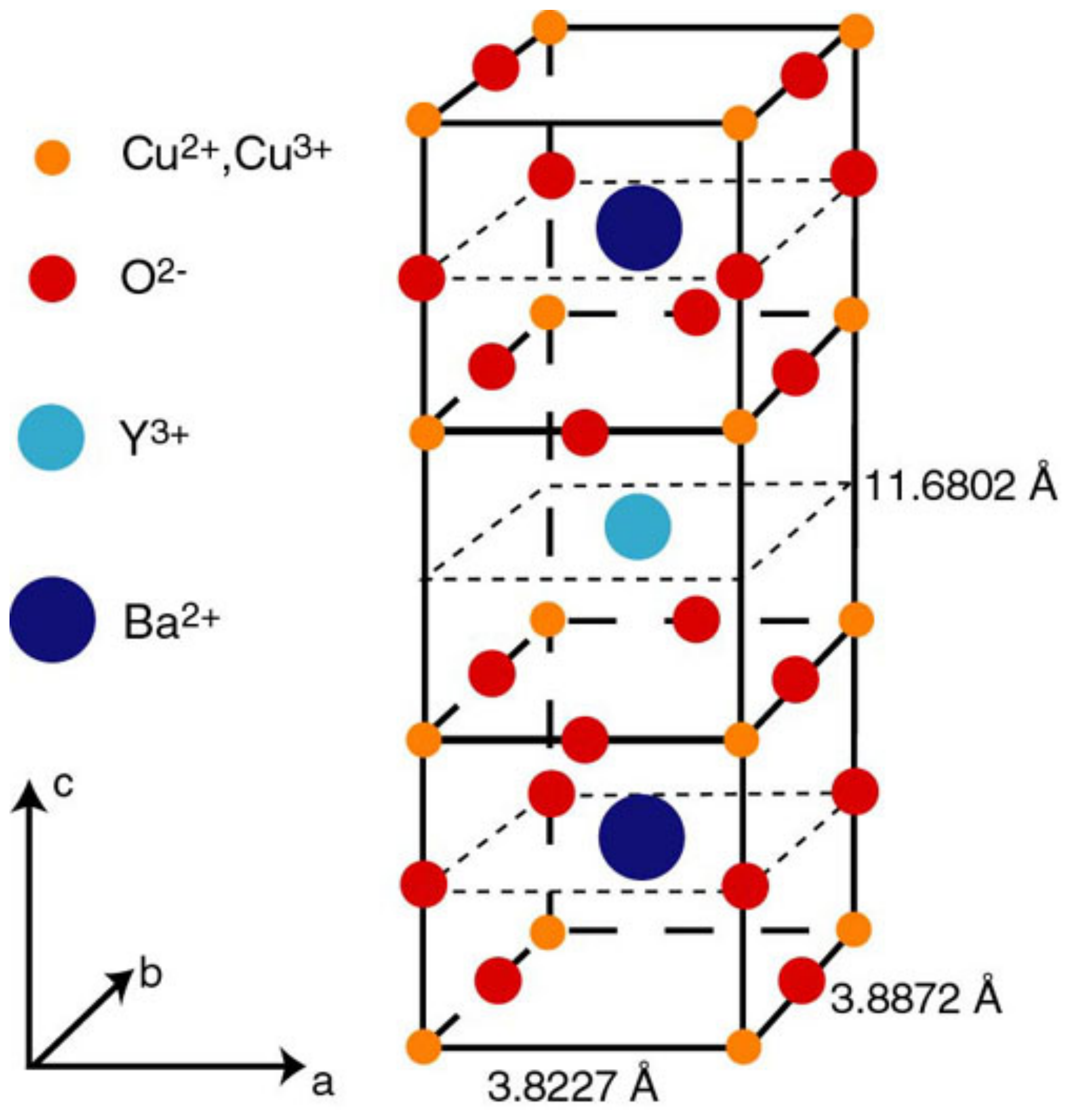

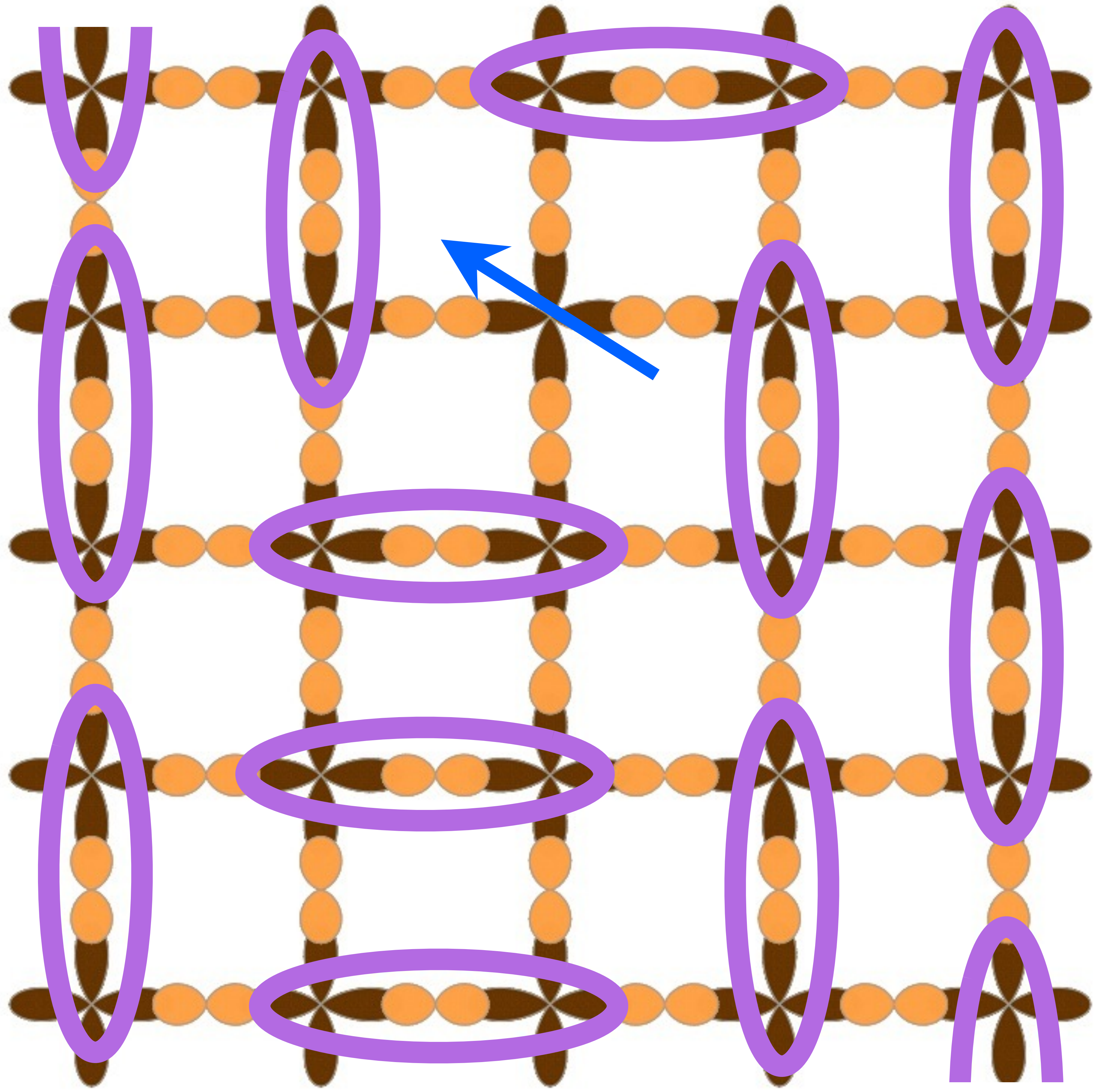

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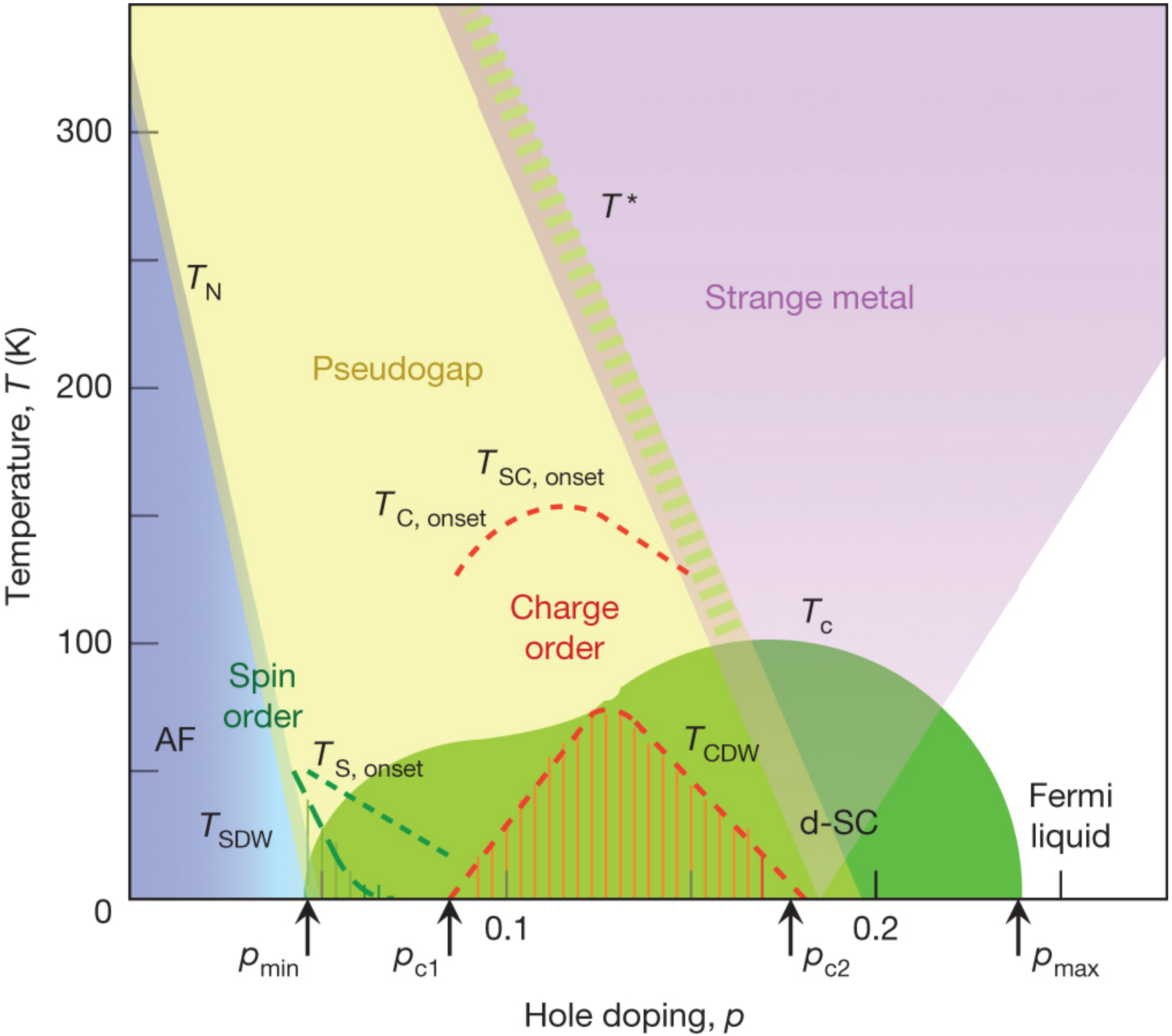
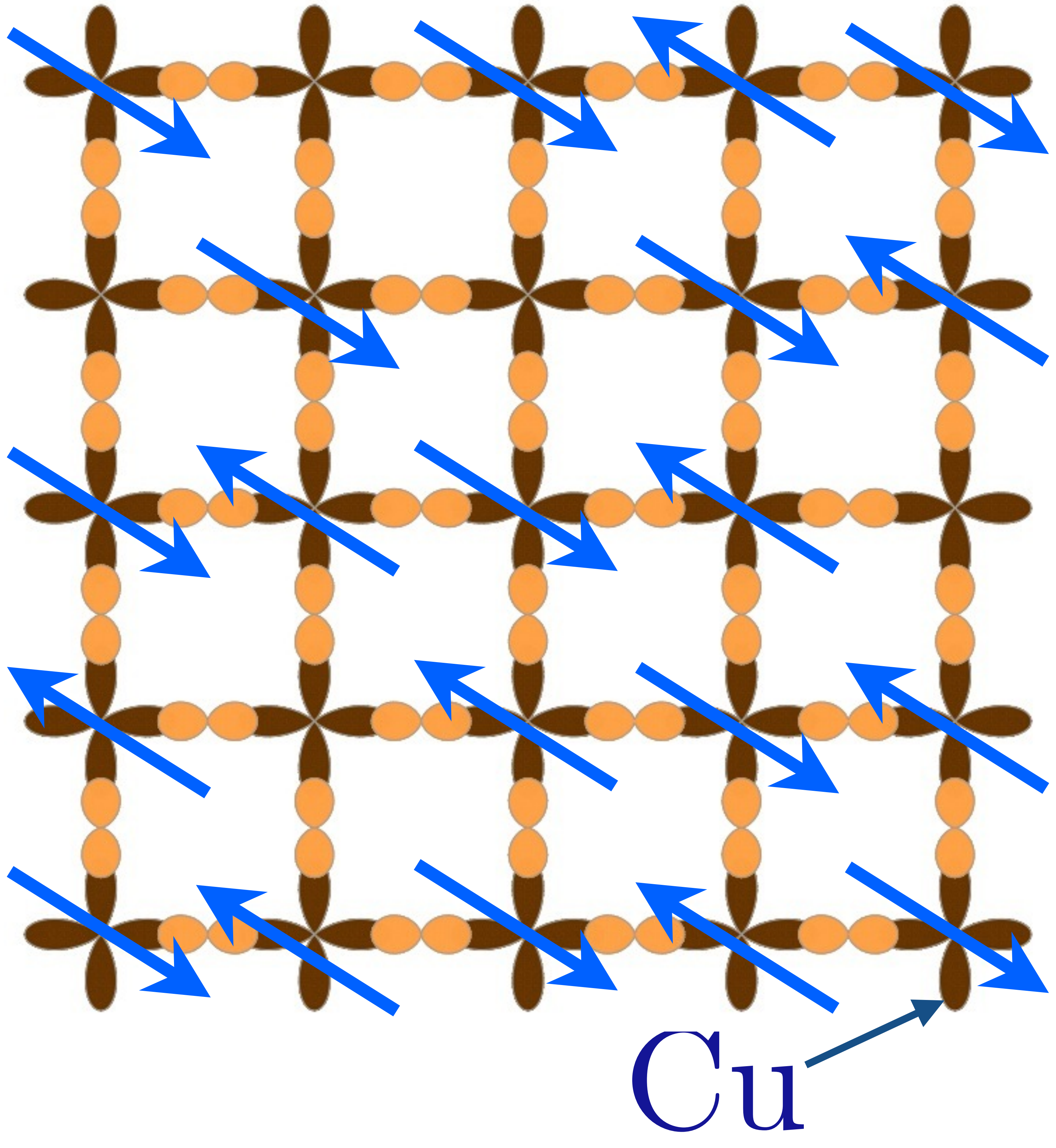
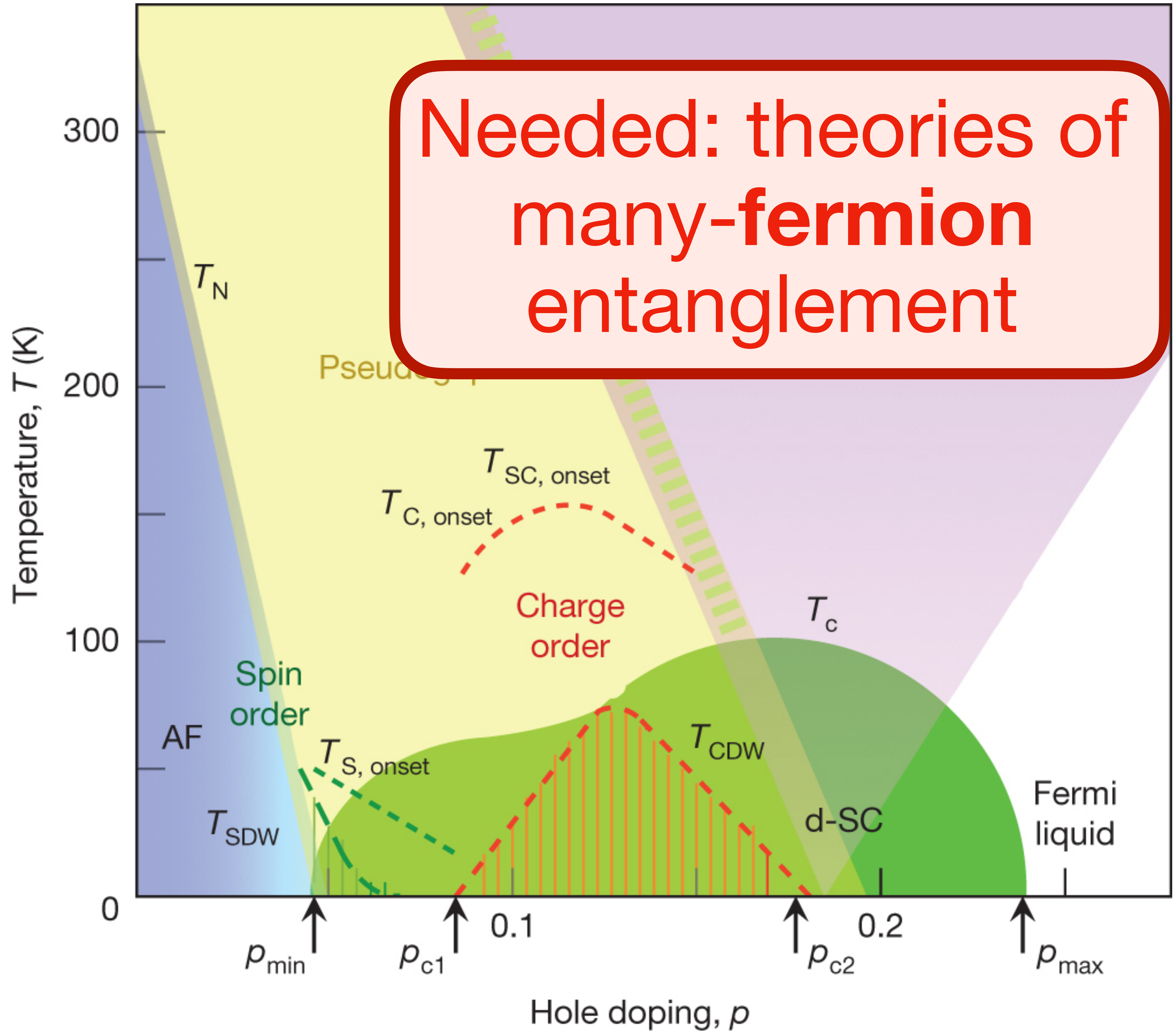
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



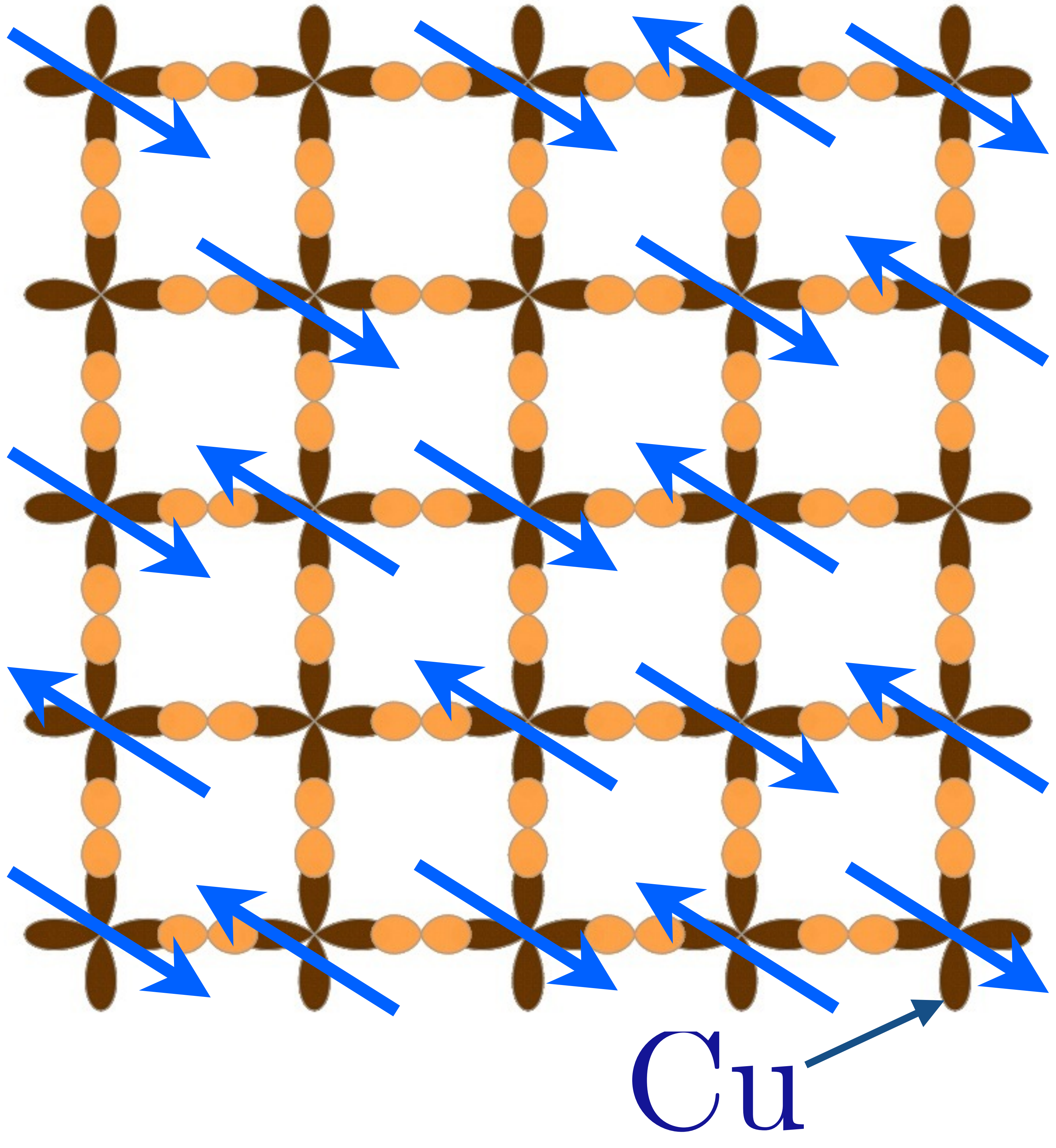


d-SC obtained upon doping AF with density p holes.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Needed: theories of many-**fermion** entanglement



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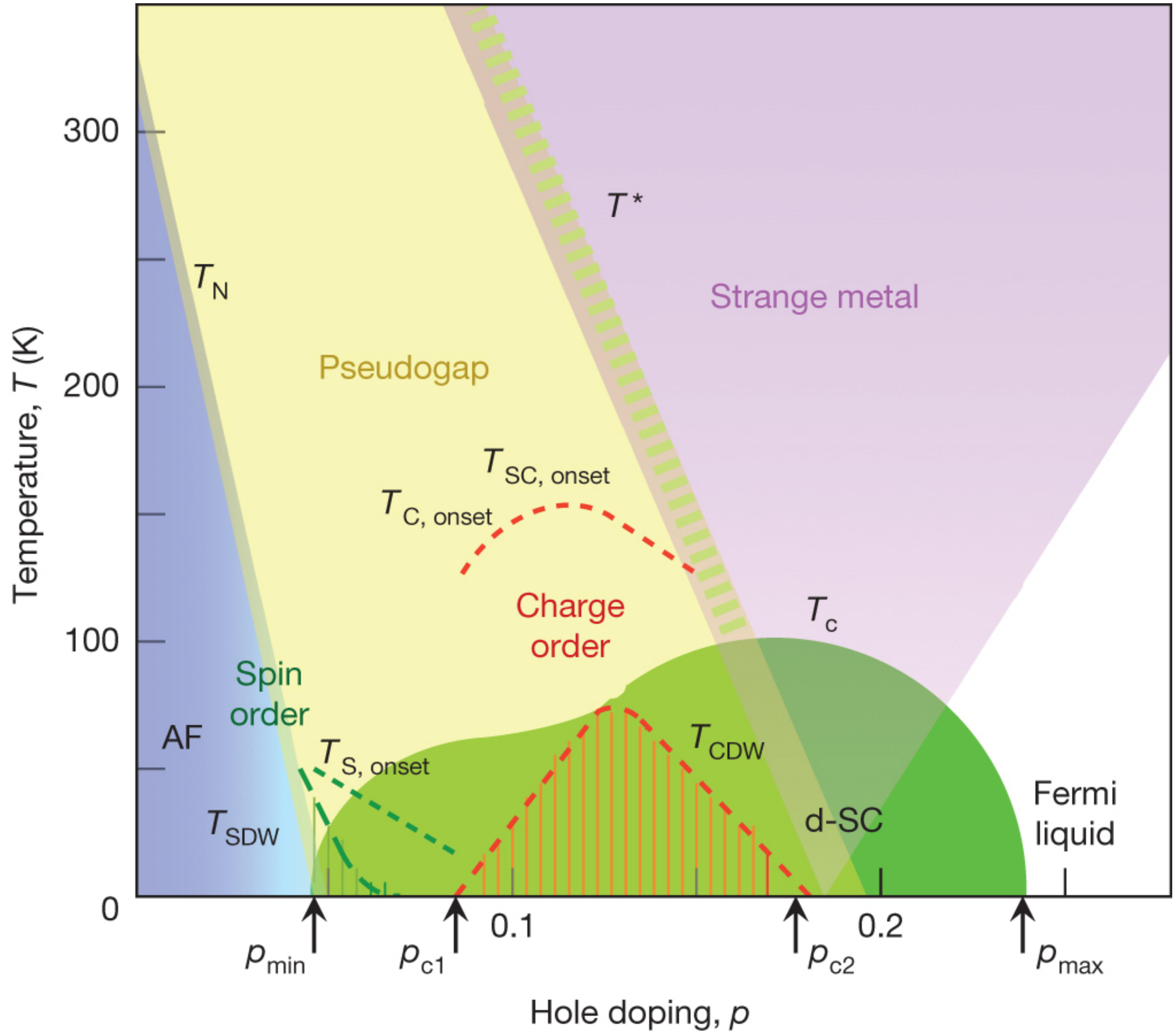
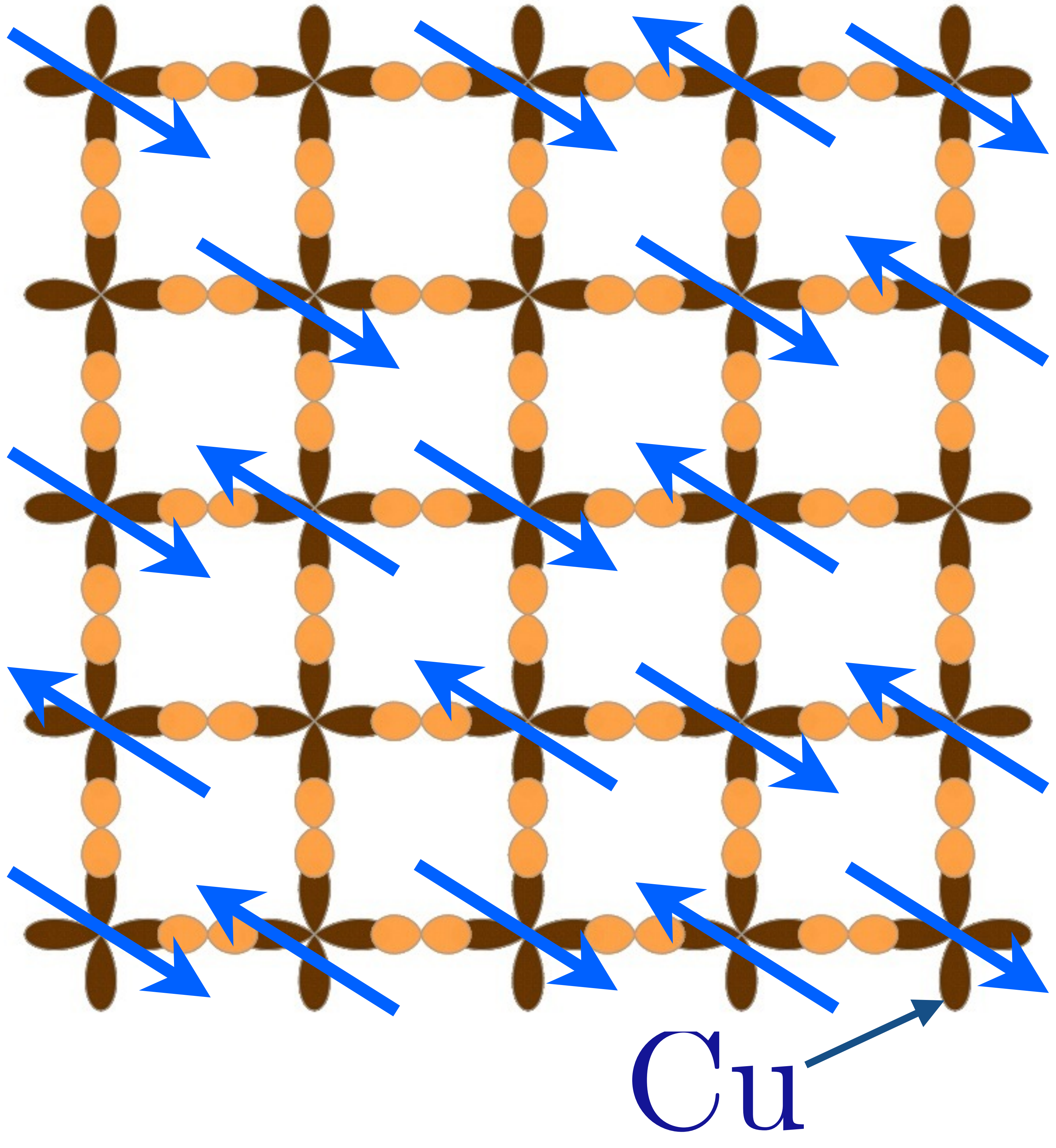


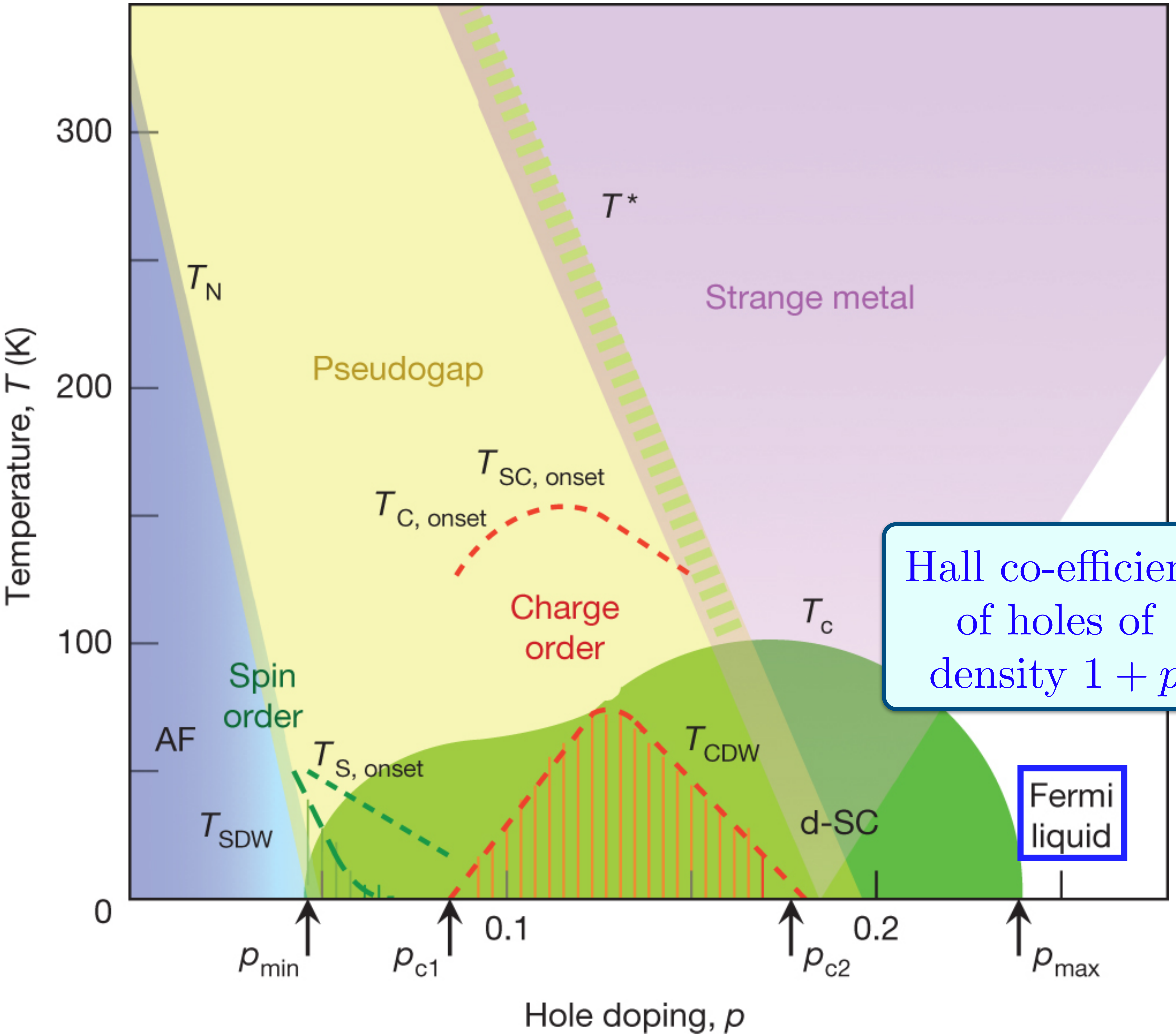


d-SC obtained upon doping AF with density p holes.

Hole density relative to the filled band $\rho = 1 + p$.

Electron density relative to the empty band $\rho_e = 1 - p$.

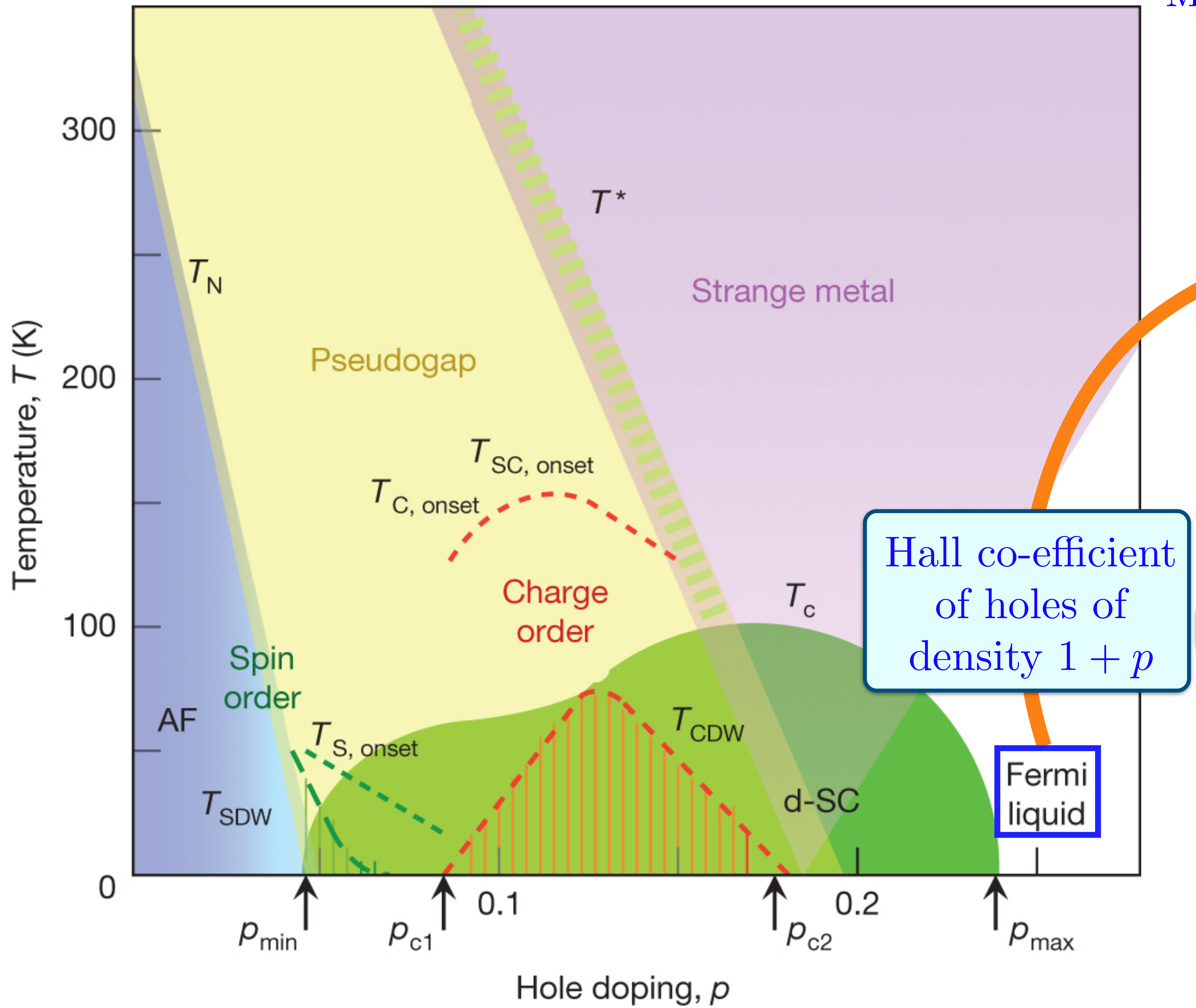
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

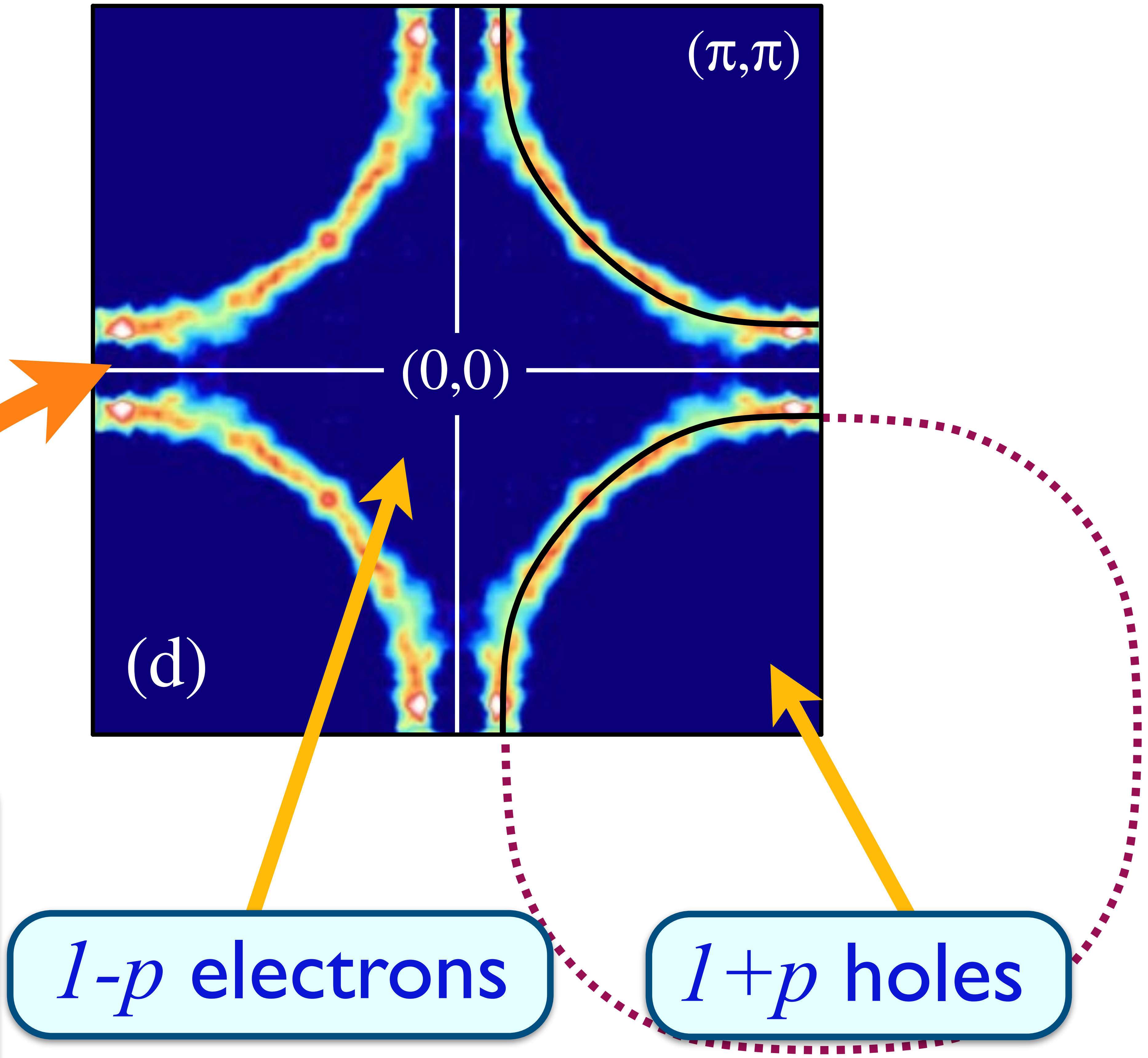


Hall co-efficient of holes of density $1 + p$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

M. Platé *et al.*, PRL **95**, 077001 (2005)





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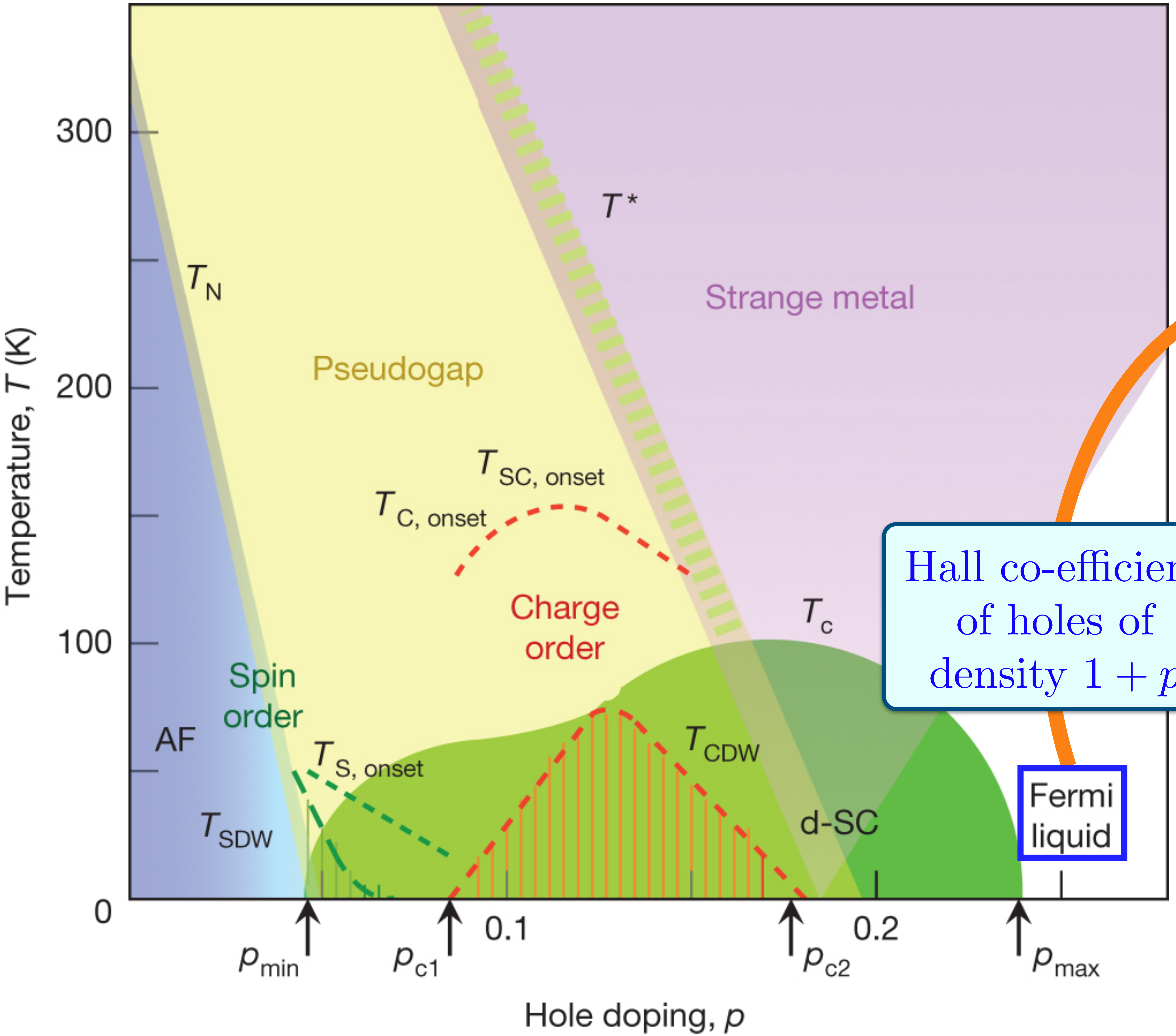

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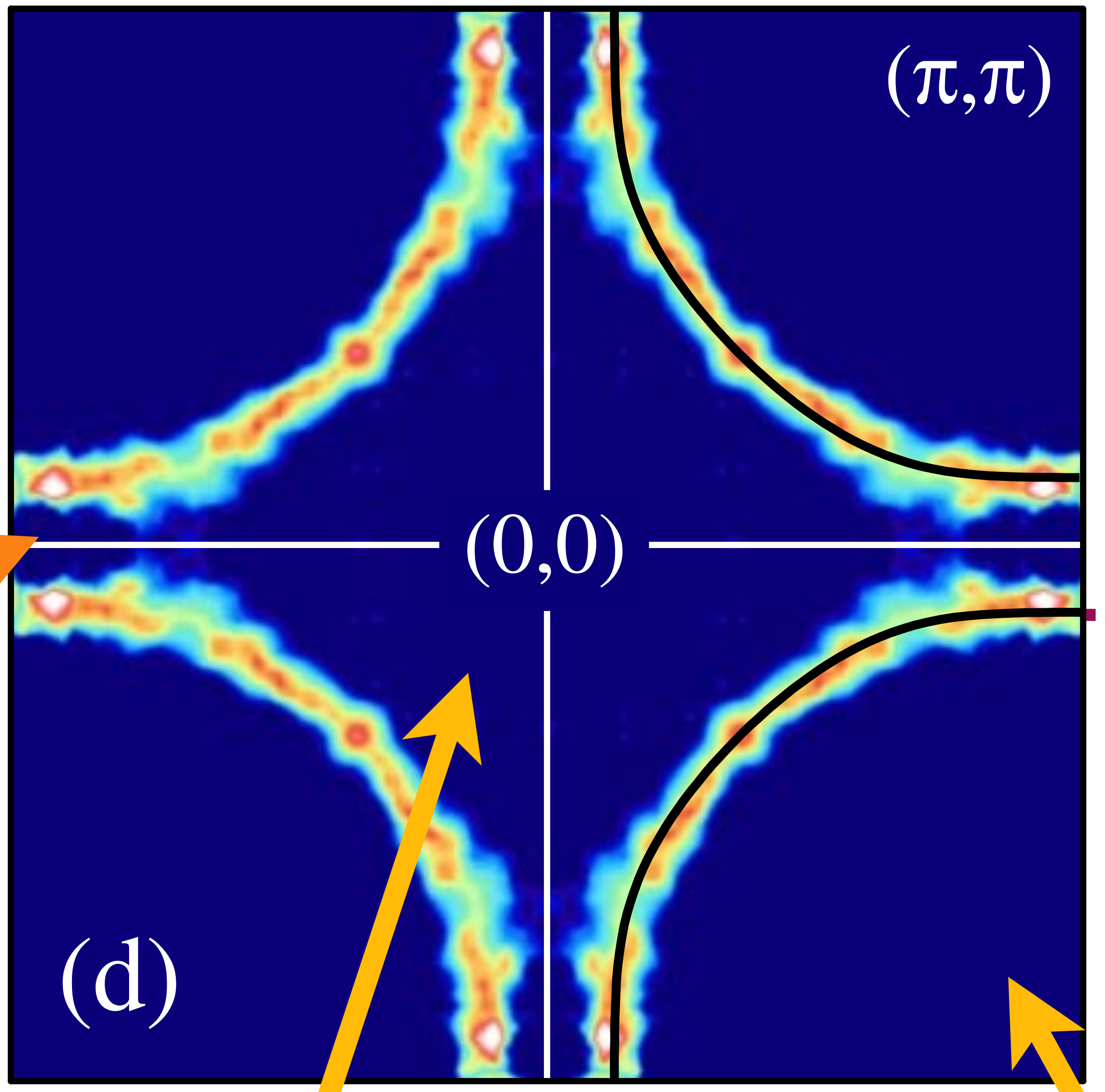


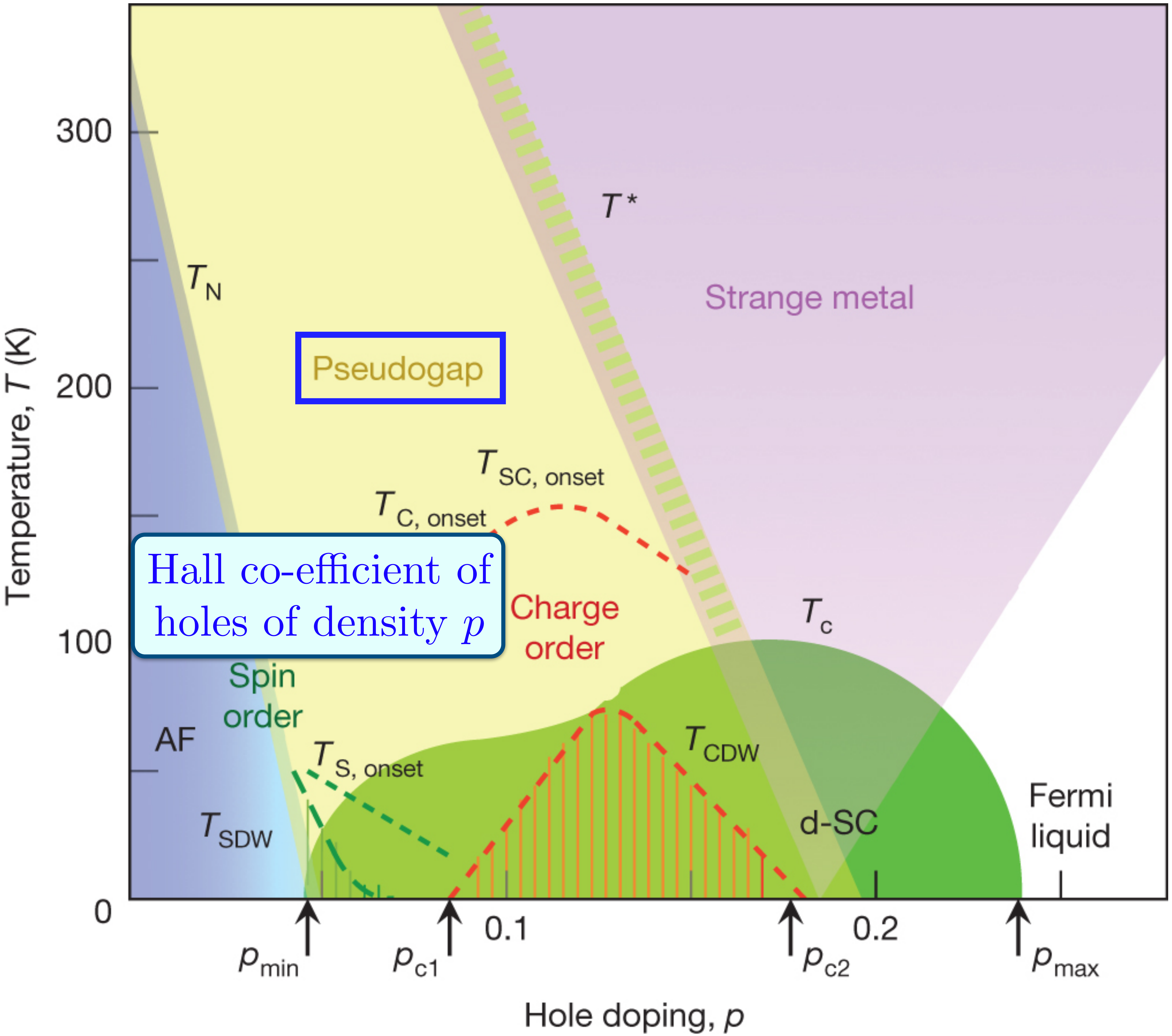
Hall co-efficient of holes of density $1+p$

1-p electrons

1+p holes

Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

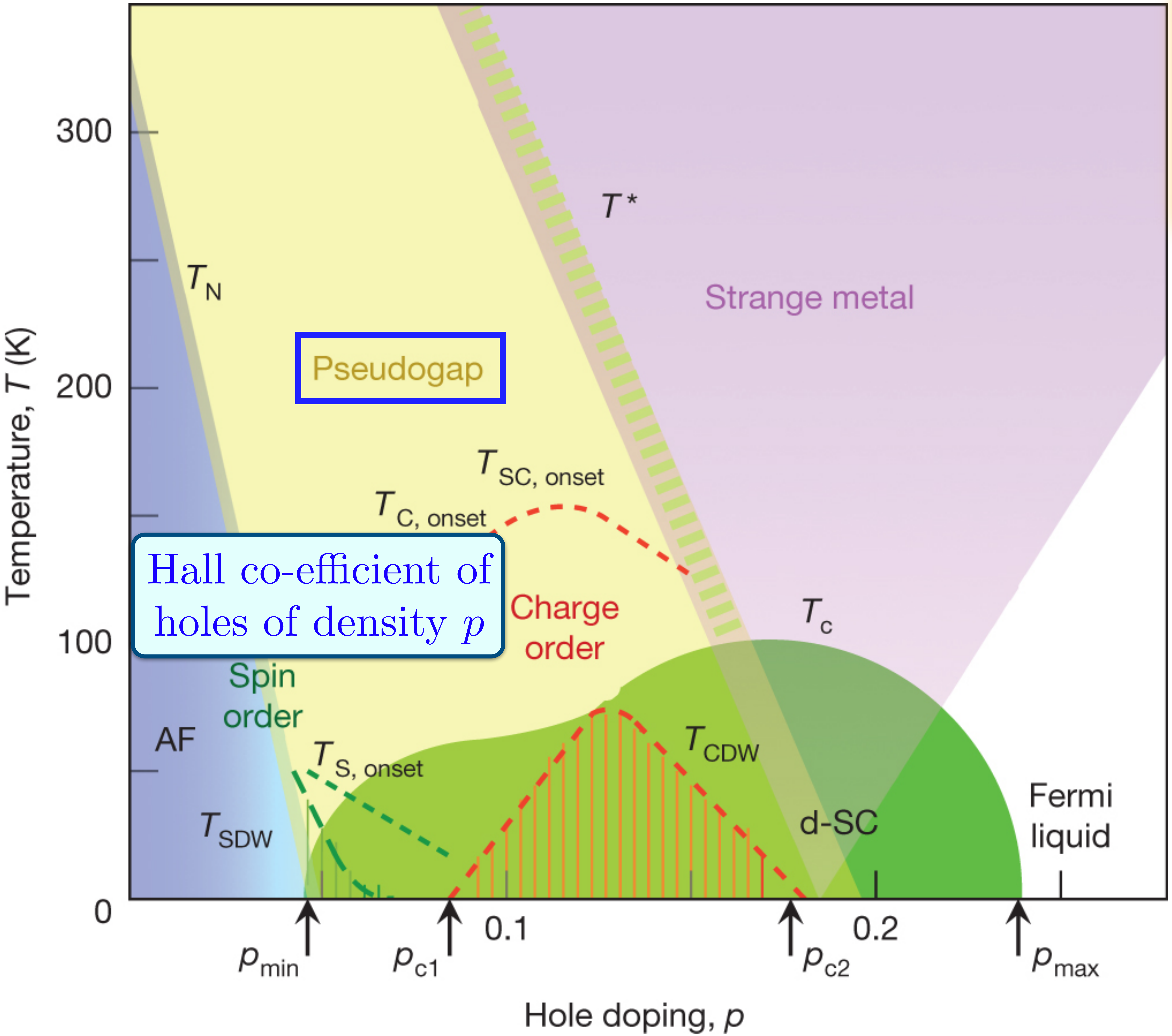
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



But there is no antiferromagnetic order to justify carrier density p

Many theories with fluctuating and intertwined AFM, d-SC and charge orders.

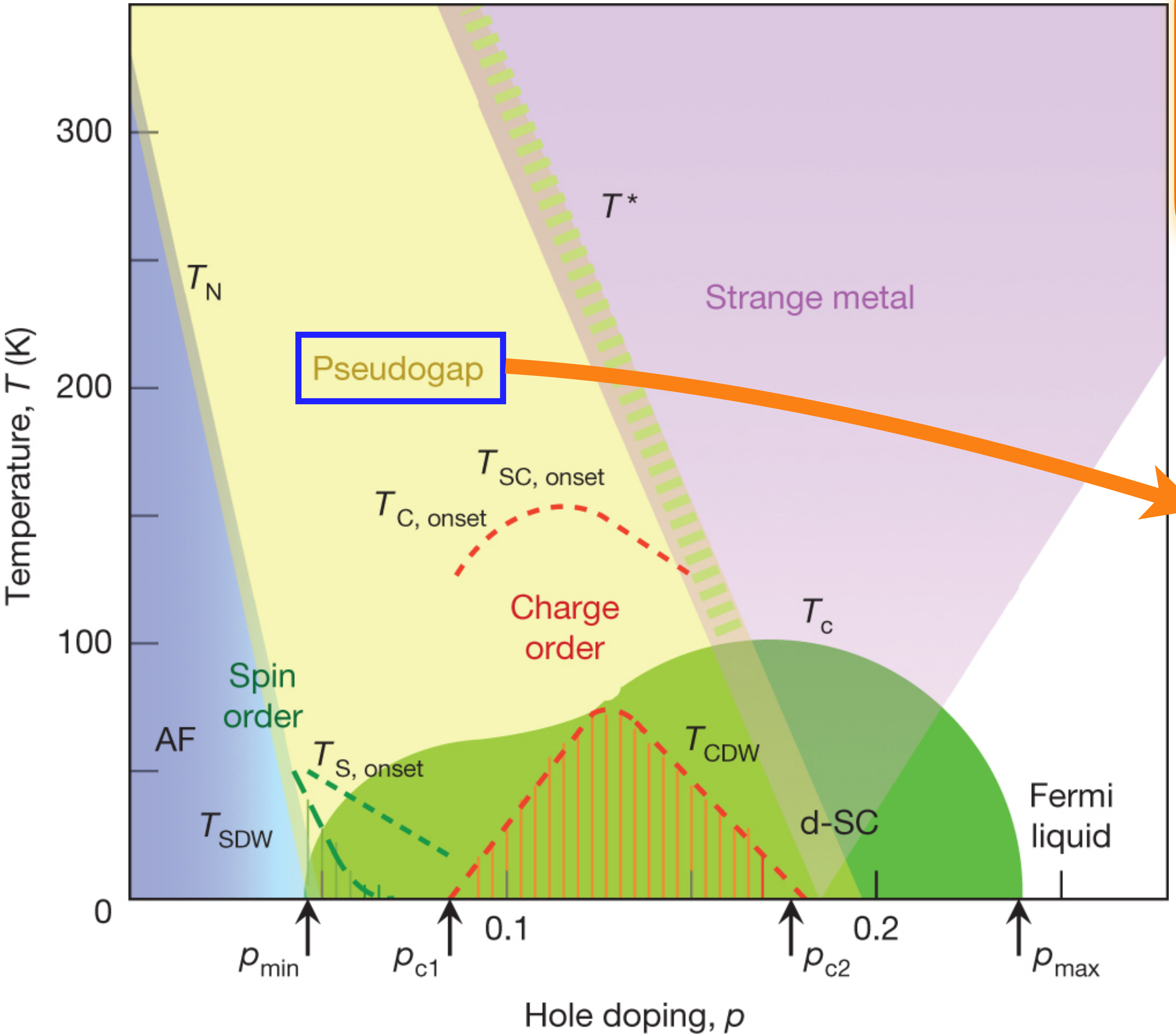
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Quantum entanglement of mobile fermions without an energy gap

I argue that a better starting point is a novel quantum ground state with no broken symmetry.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

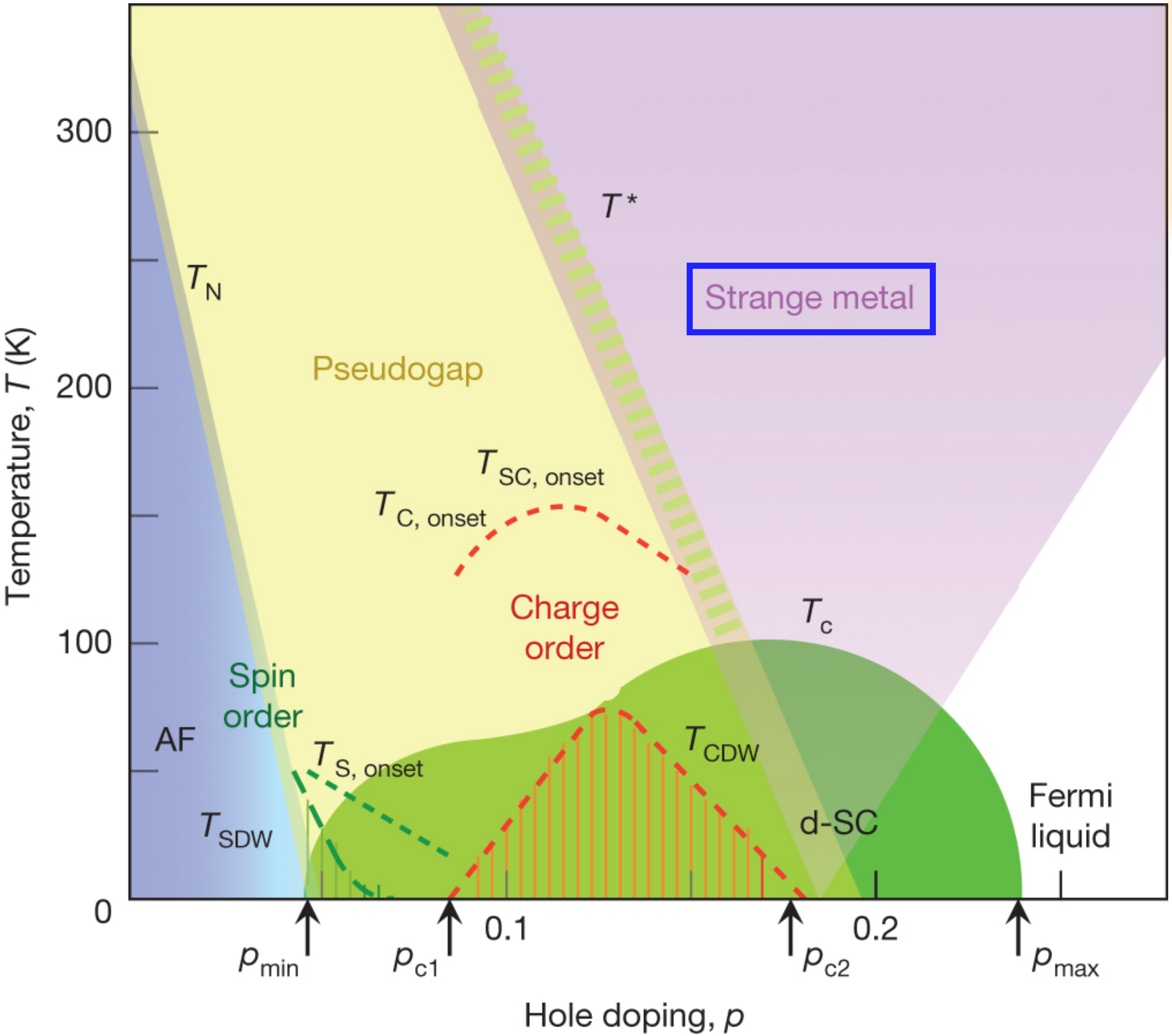


Quantum entanglement of mobile fermions without an energy gap

I. Fractionalized Fermi Liquid (FL*)

Fractionalized (anyonic) spinon excitations co-existing with electron-like quasiparticles on a Fermi surface.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

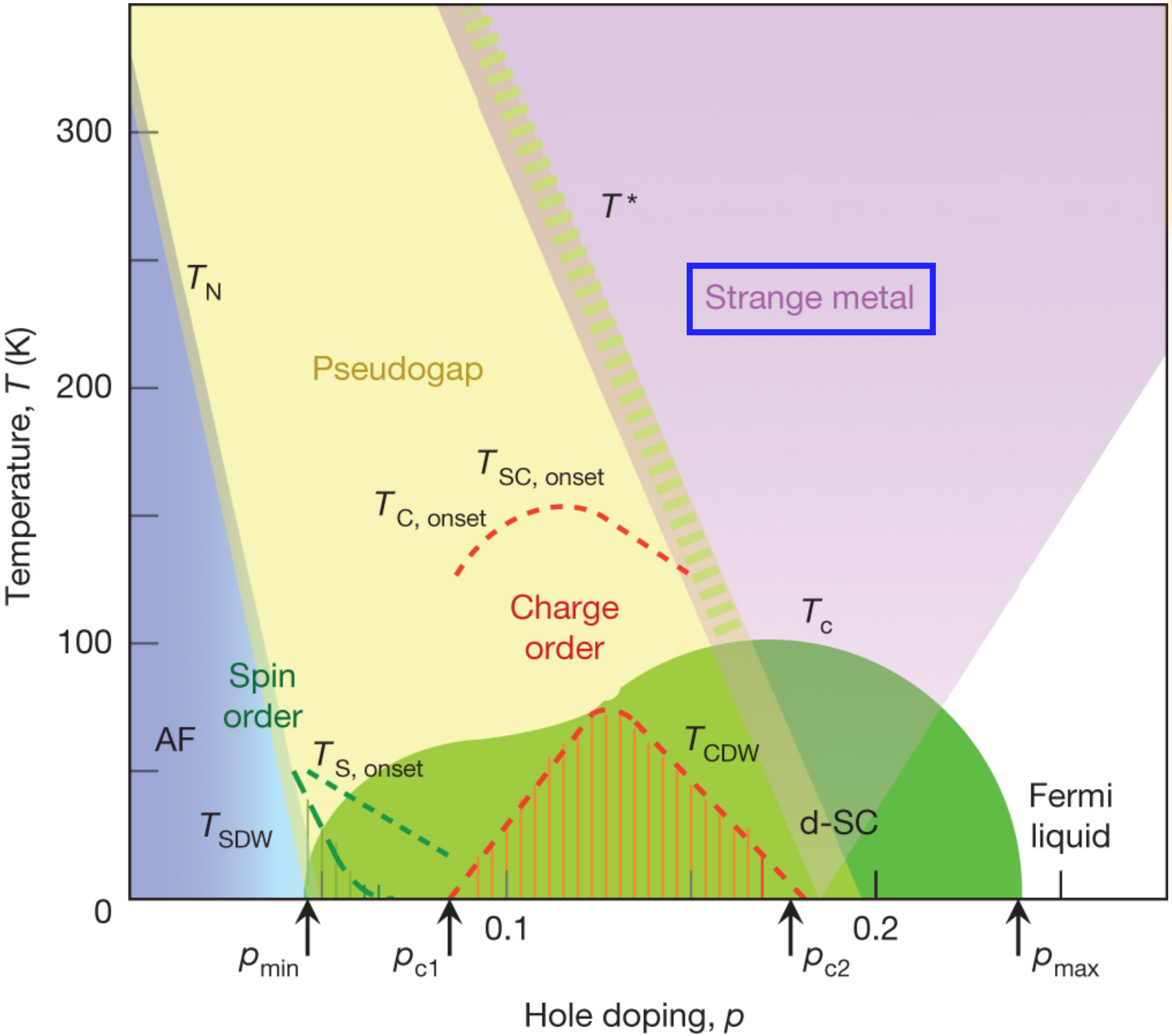


Quantum entanglement of mobile fermions without an energy gap

II. Sachdev-Ye-Kitaev (SYK) liquid

- Compressible state with no quasiparticles.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Quantum entanglement of mobile fermions without an energy gap

II. Sachdev-Ye-Kitaev (SYK) liquid

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- SYK: low energy theory of generic charged black holes in asymptotically flat 3+1 dimensional space.

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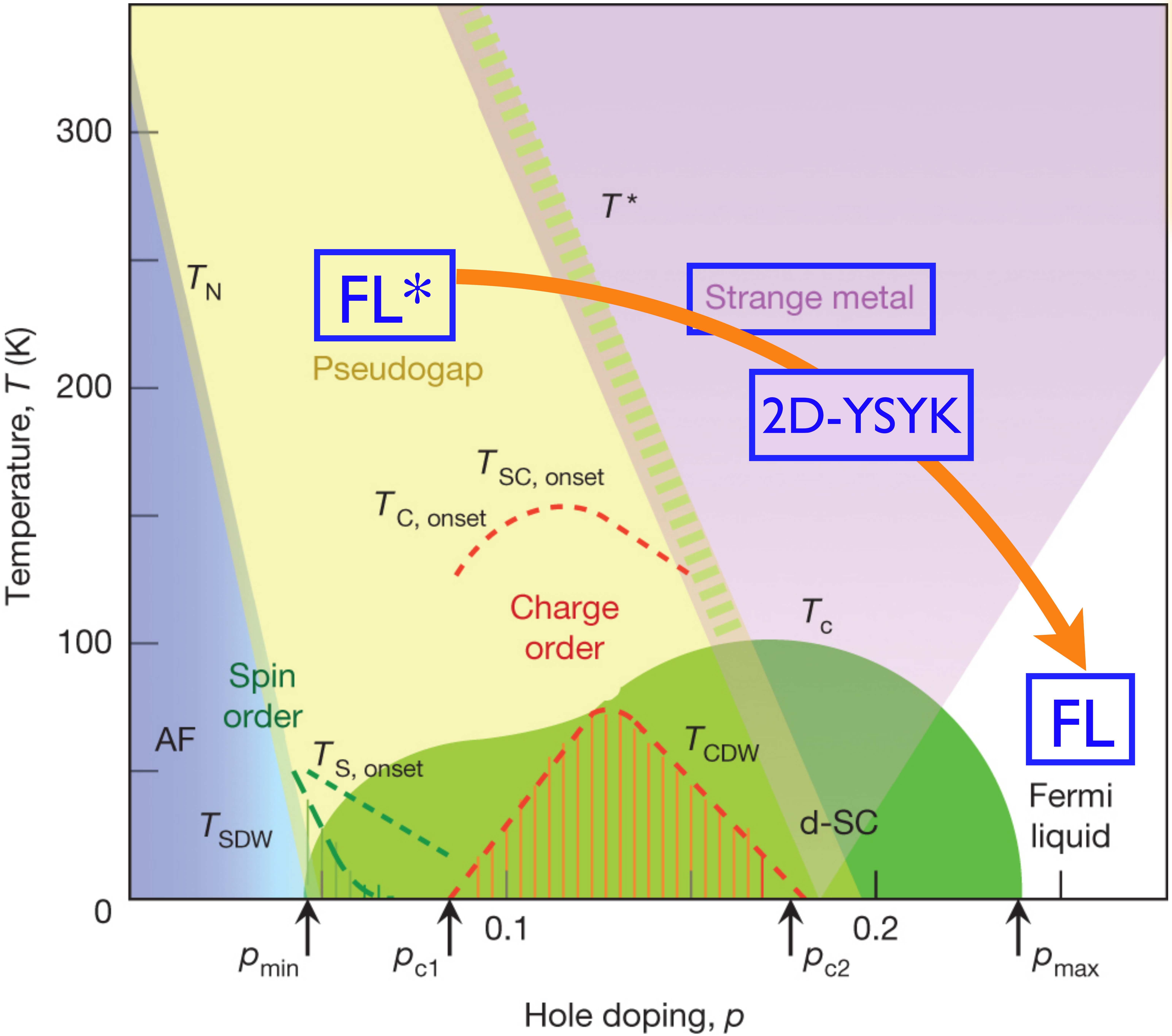

Quantum entanglement of mobile fermions without an energy gap

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- 2D-YSYK: universal theory of strange metals: FL*-FL transition in cuprates.

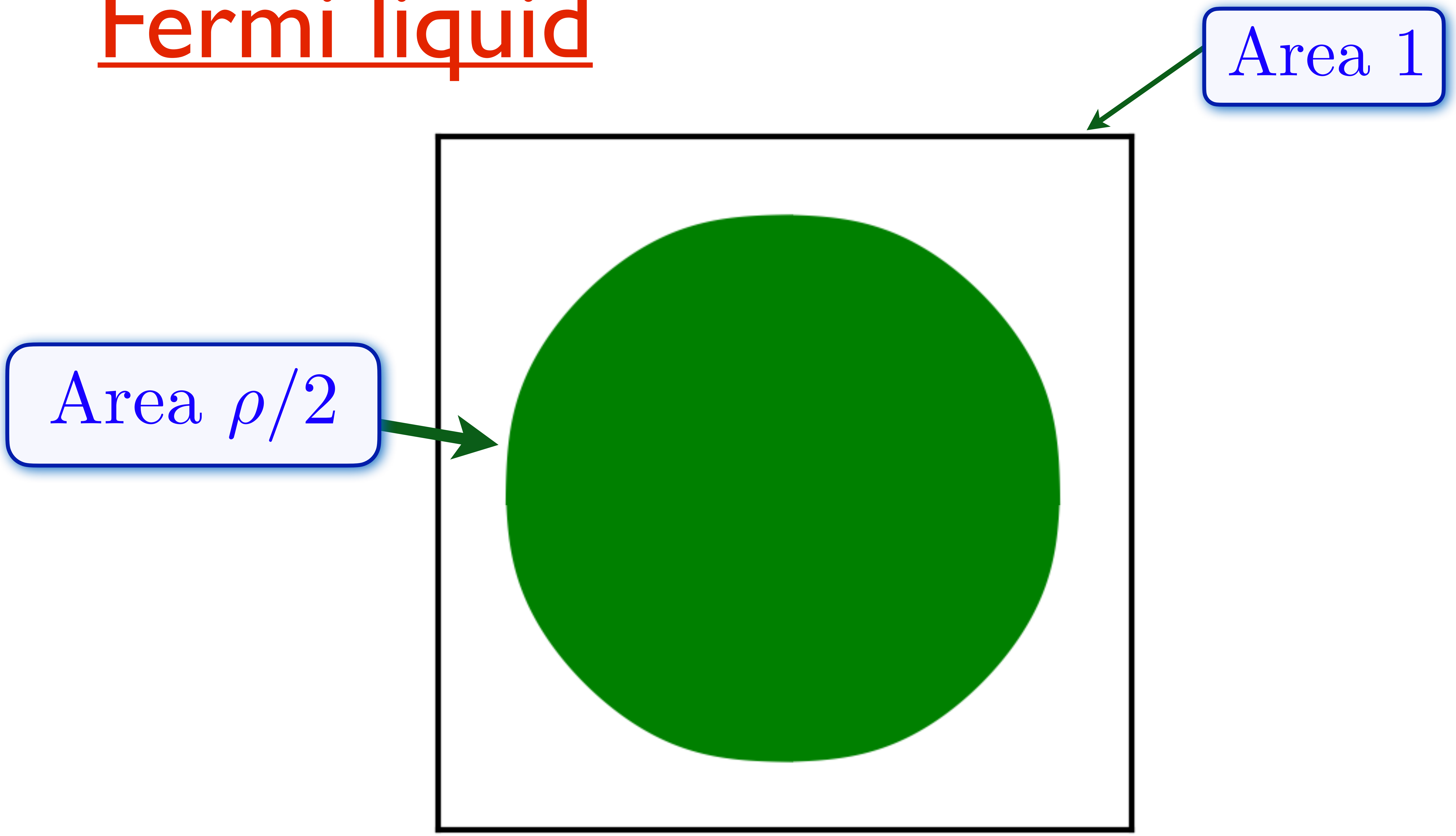
Many fermion entanglement I:

Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density ρ



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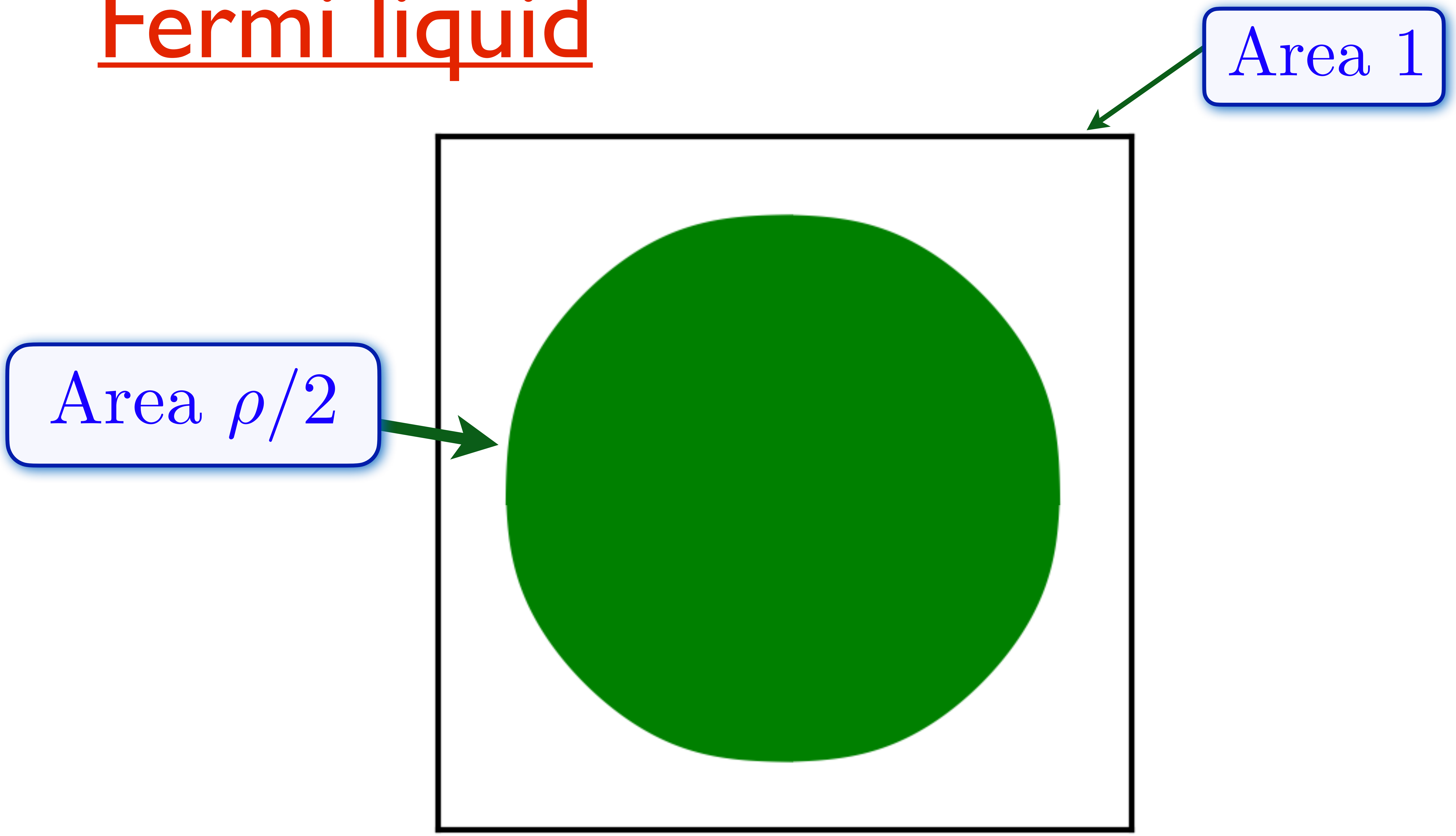
Fermi liquid

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient of carrier density ρ



Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Oshikawa, 2000: Area constrained by an anomaly-argument of global U(1) and translations

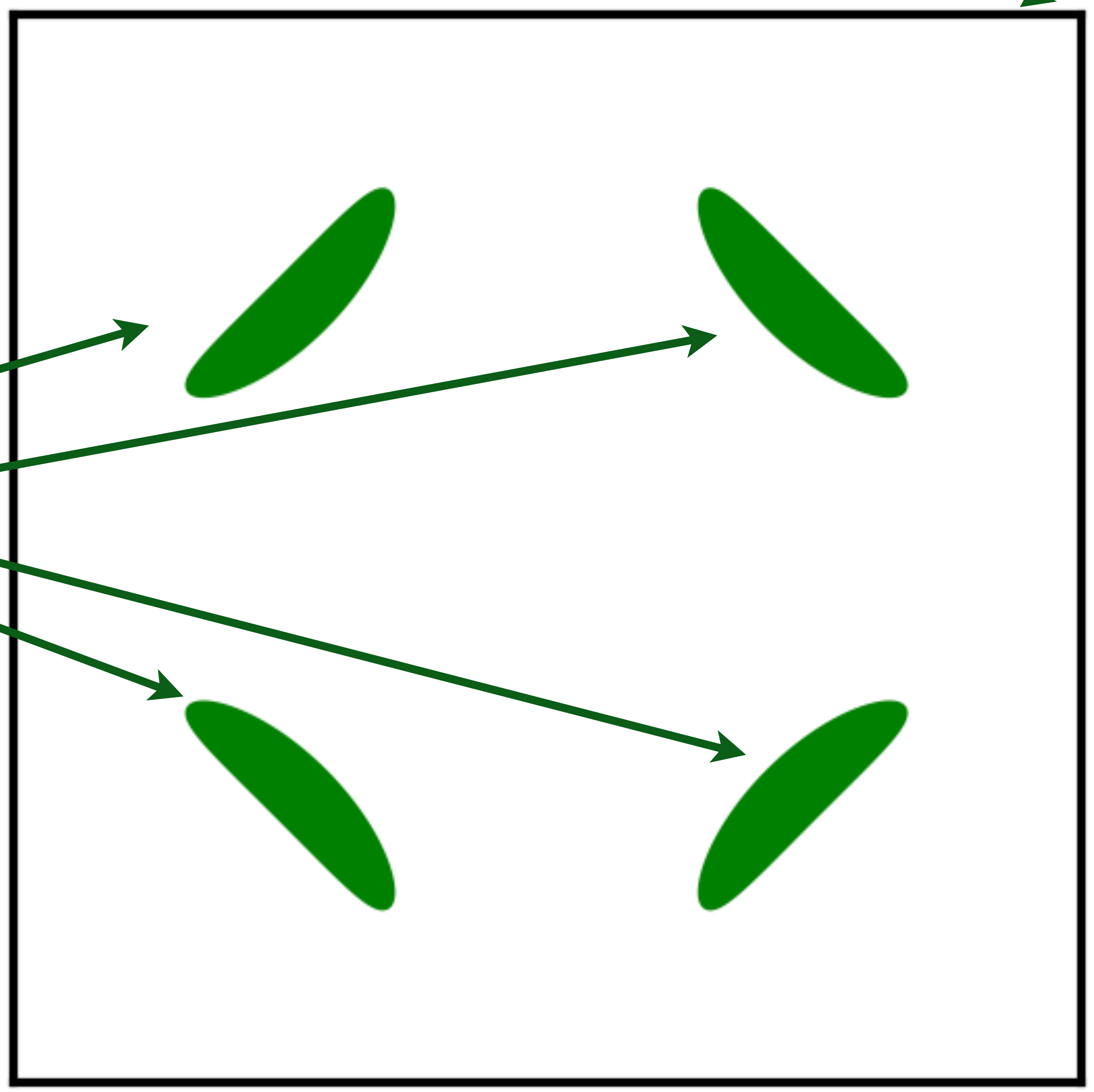
Fractionalized Fermi liquid (FL*)

Area 1

Spin-1/2 holes of density

$$\rho = 1 + p$$

Total area $(\rho - 1)/2$



No broken symmetry

Positive Hall coefficient of carrier density $\rho - 1$

Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

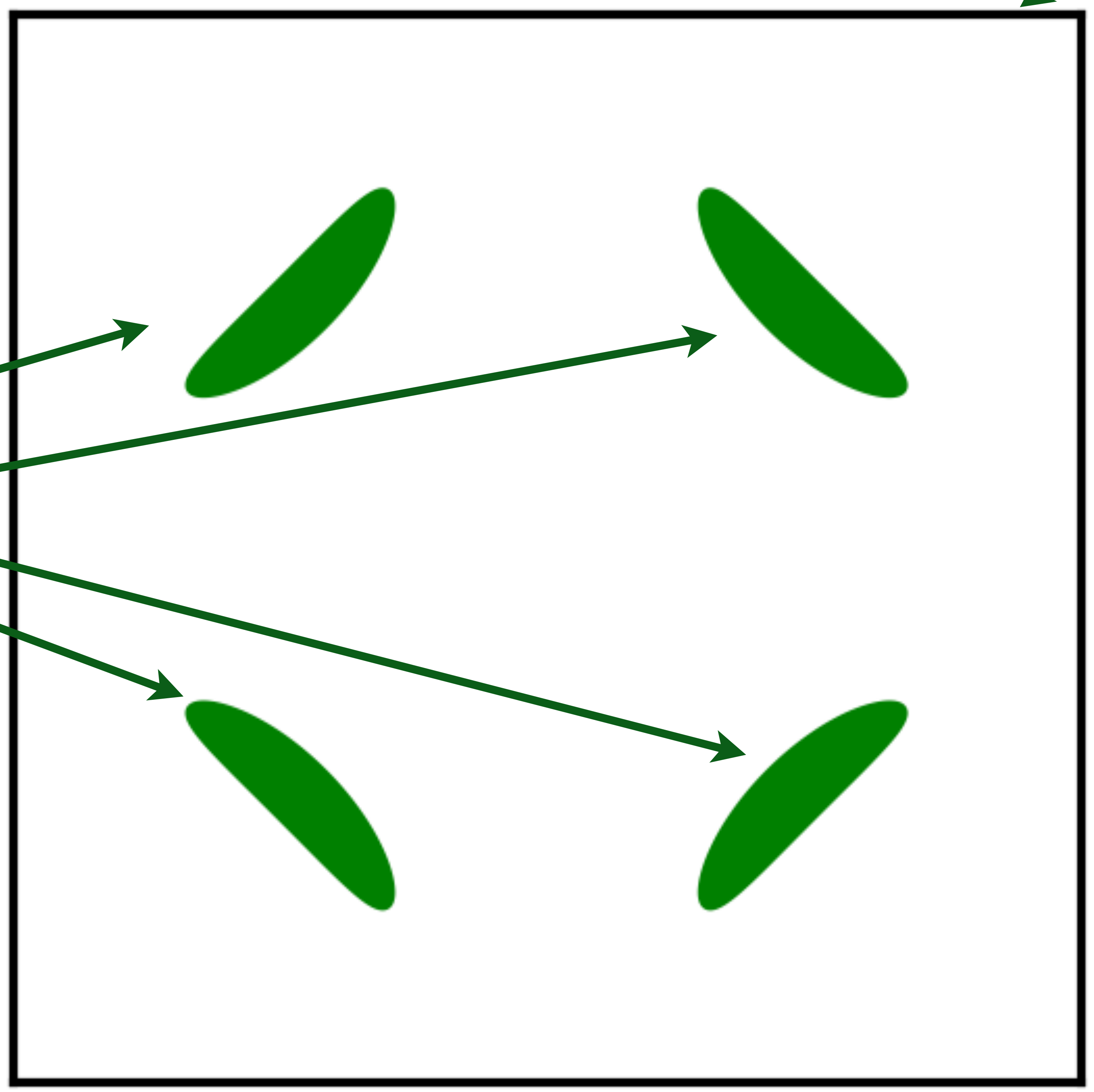
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density

$$\rho = 1 + p$$

Positive Hall coefficient of carrier density $\rho - 1$



The density deficit (1) in the area is **quantized** by rigid structure of the spin liquid.

Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
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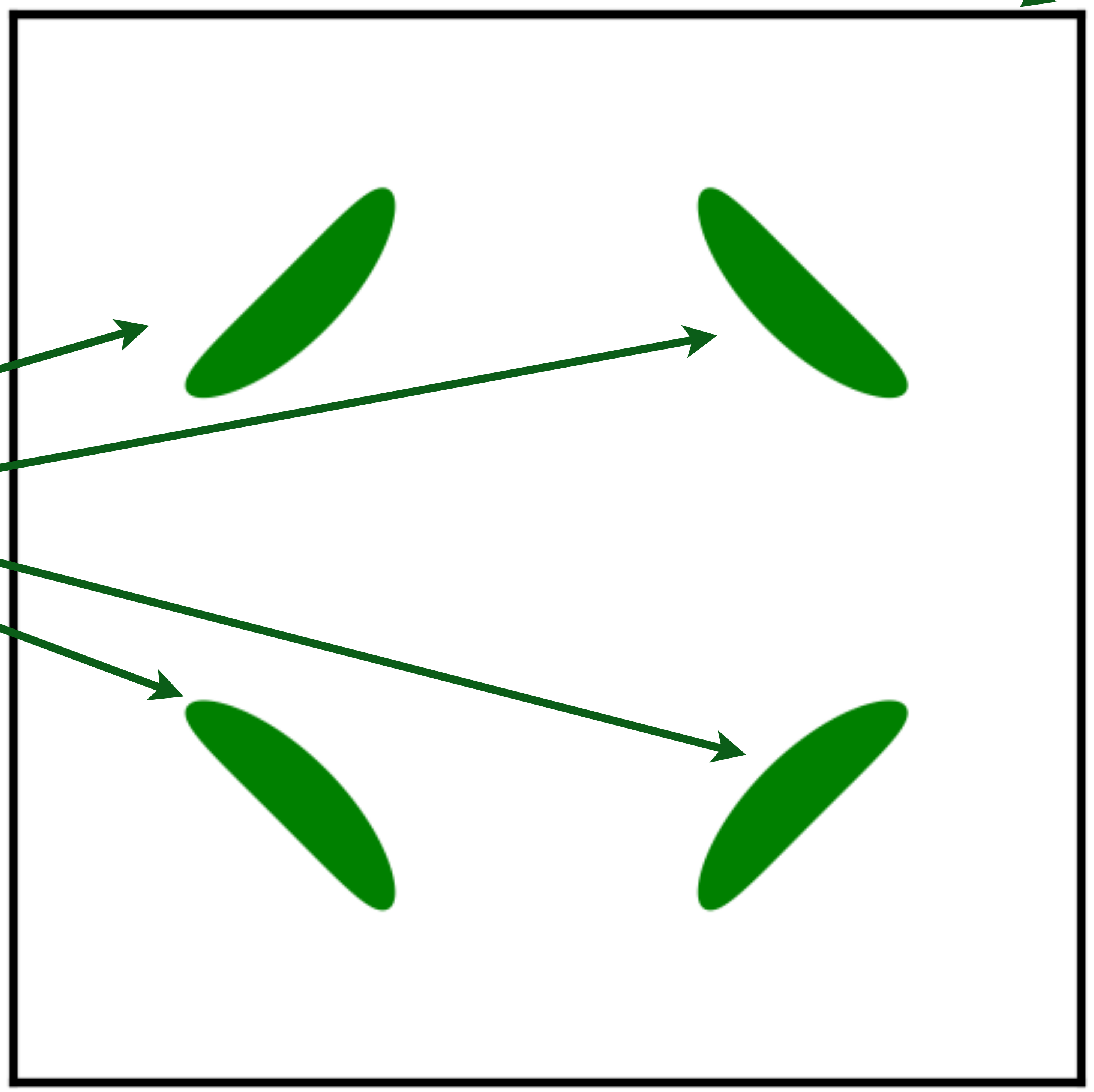
Total area $(\rho - 1)/2$

Area 1

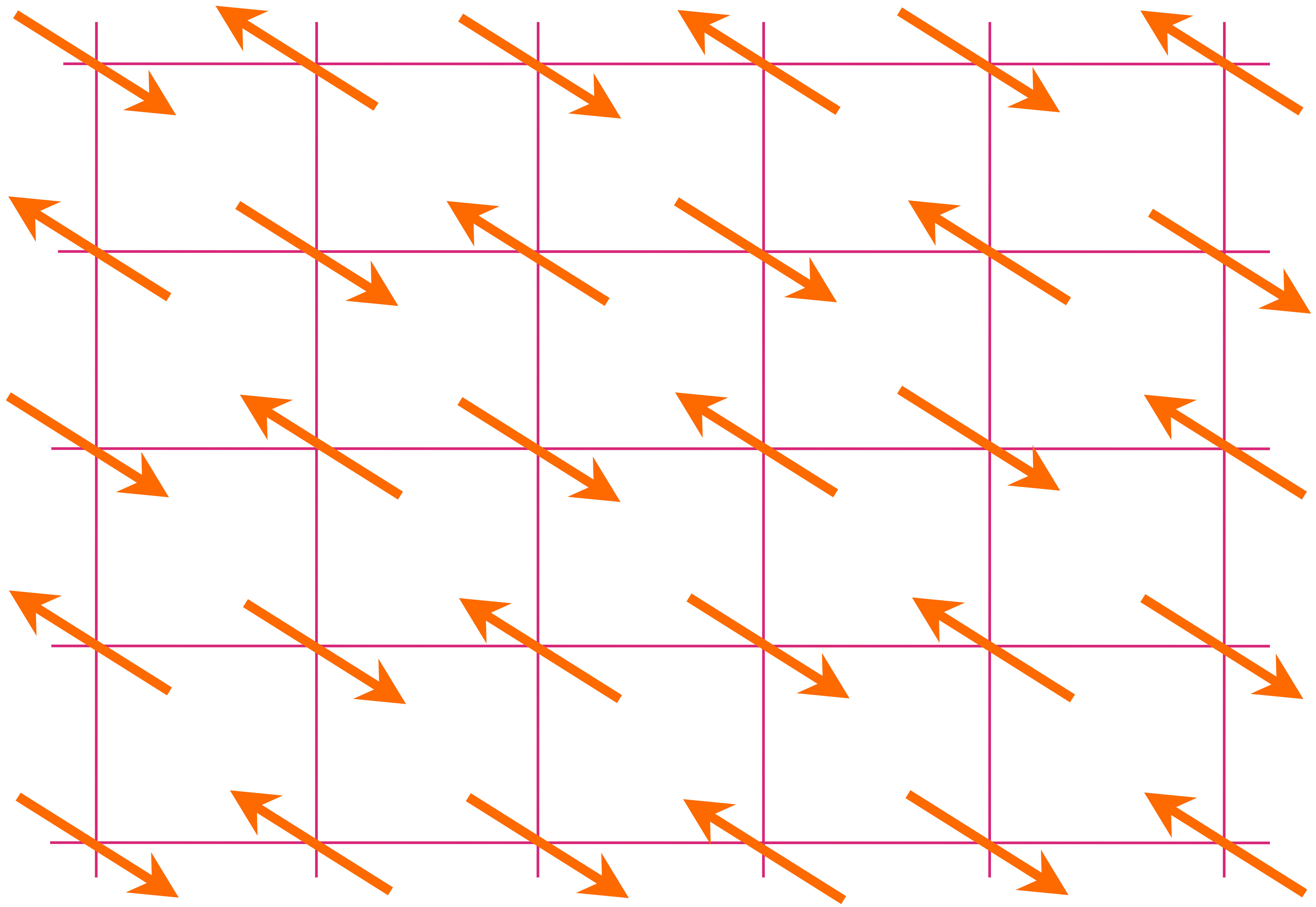


Measuring non-Luttinger Fermi surface area is direct evidence for multi-fermion quantum entanglement.

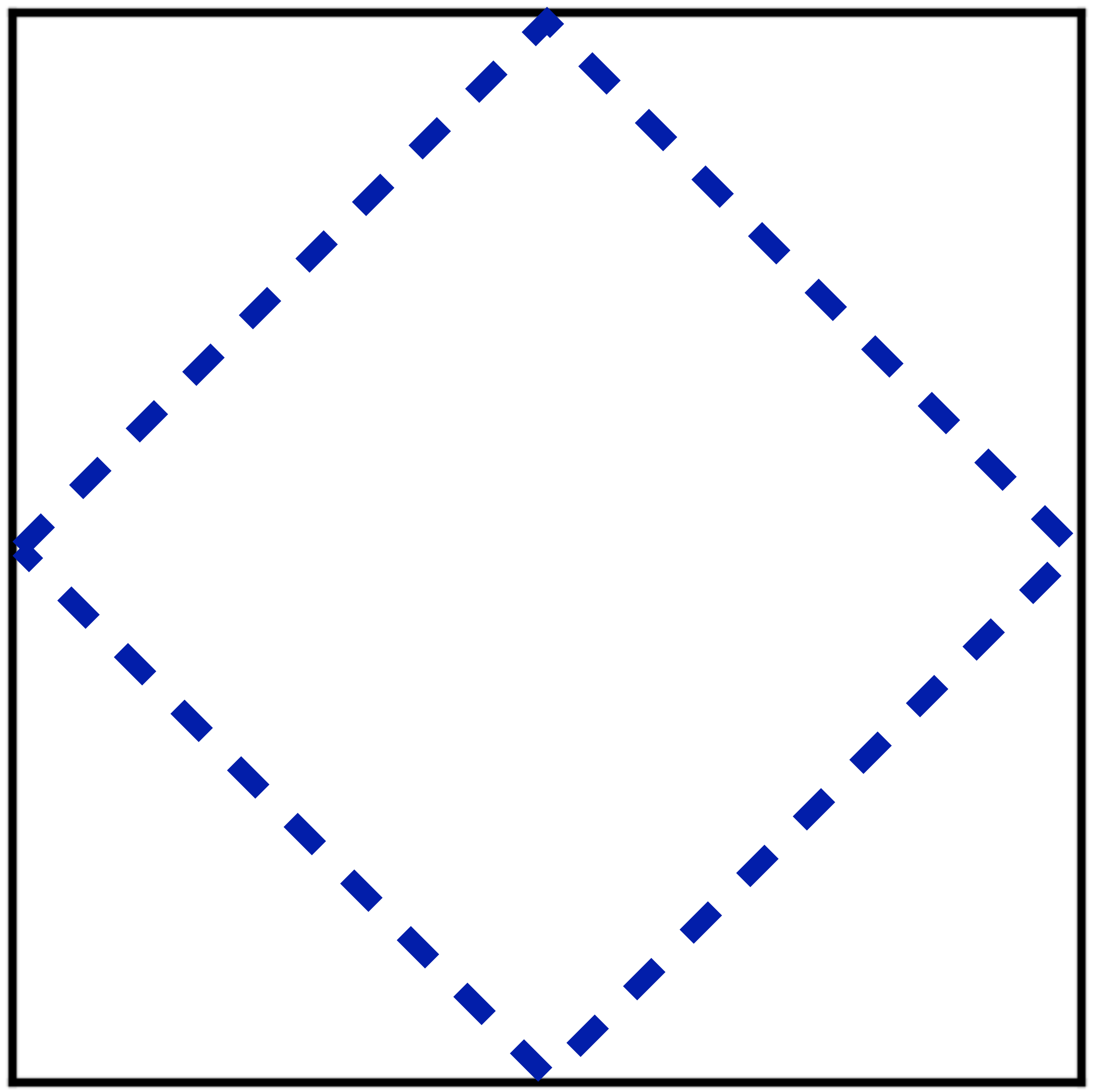
Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Insulating antiferromagnet



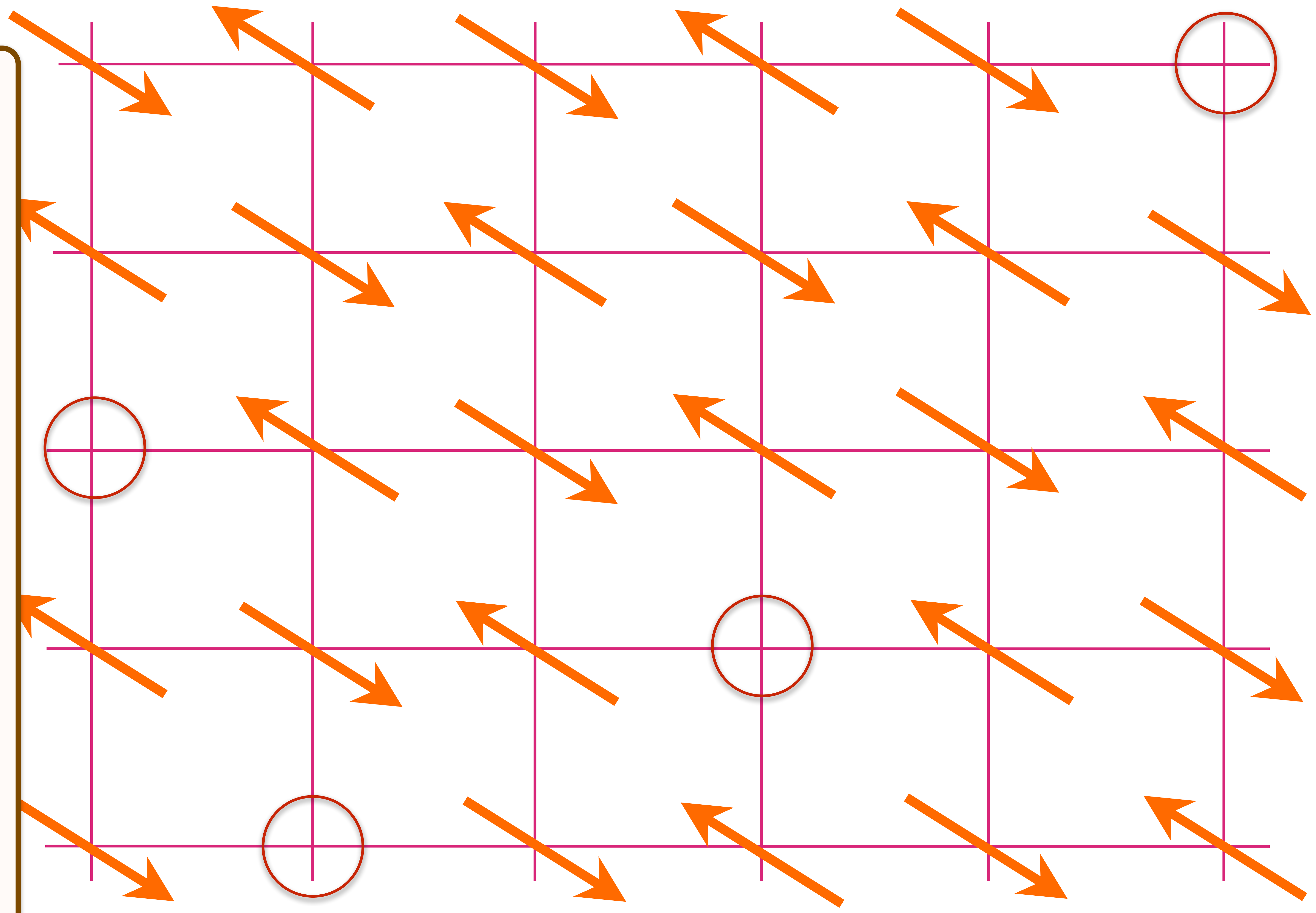
Reduced Brillouin
Zone.
Broken symmetry

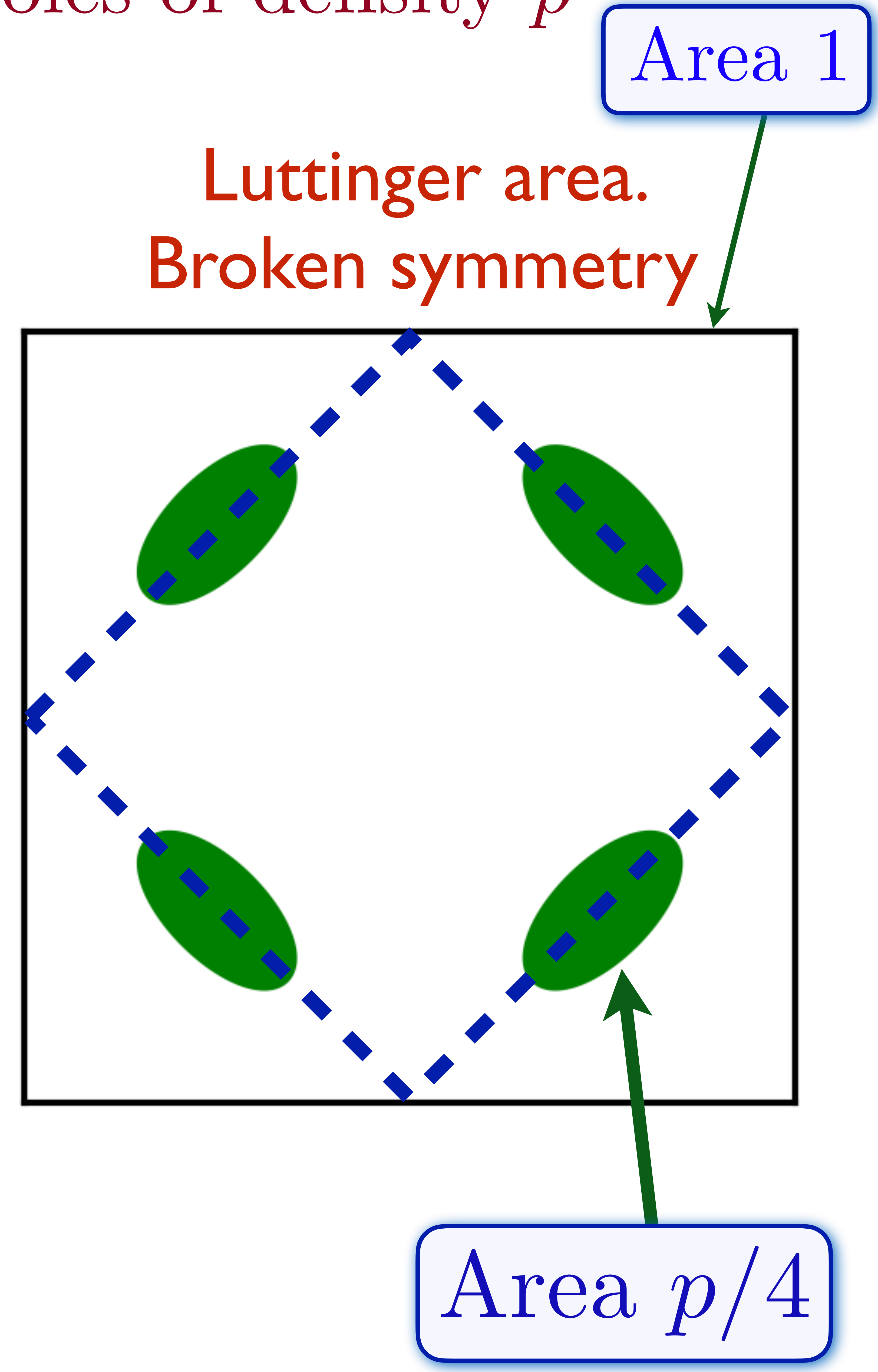


Doping an insulating antiferromagnet with holes of density p

AF metal

Fermi liquid with density p of spin $1/2$, charge $+e$ holes. Coherent inter-layer transport requires inter-layer spin correlations.



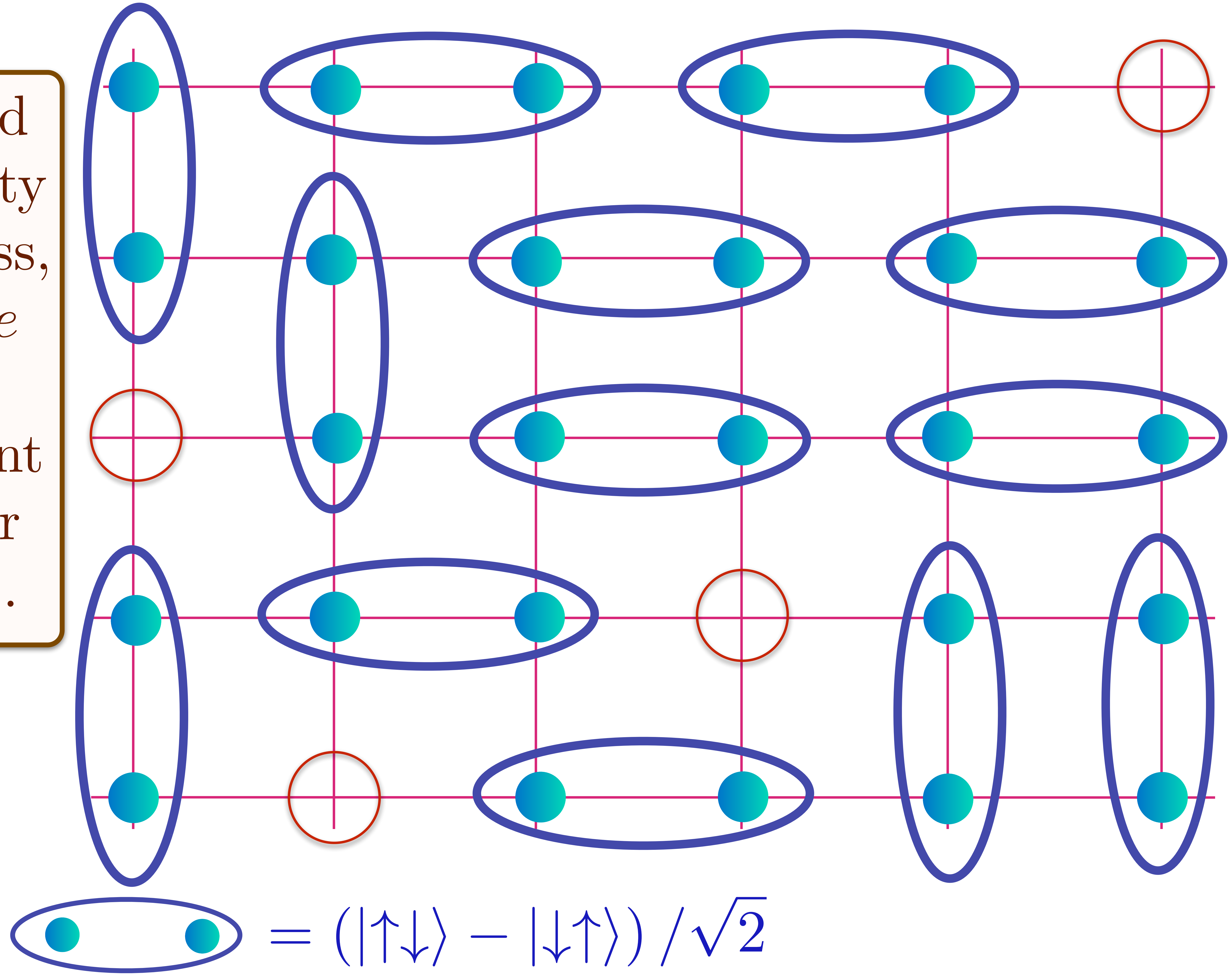


Doping an insulating antiferromagnet with holes of density p

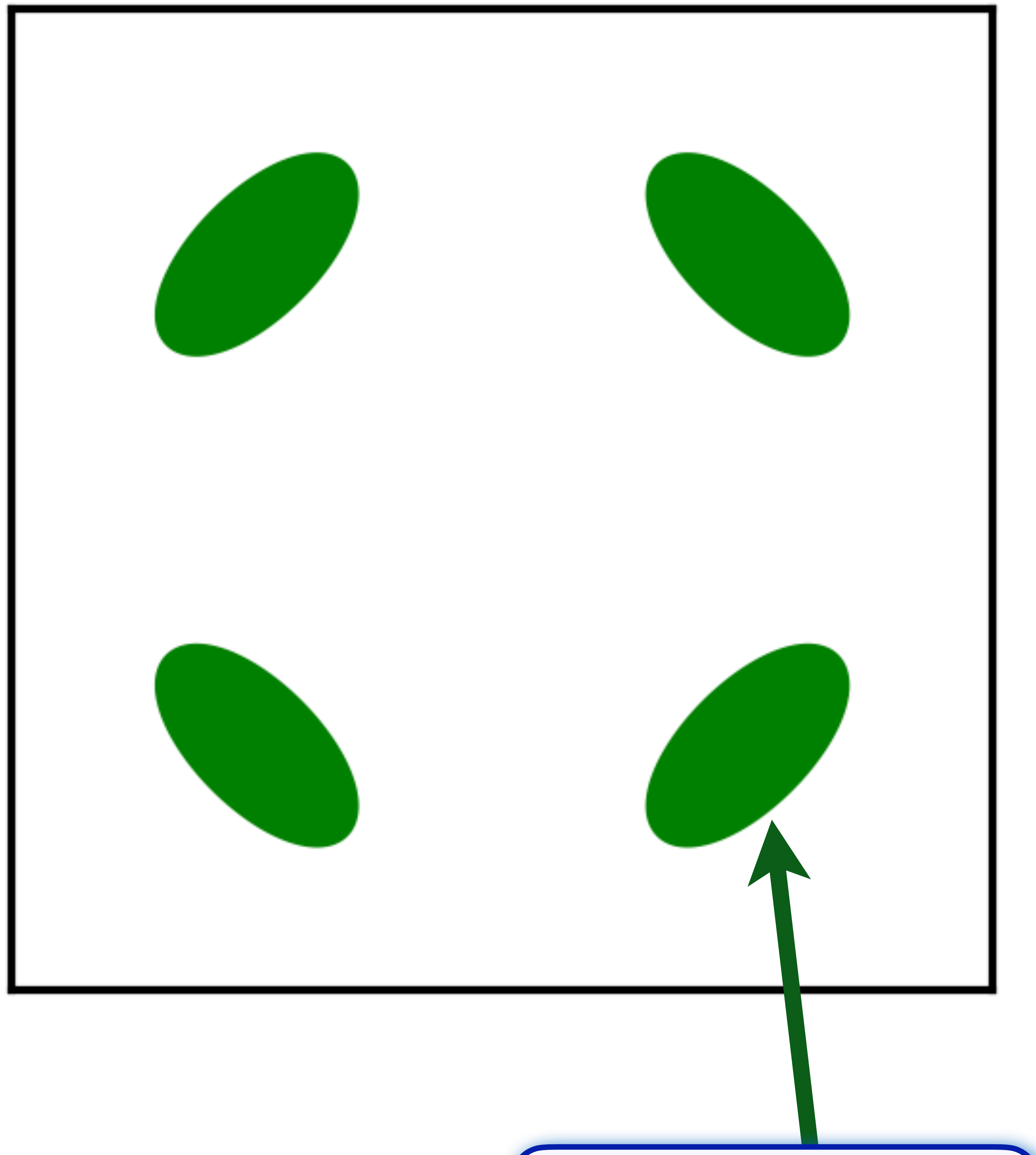
Holon metal

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Spin liquid with density p of spinless, charge $+e$ holons.
No coherent inter-layer transport.



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

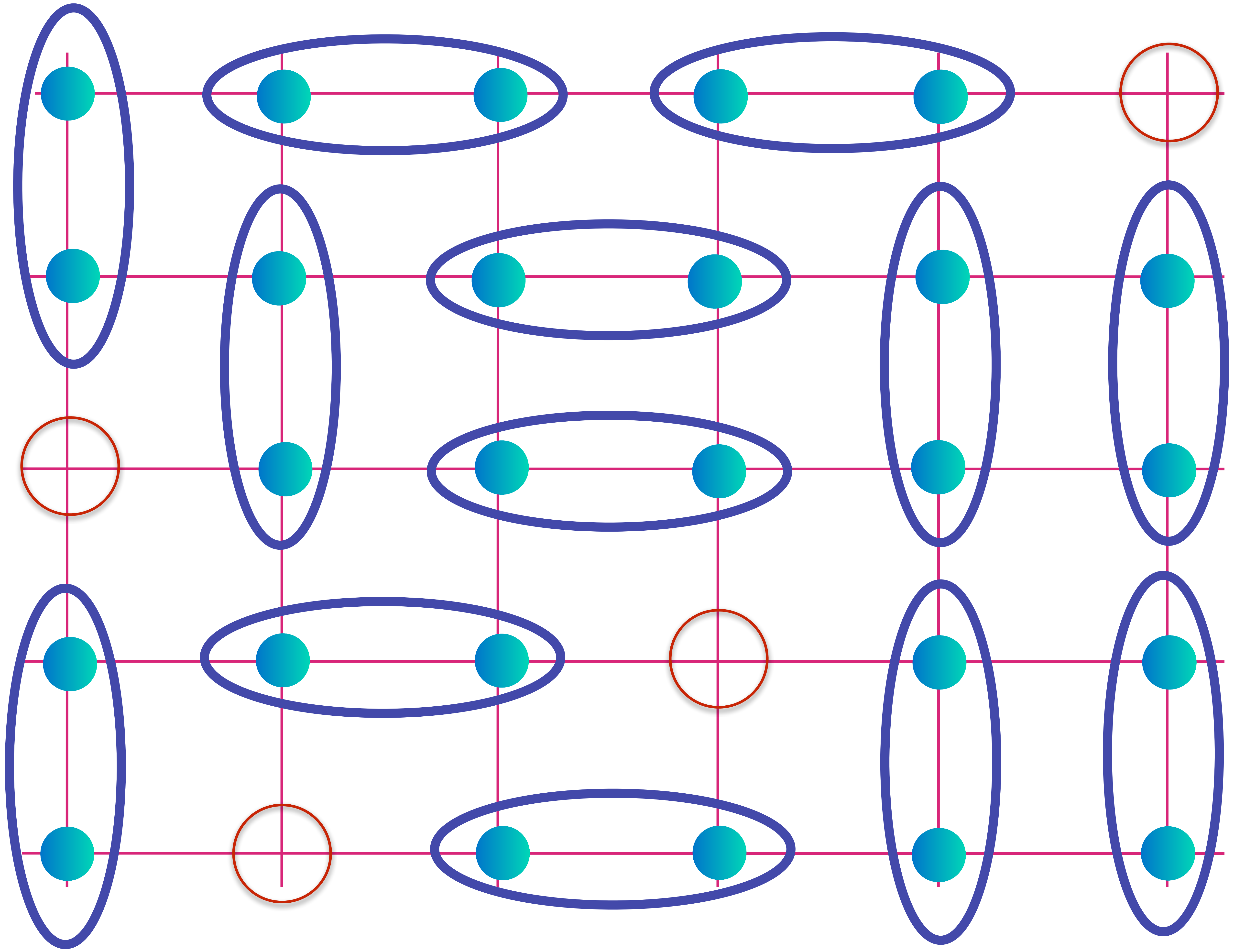
R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

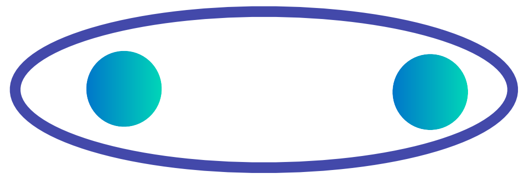
Doping an insulating antiferromagnet with holes of density p

<u>Holon metal</u>

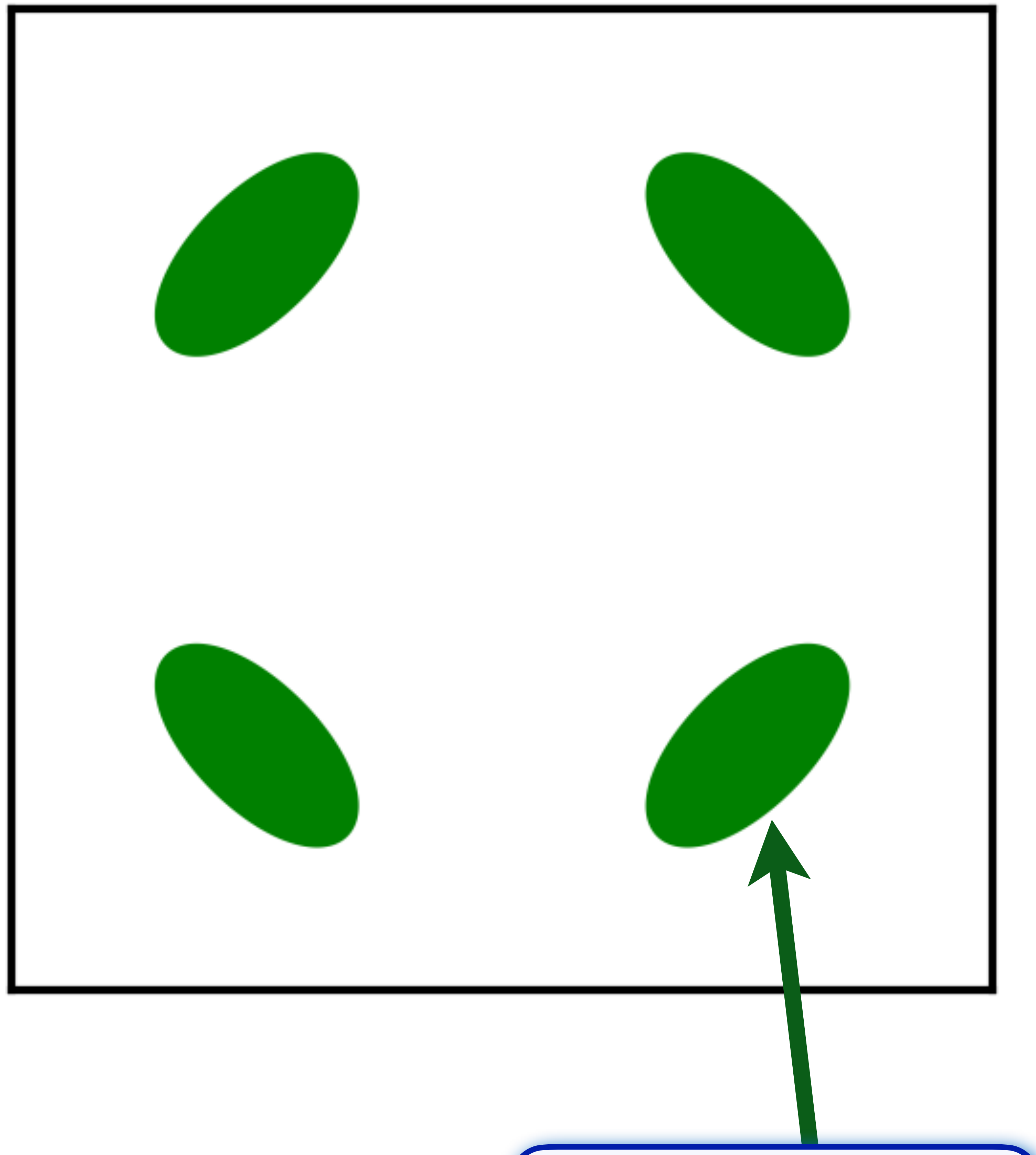
Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Spin liquid with density p of spinless, charge $+e$ <u>holons</u>.
No coherent inter-layer transport.





$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



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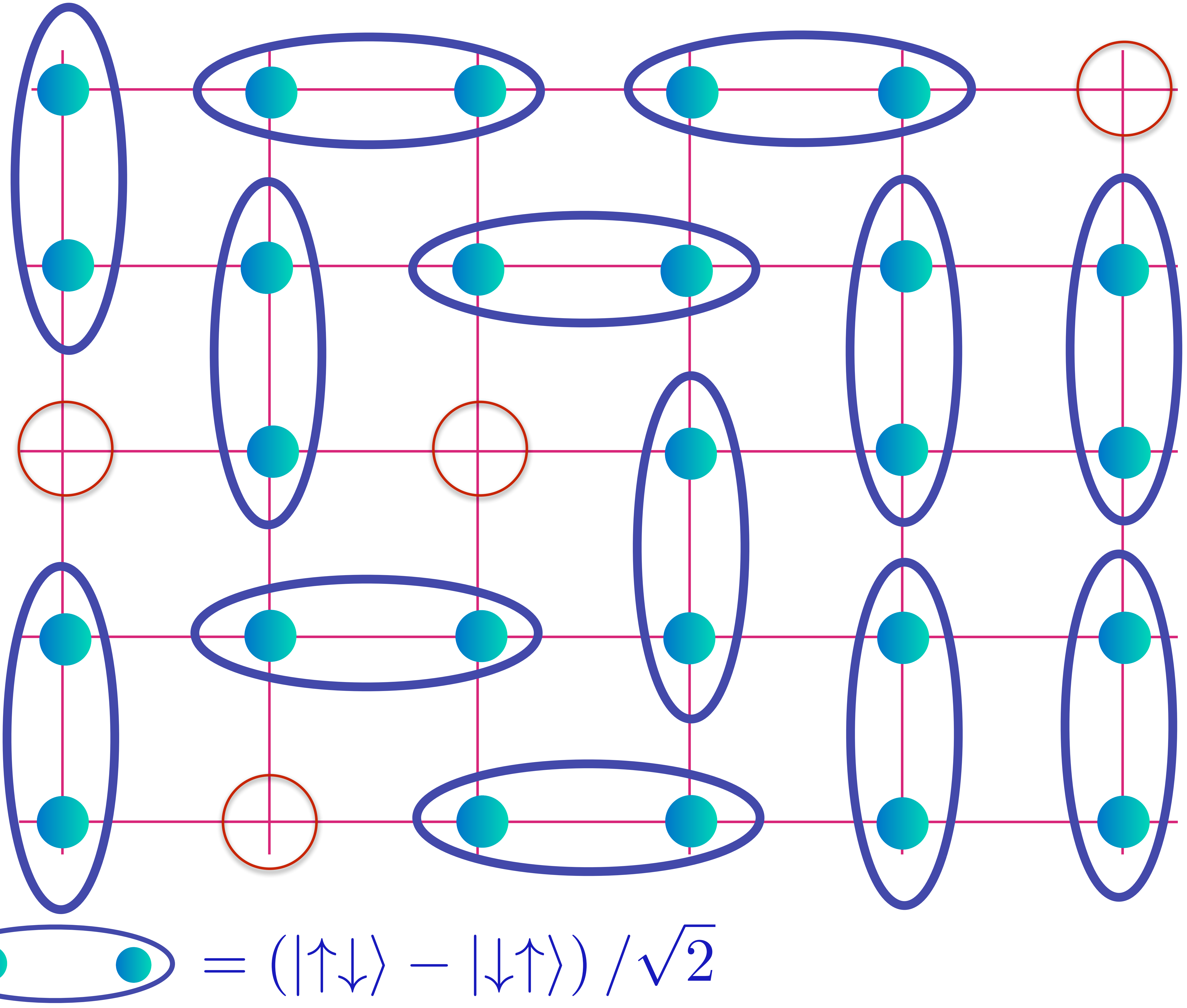

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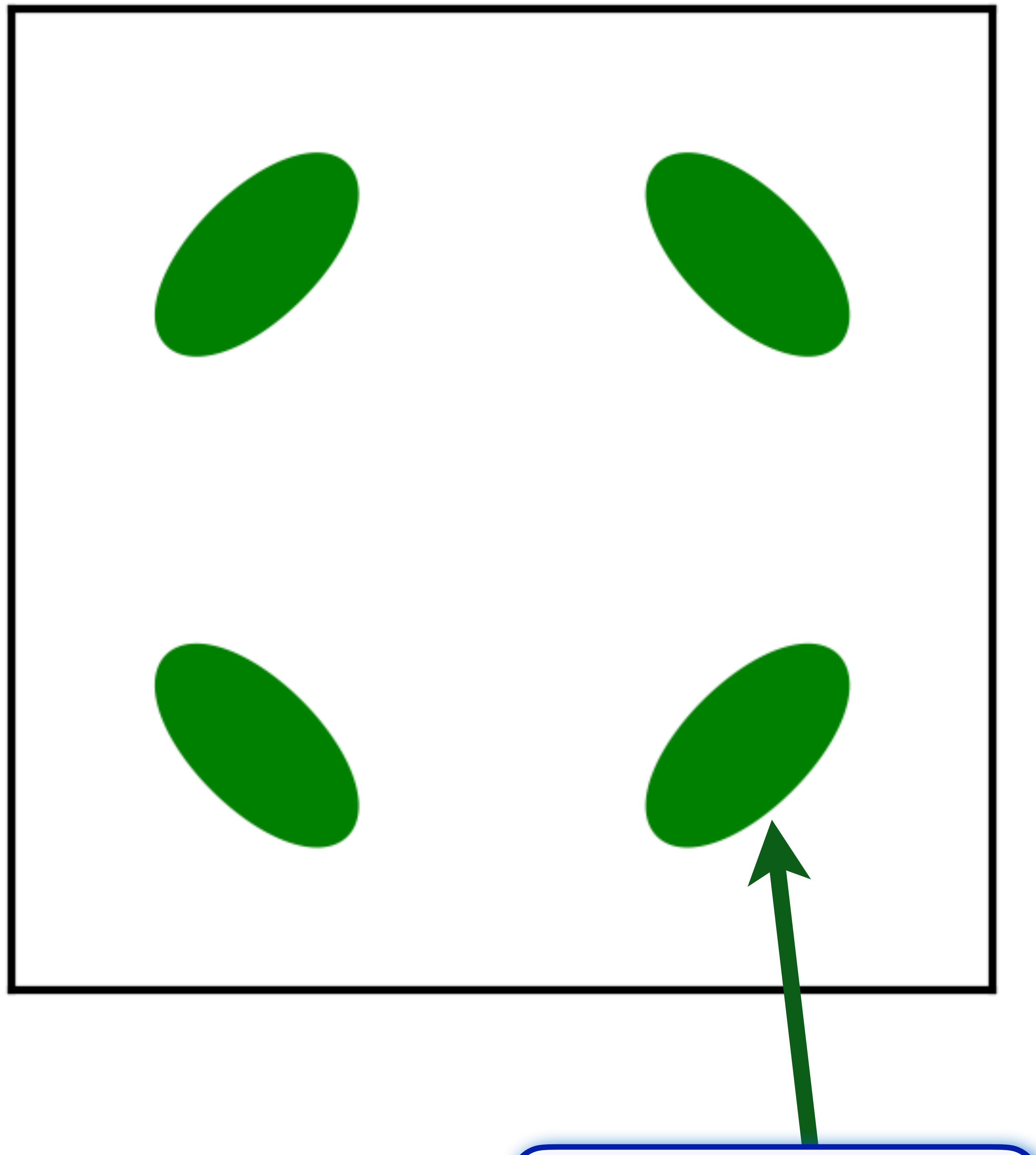

Area $p/4$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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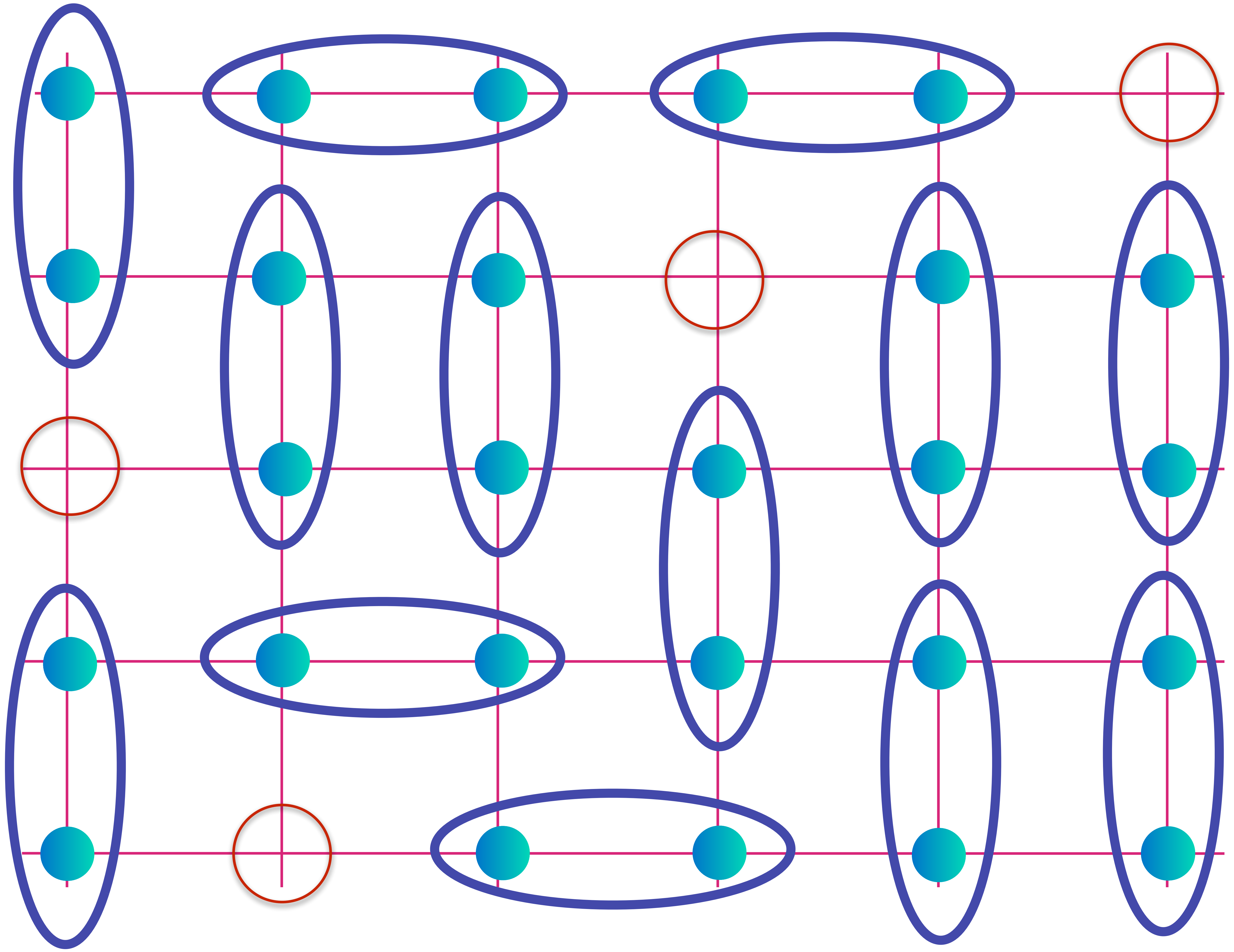
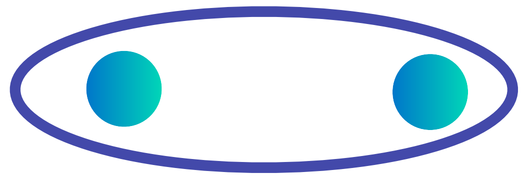
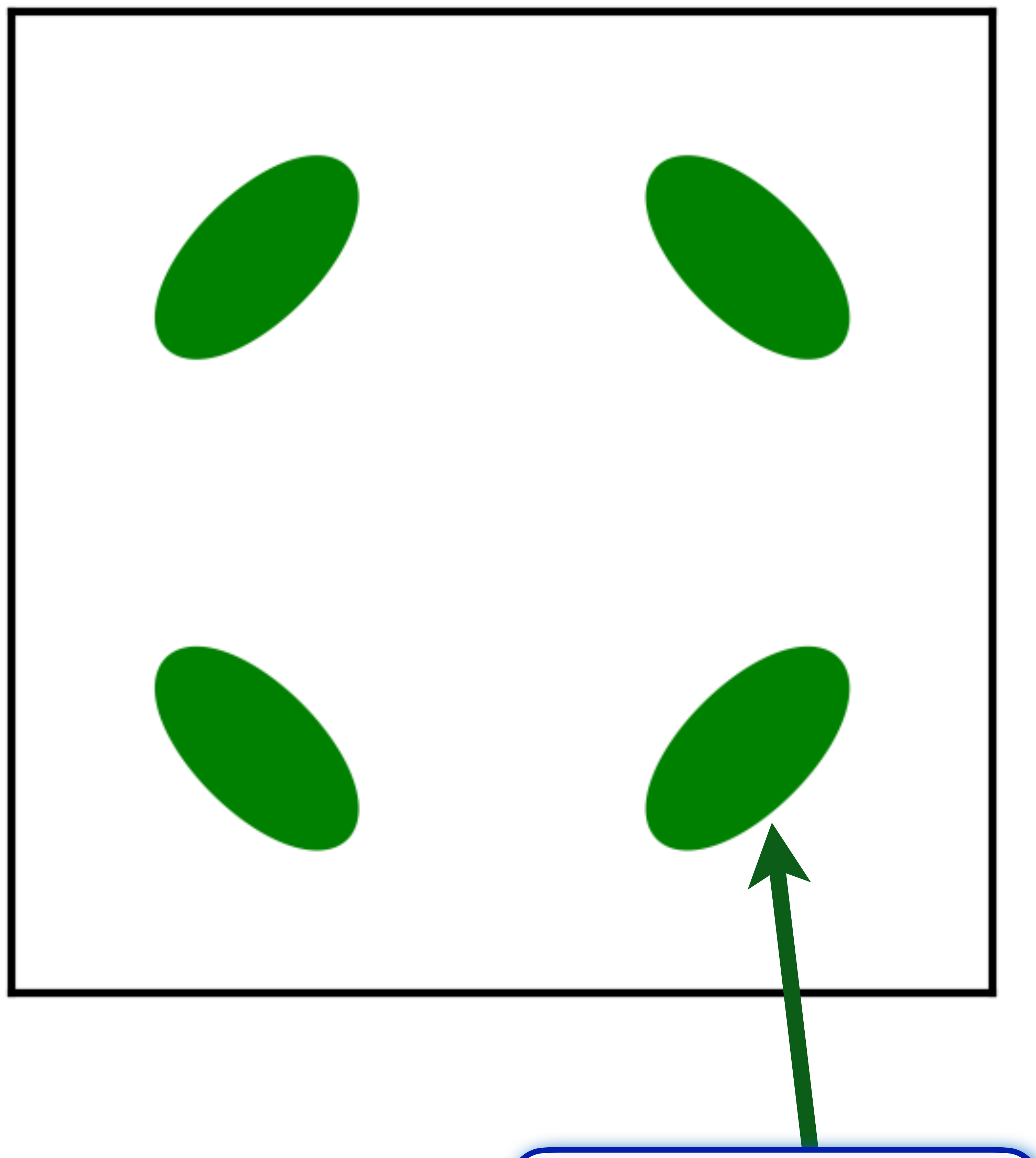


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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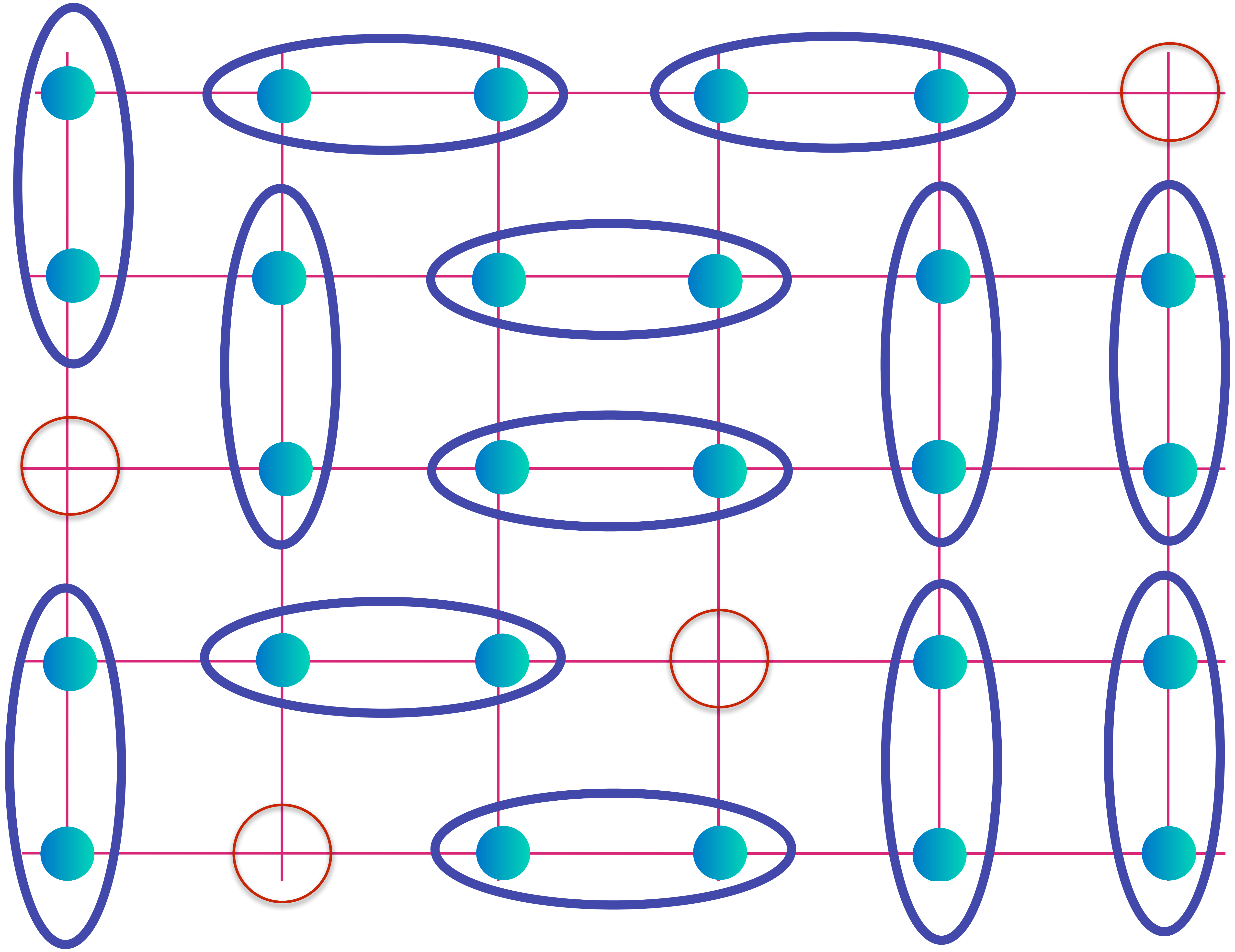
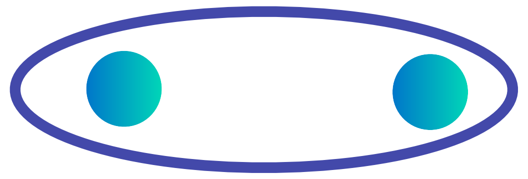
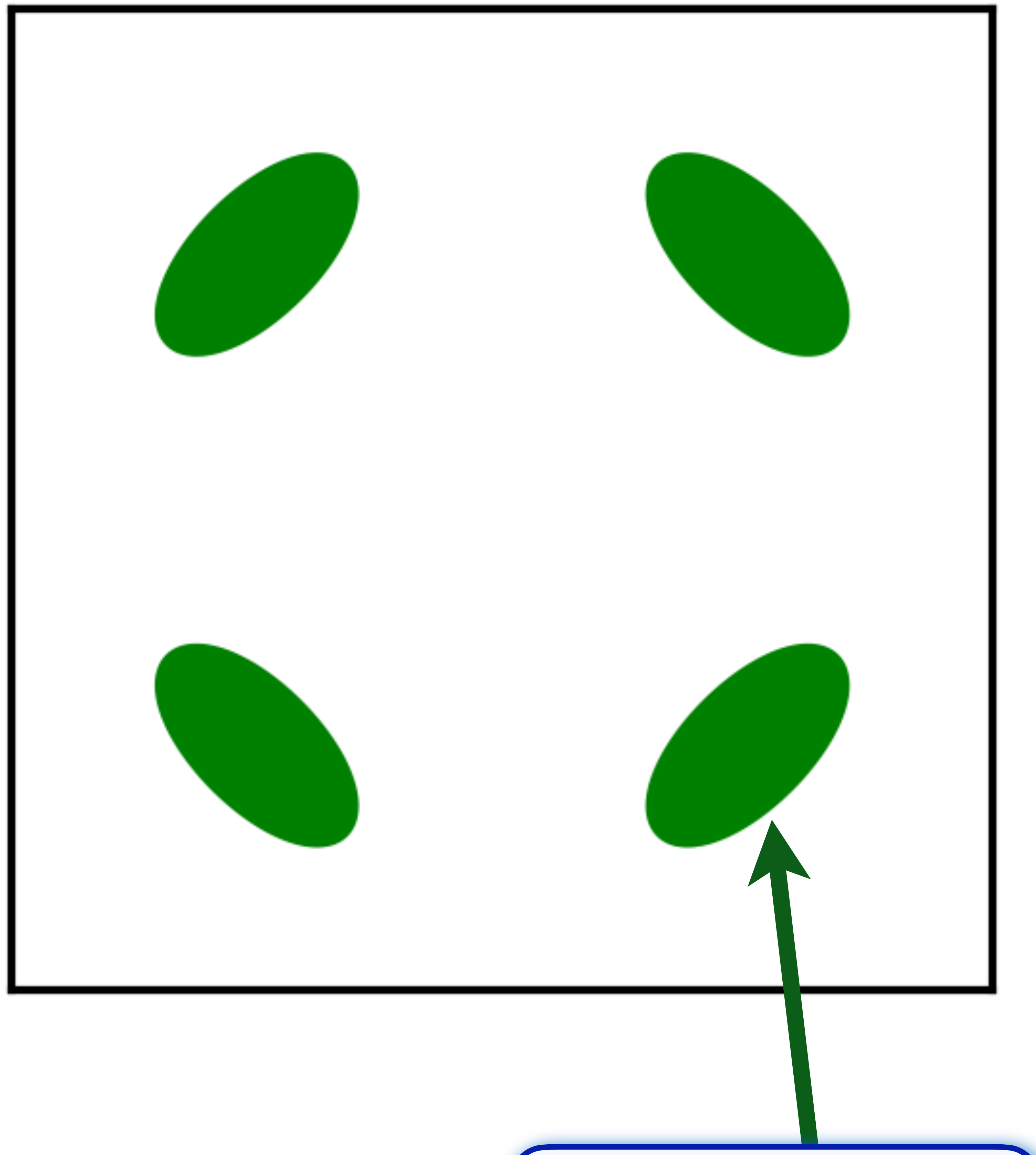


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



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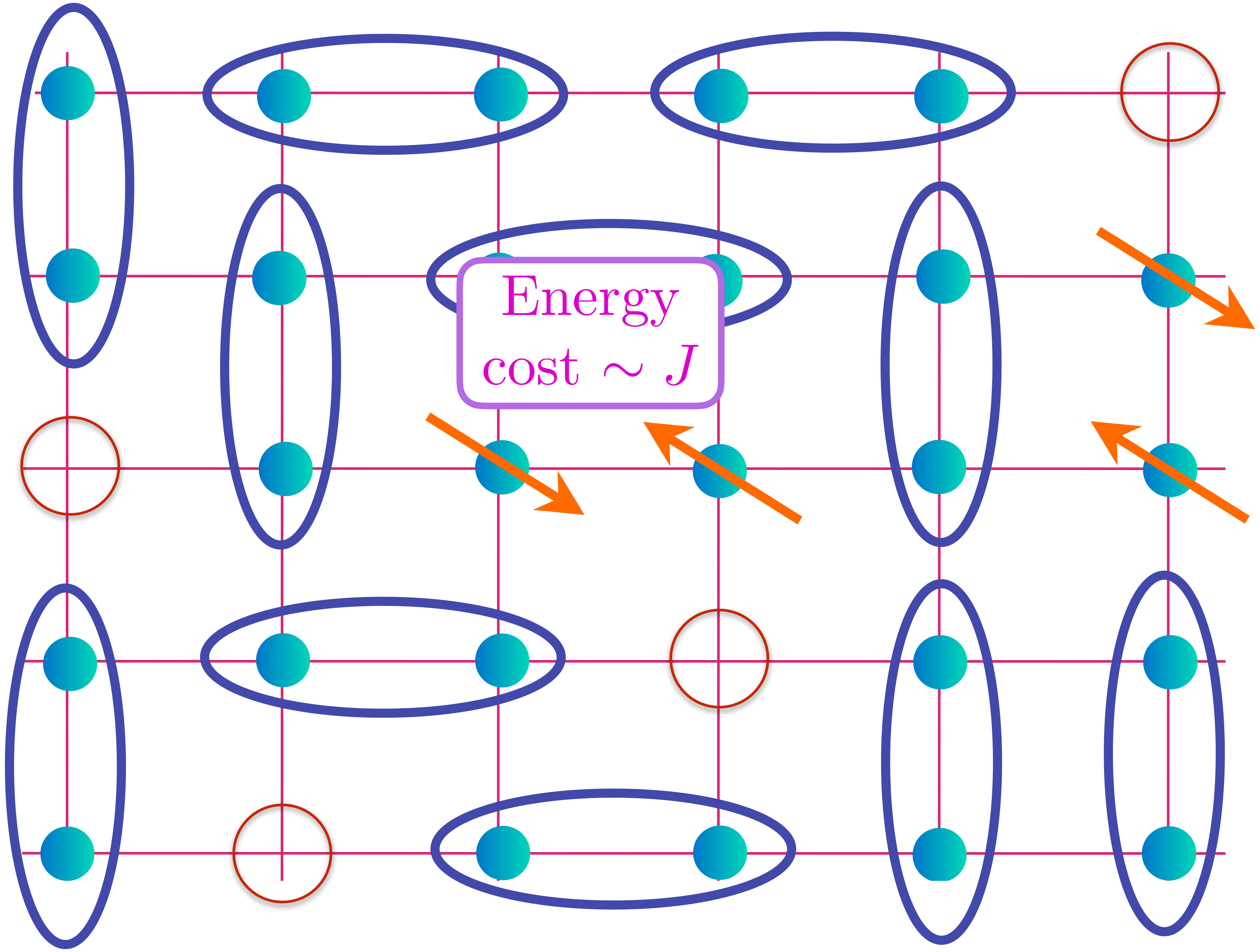
Doping an insulating antiferromagnet with holes of density p

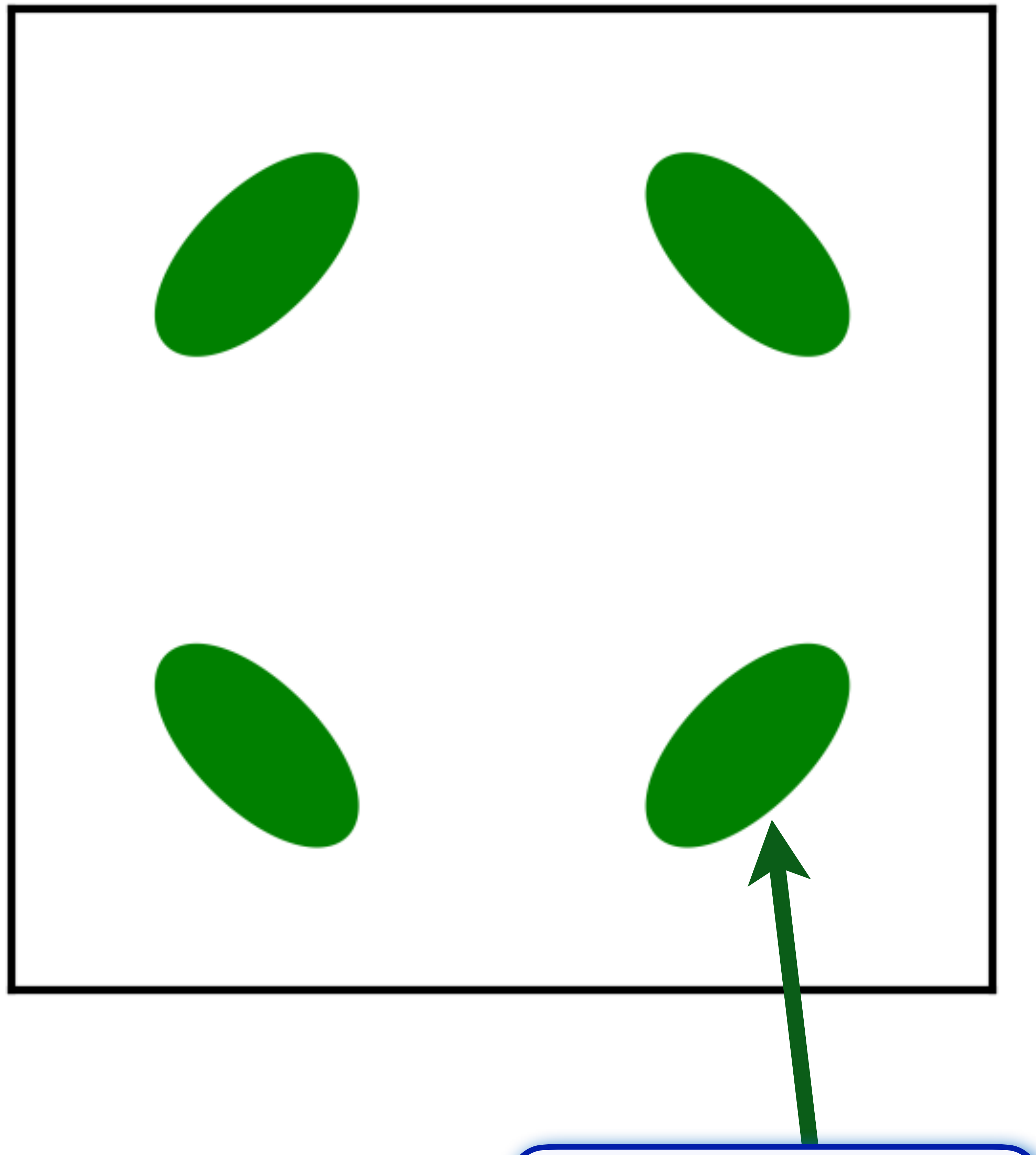
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

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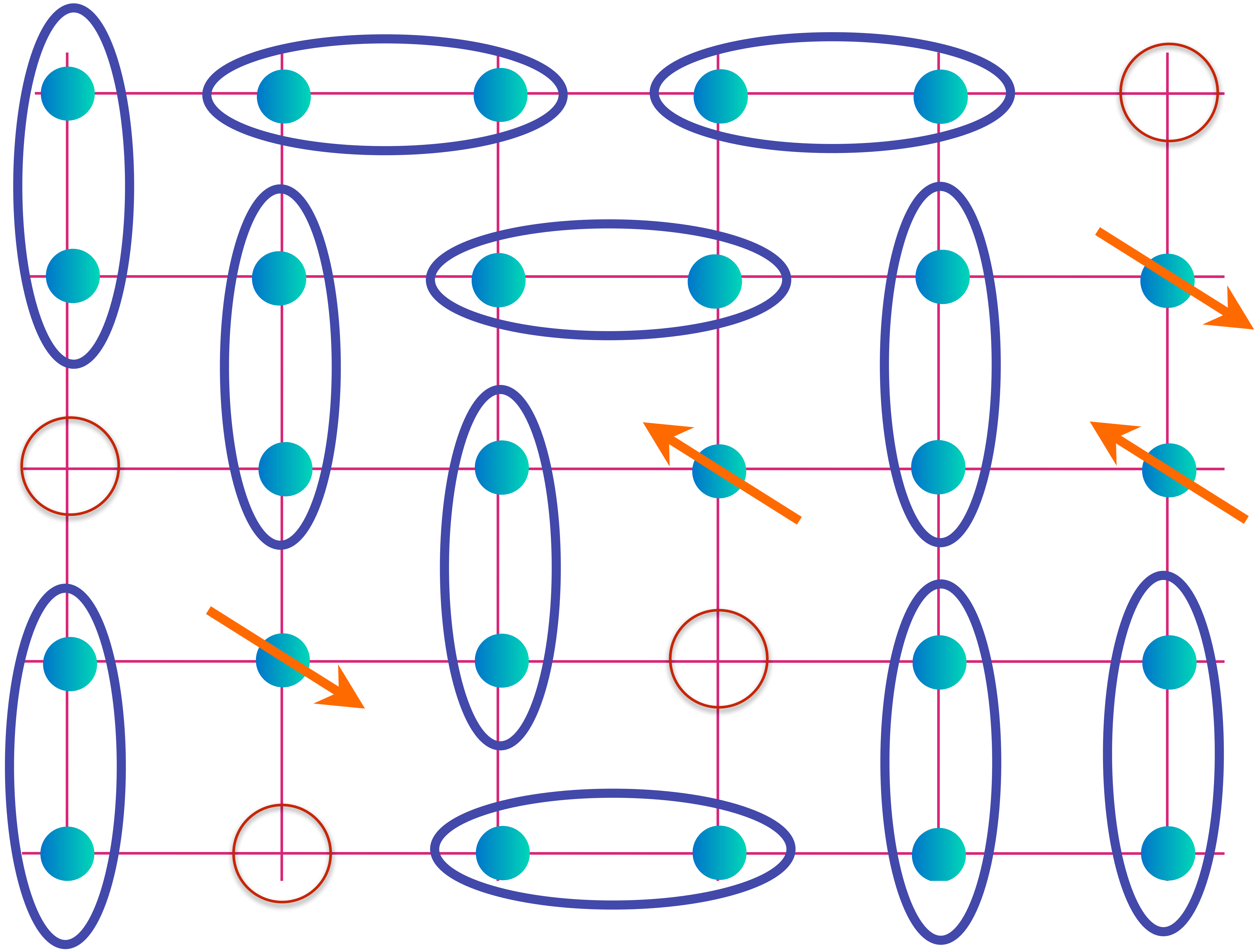
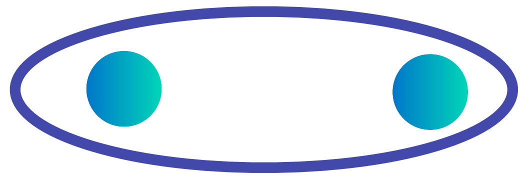
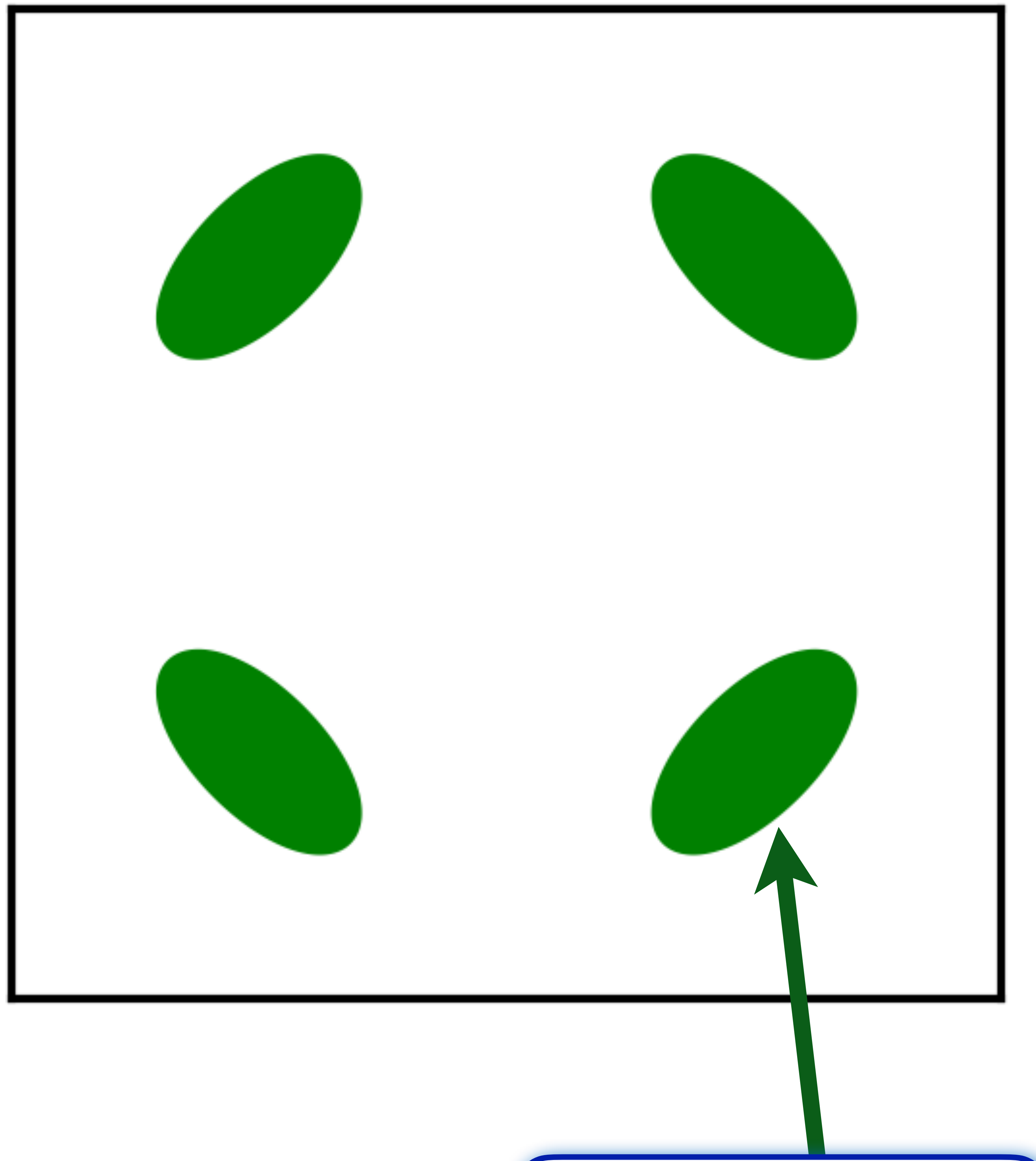


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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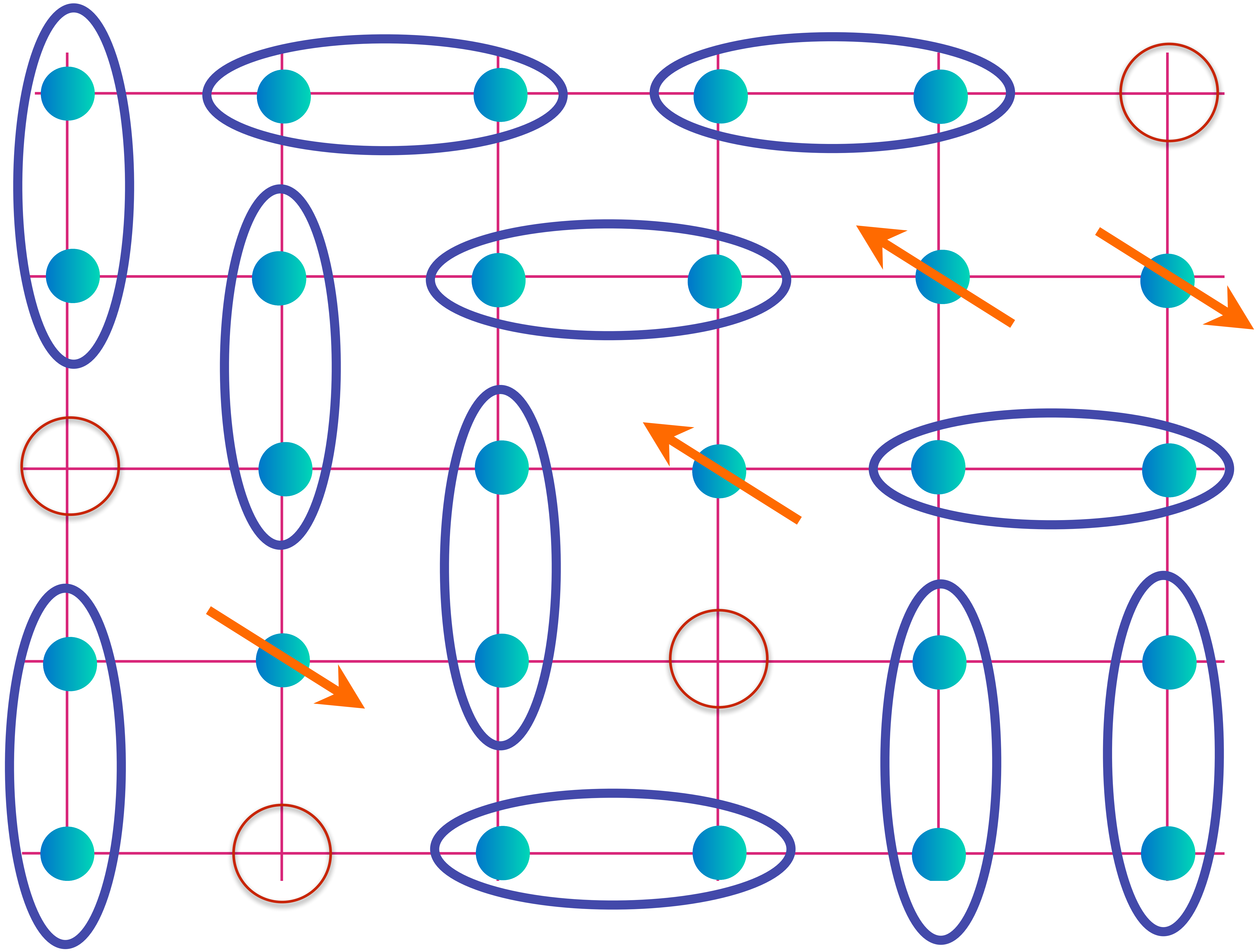
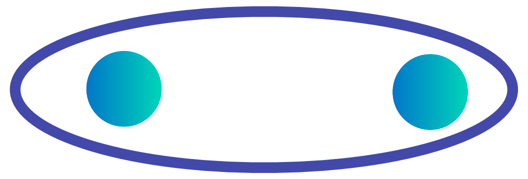
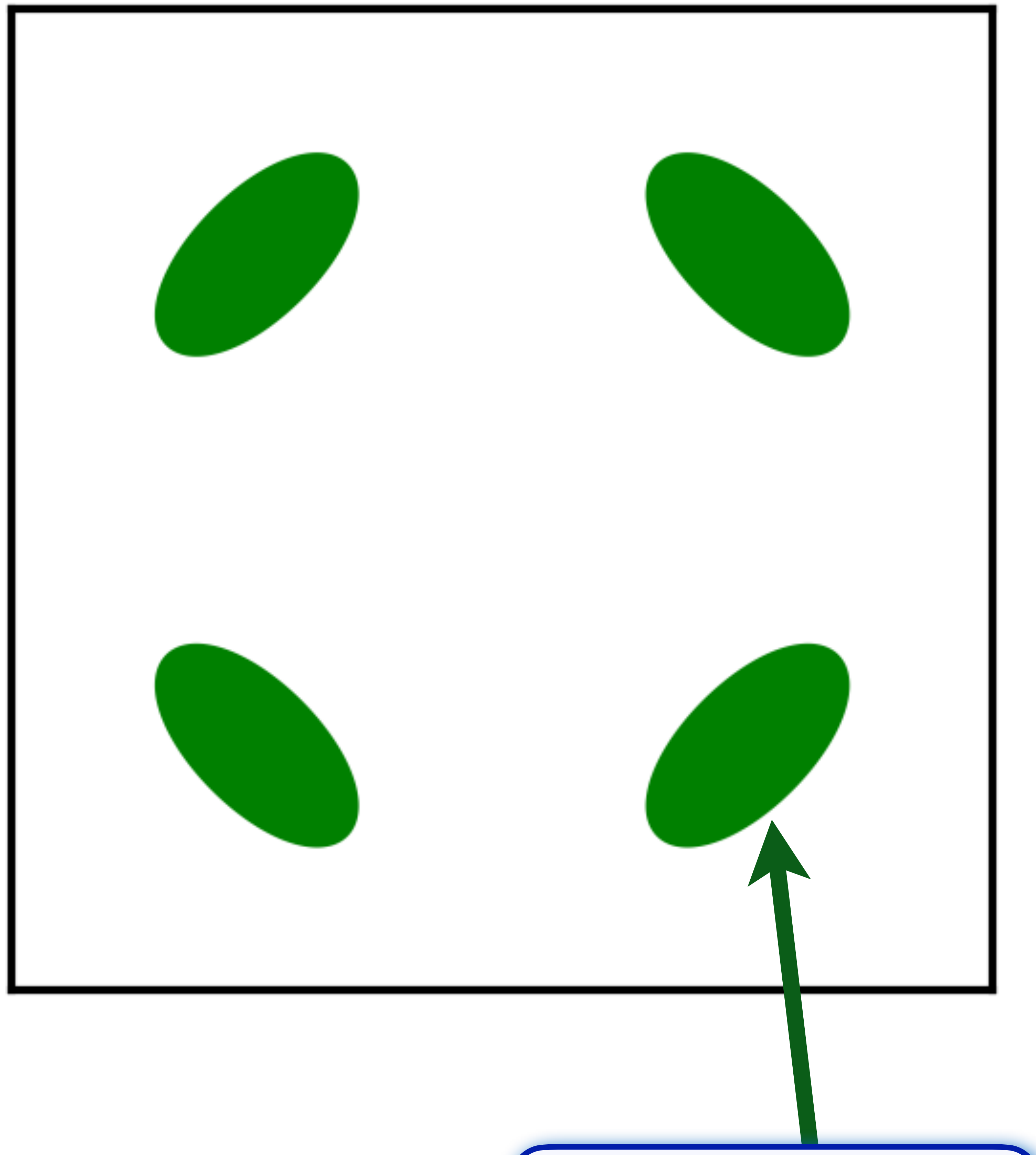


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

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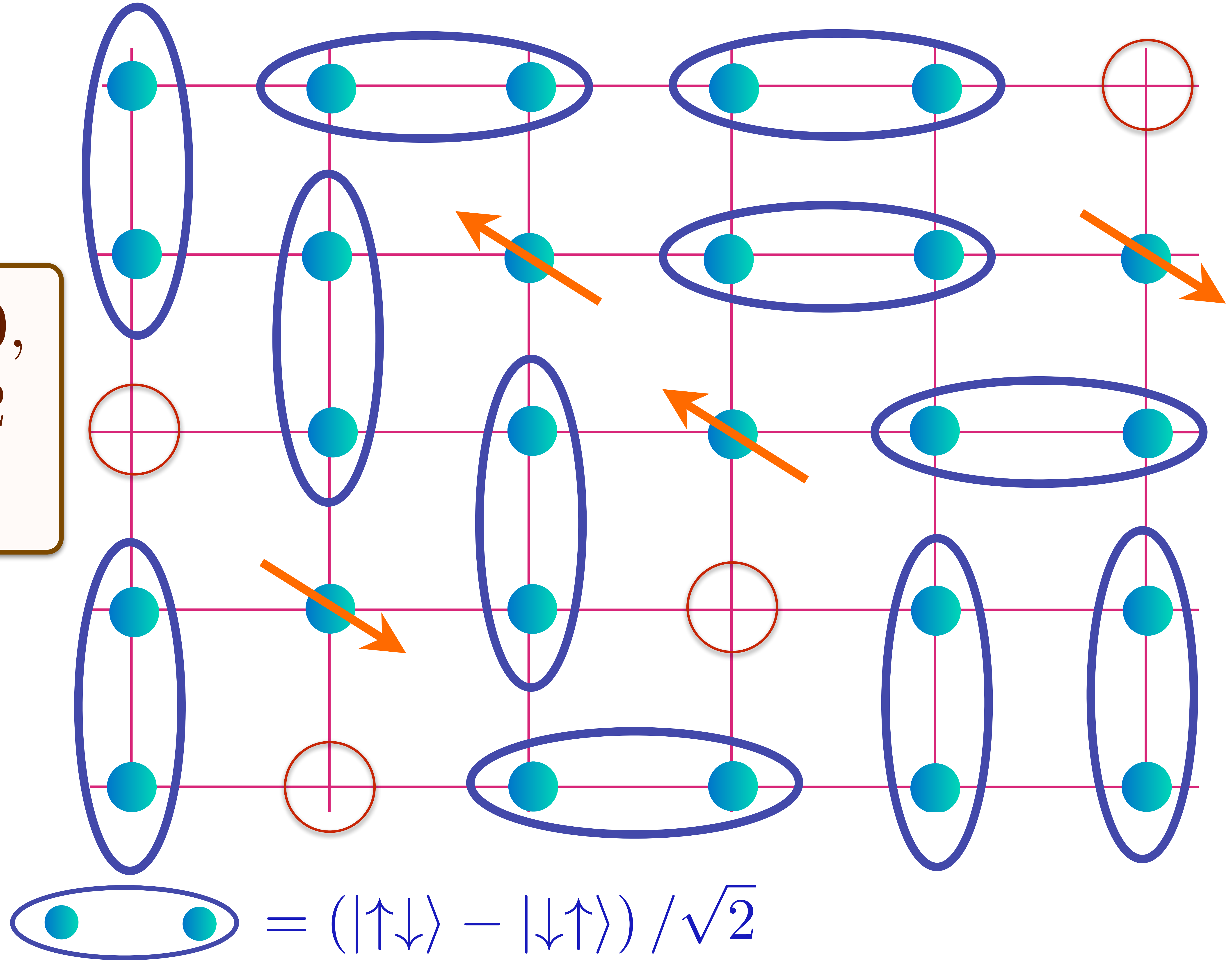
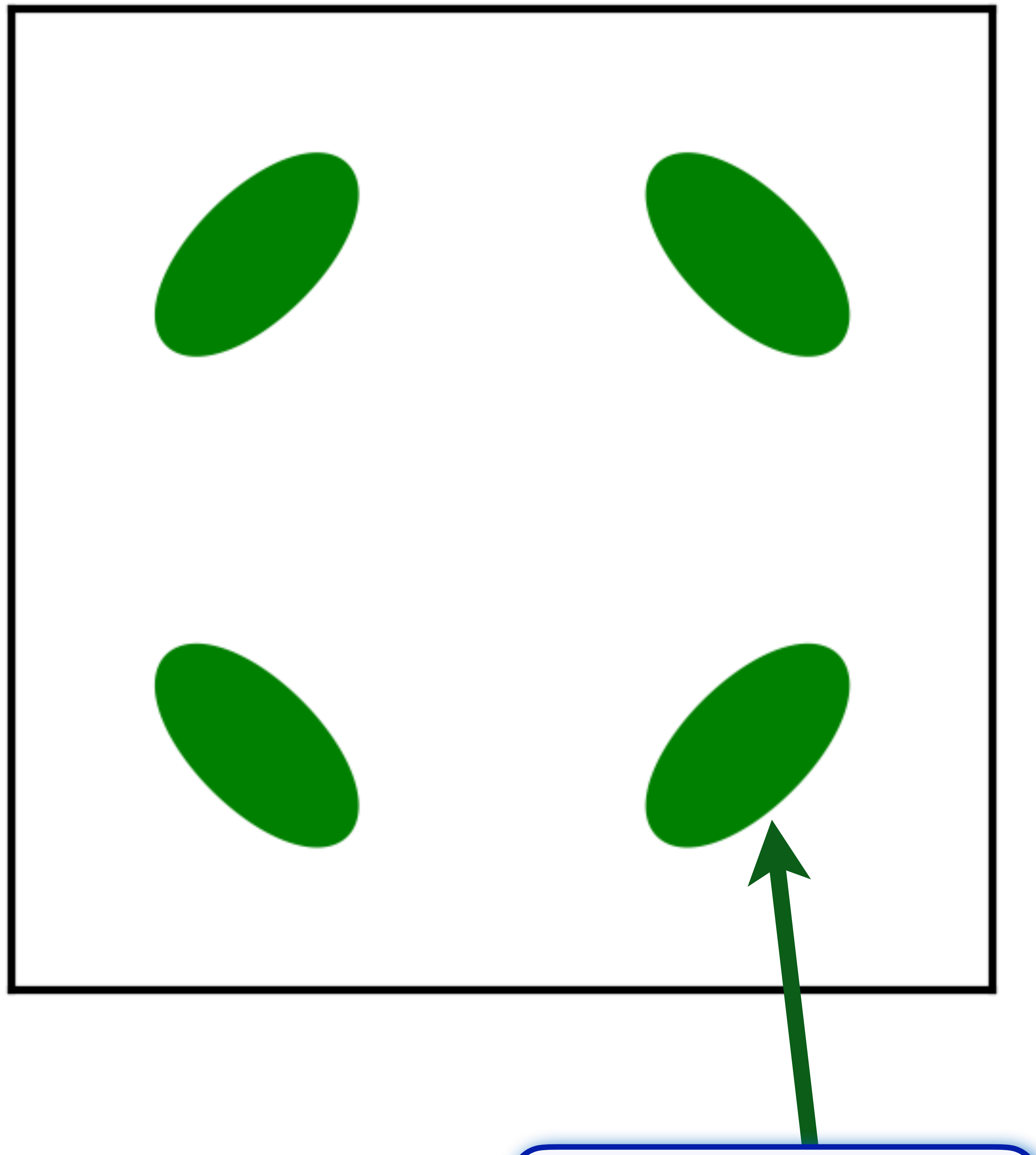


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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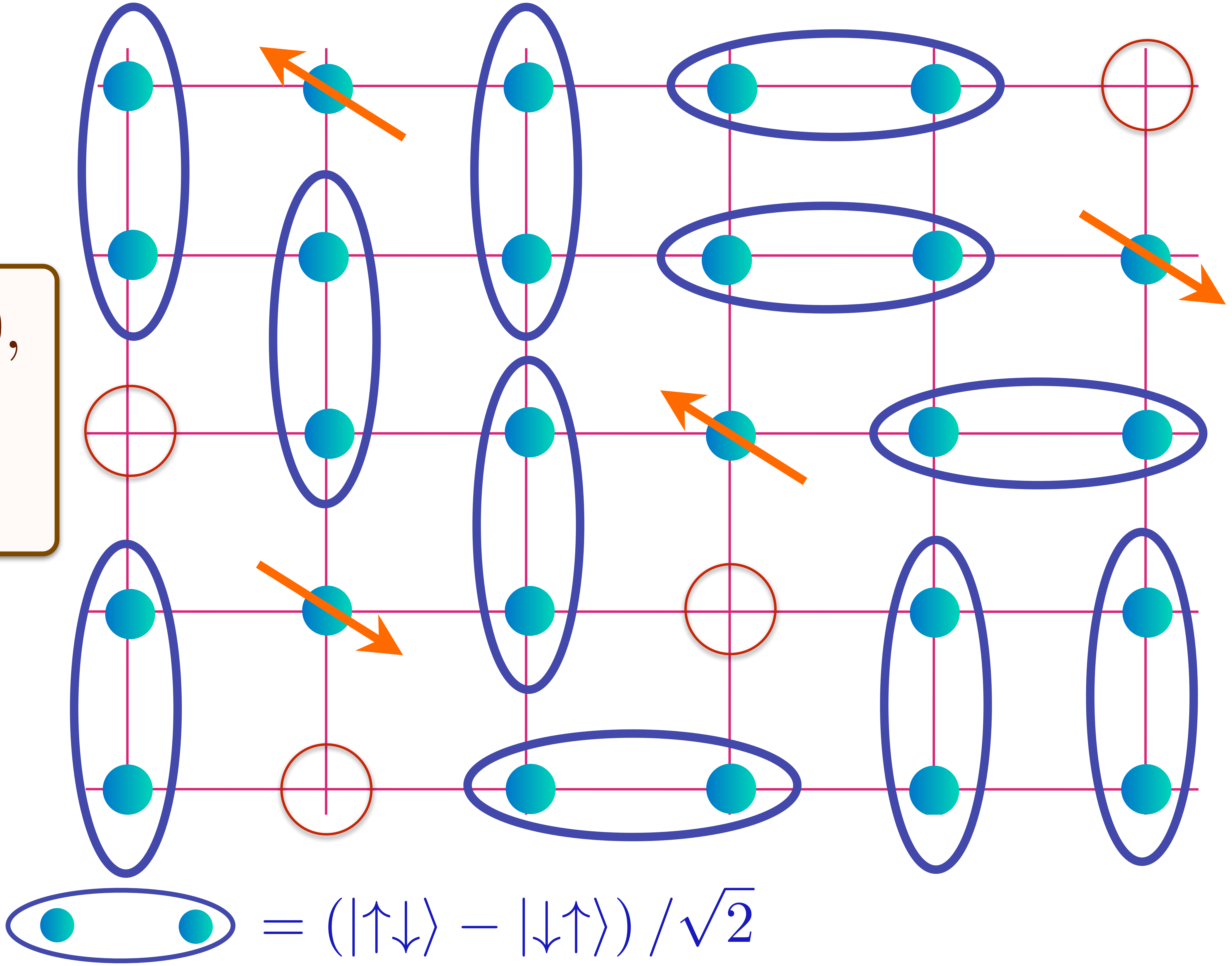

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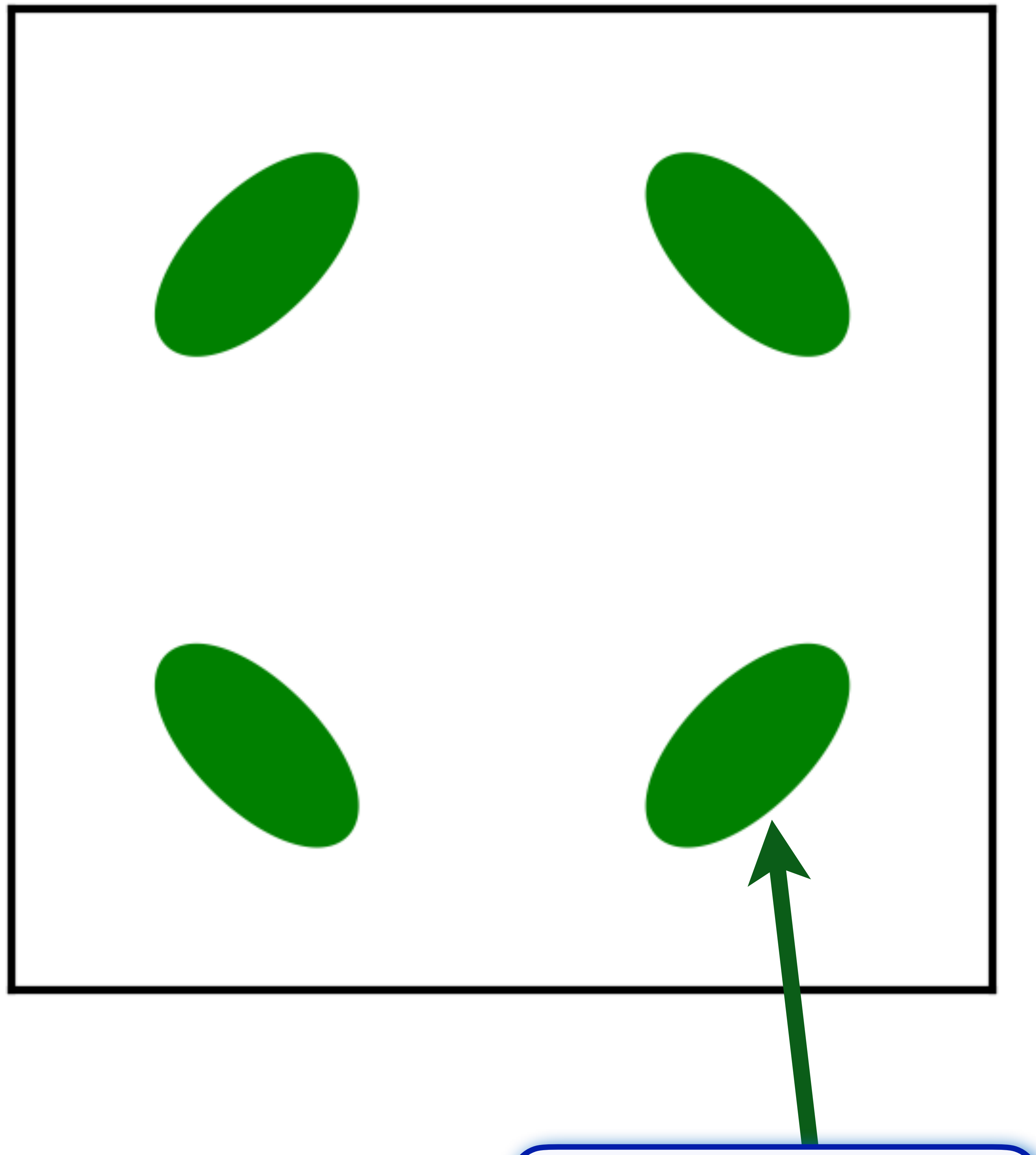

Area $p/4$

Doping an insulating antiferromagnet with holes of density p

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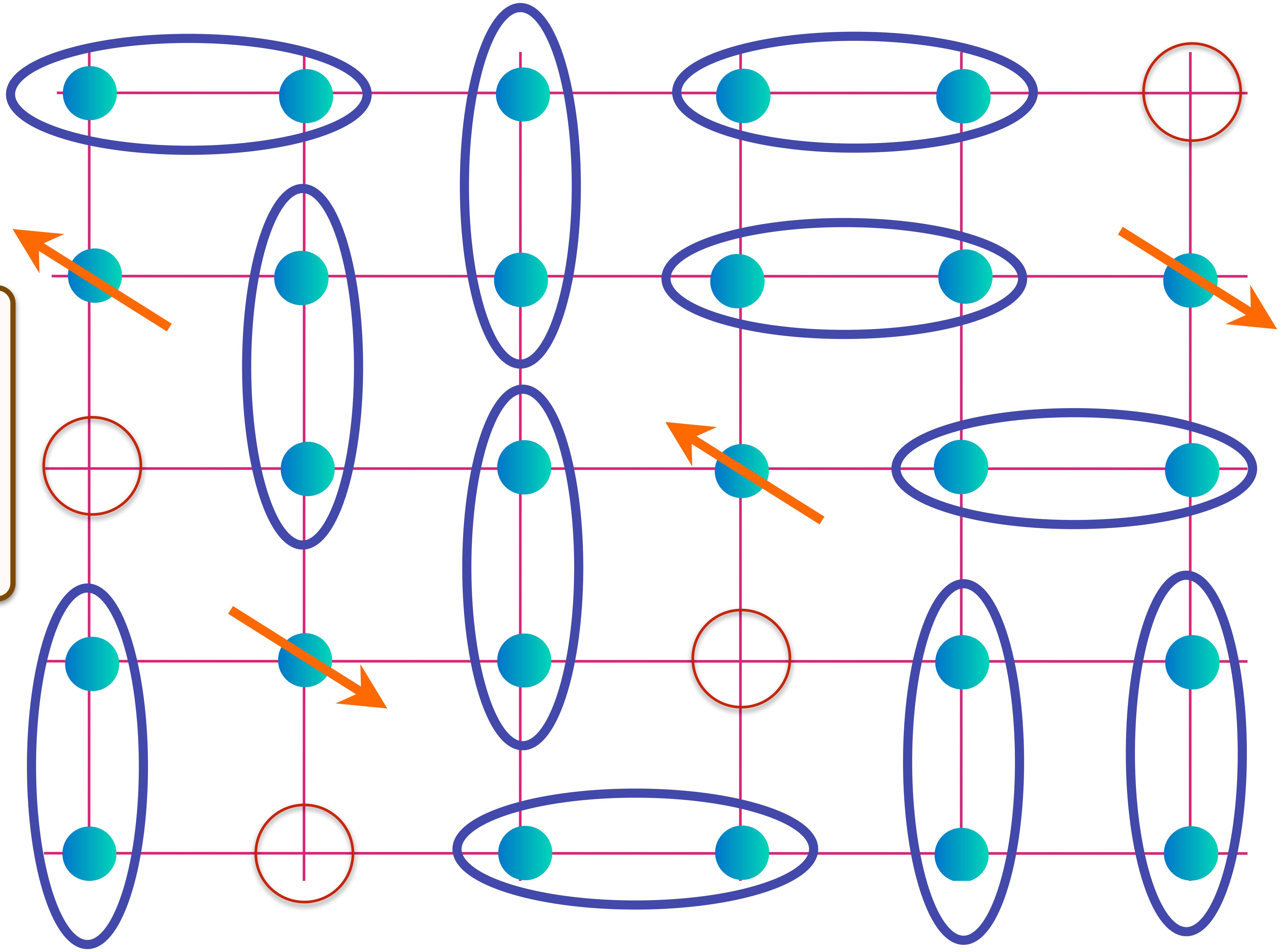
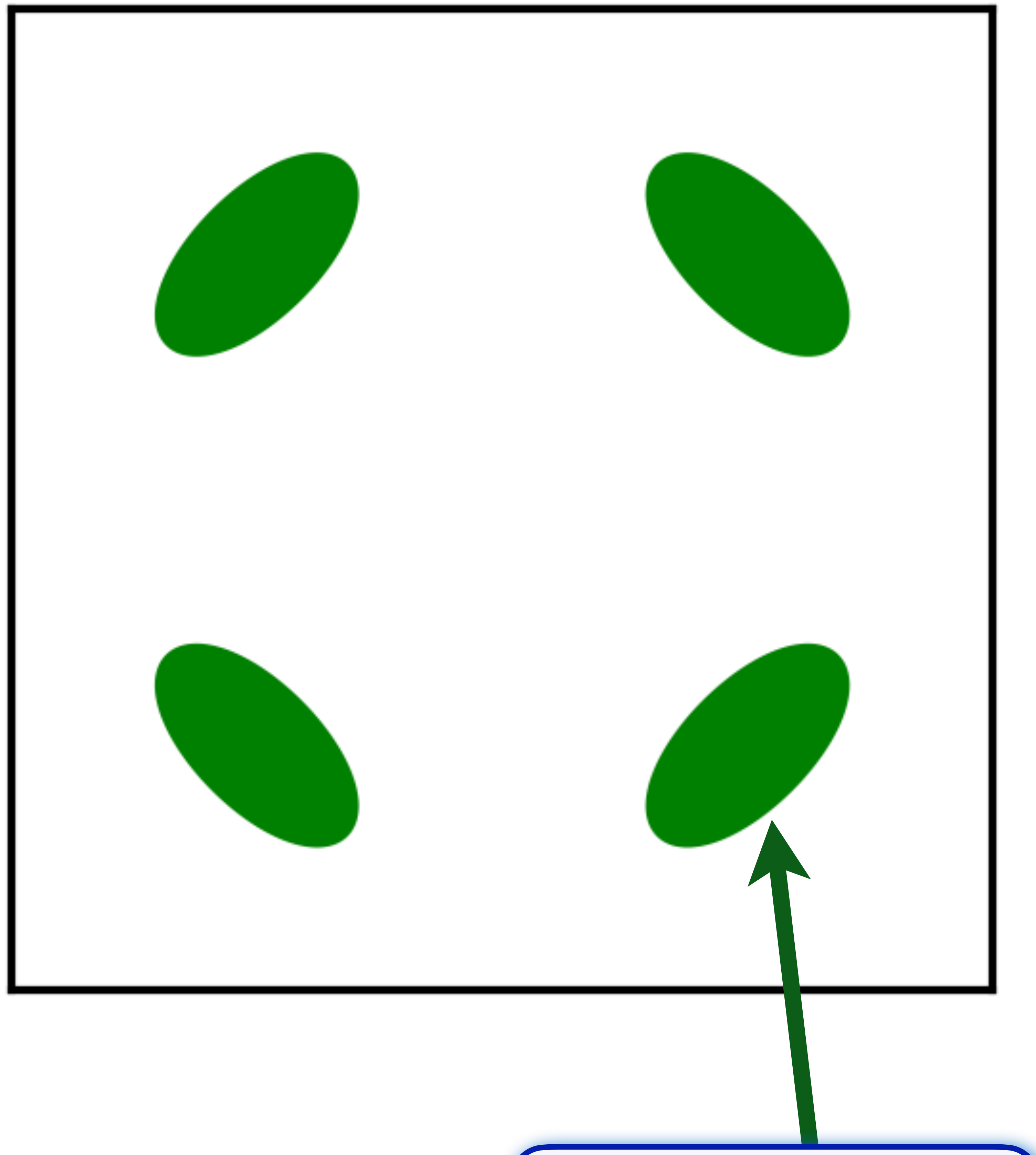


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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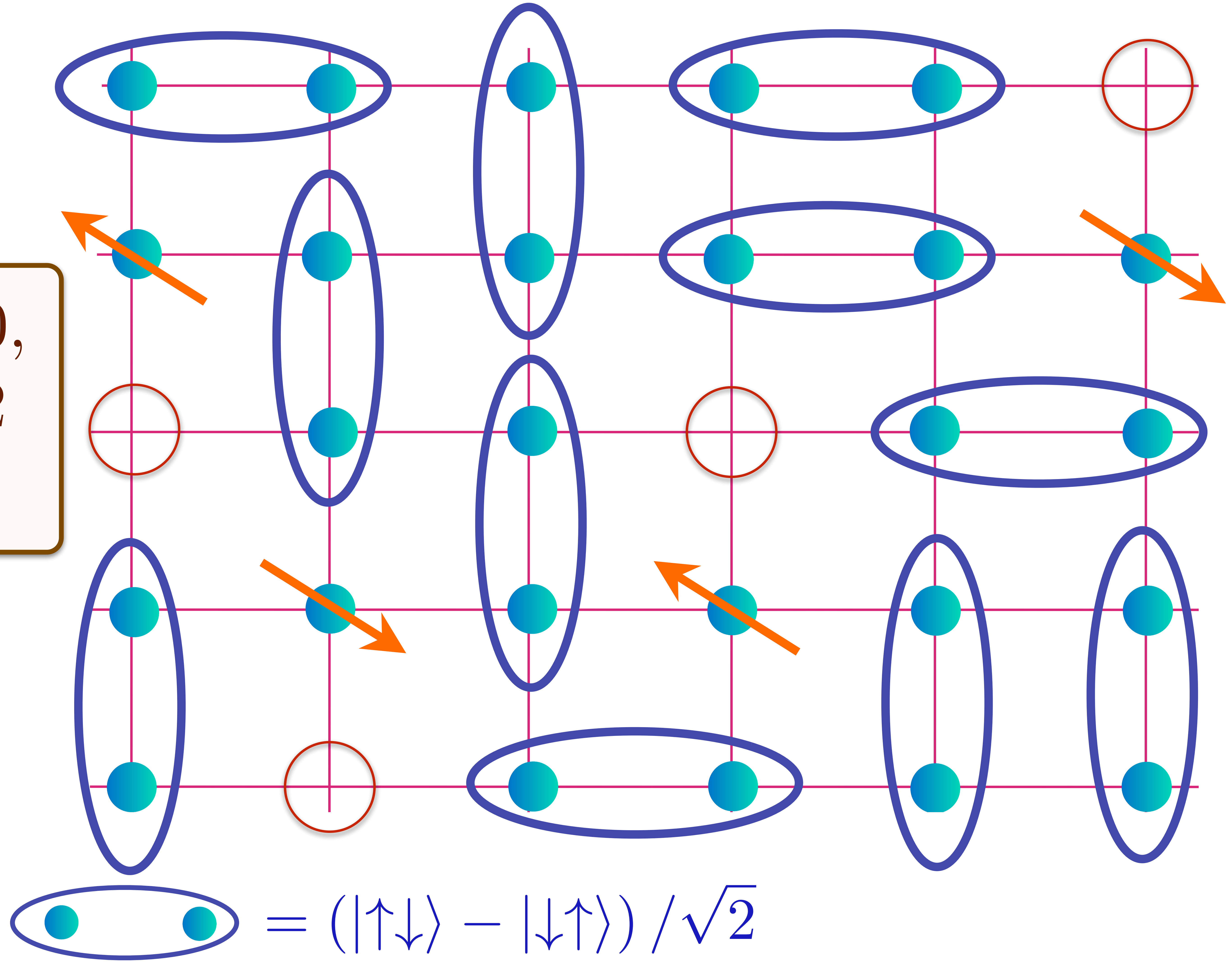
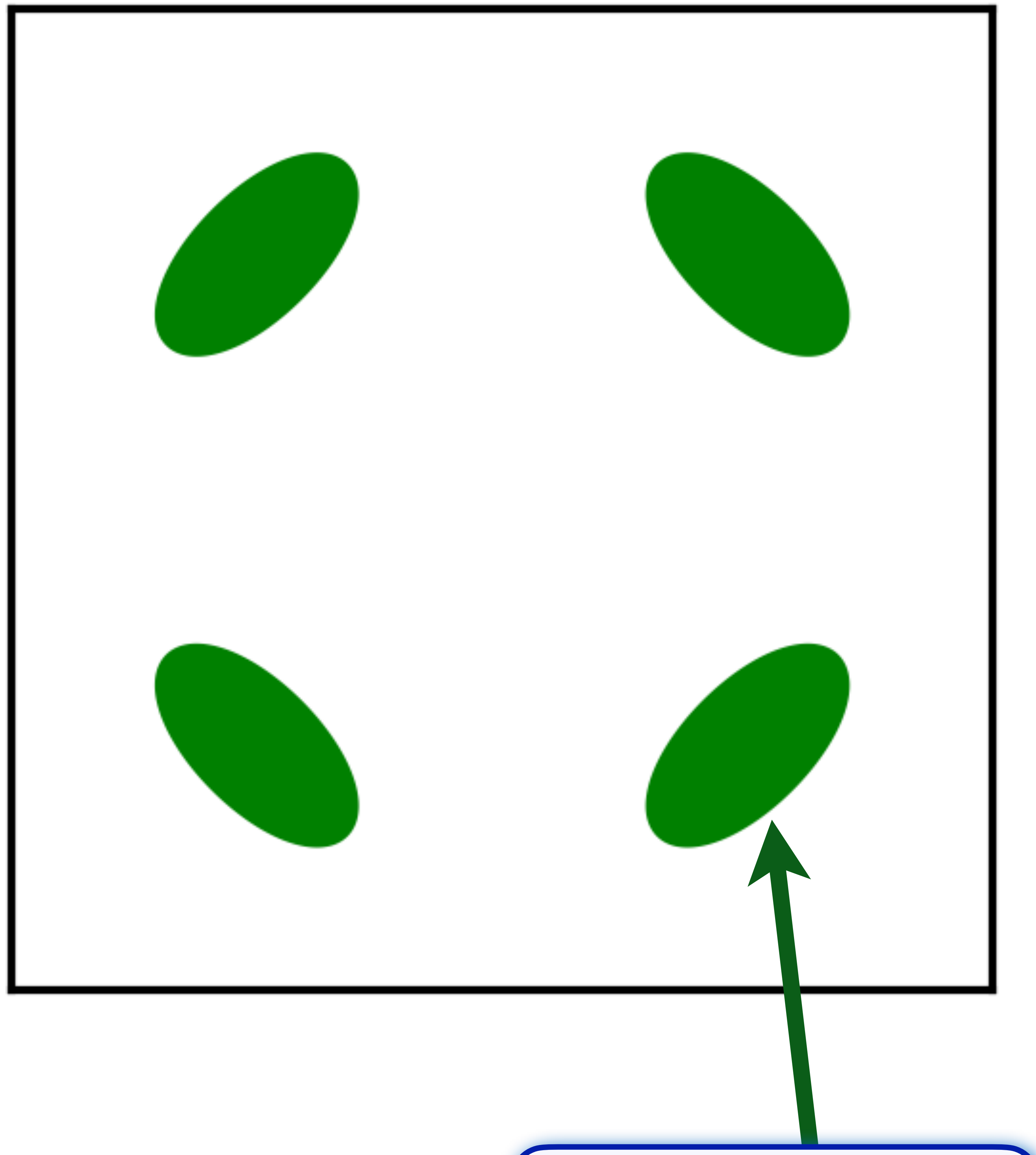
Charge 0, spin-1/2 spinons





Area $p/4$

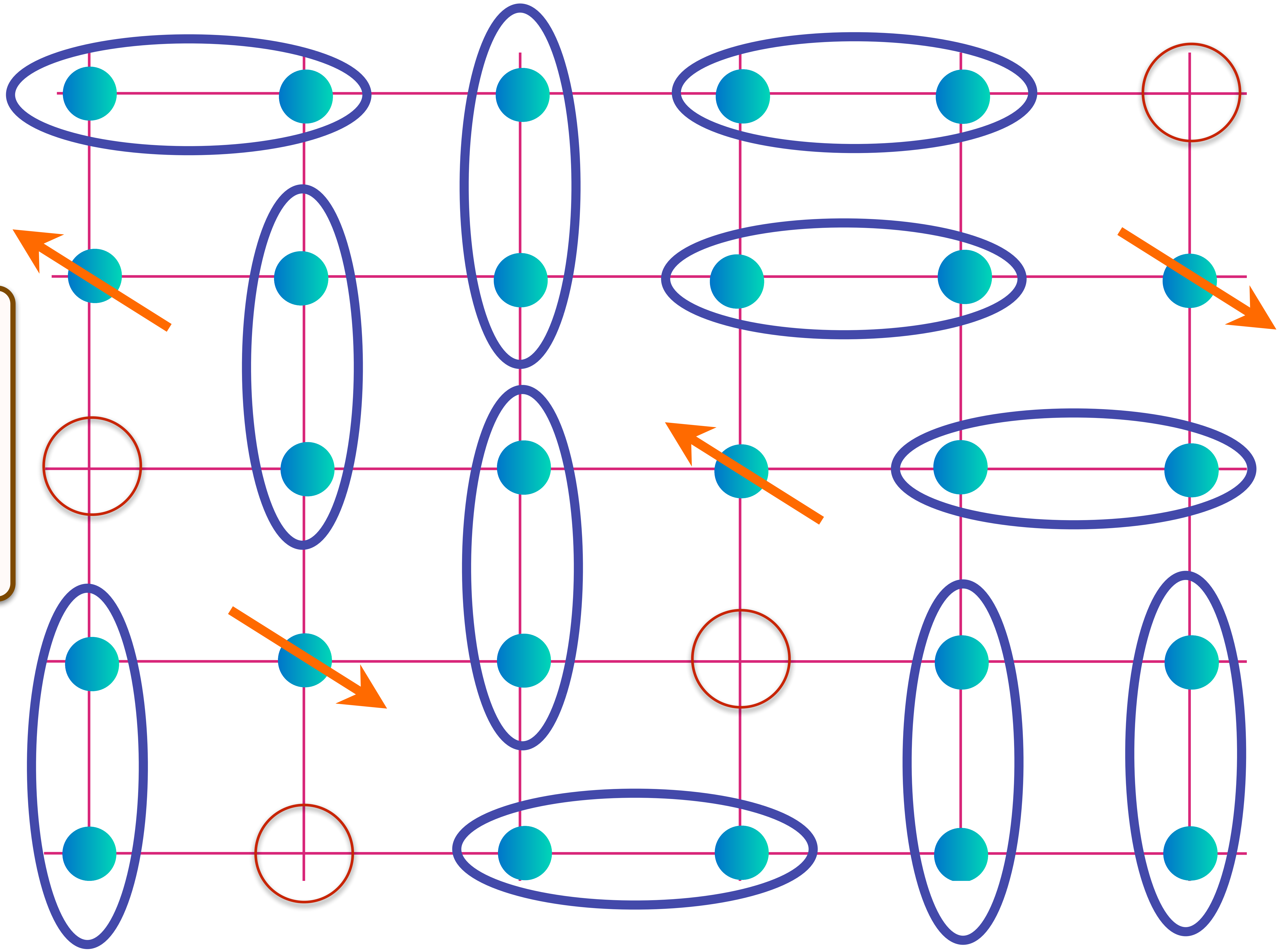
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Doping an insulating antiferromagnet with holes of density p

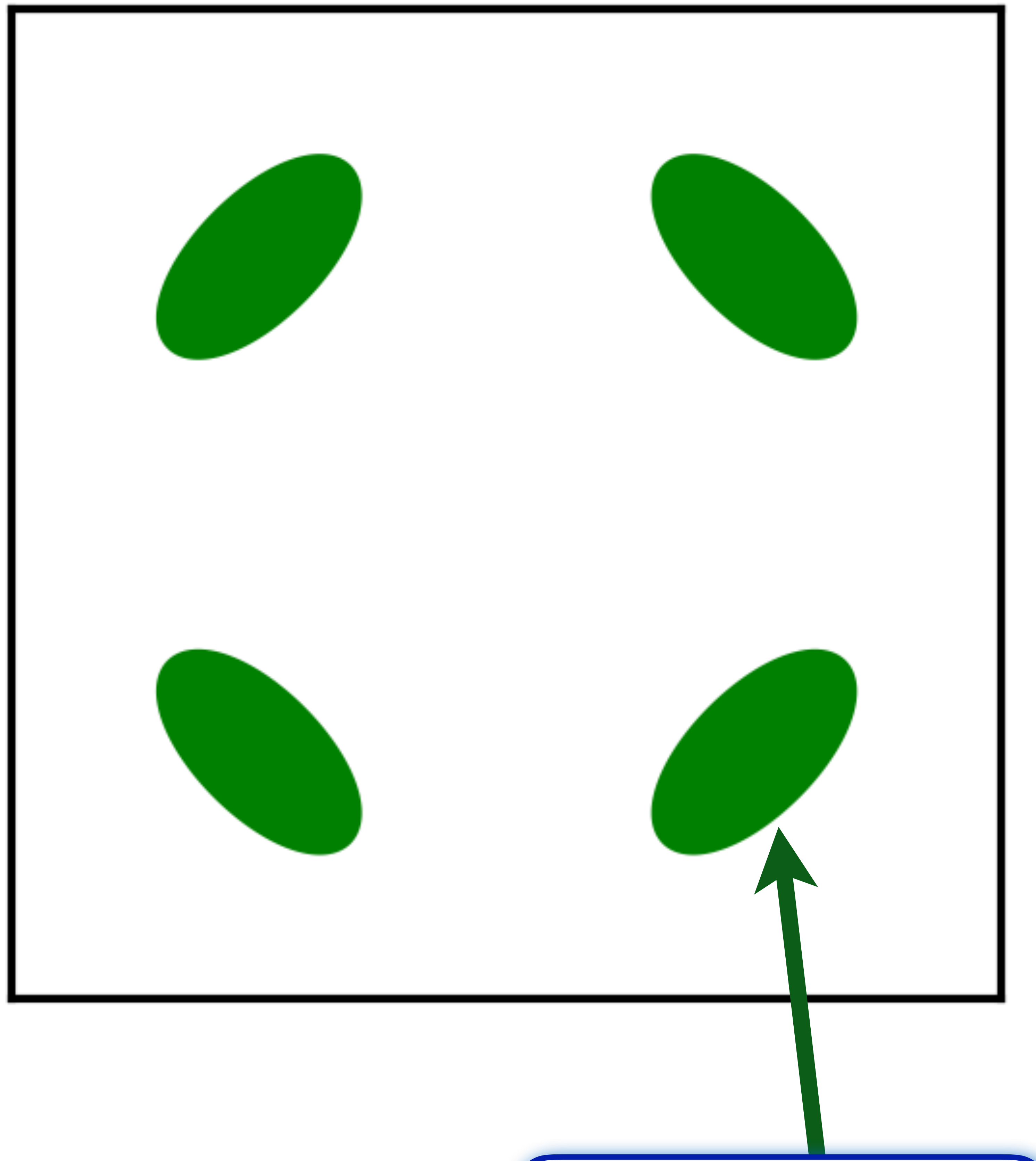
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

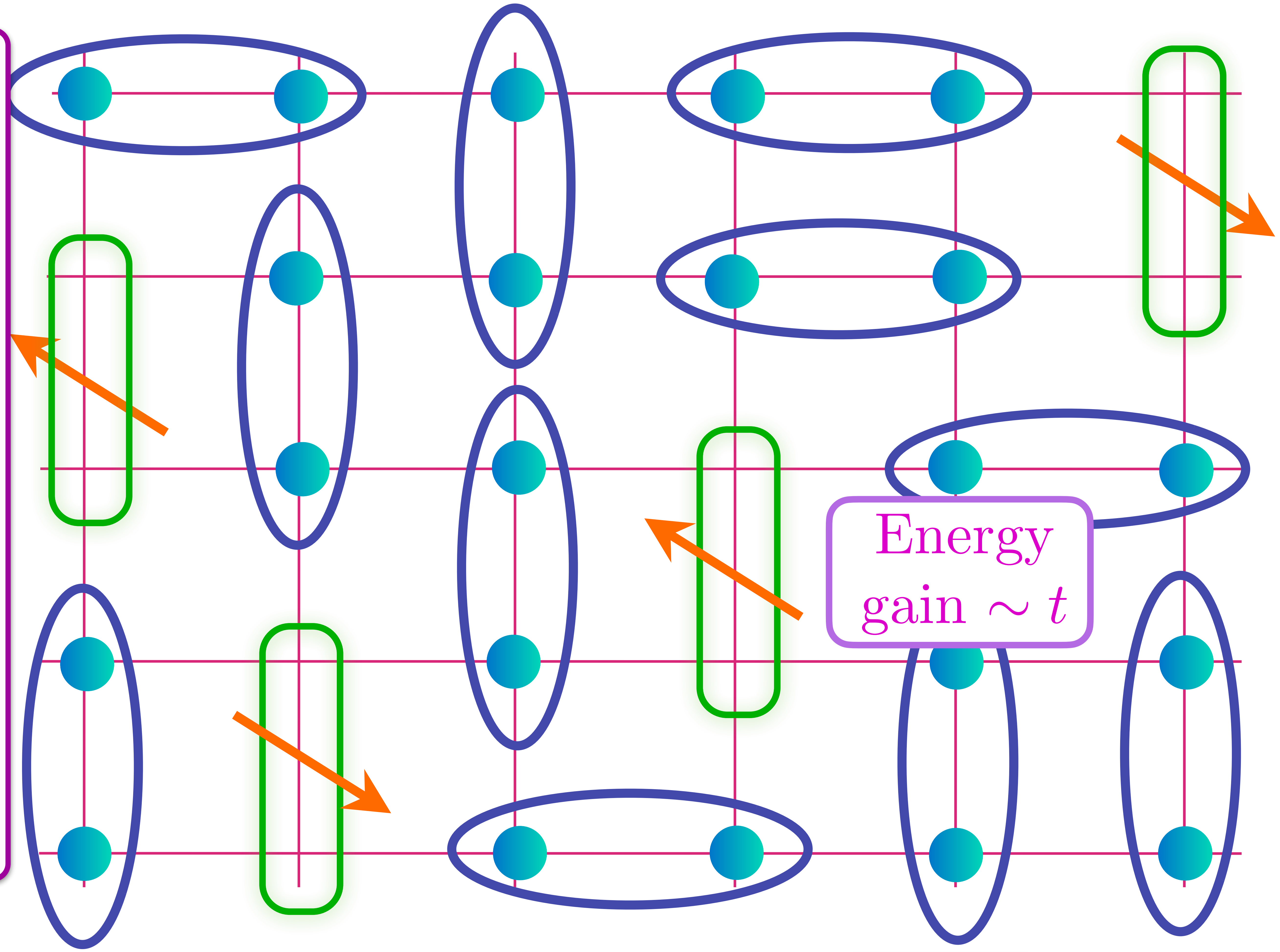


Area $p/4$

Doping an insulating antiferromagnet with holes of density p

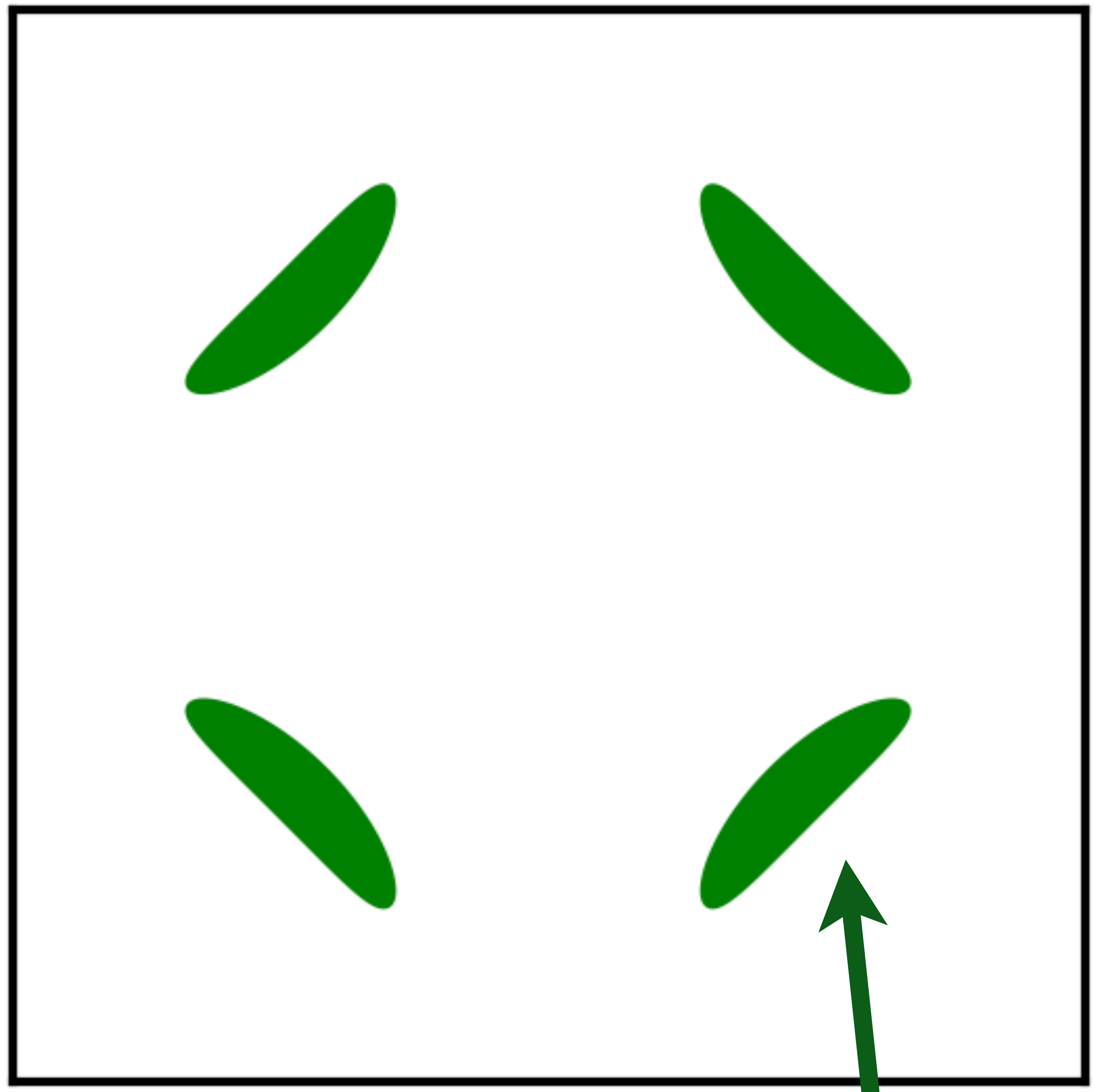
FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.

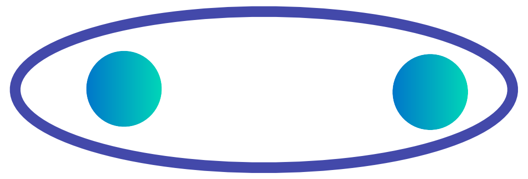
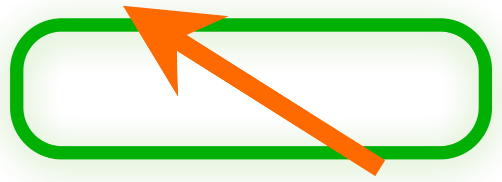


Energy gain $\sim t$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/8$

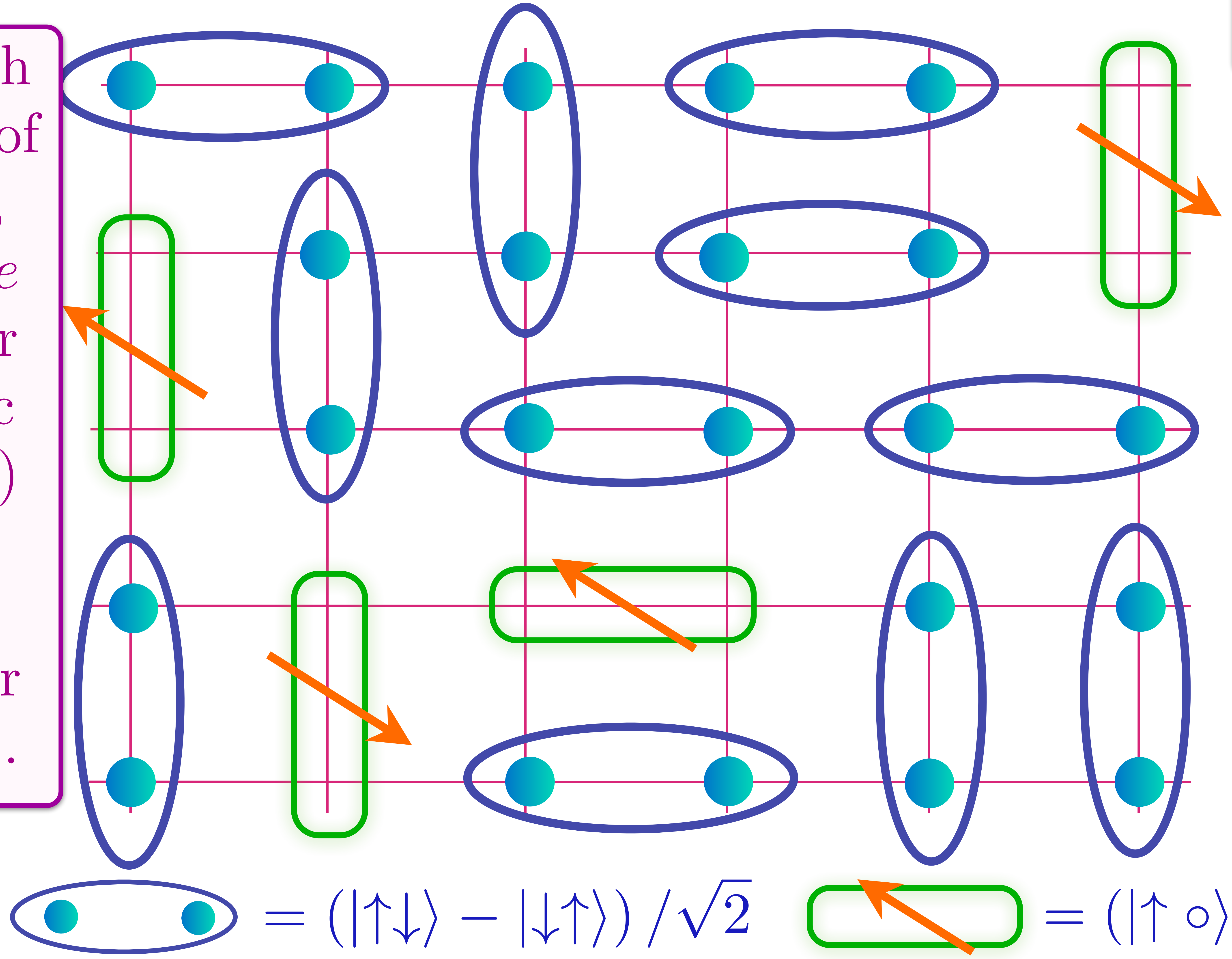
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

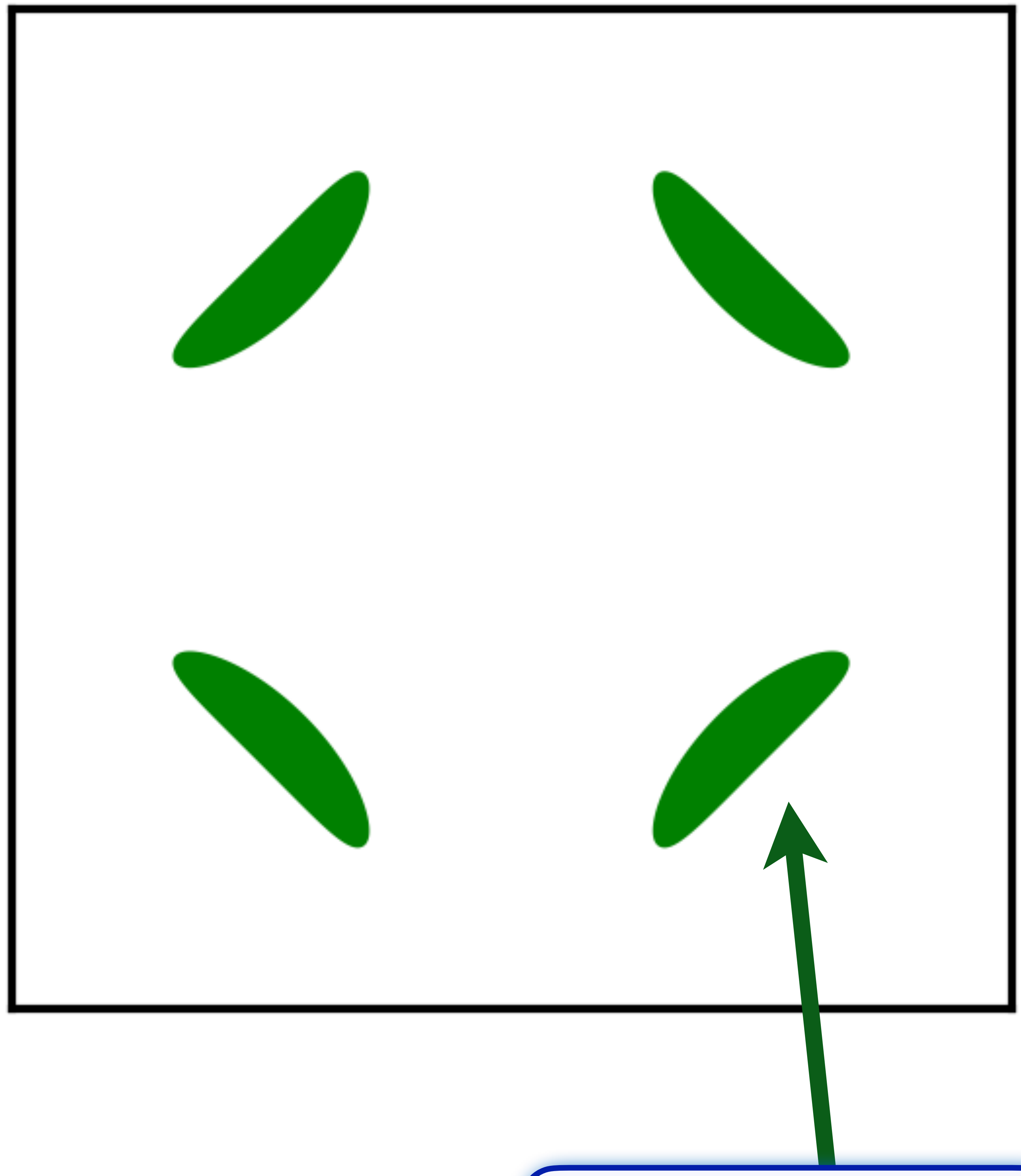
FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2} \qquad = \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/8$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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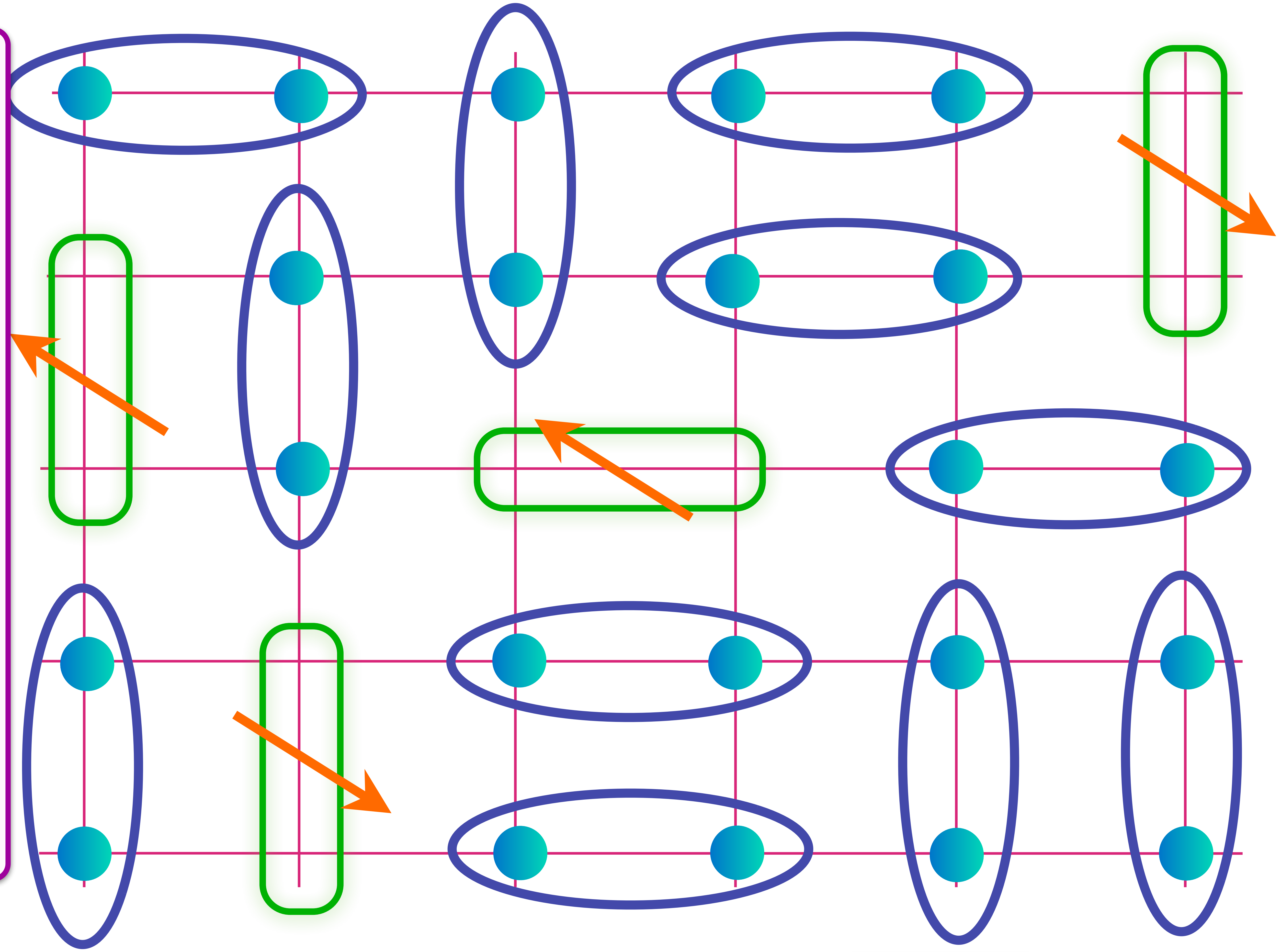
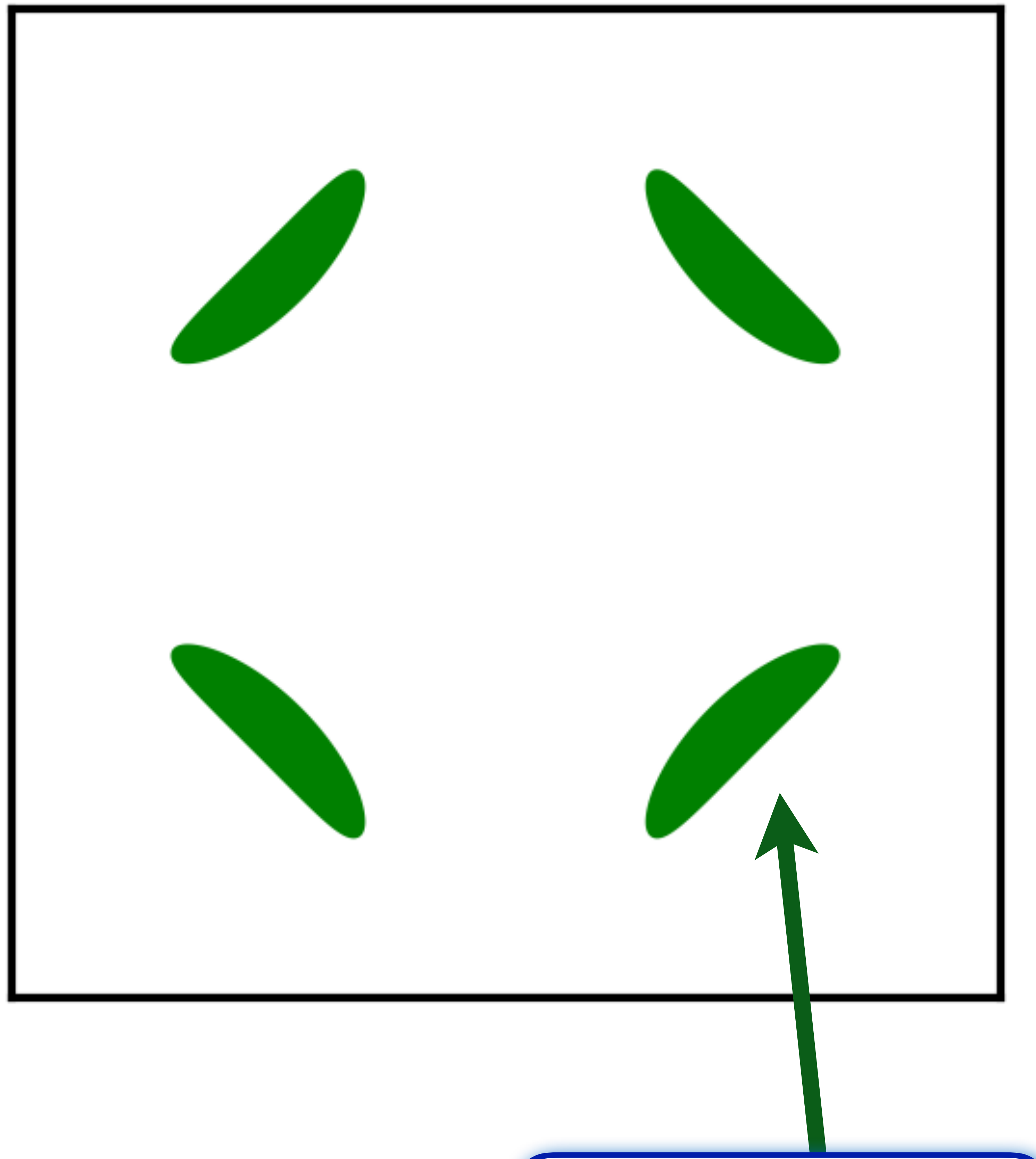
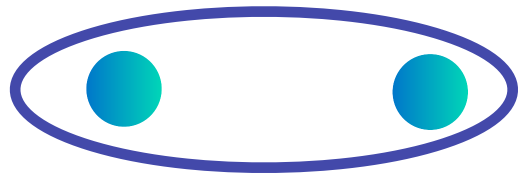
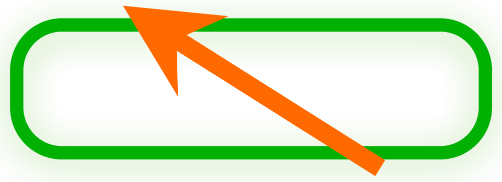




Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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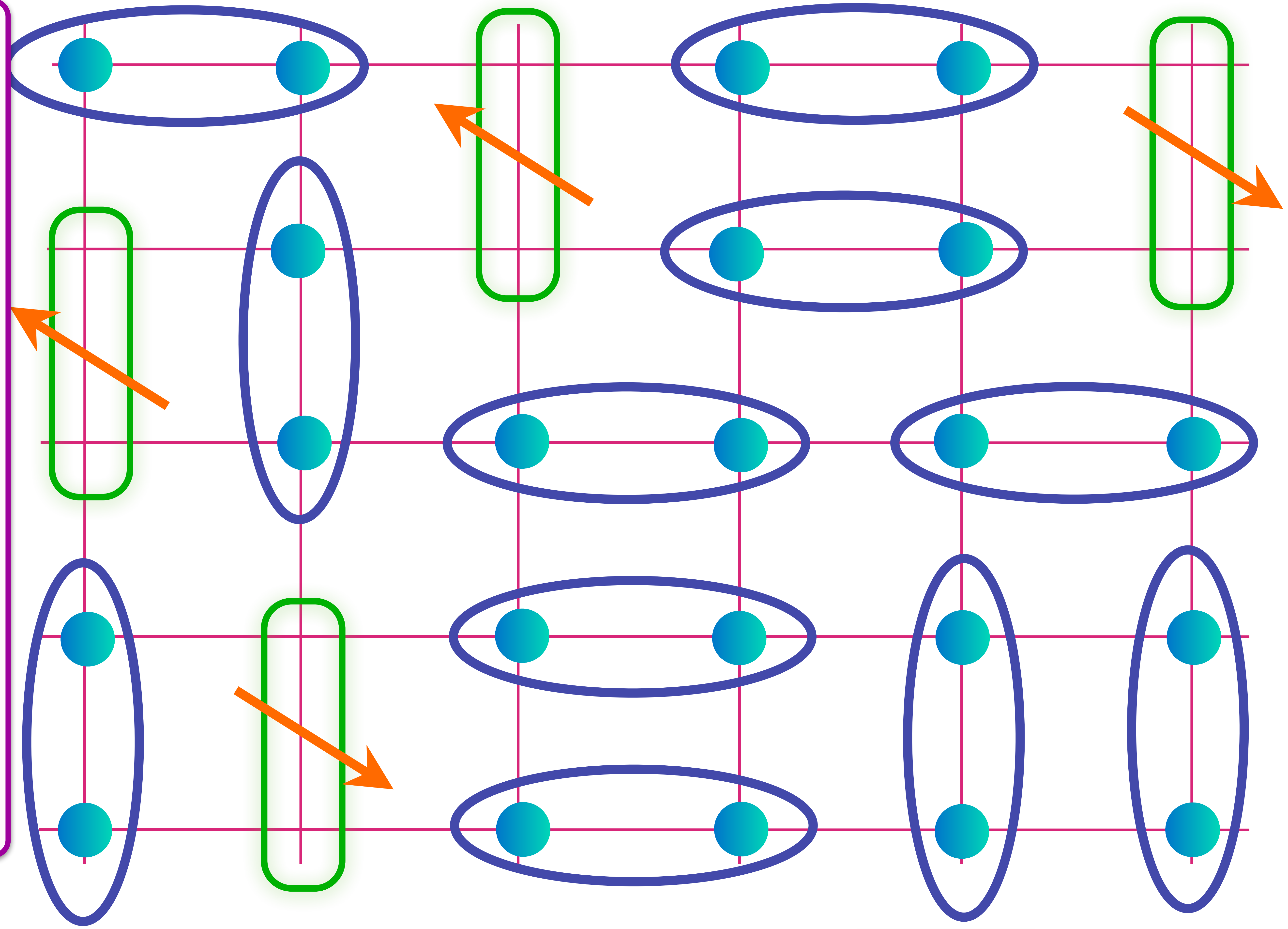
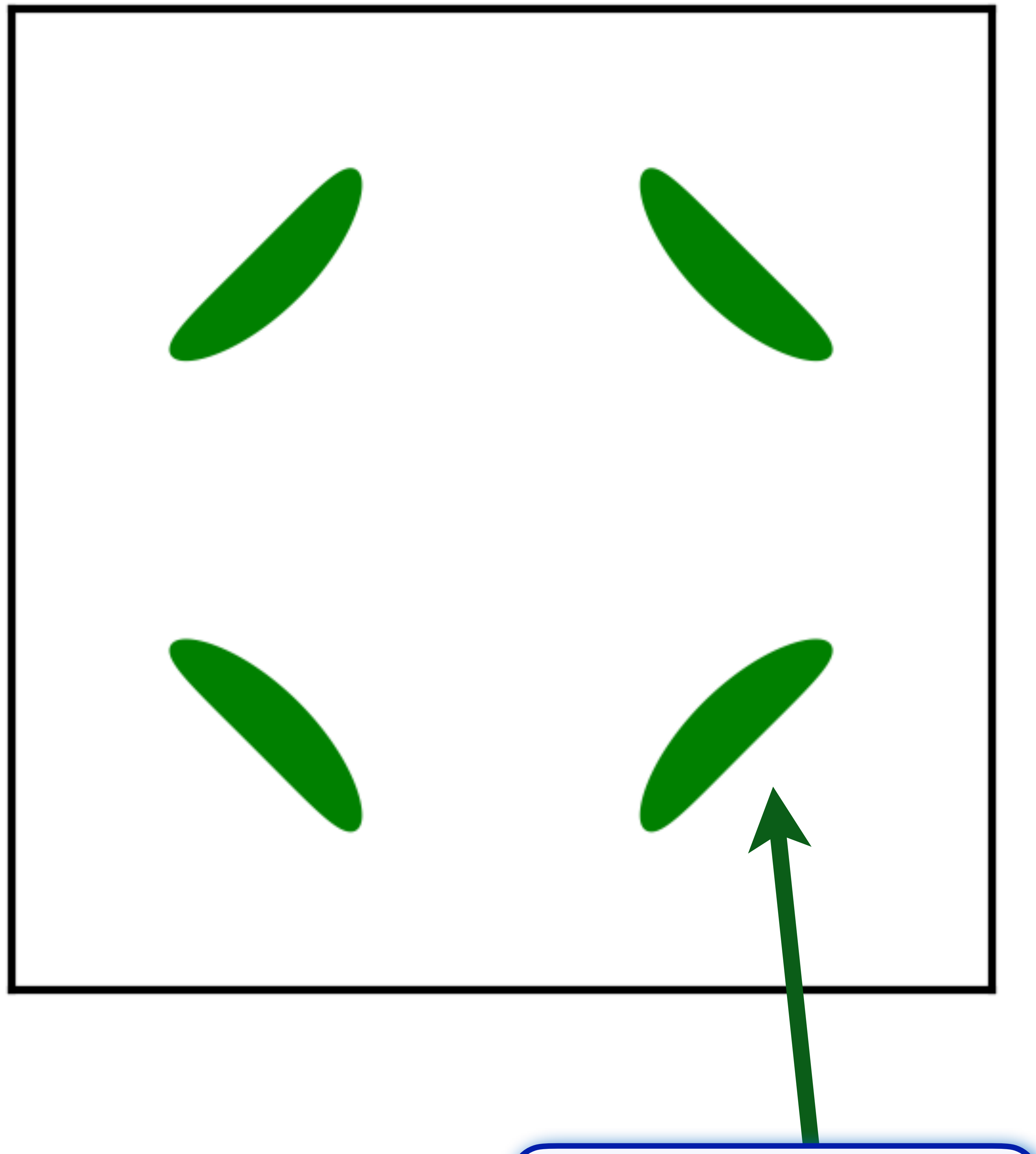
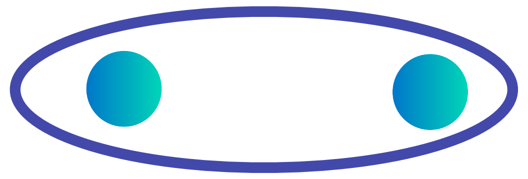
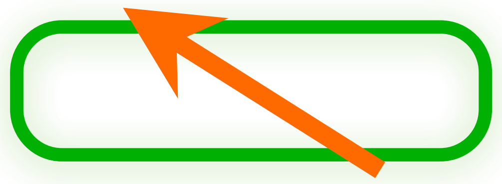


Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

FL*

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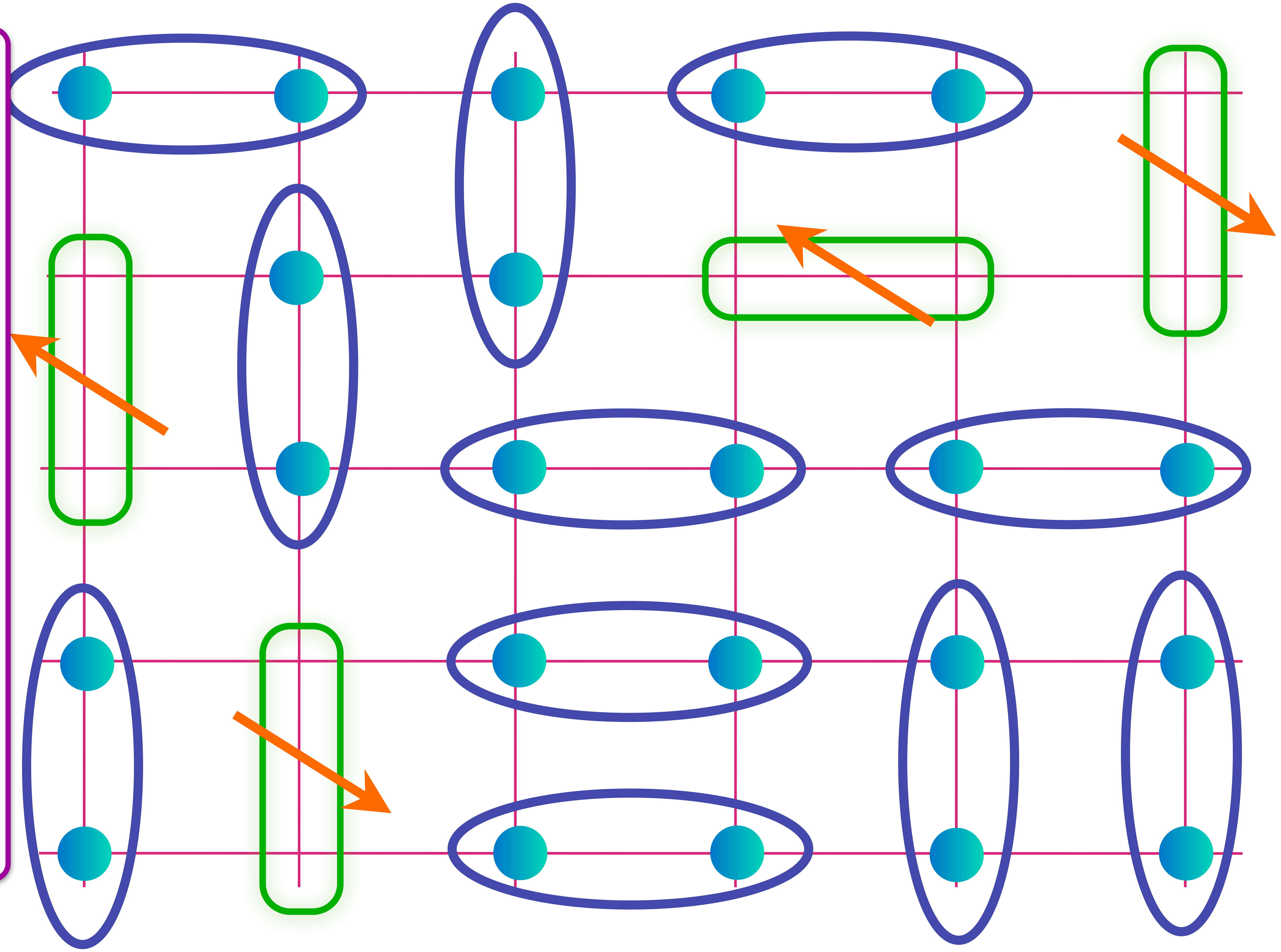
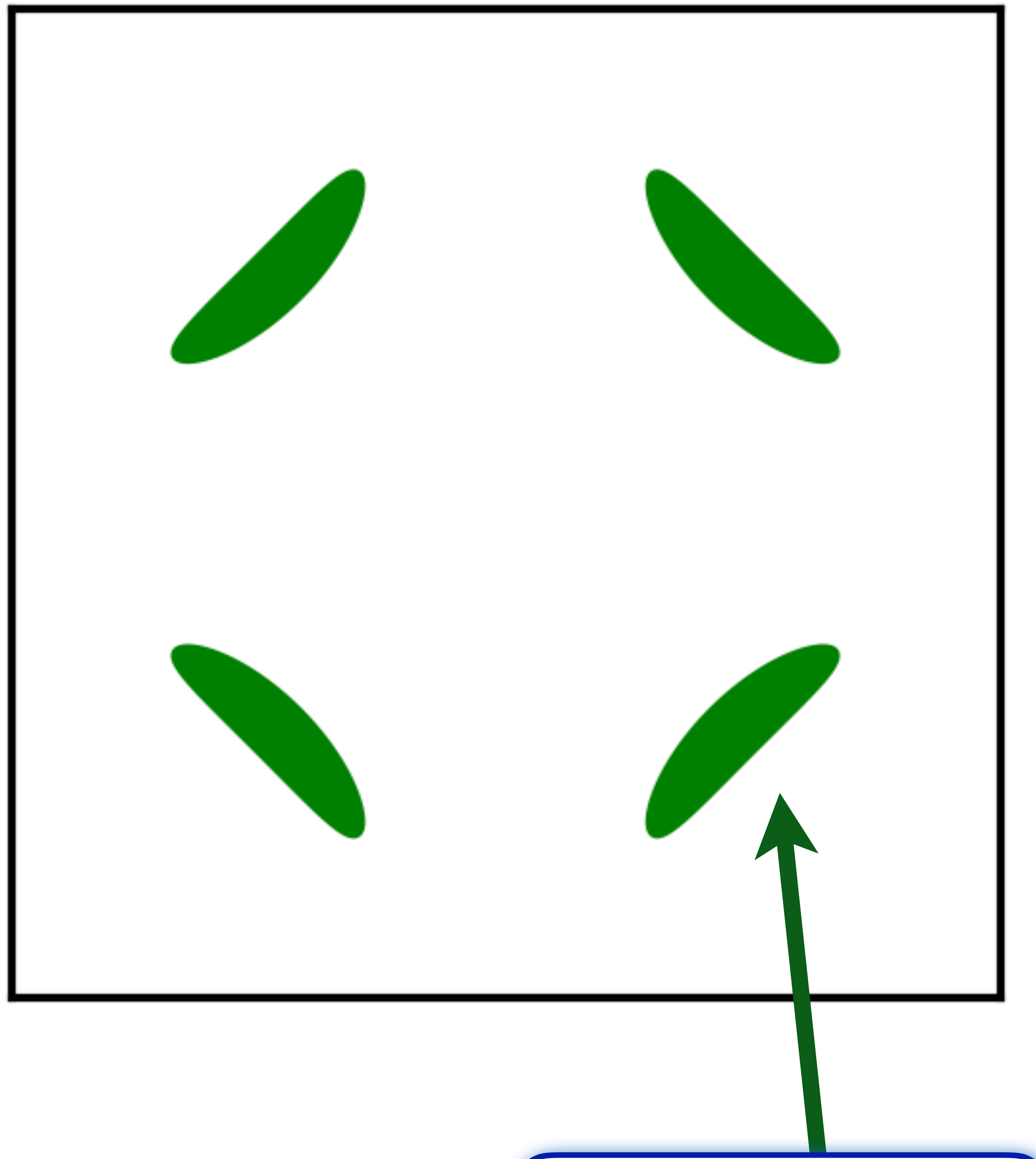
Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.





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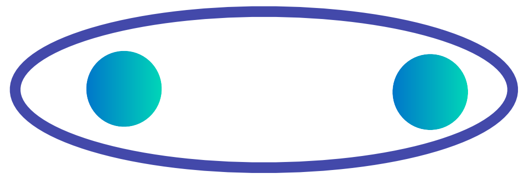
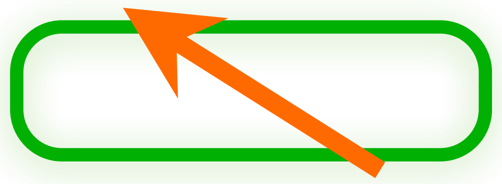
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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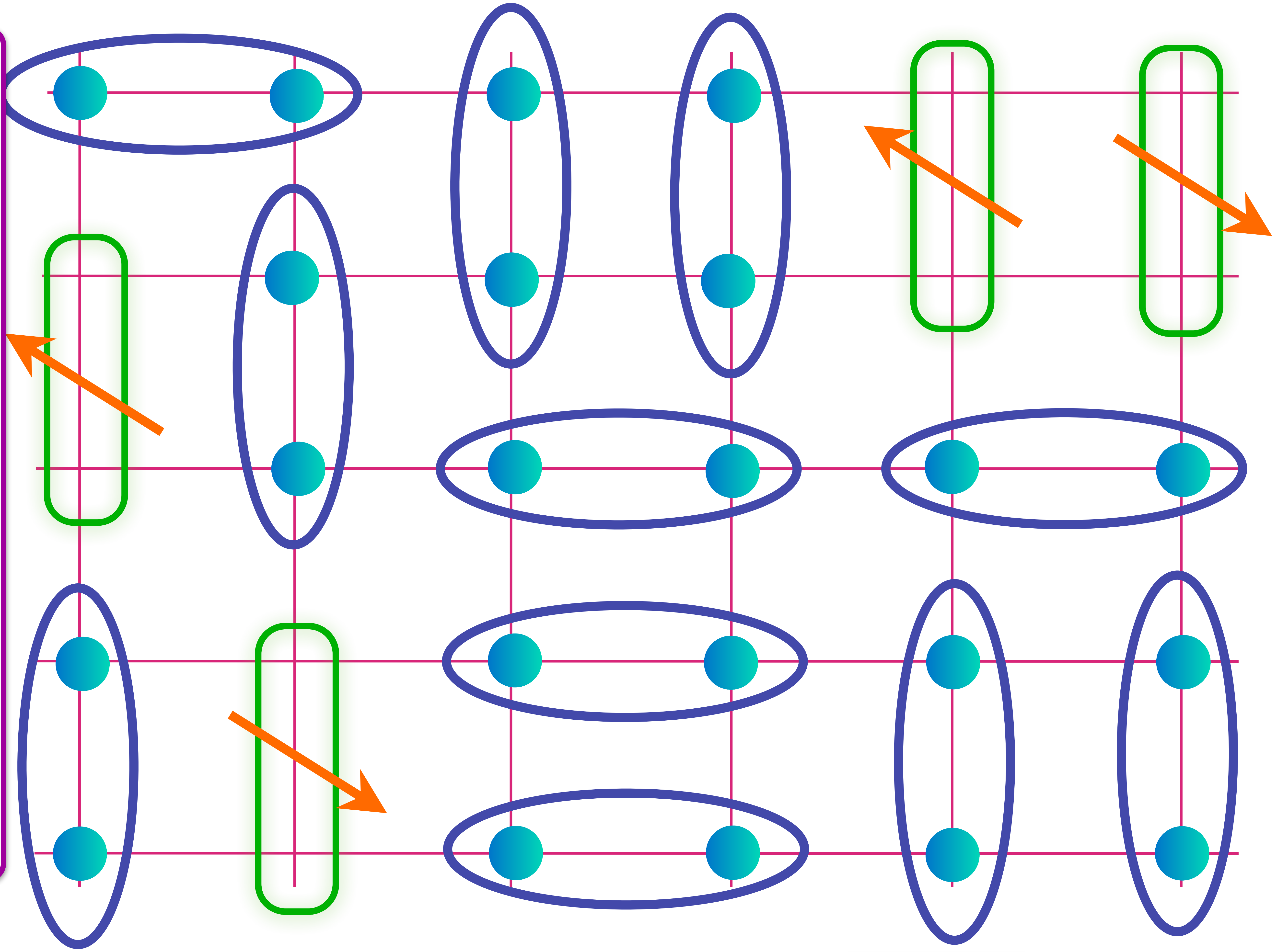

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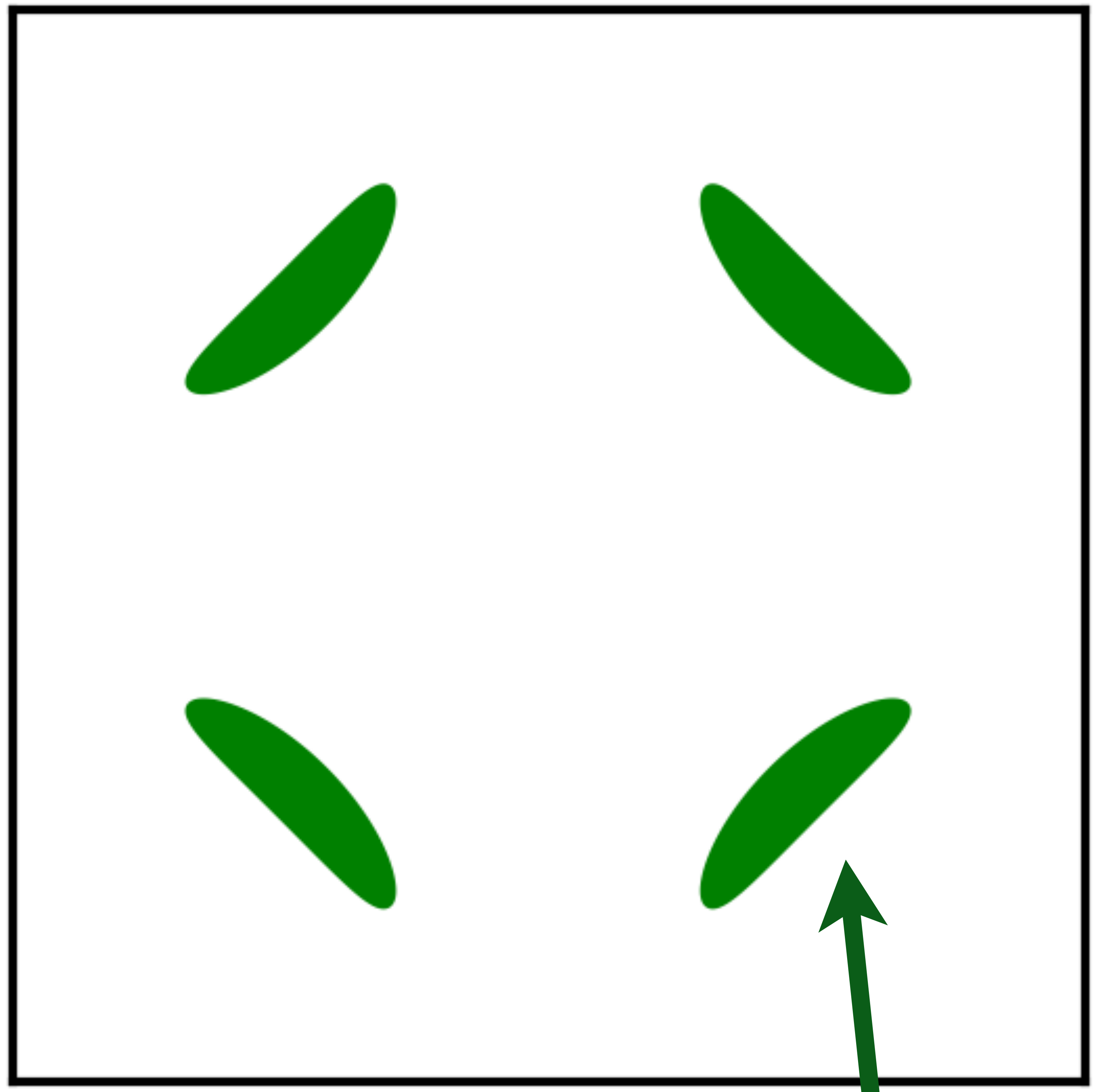

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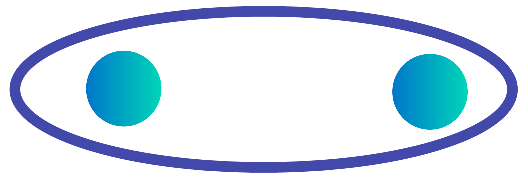
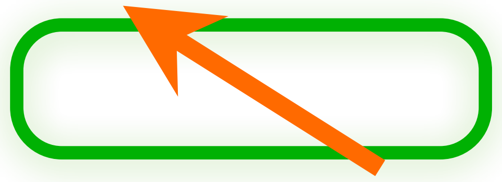
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right)/\sqrt{2}$

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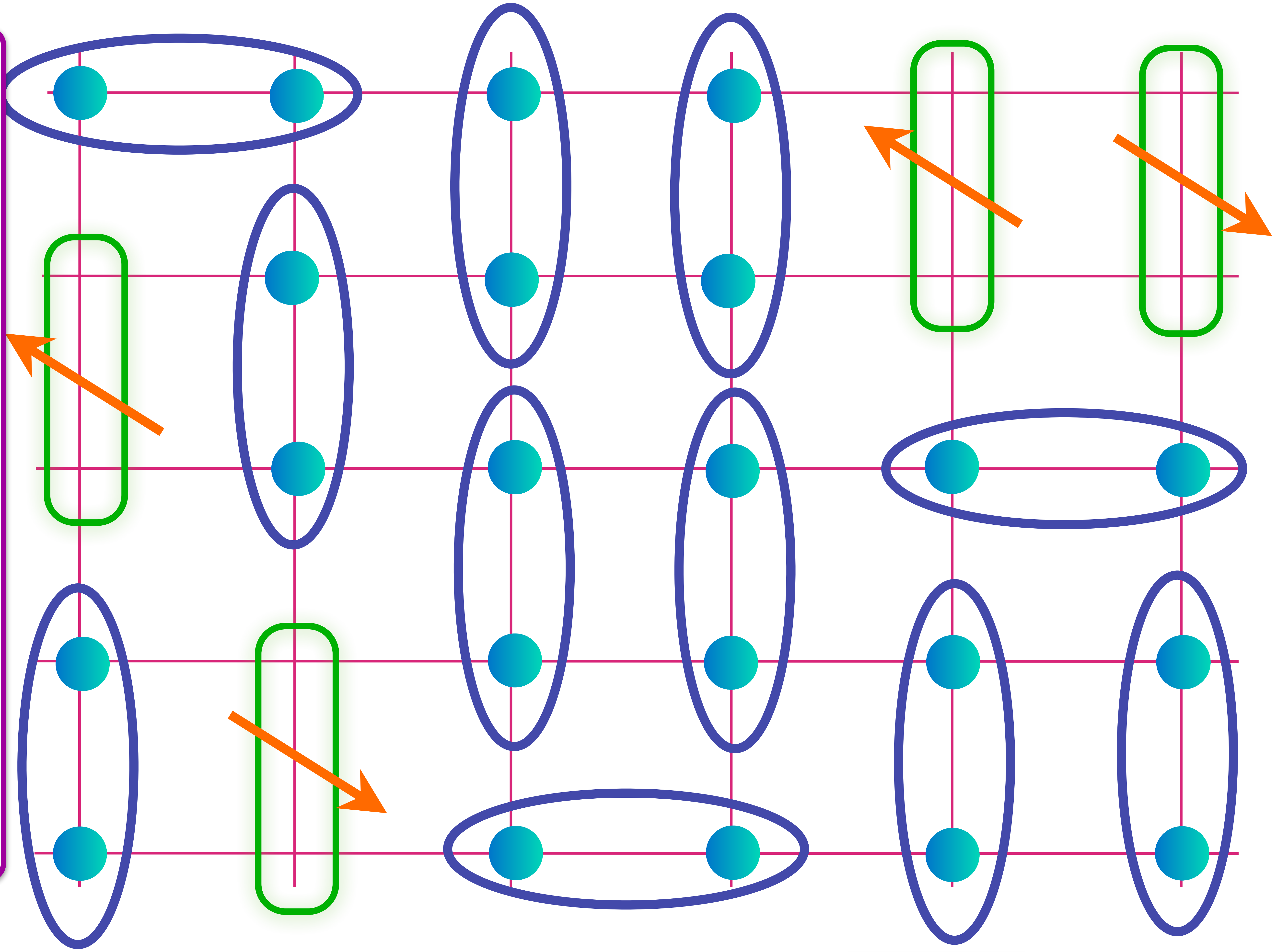

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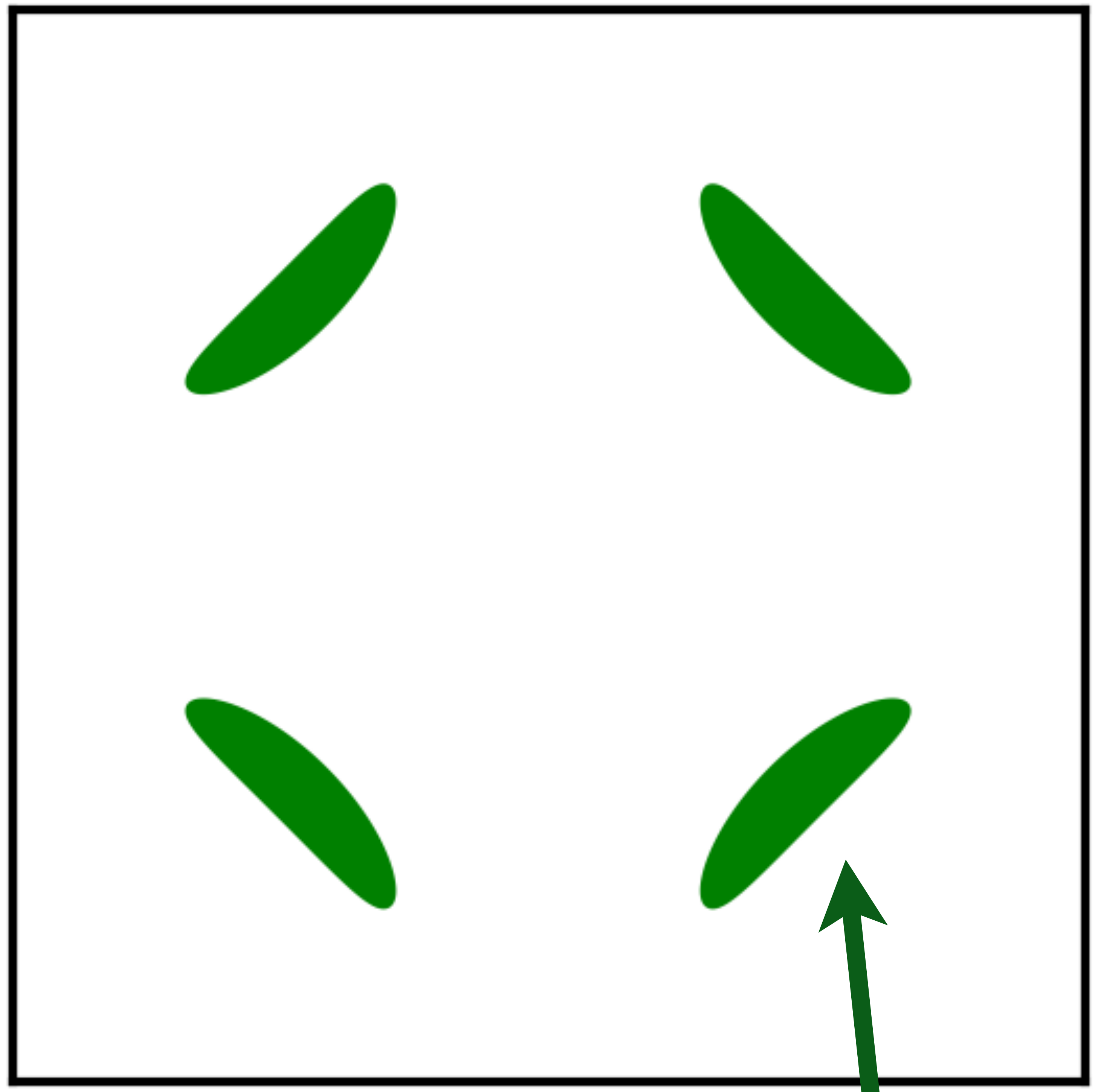

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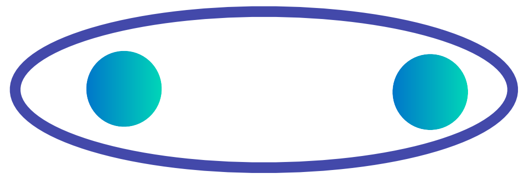
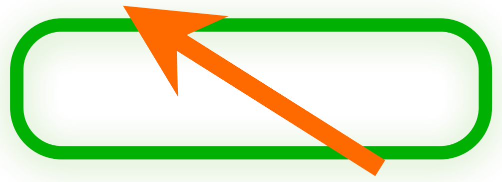
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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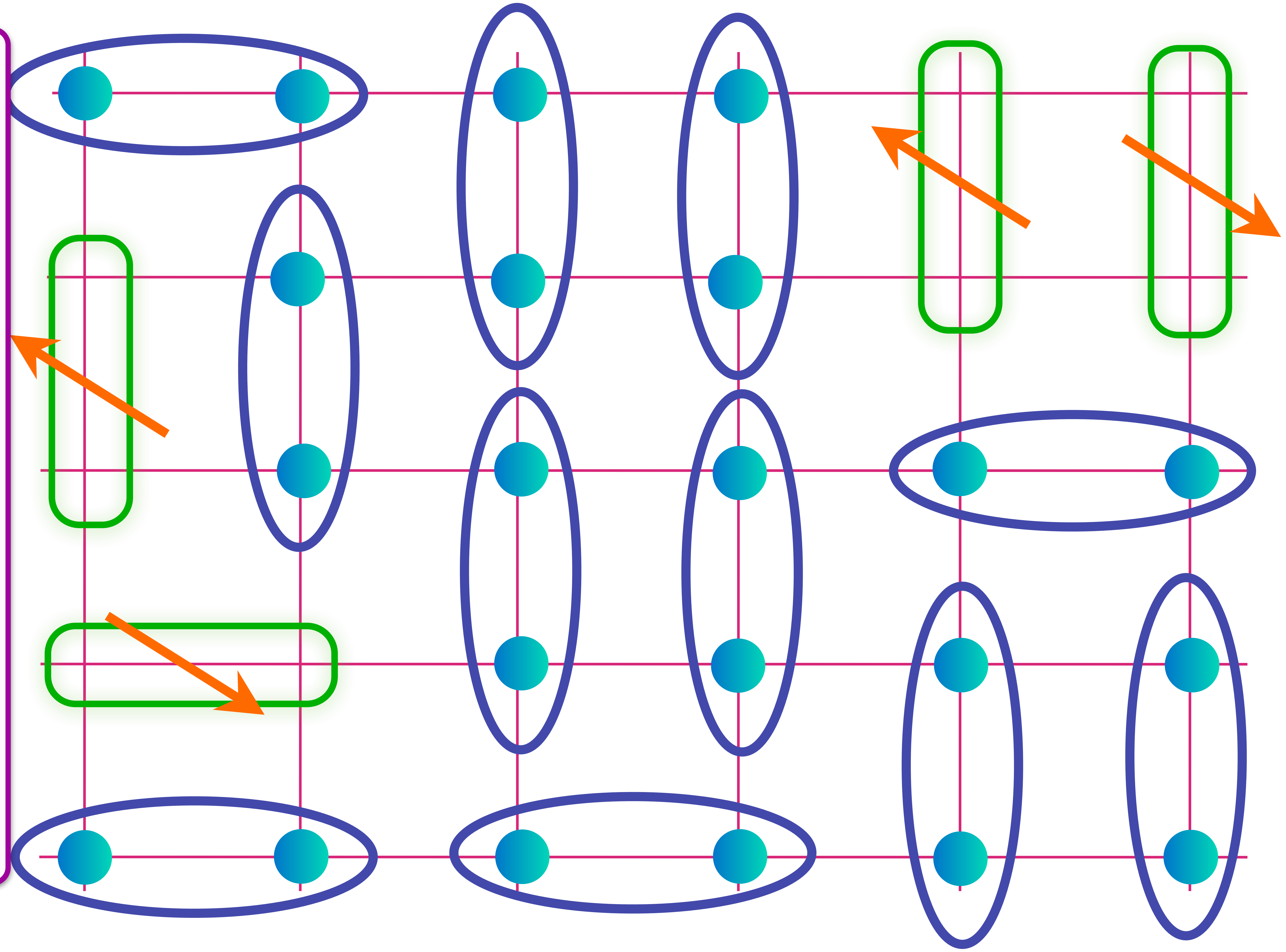

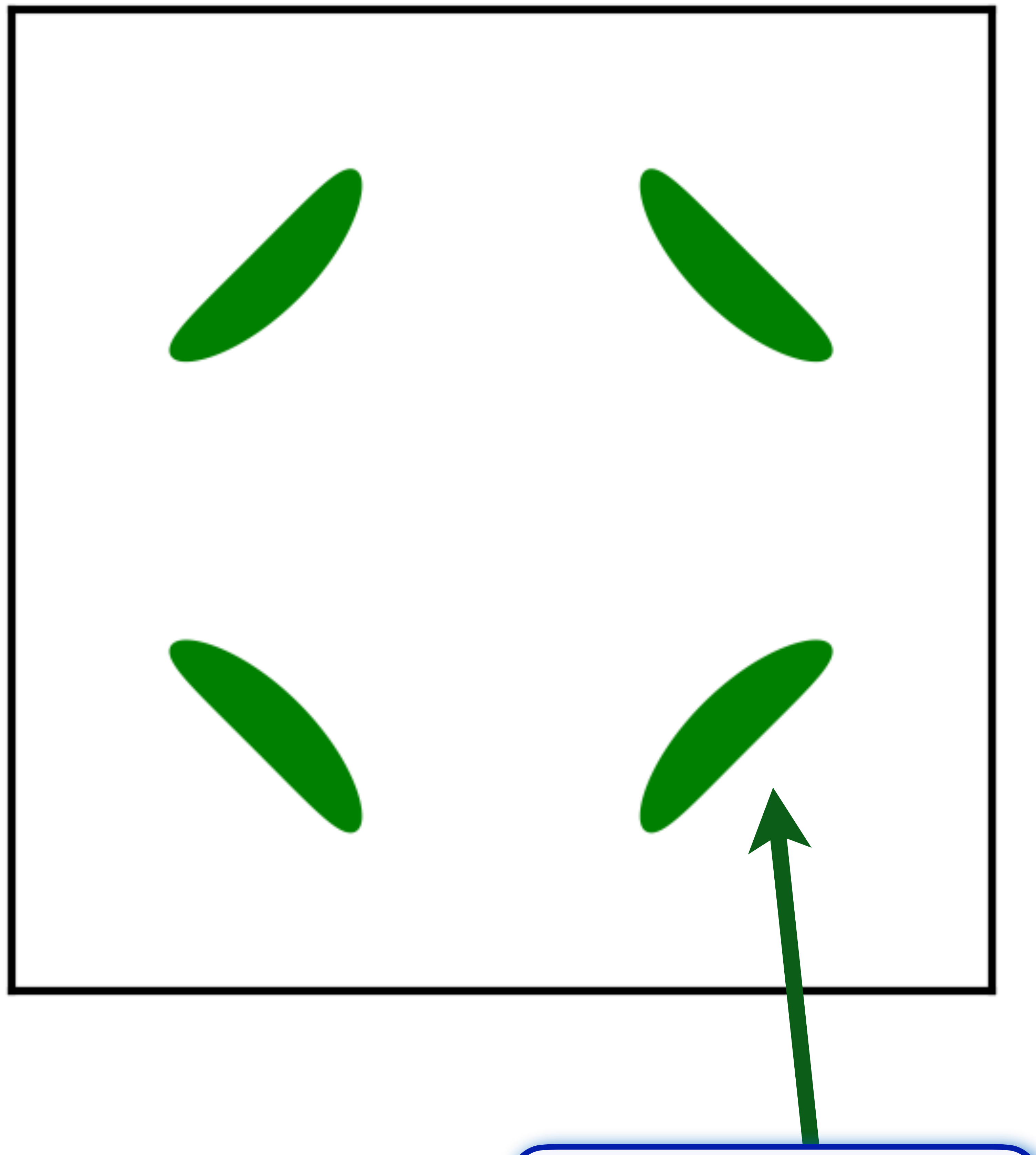

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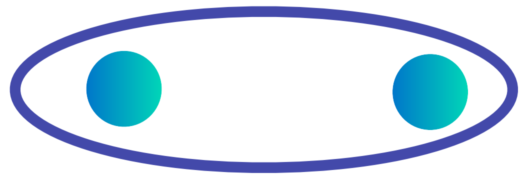
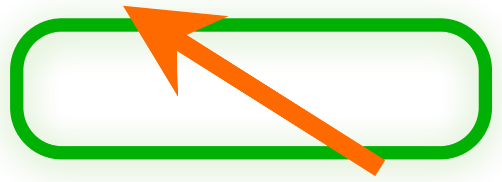
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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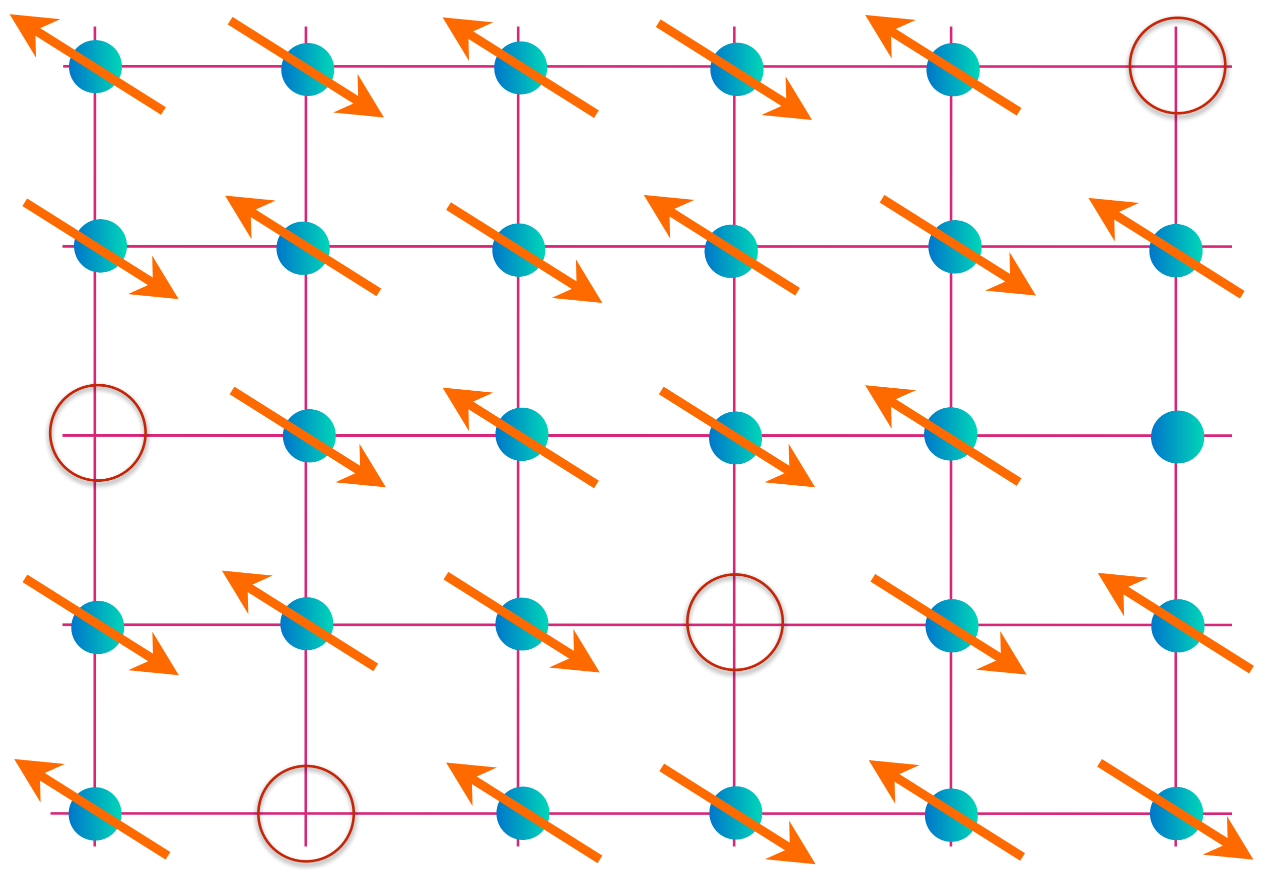
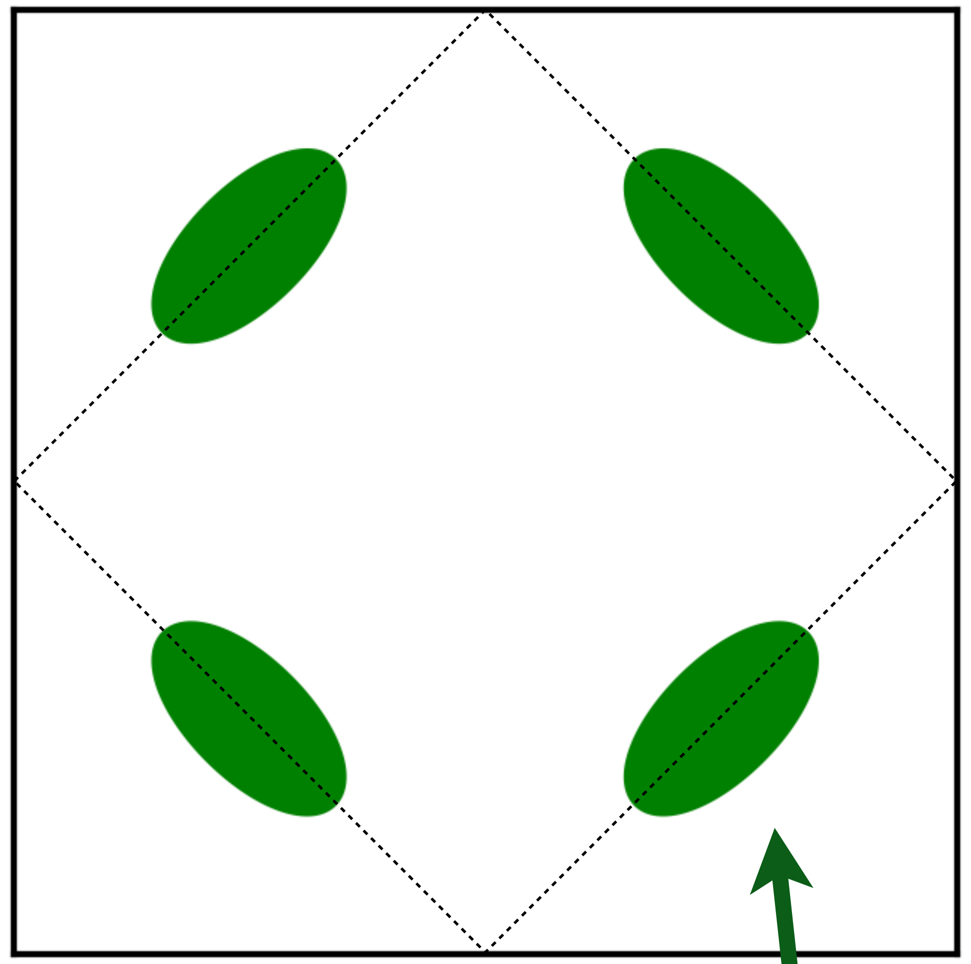
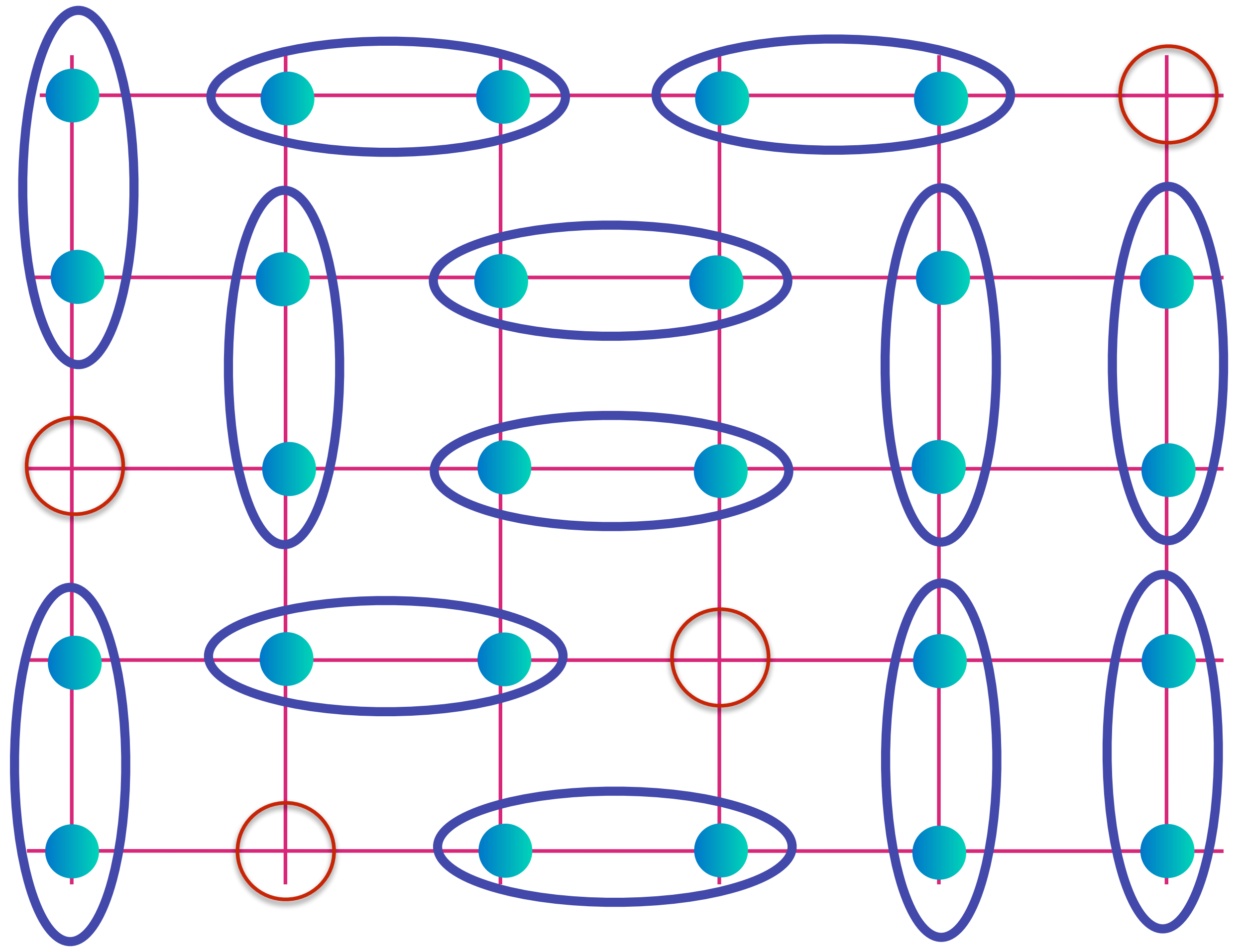
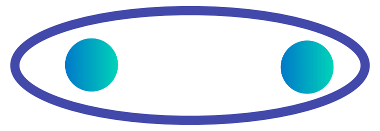


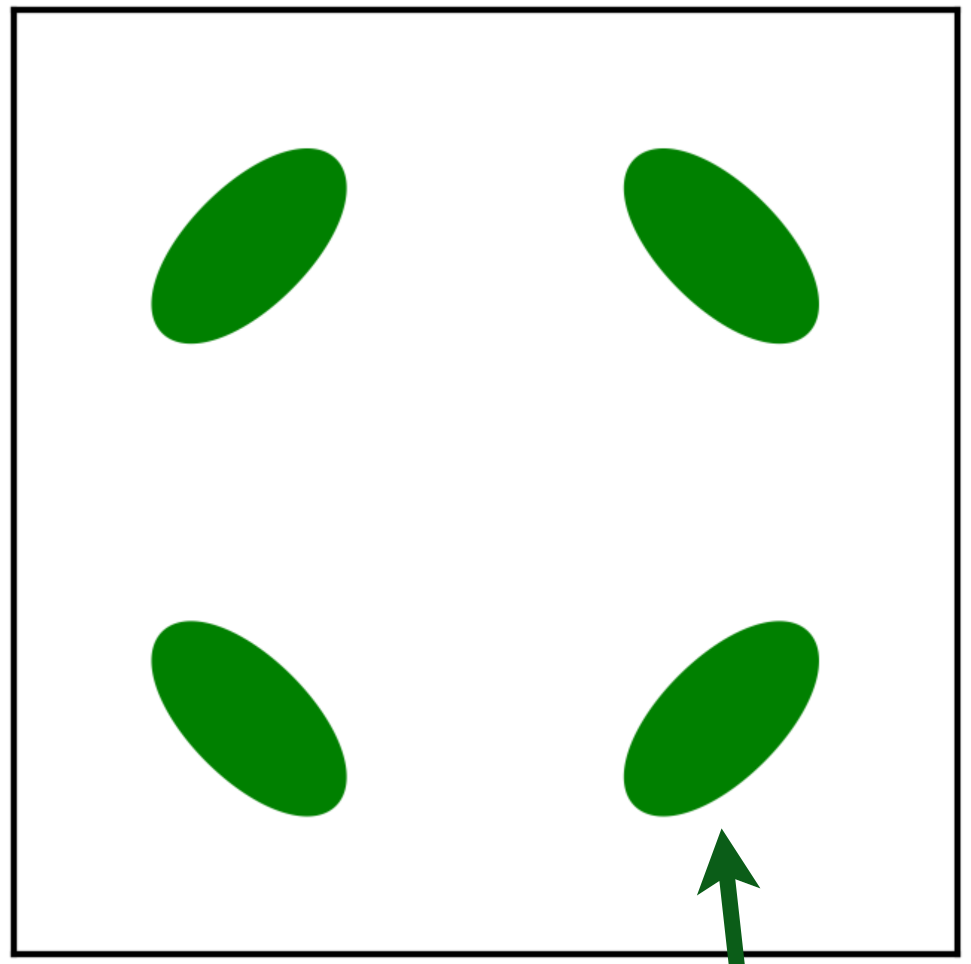


Area $p/4$

AF metal and SDW fluctuation



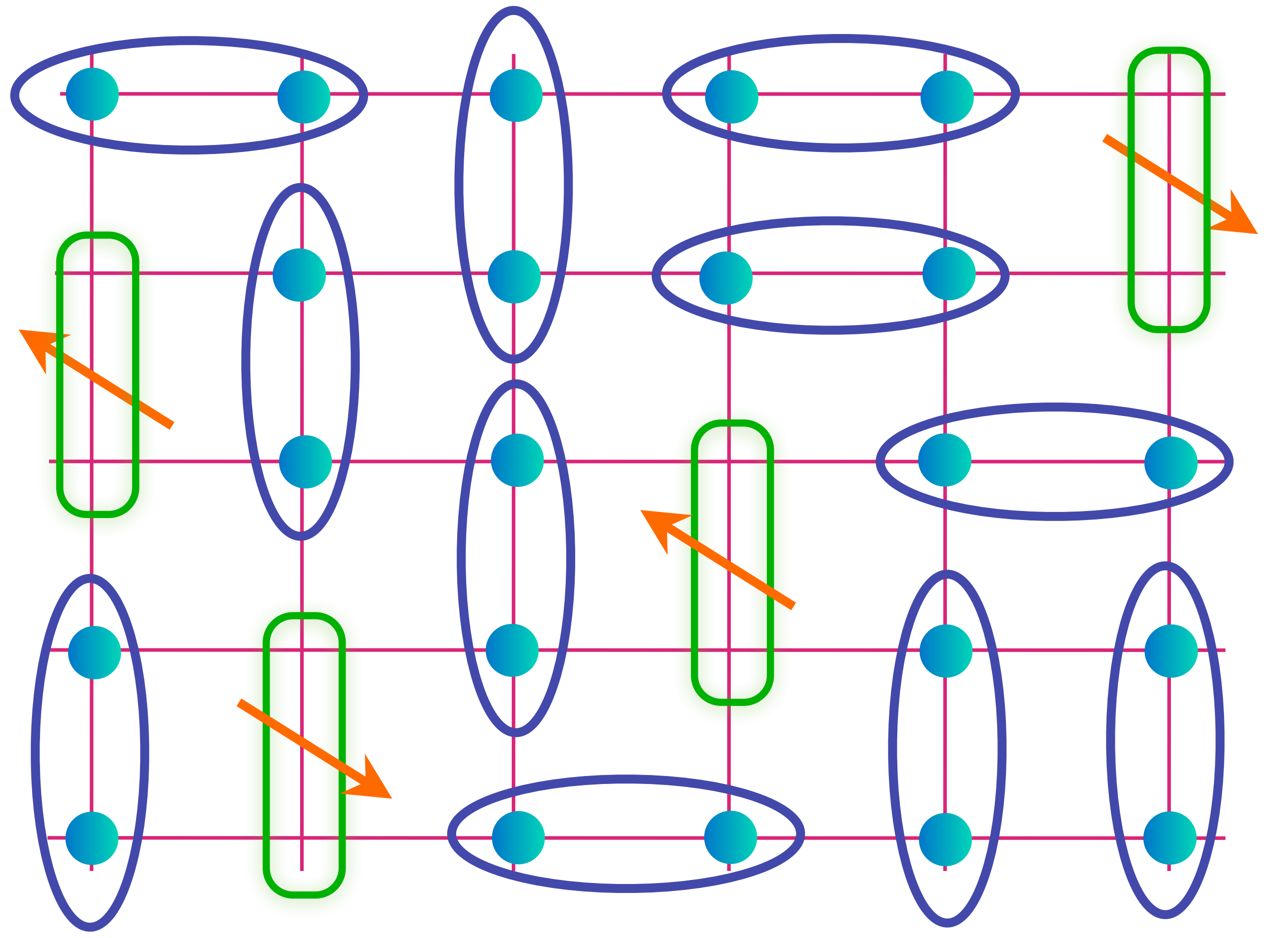
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

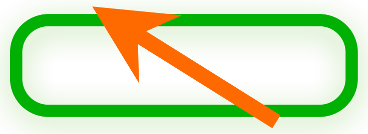


Area $p/4$

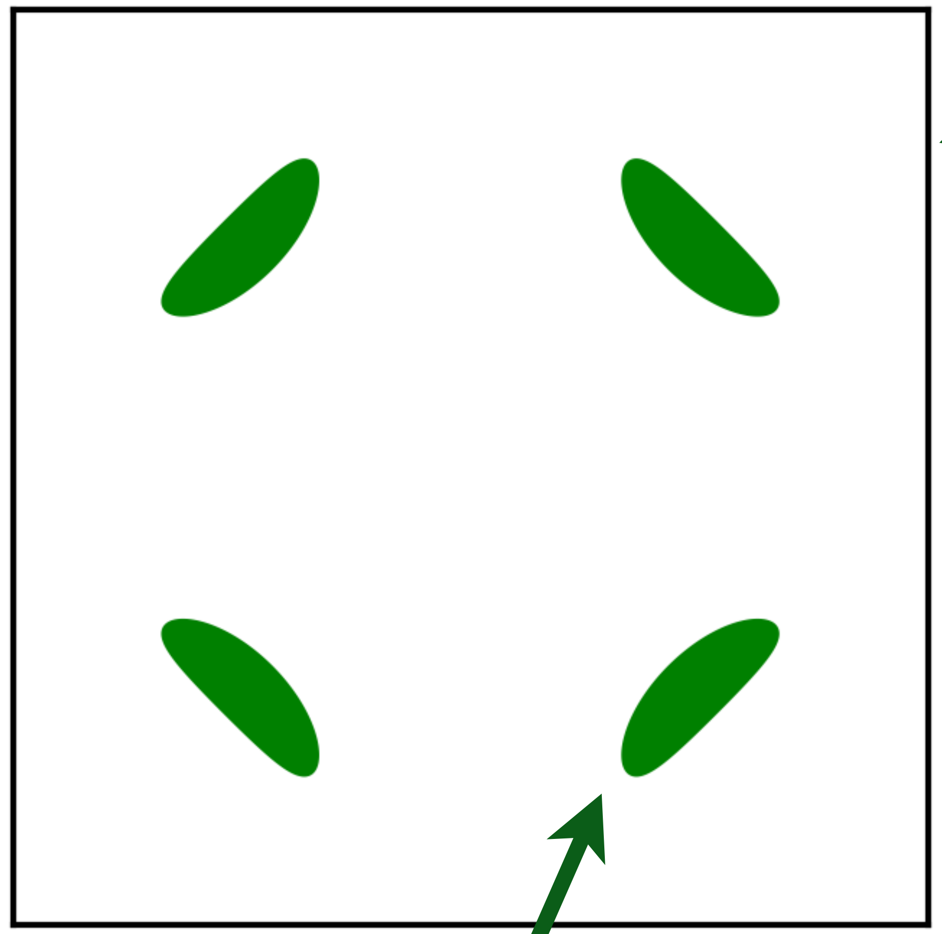
Holon metal

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

FL*



Area 1

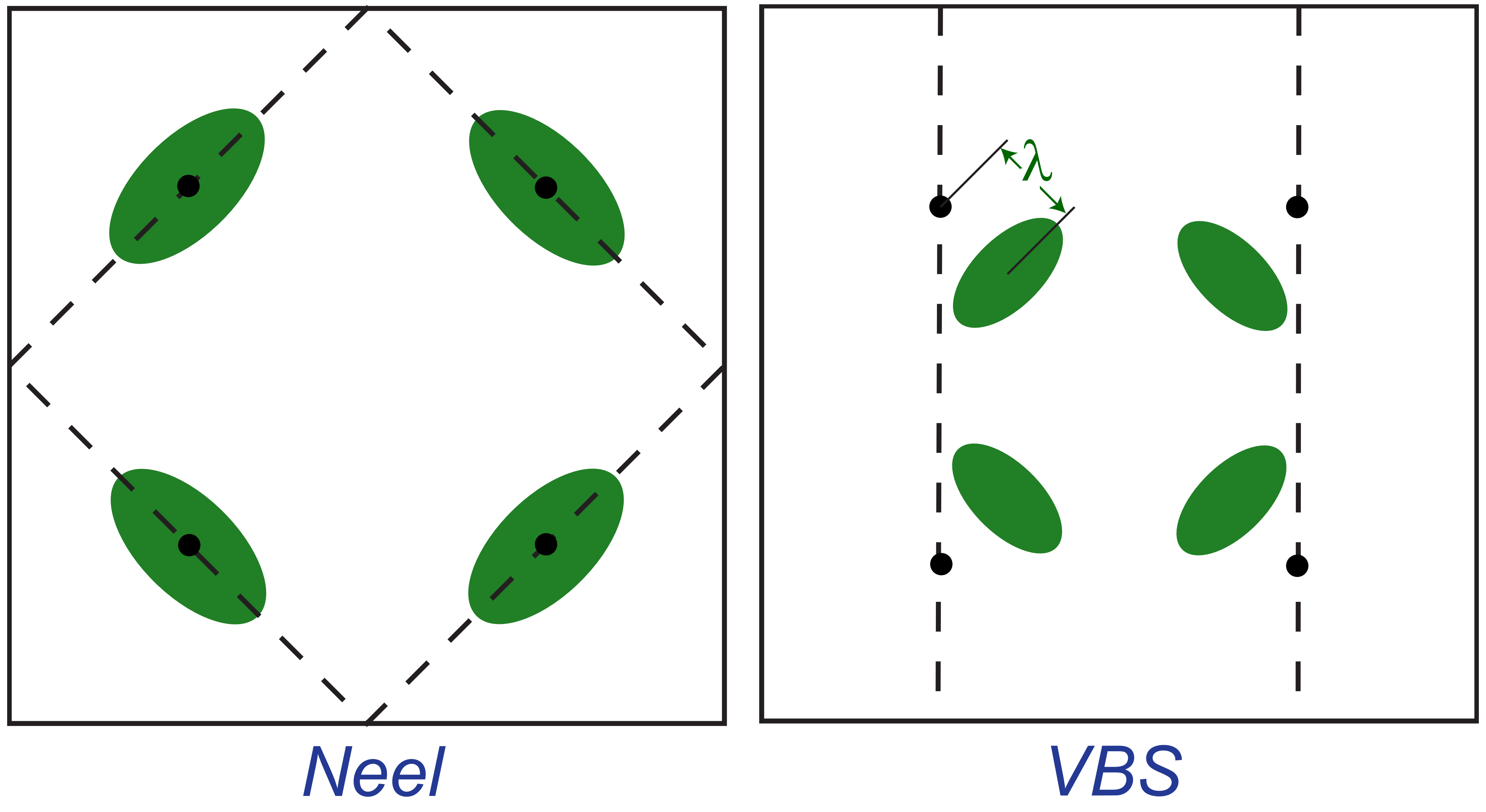
Area $p/8$

Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

Factor of 2 between SDW fluctuation and FL*

Many fermion entanglement I:

Observation of the Yamaji effect in the cuprate pseudogap

See also:

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

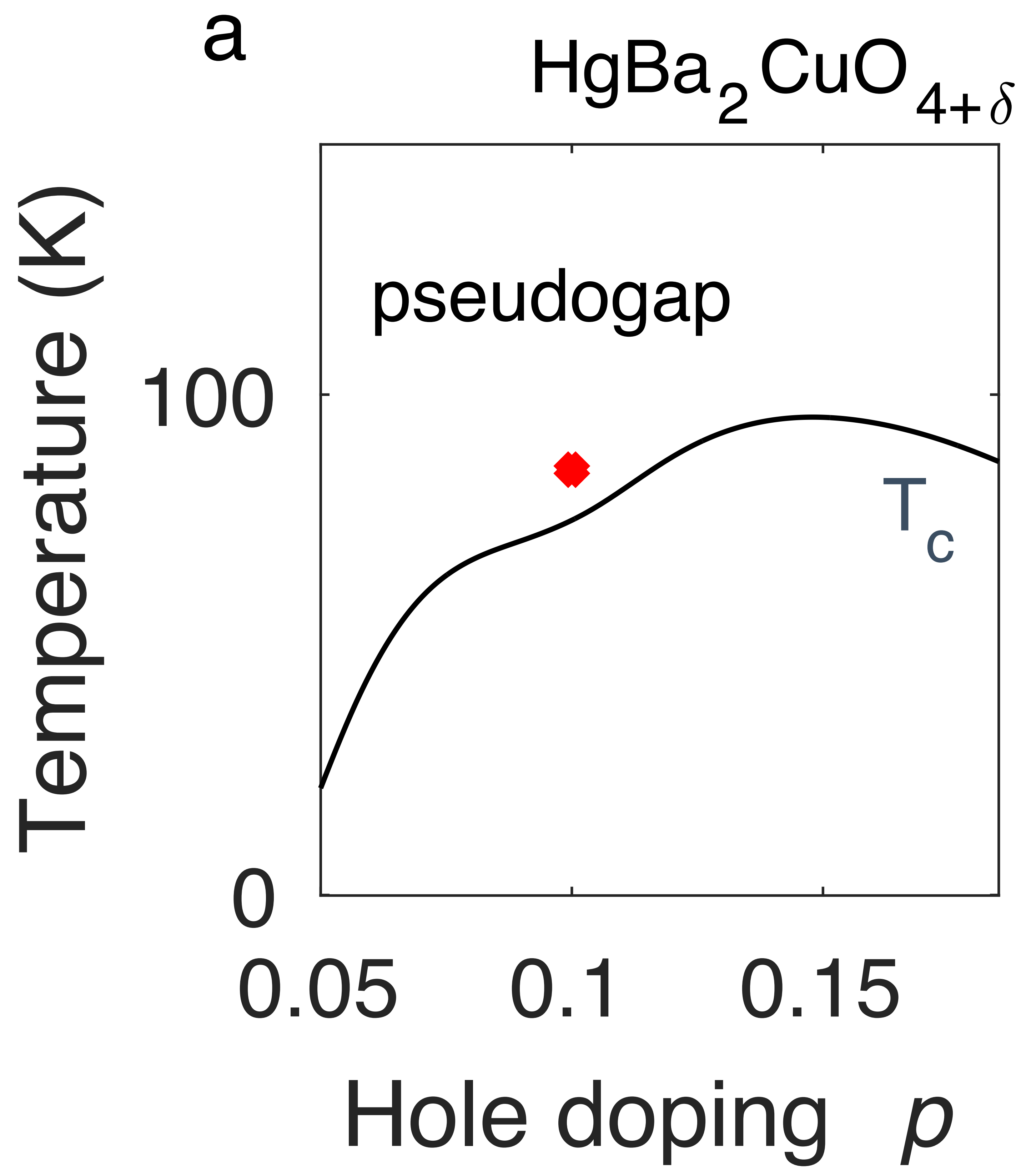
Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

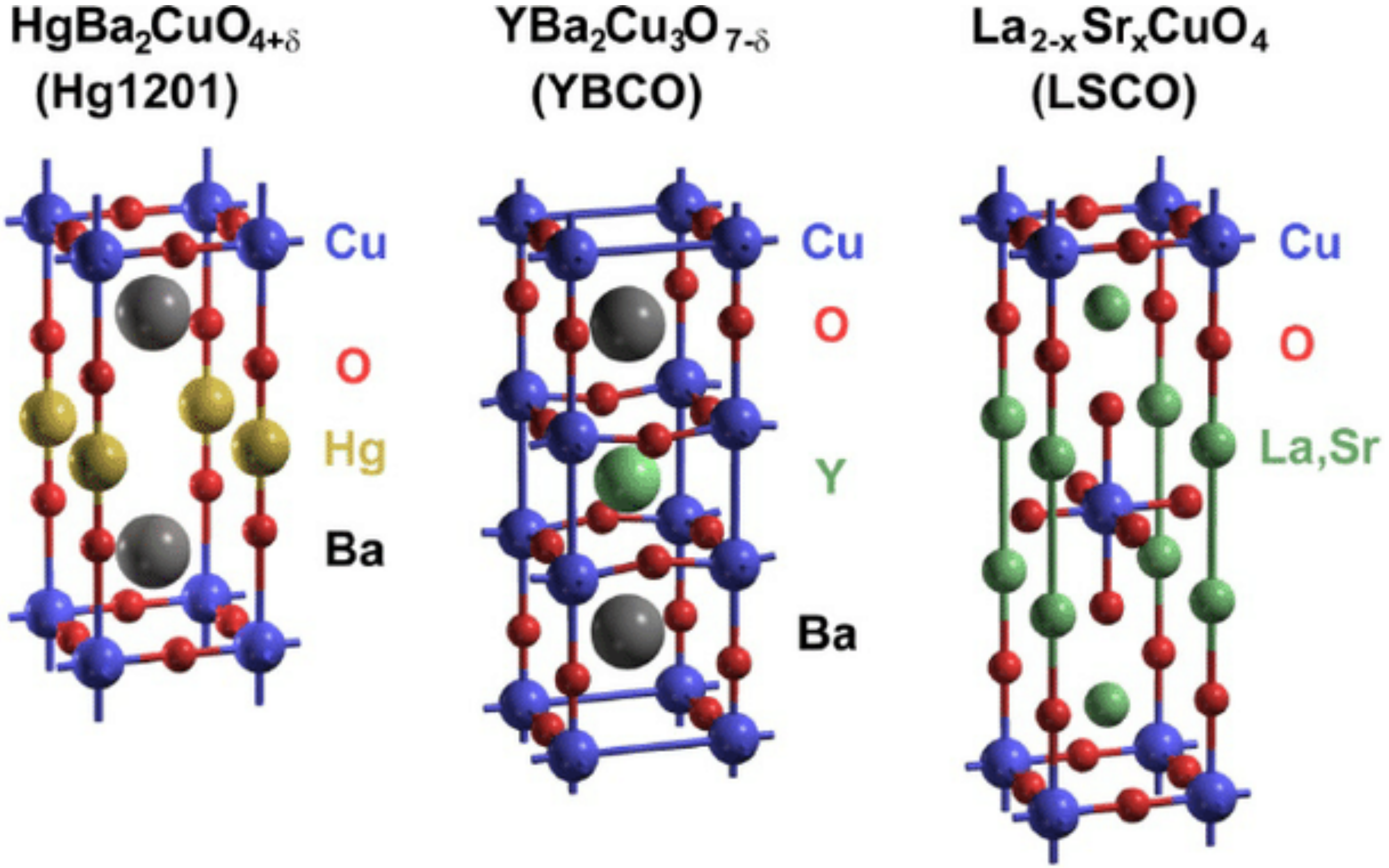
Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$

Observation of the Yamaji effect in a cuprate superconductor

nature physics
21, 1753 (2025)
Published online: 16 September 2025

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]





Observation of the Yamaji effect in a cuprate superconductor

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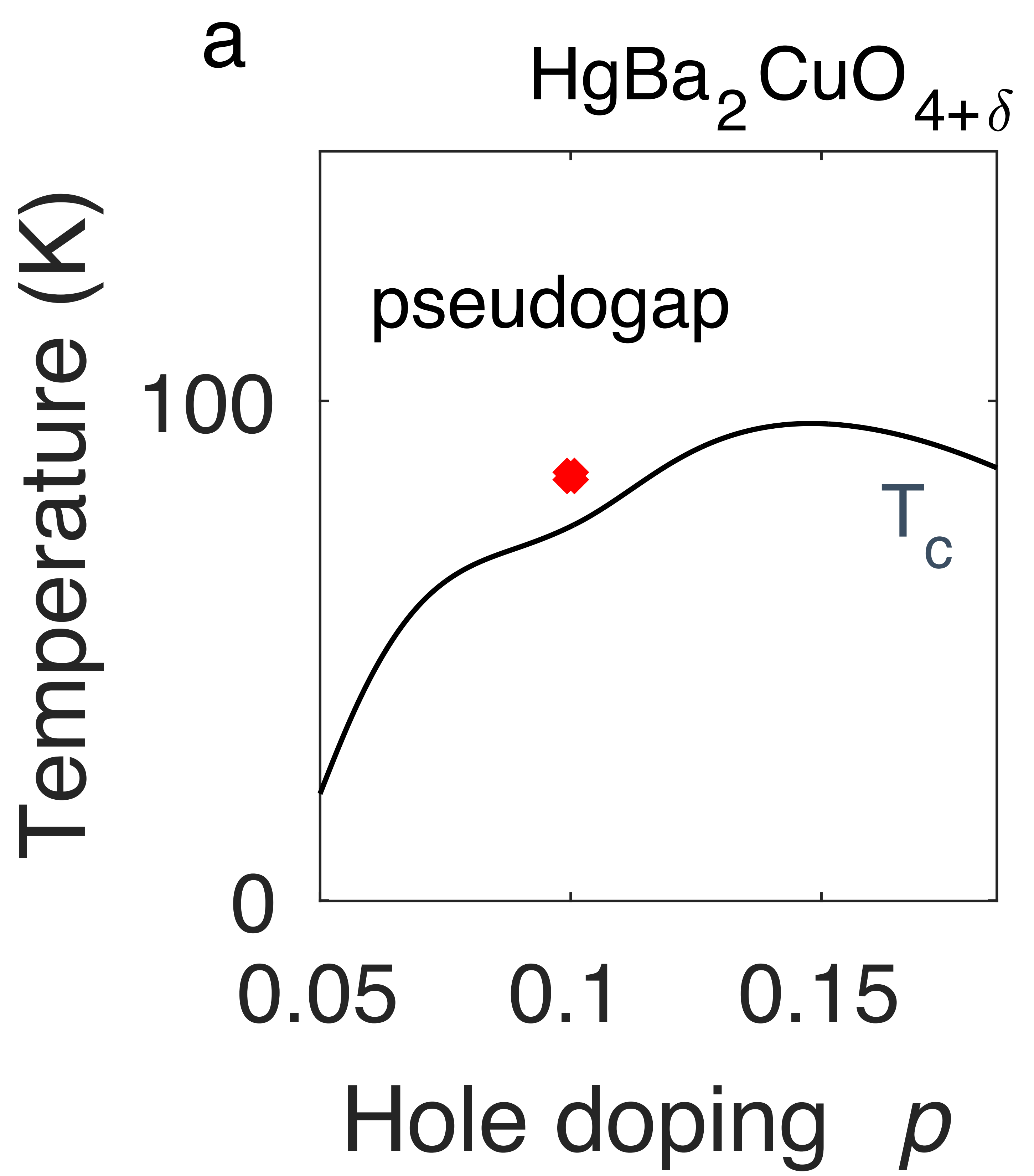

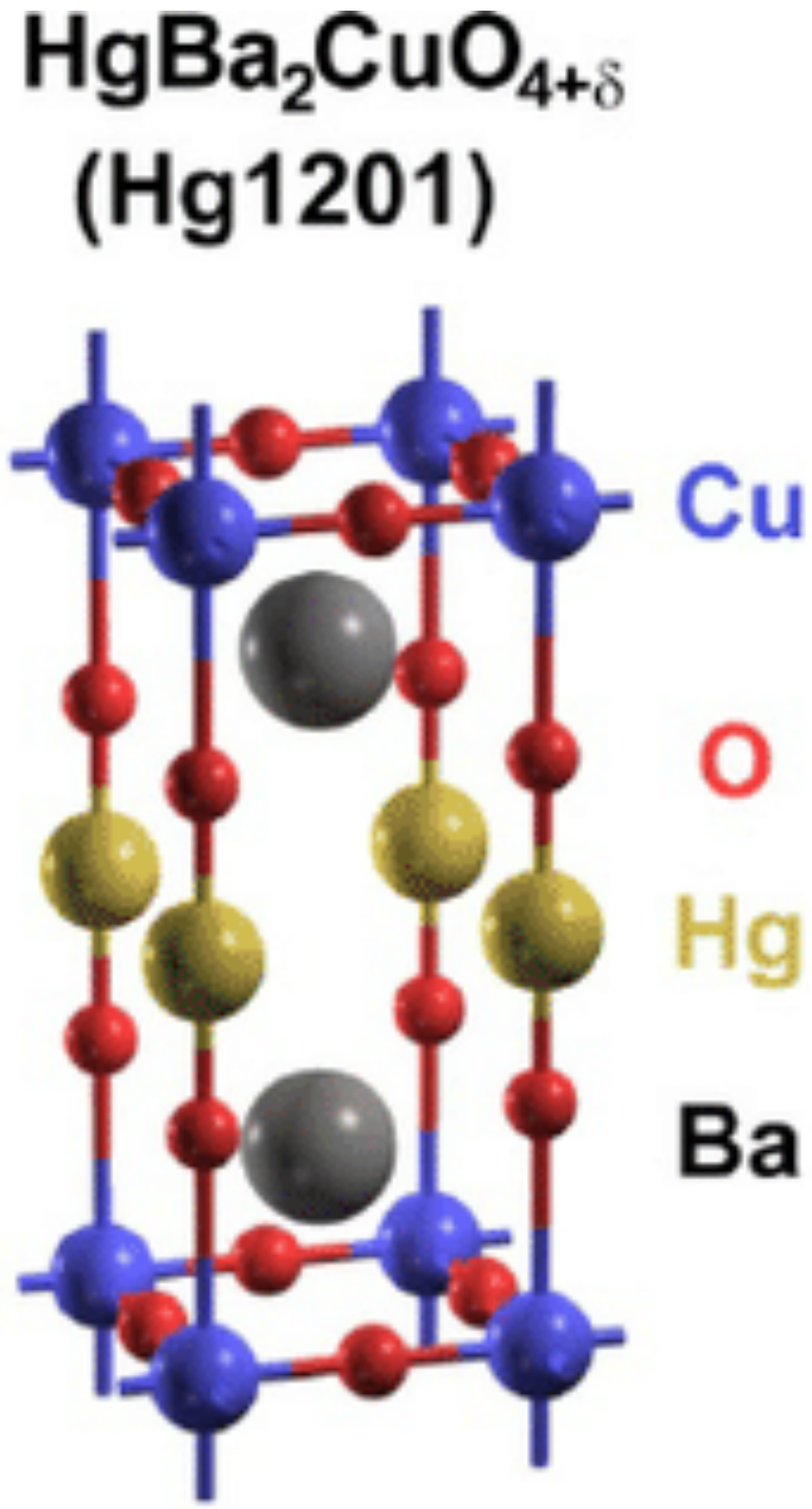

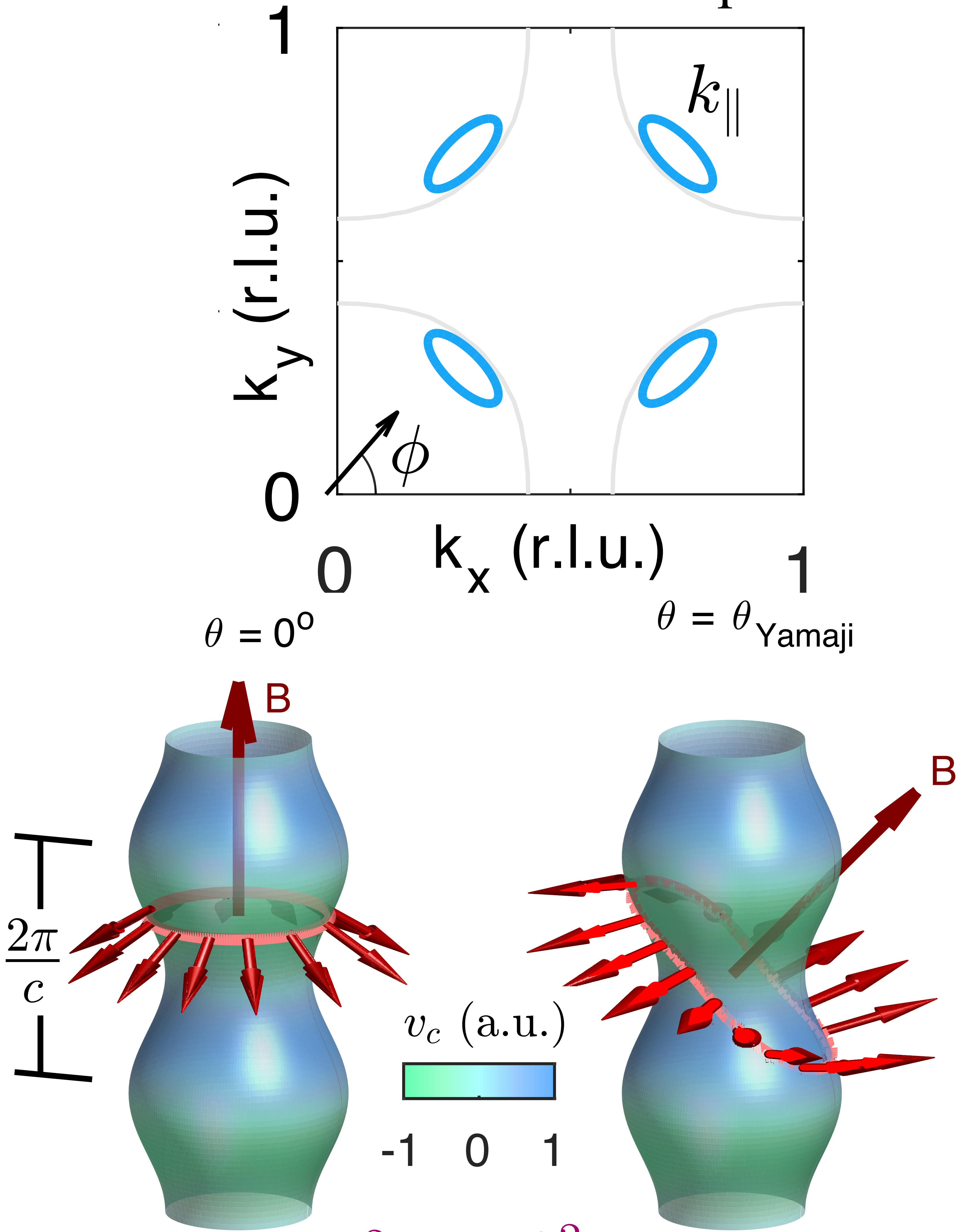

At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

$$\epsilon(\mathbf{k}) = \frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2} - 2t_\perp \cos(k_z c)$$

K. Yamaji JPSJ **58**, 1520 (1989)

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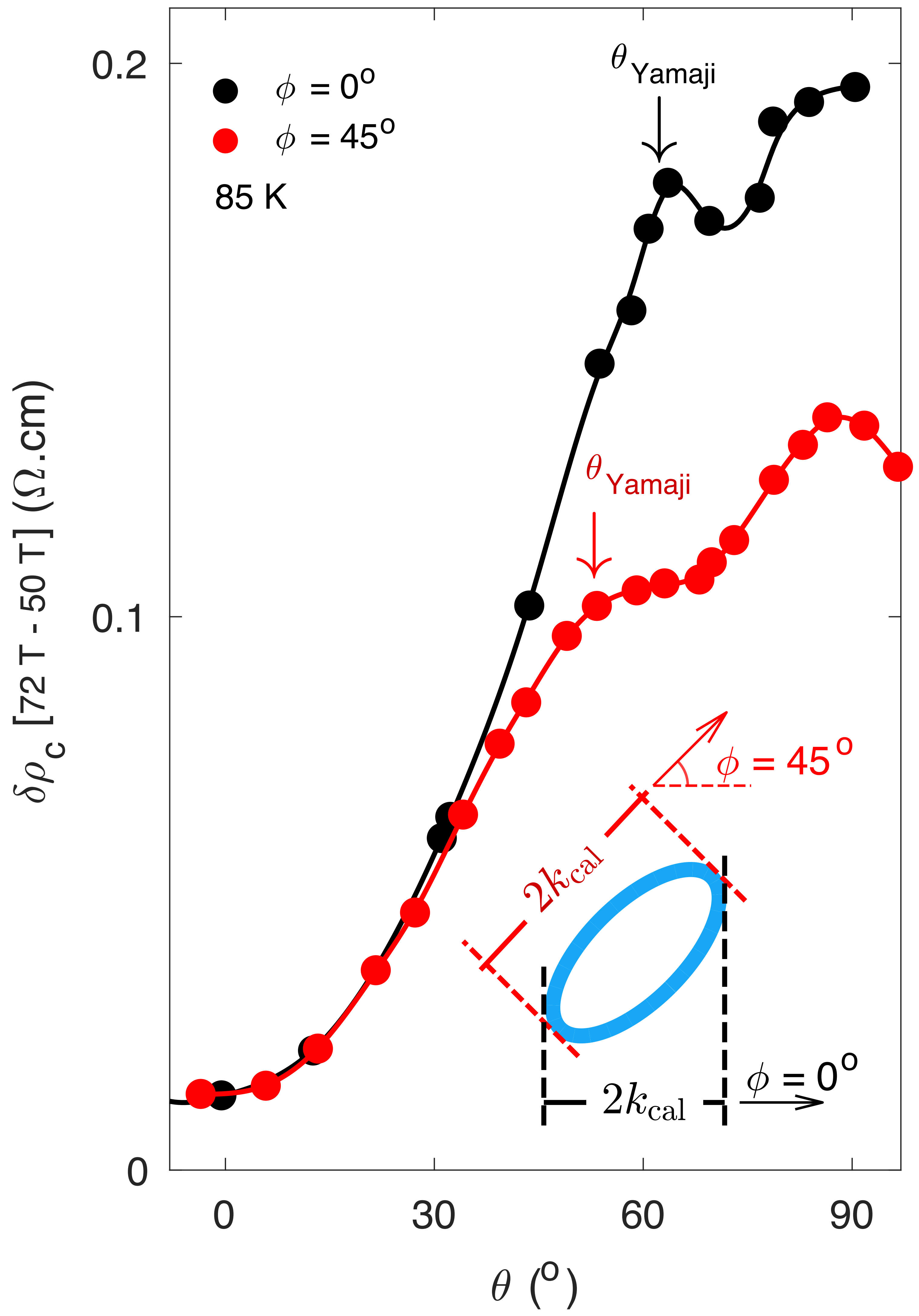

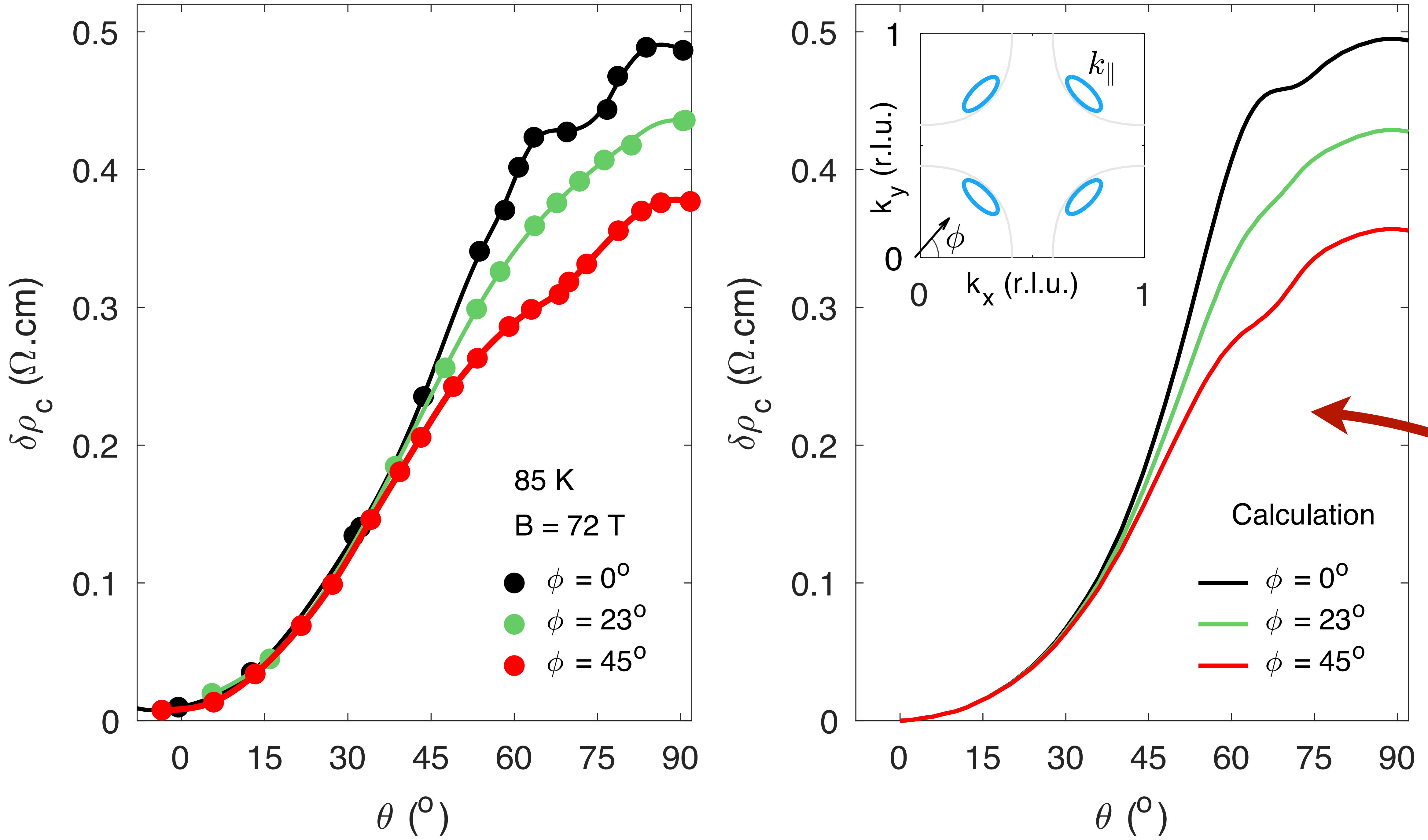

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

$$\frac{\partial f}{\partial t} + e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} \left(-\frac{\partial f}{\partial \epsilon} \right) = -\frac{f - f_0}{\tau}$$

$$\mathbf{v} = \nabla_{\mathbf{k}} \epsilon(\mathbf{k}) \ ; \ f_0(\epsilon) = \frac{1}{e^{(\epsilon - \mu)/T} + 1}$$

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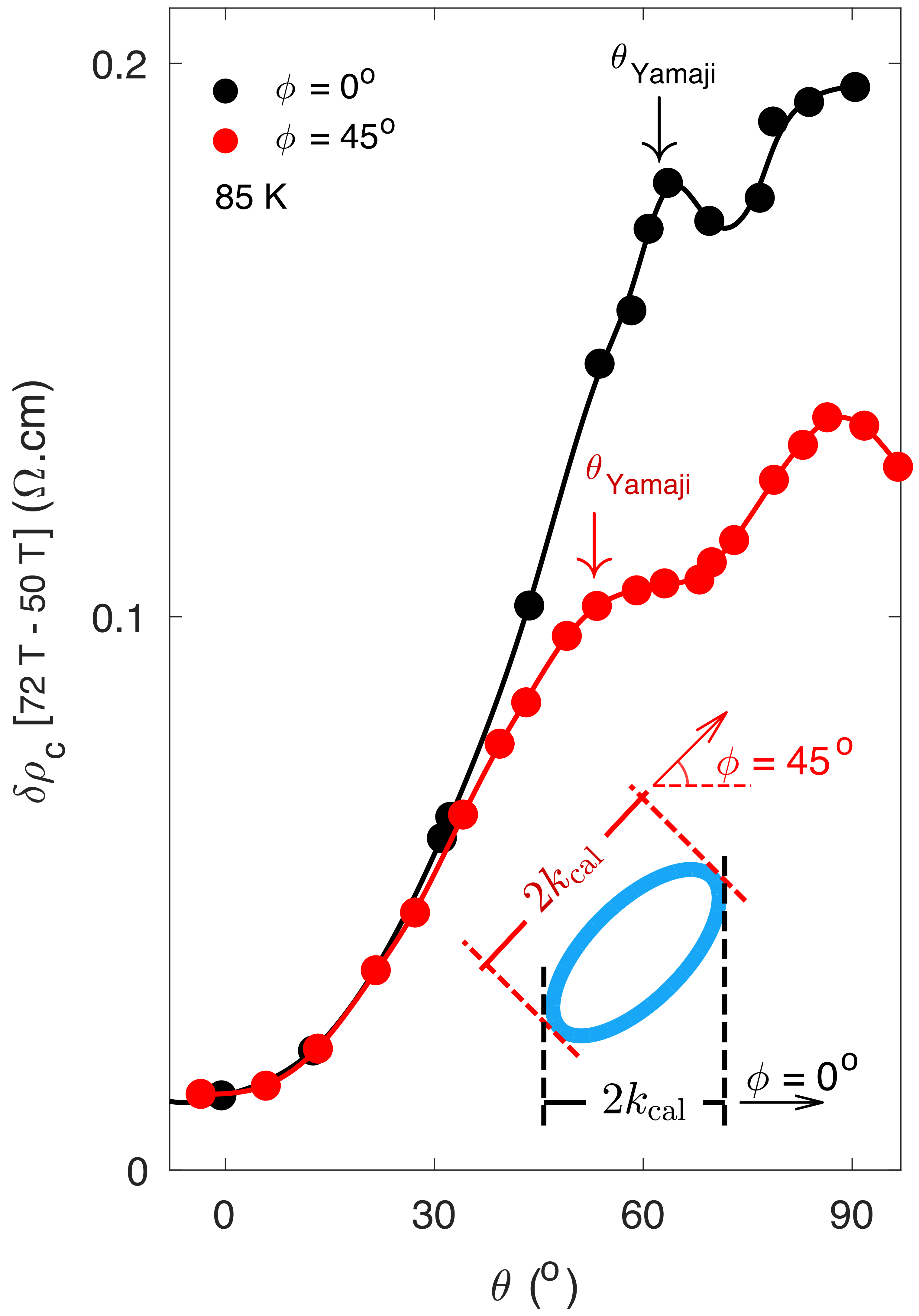

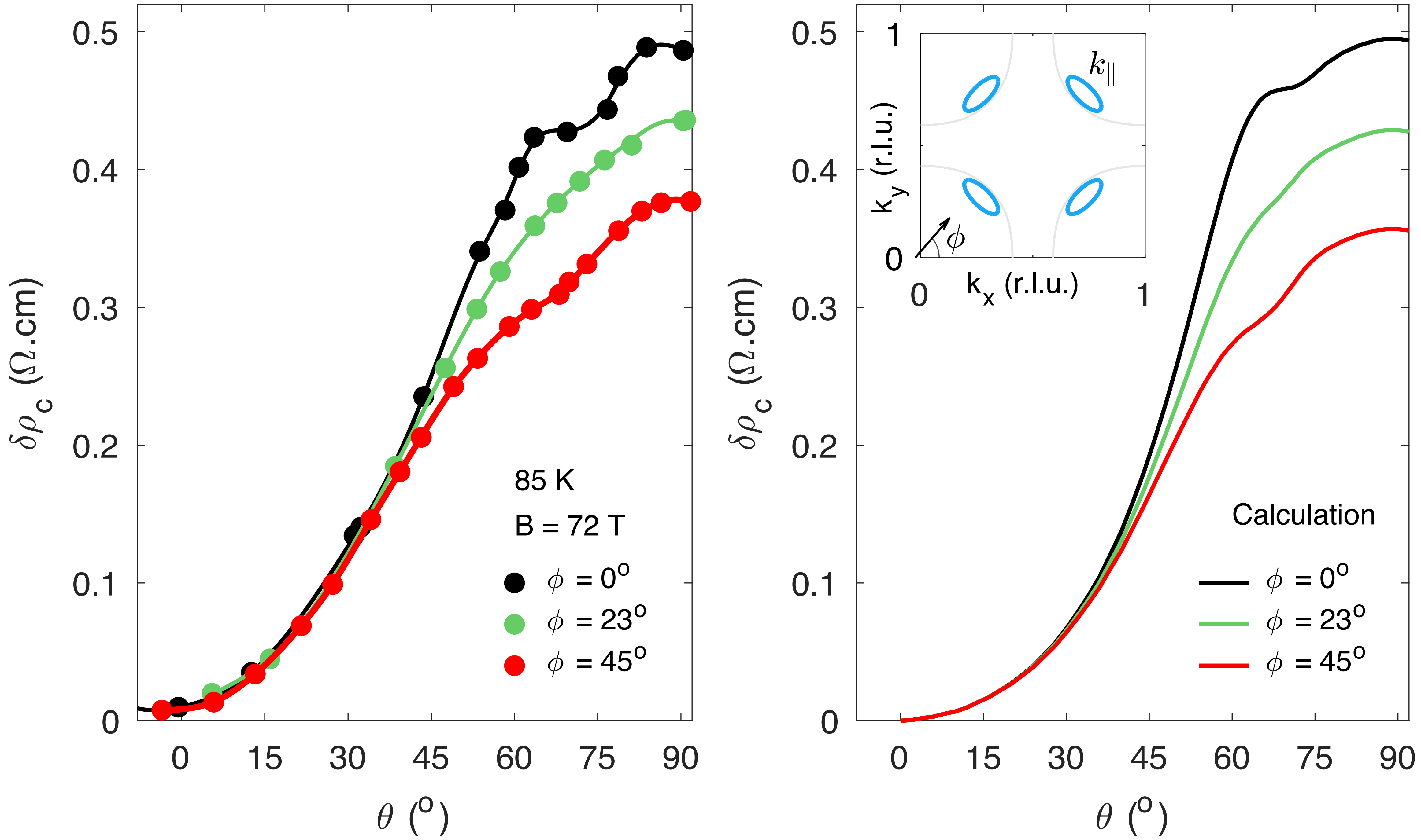

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Excellent evidence for hole pockets with coherent interlayer-transport.

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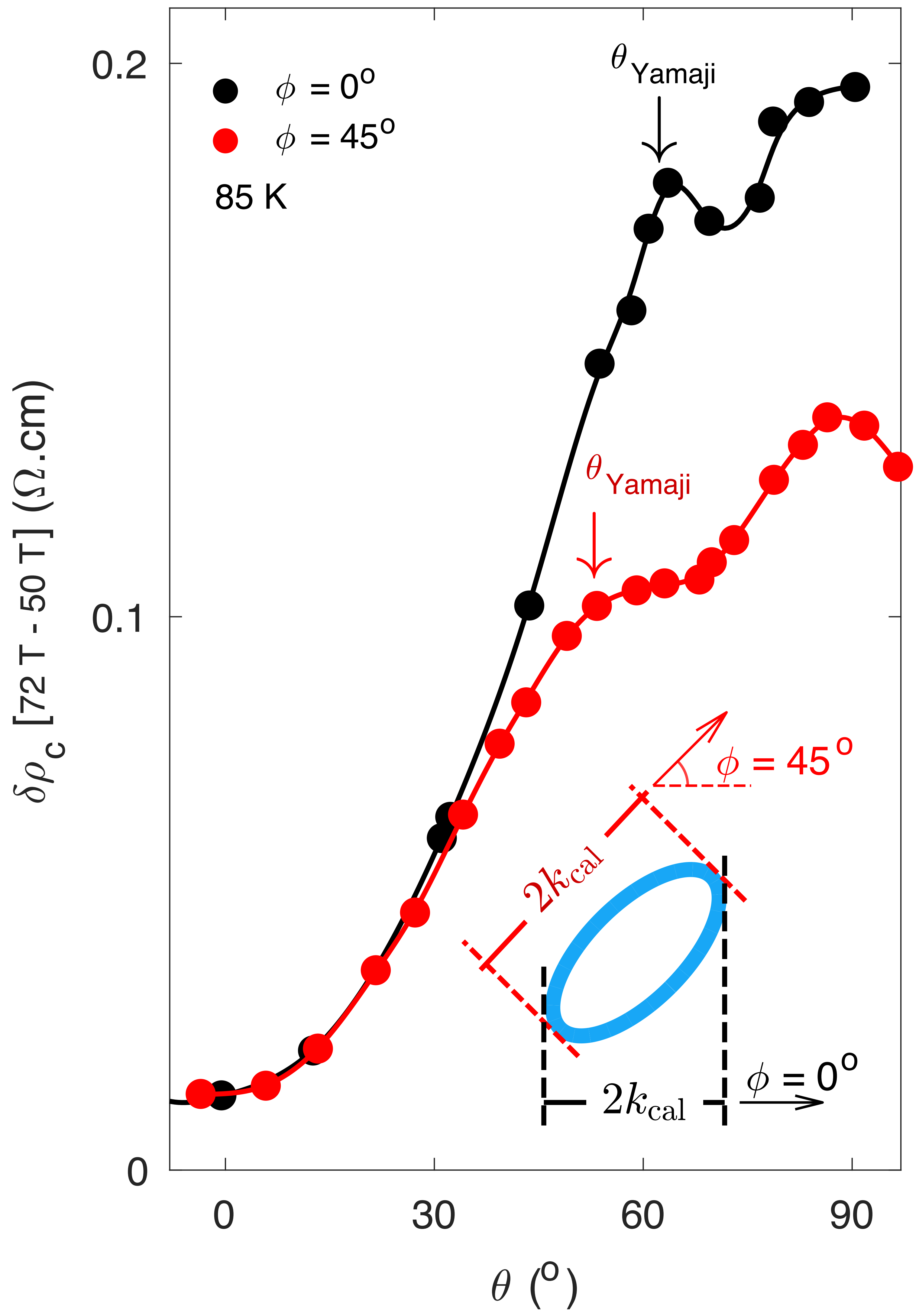

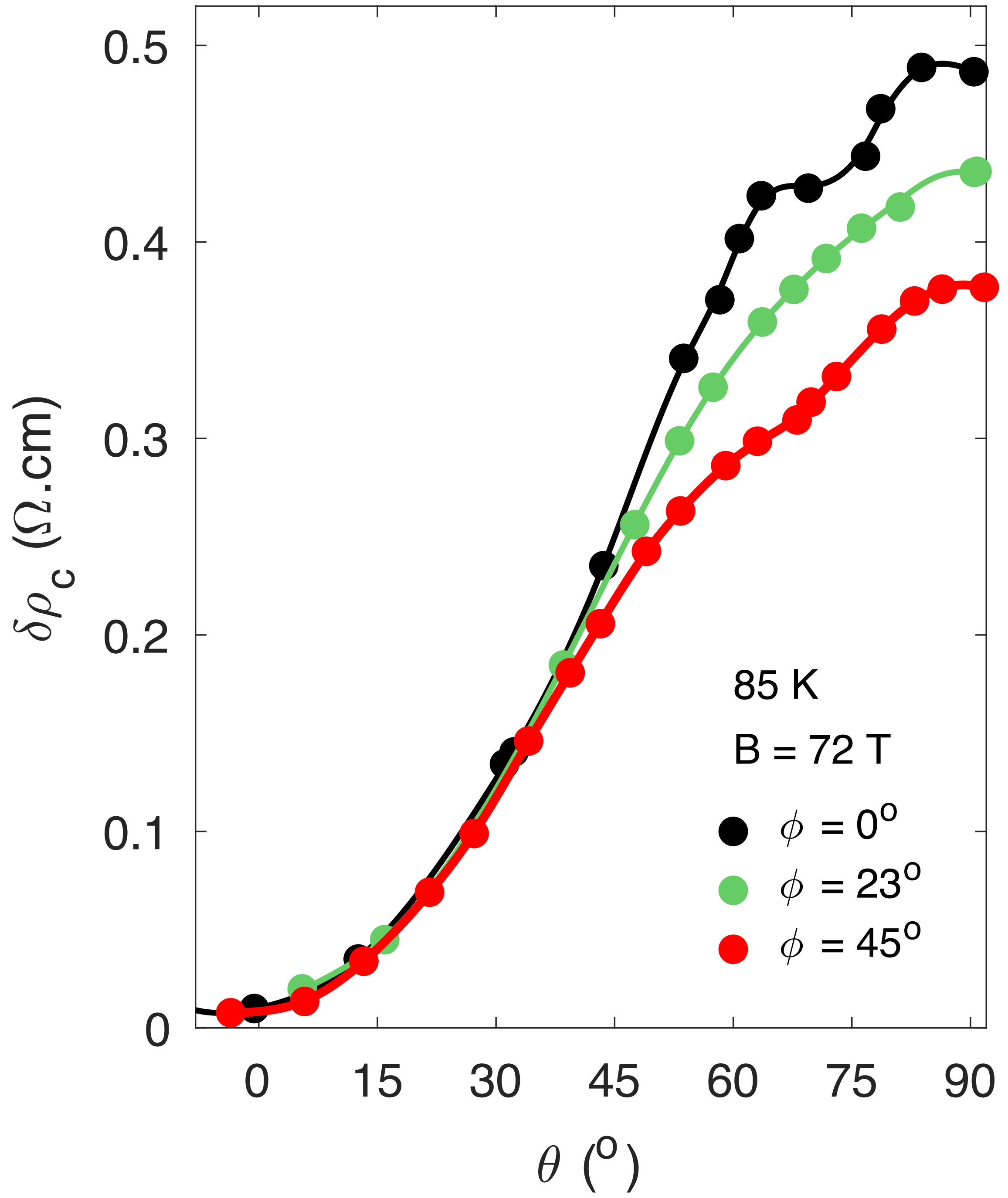

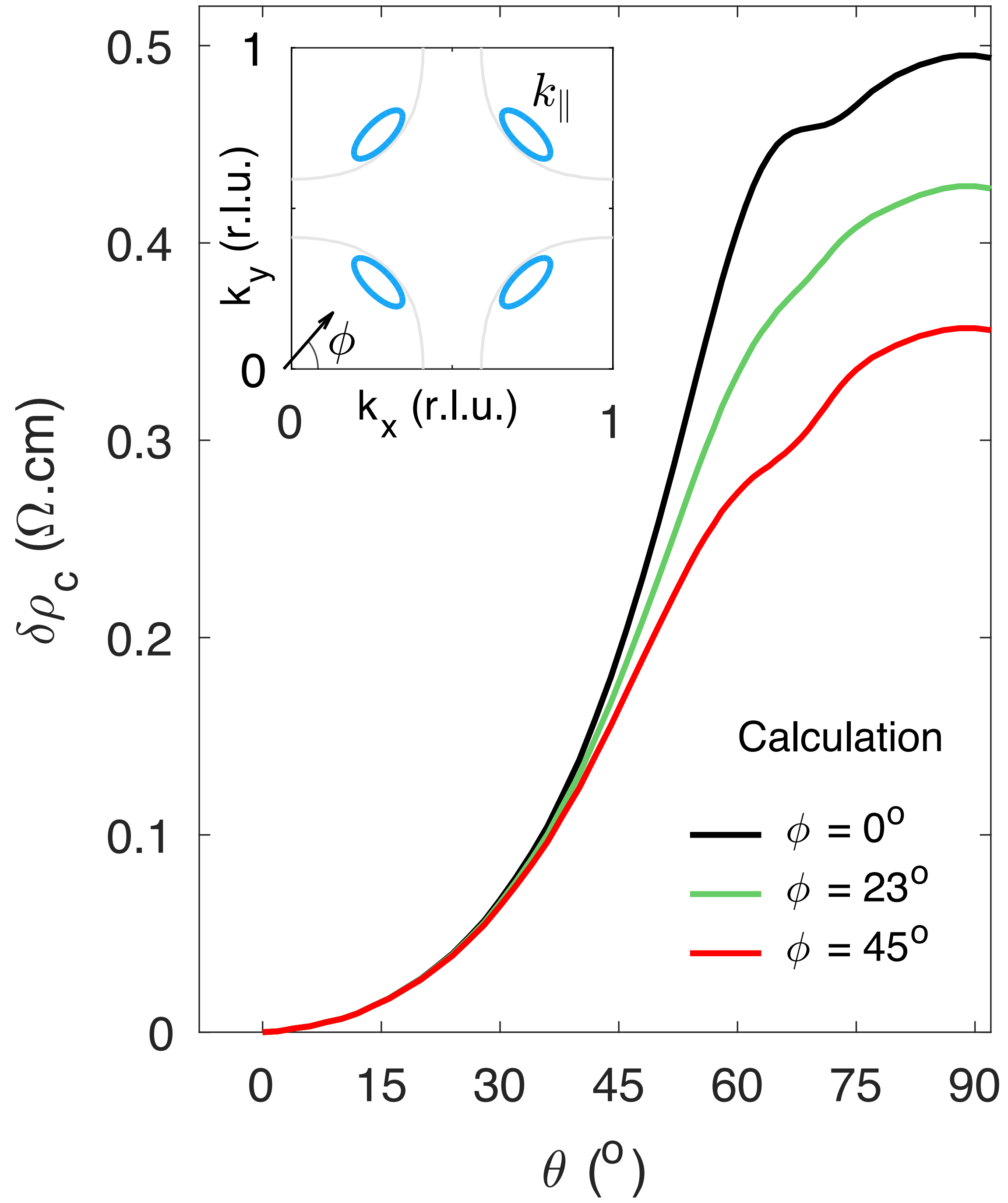

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Rules out holon metal

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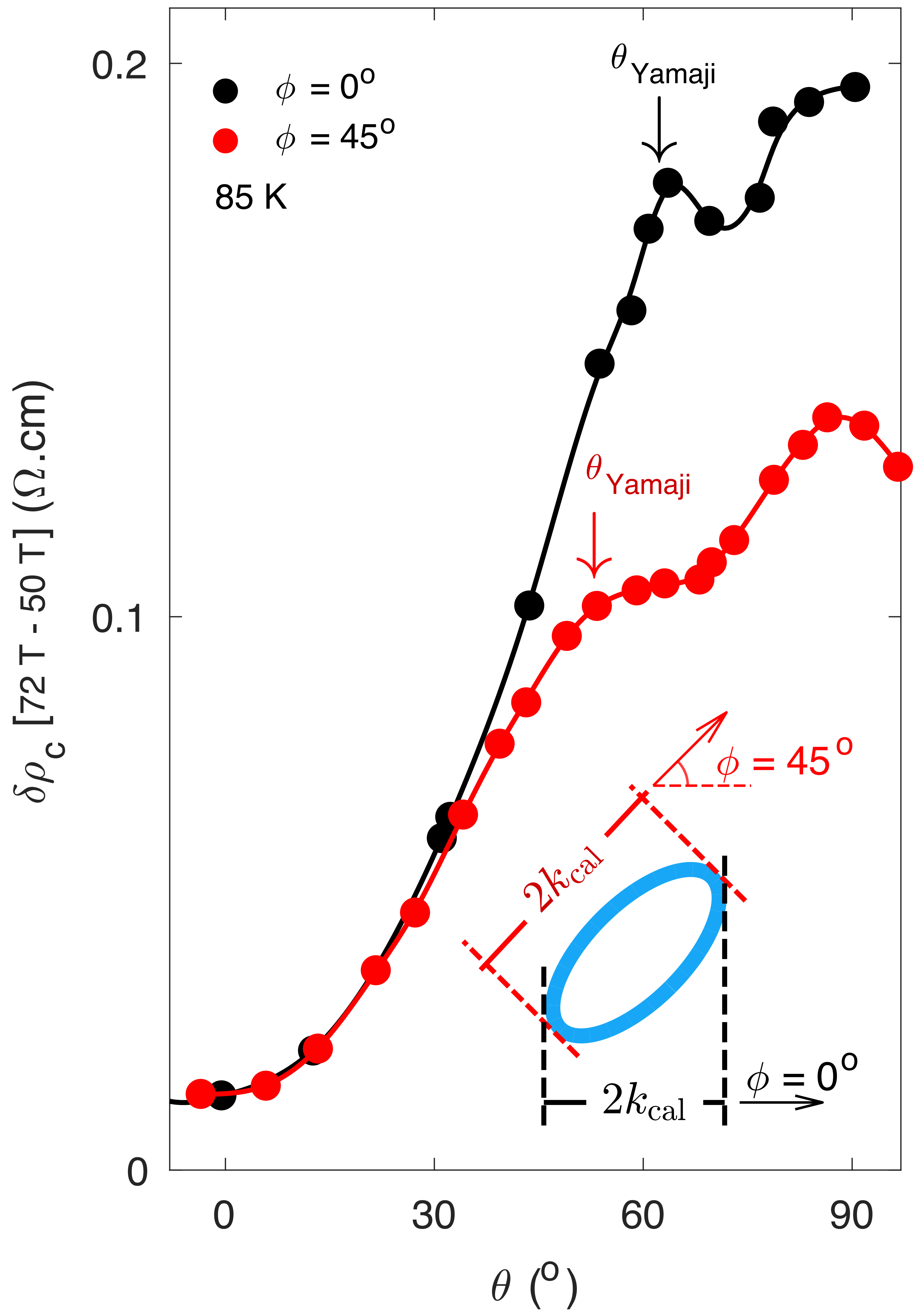

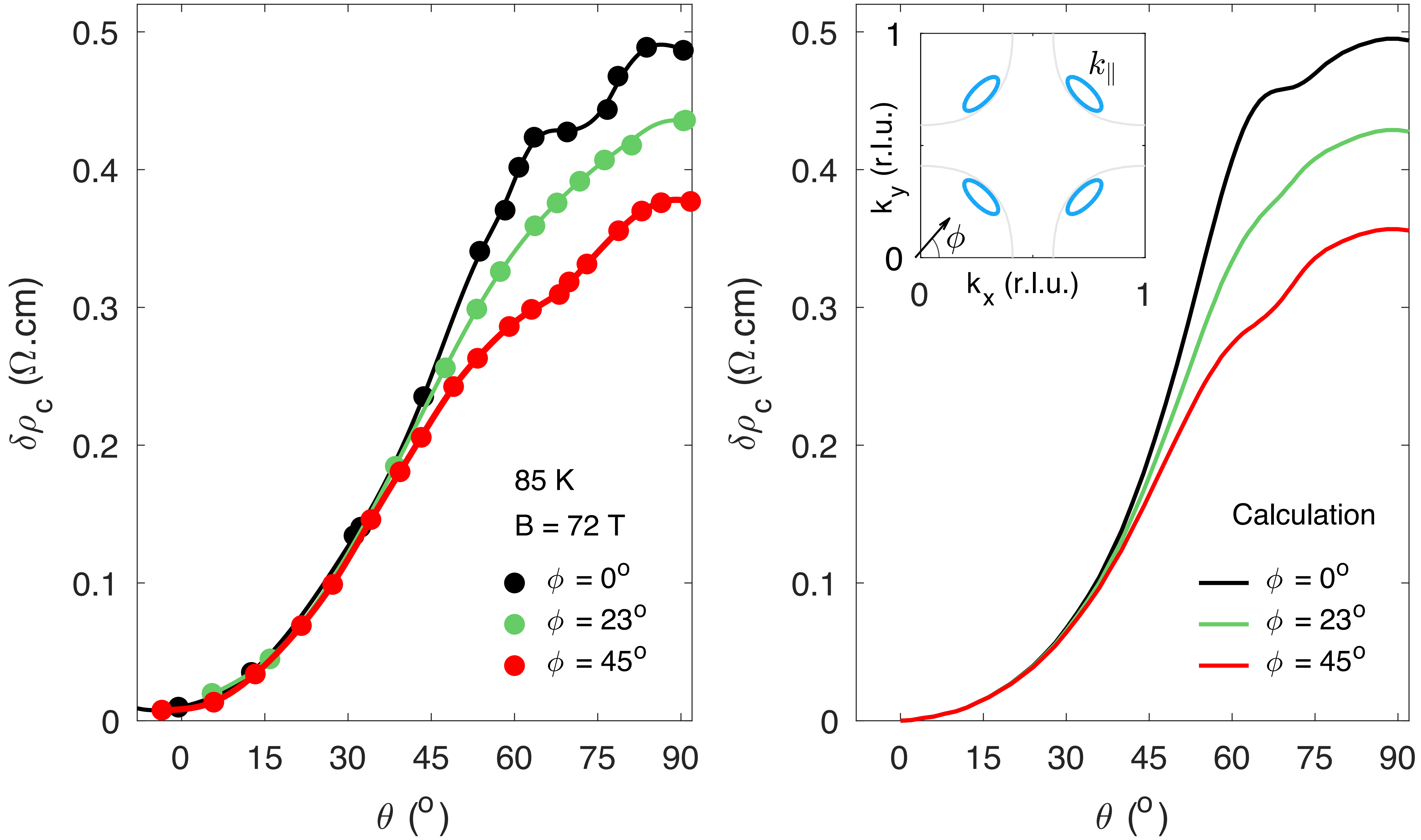

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Rules out holon metal
and possibly SDW metal

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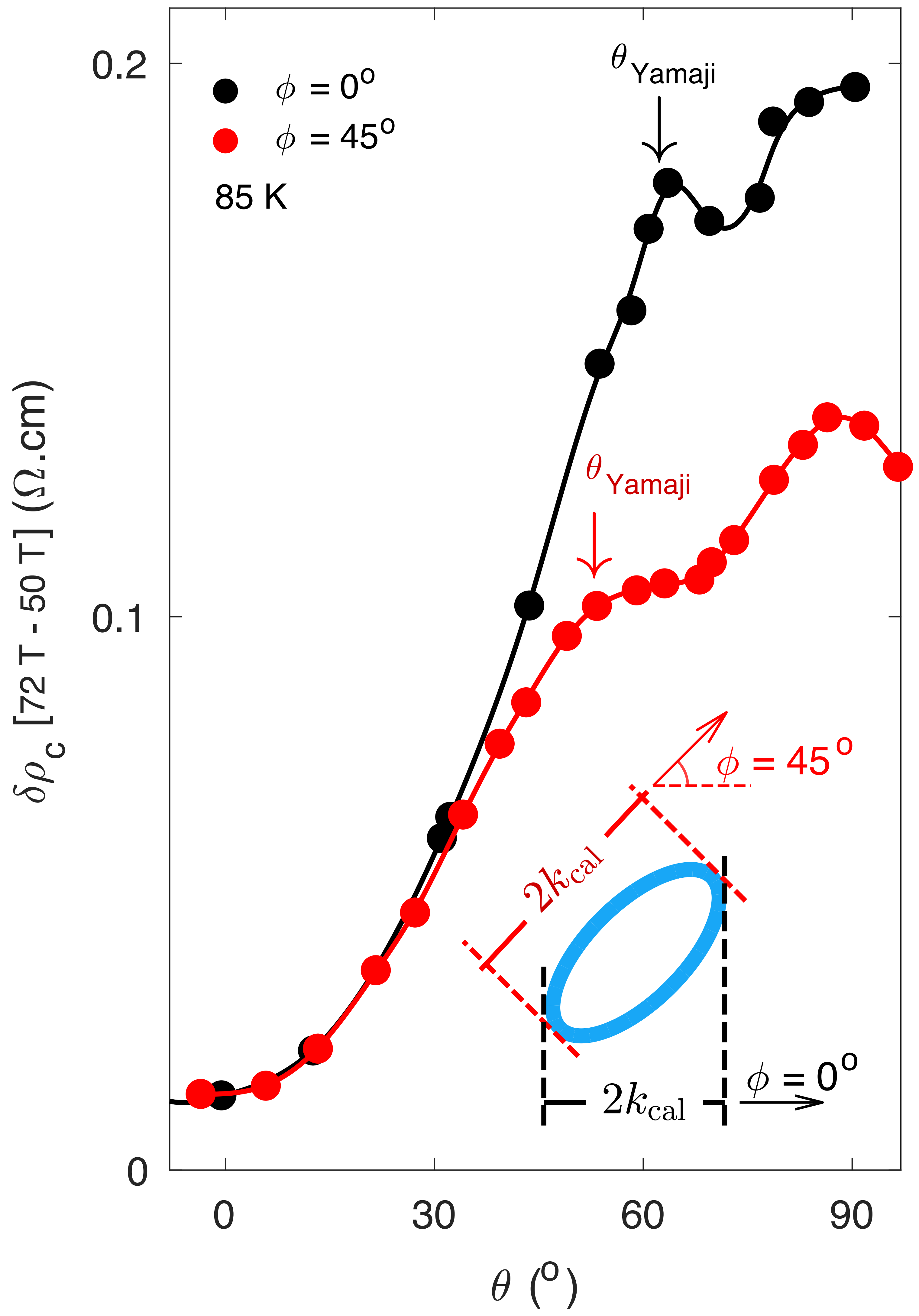

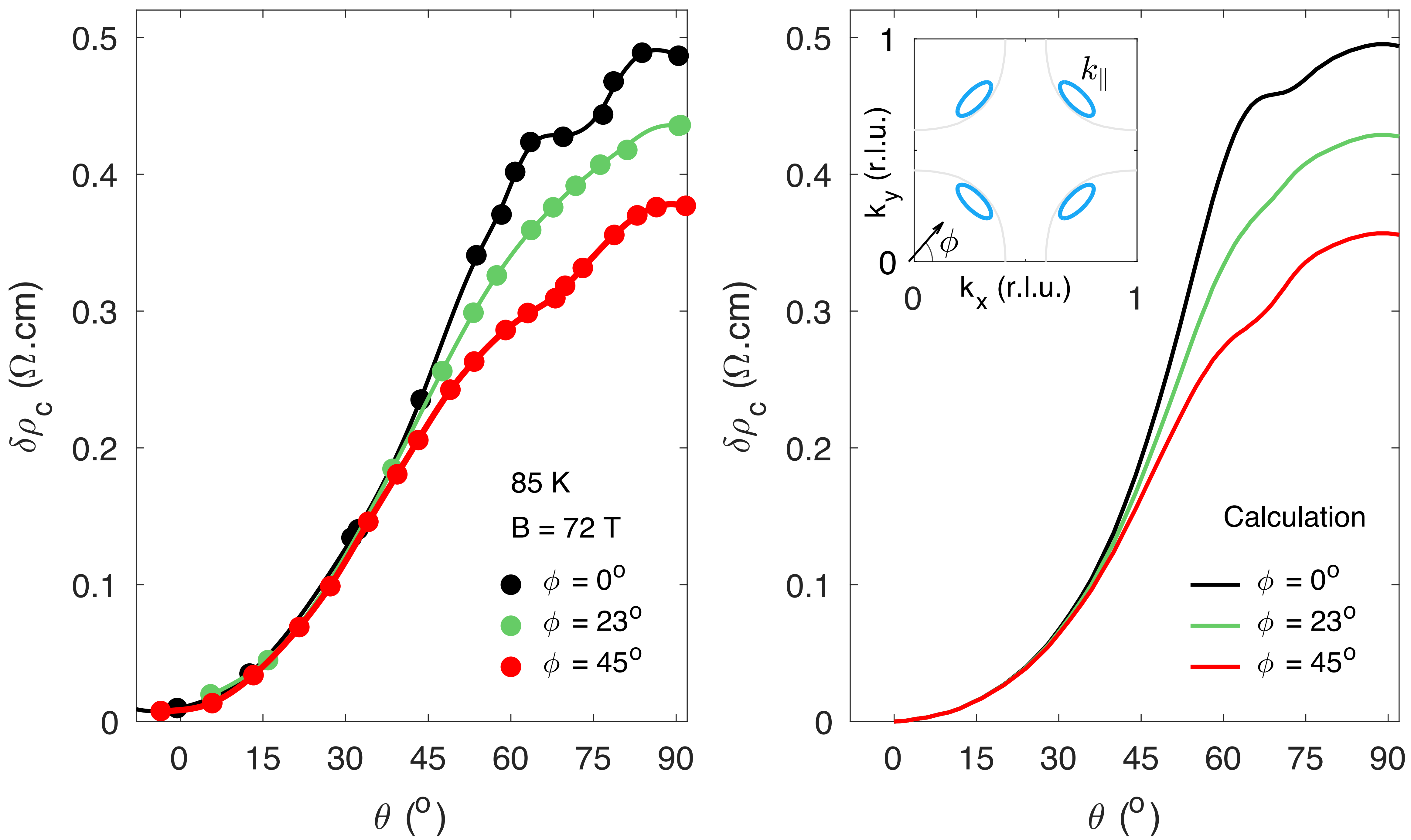

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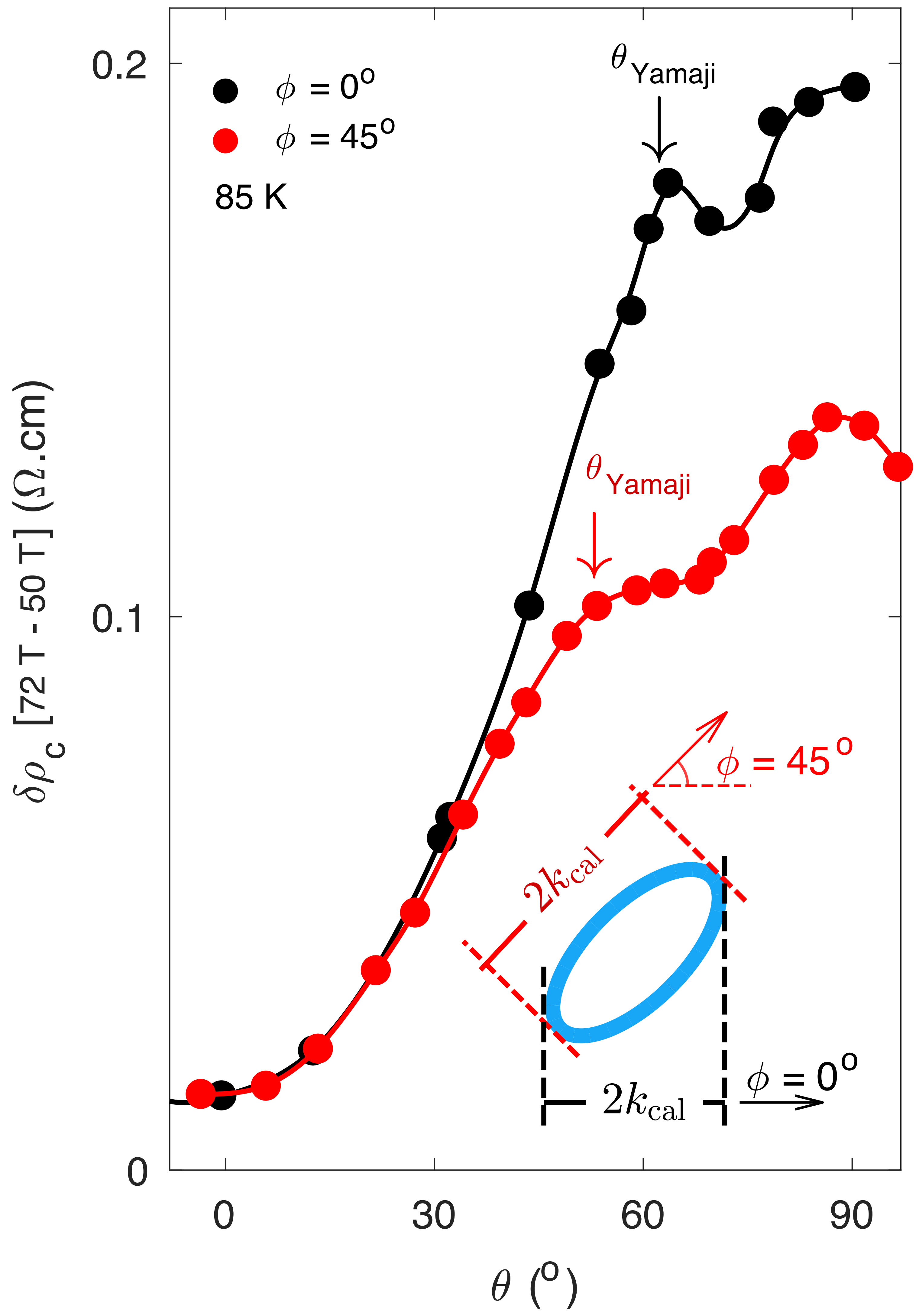

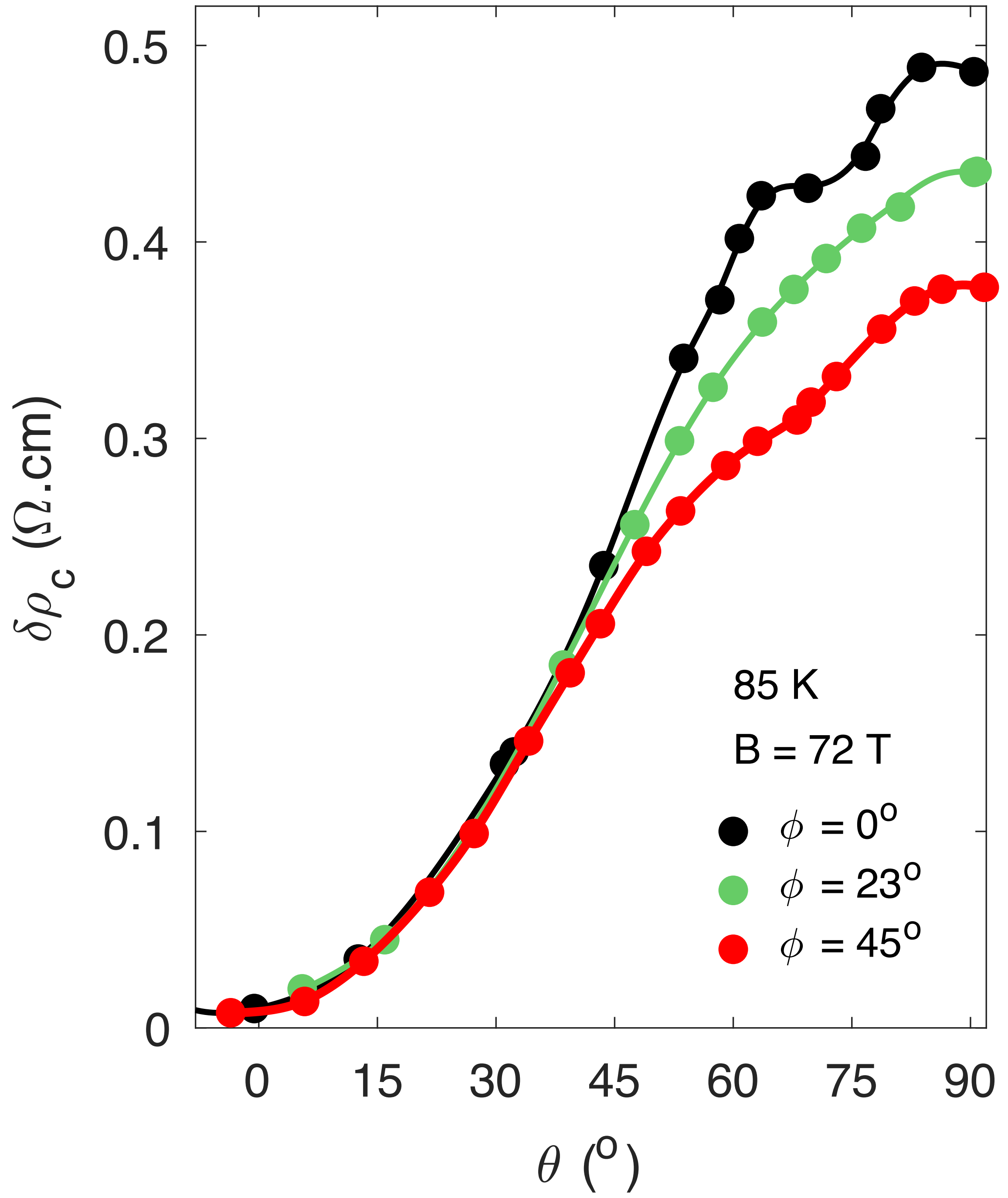

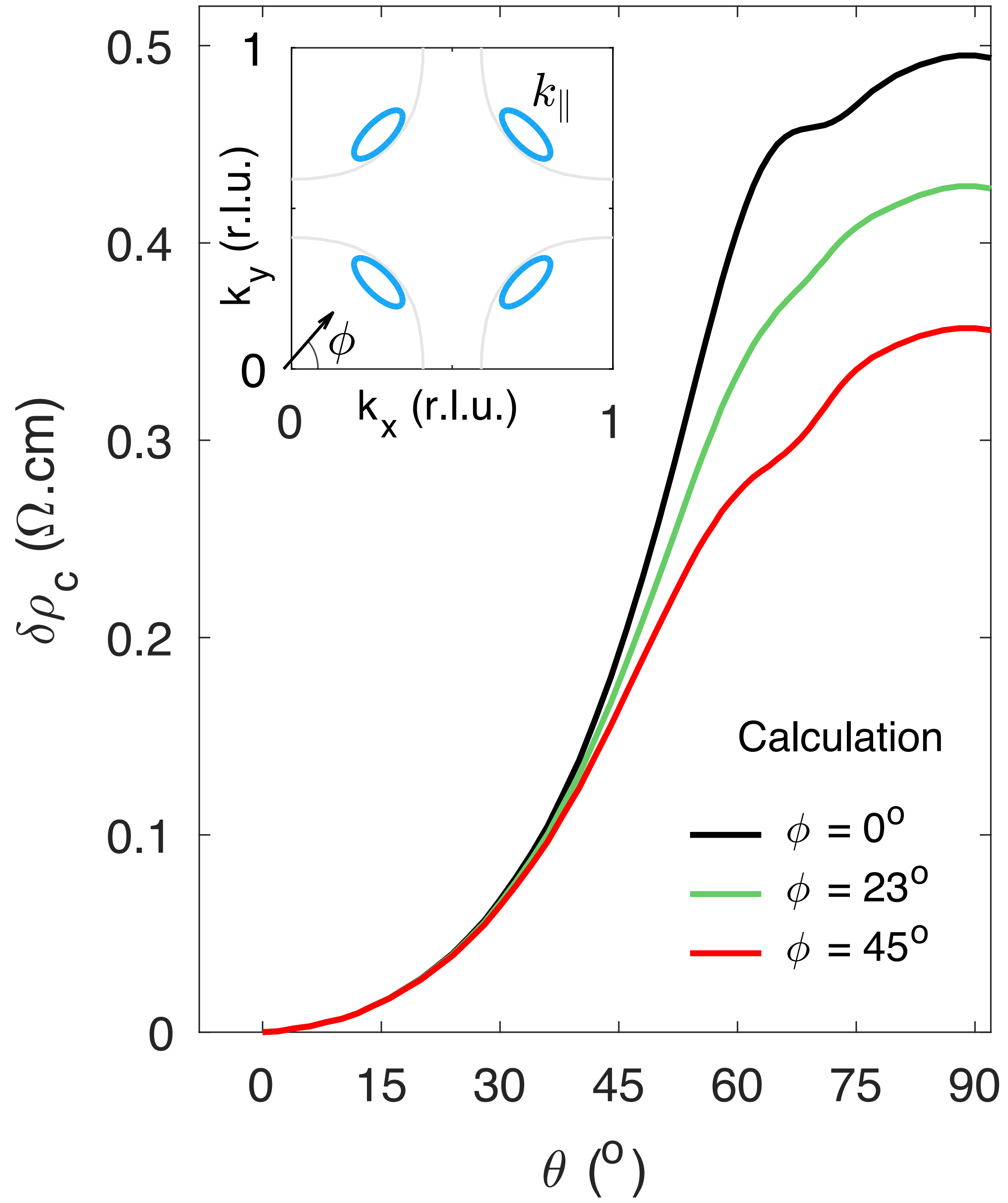

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(was expected by us!)

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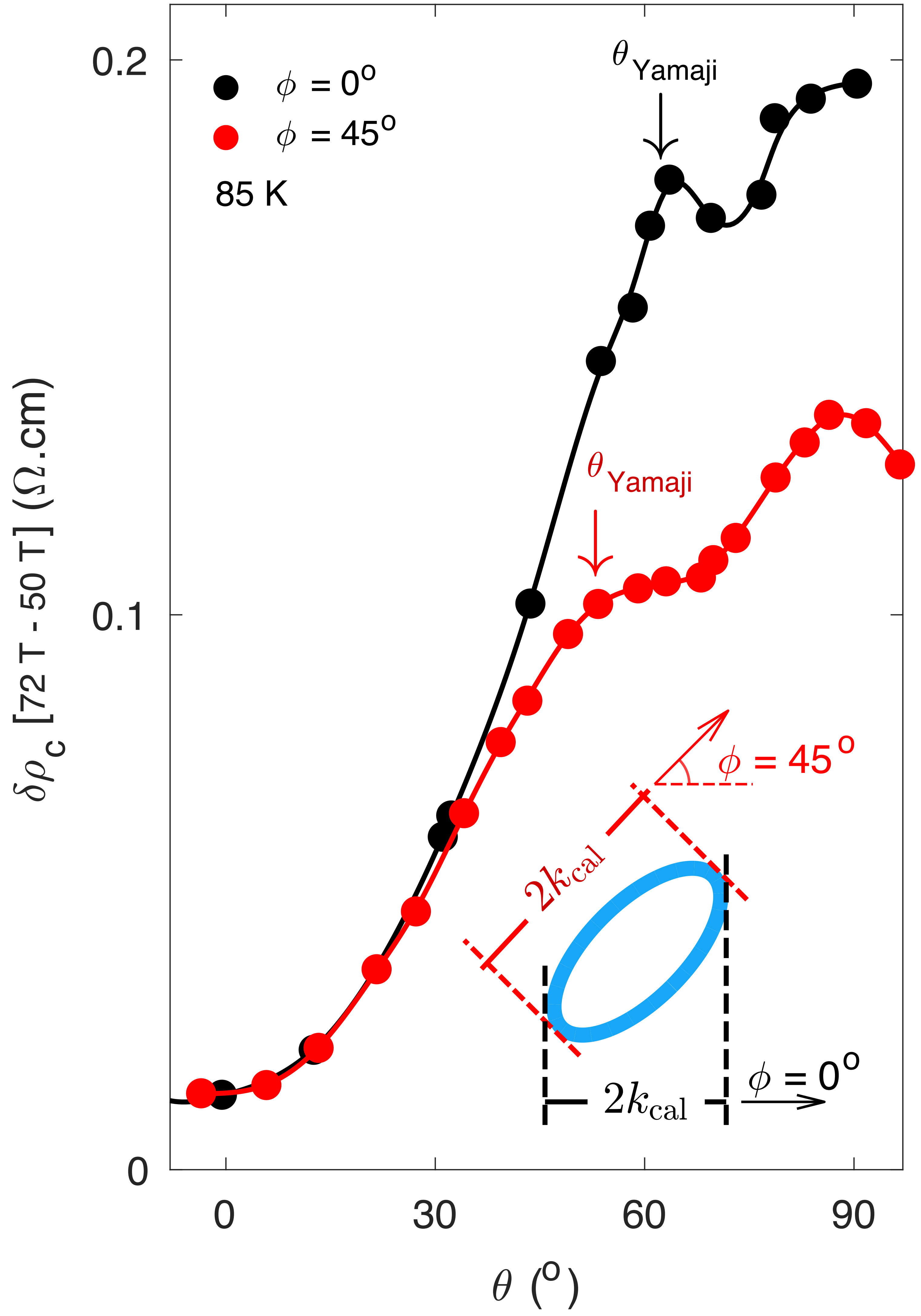

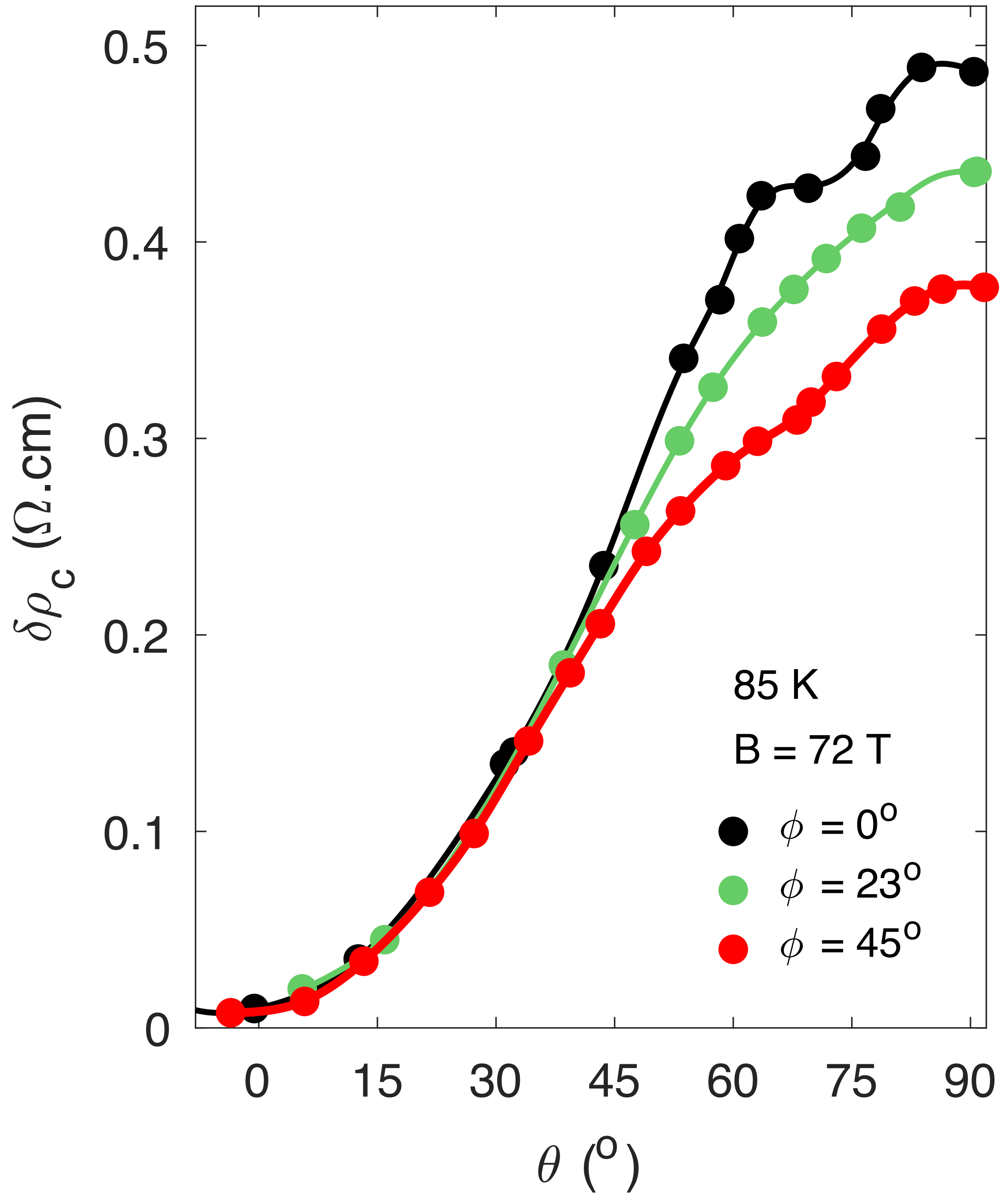

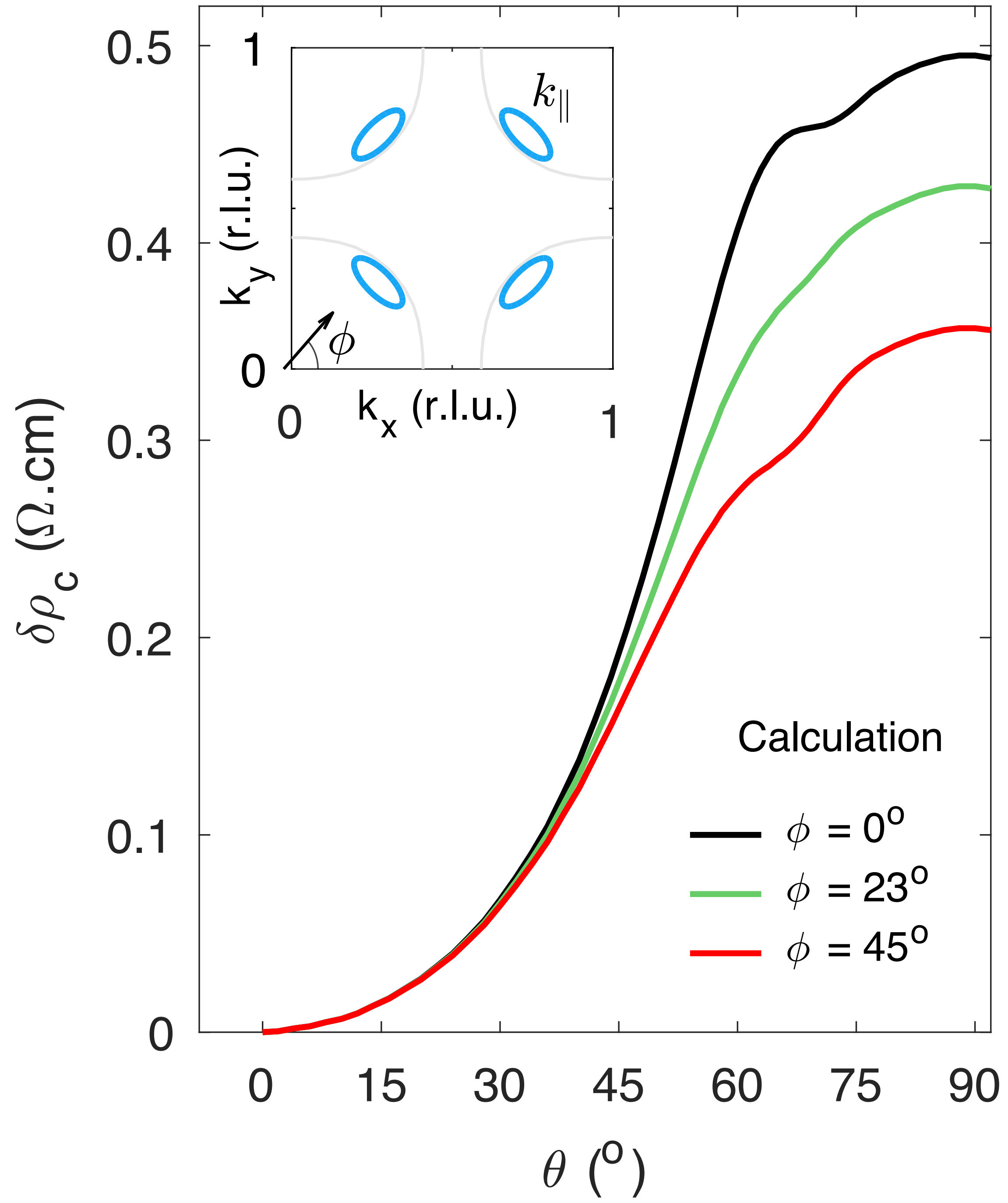

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
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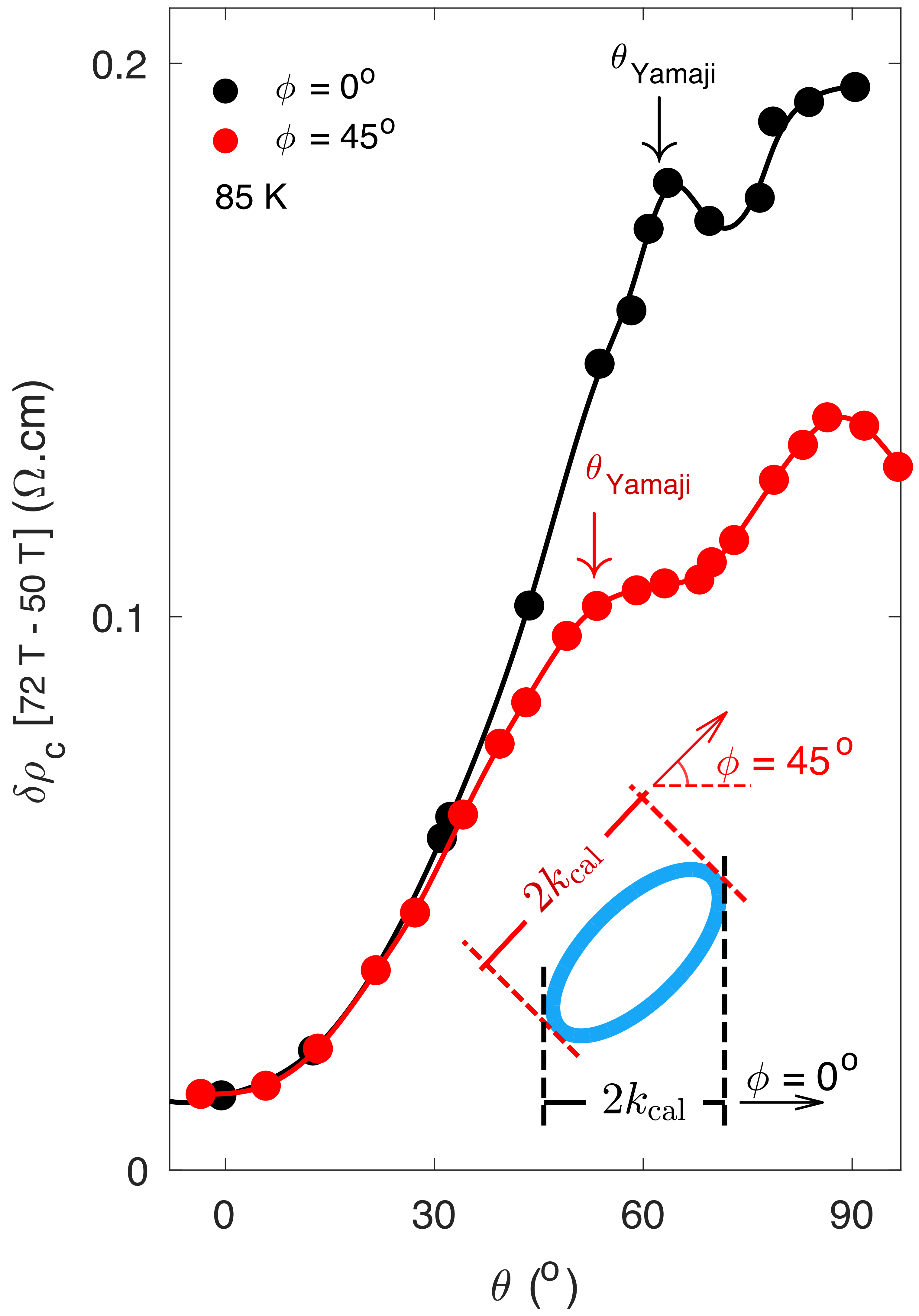

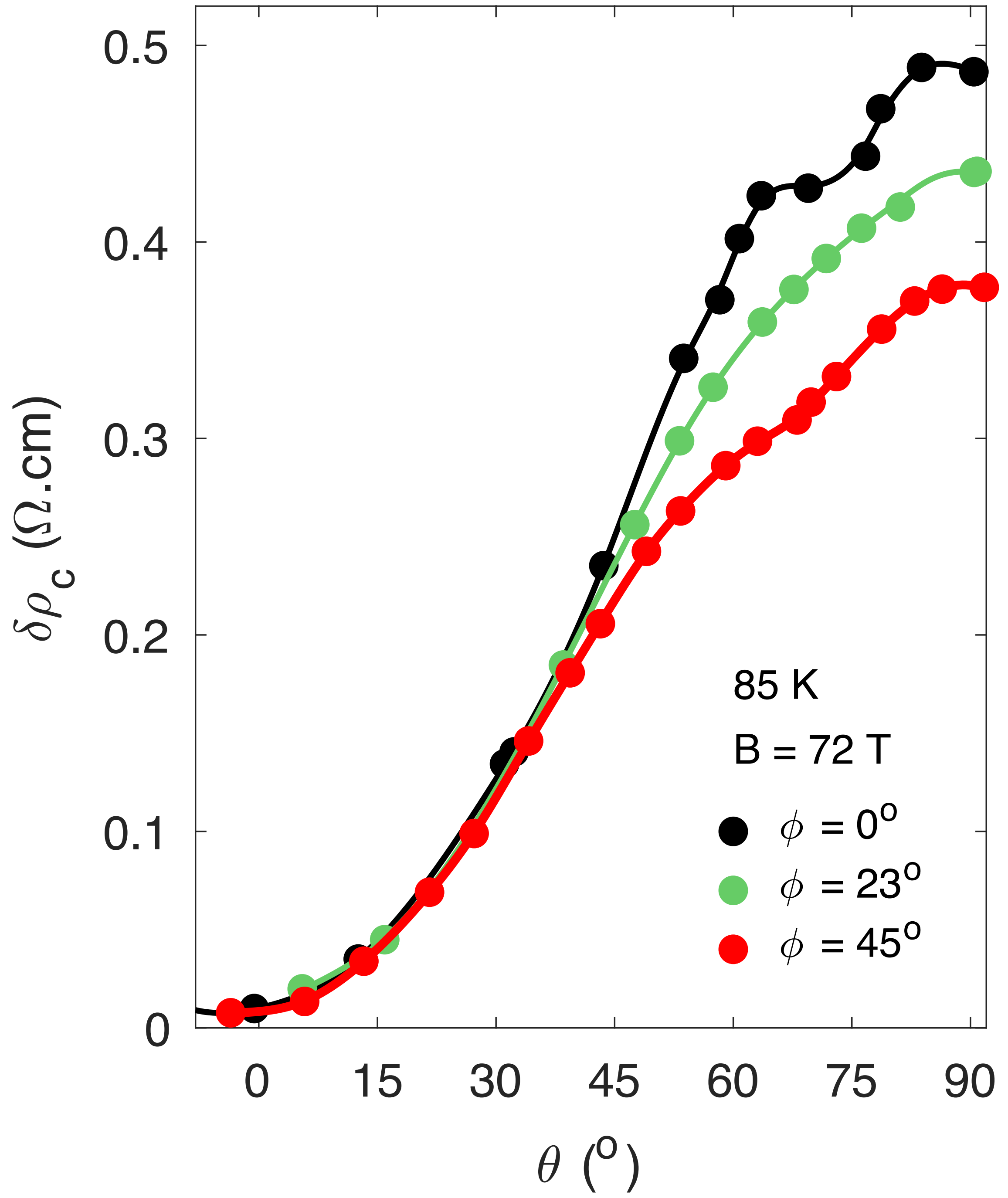

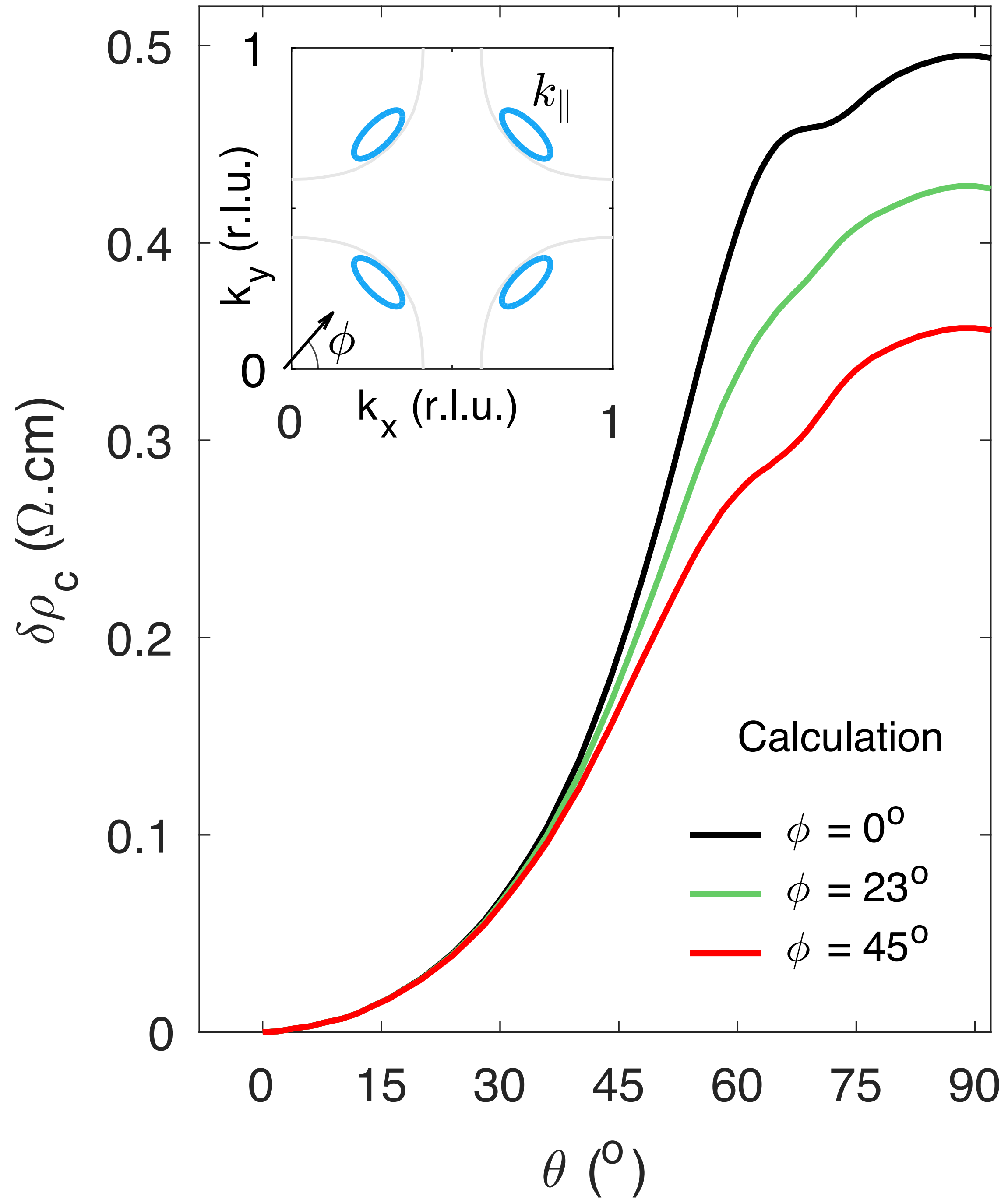

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(p/8 also in Yang-Rice-Zjang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

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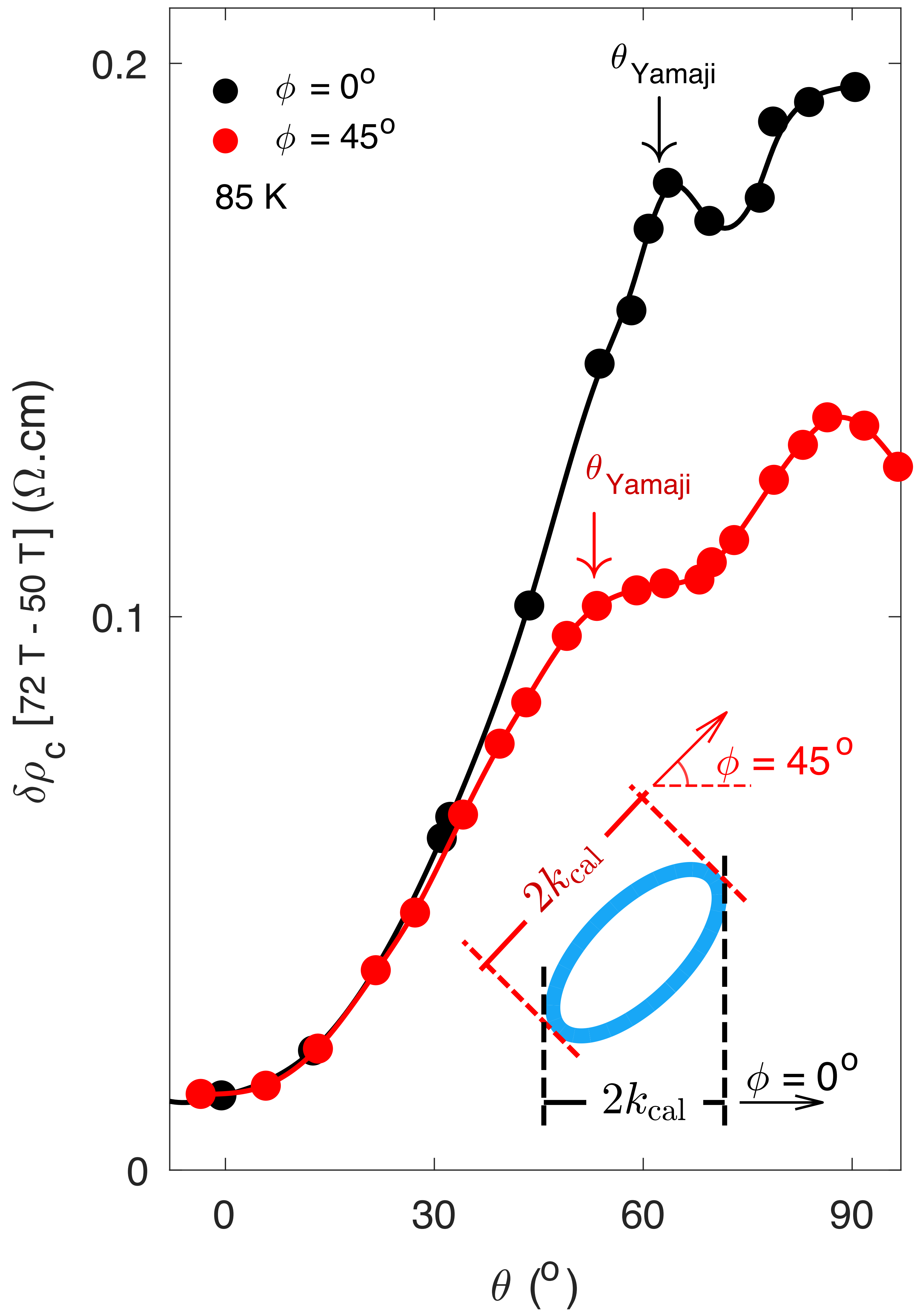

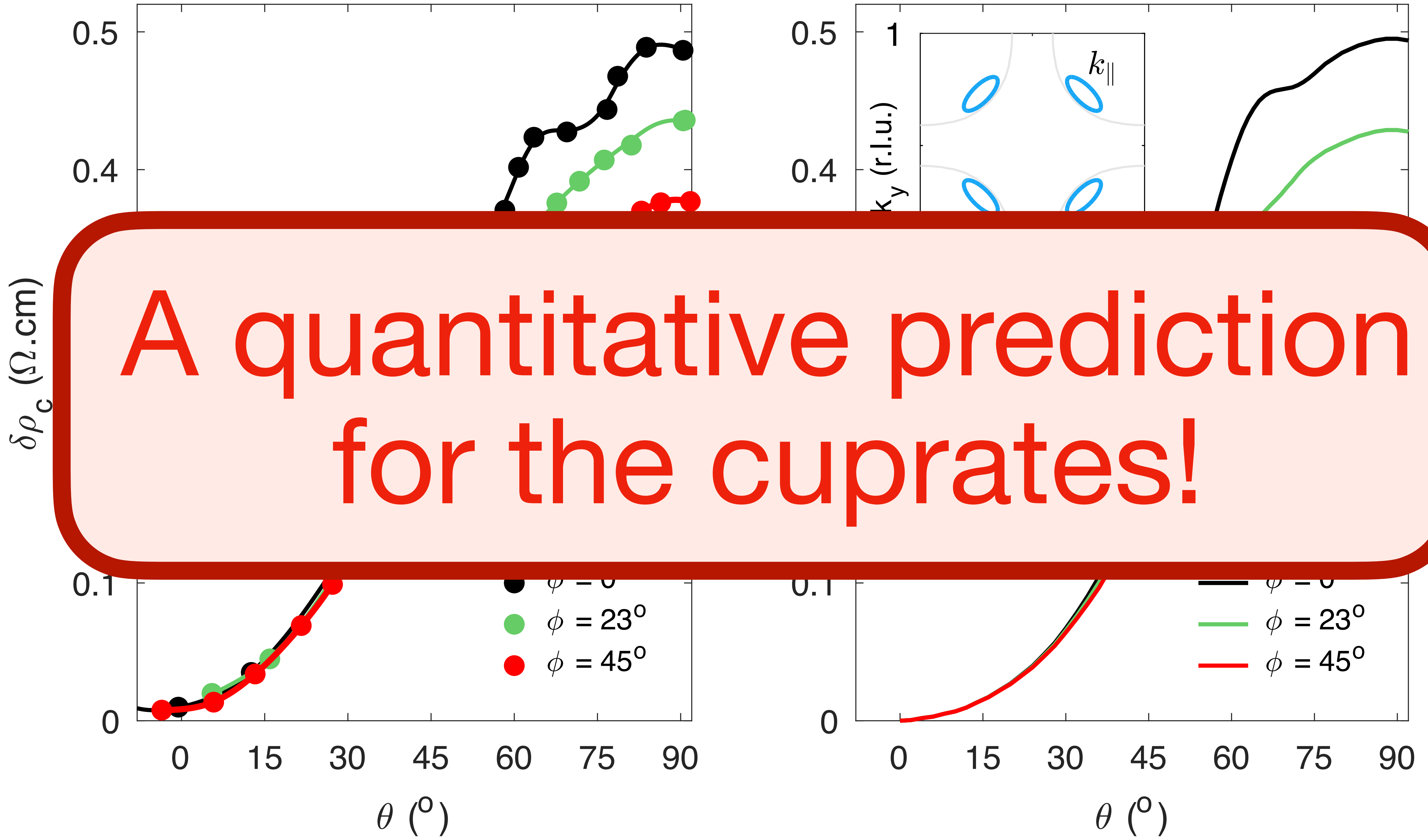

A quantitative prediction for the cuprates!

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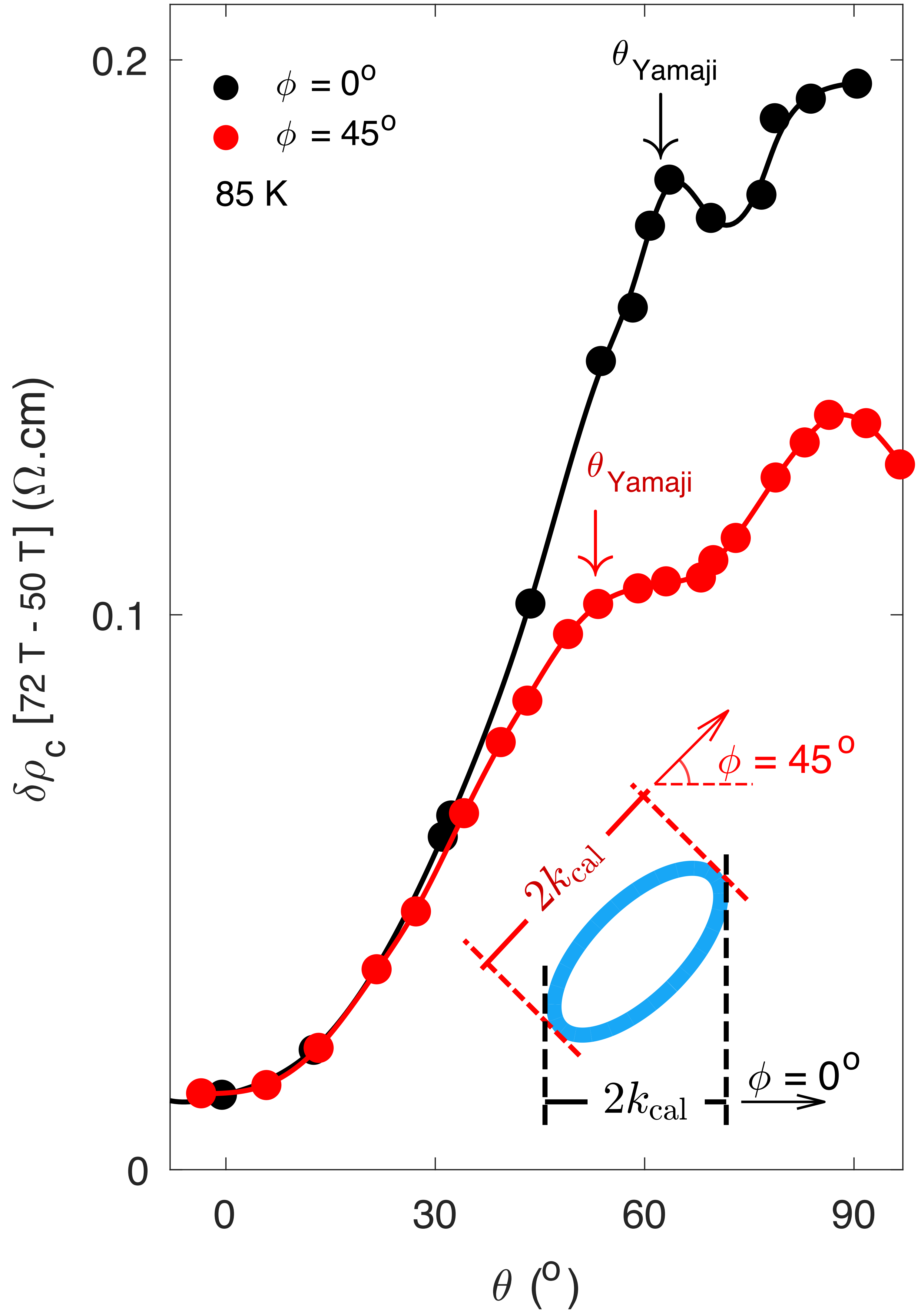

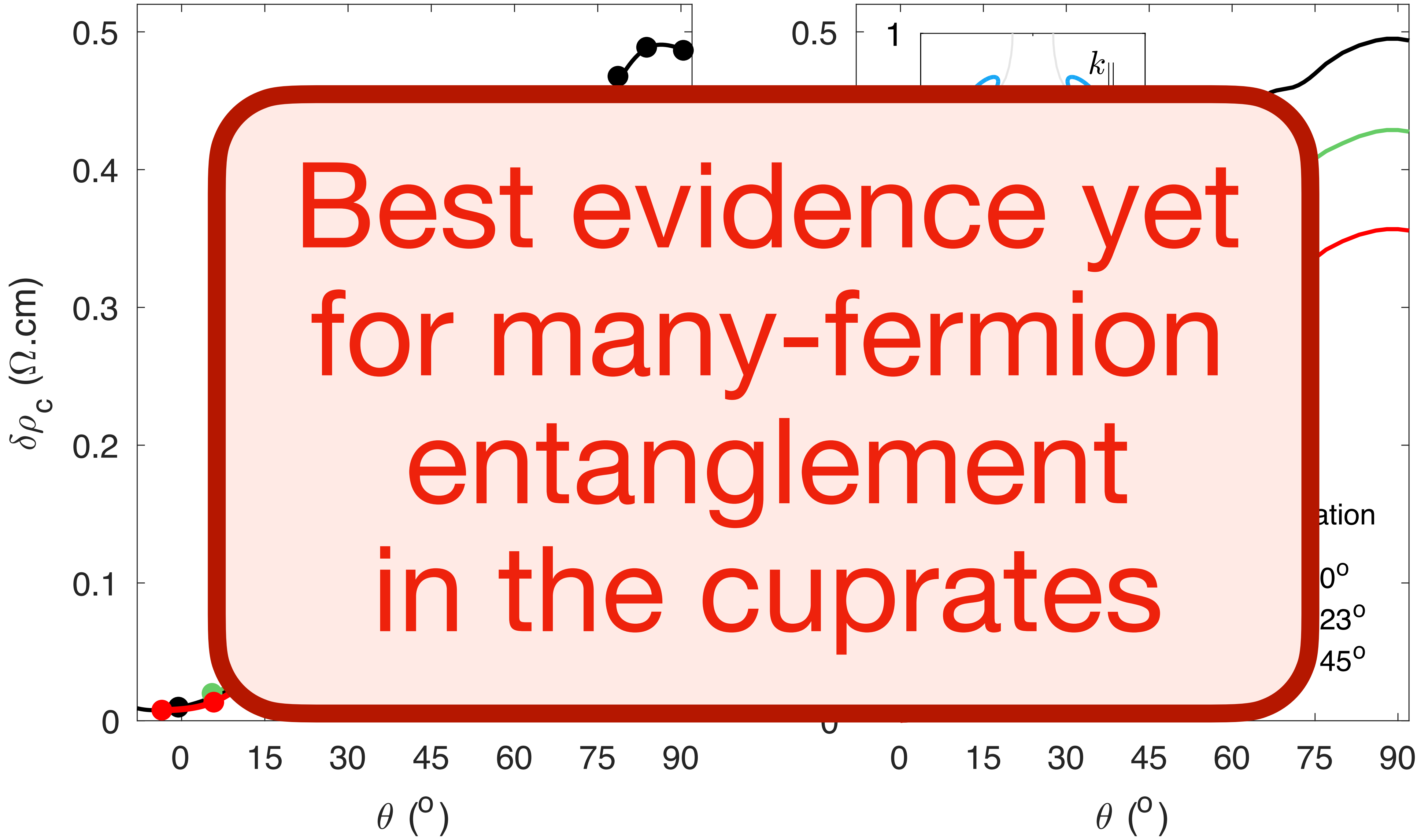

Best evidence yet for many-fermion entanglement in the cuprates

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Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Many fermion entanglement I:

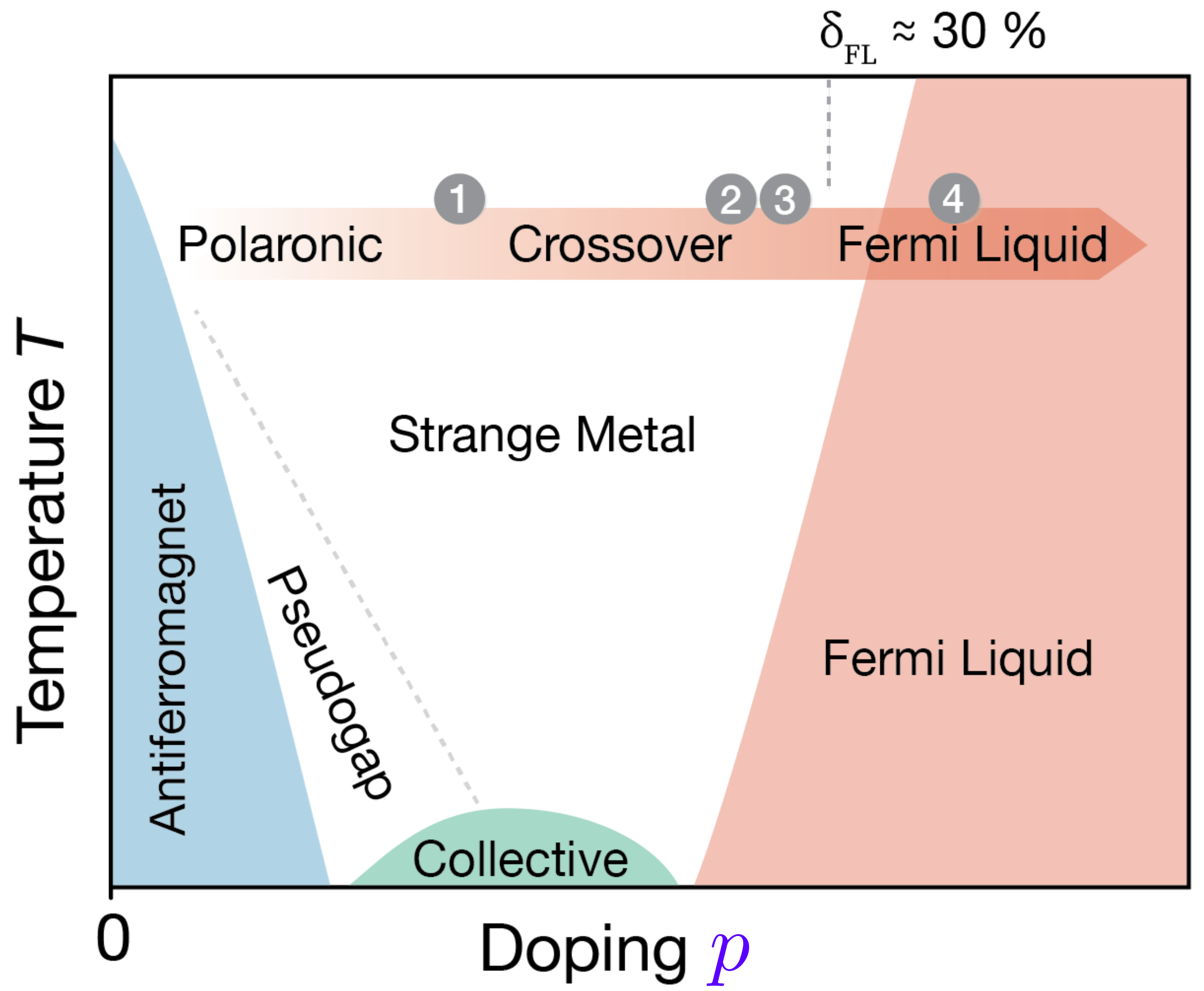
Wavefunction for FL*
and
observations on ultracold atoms

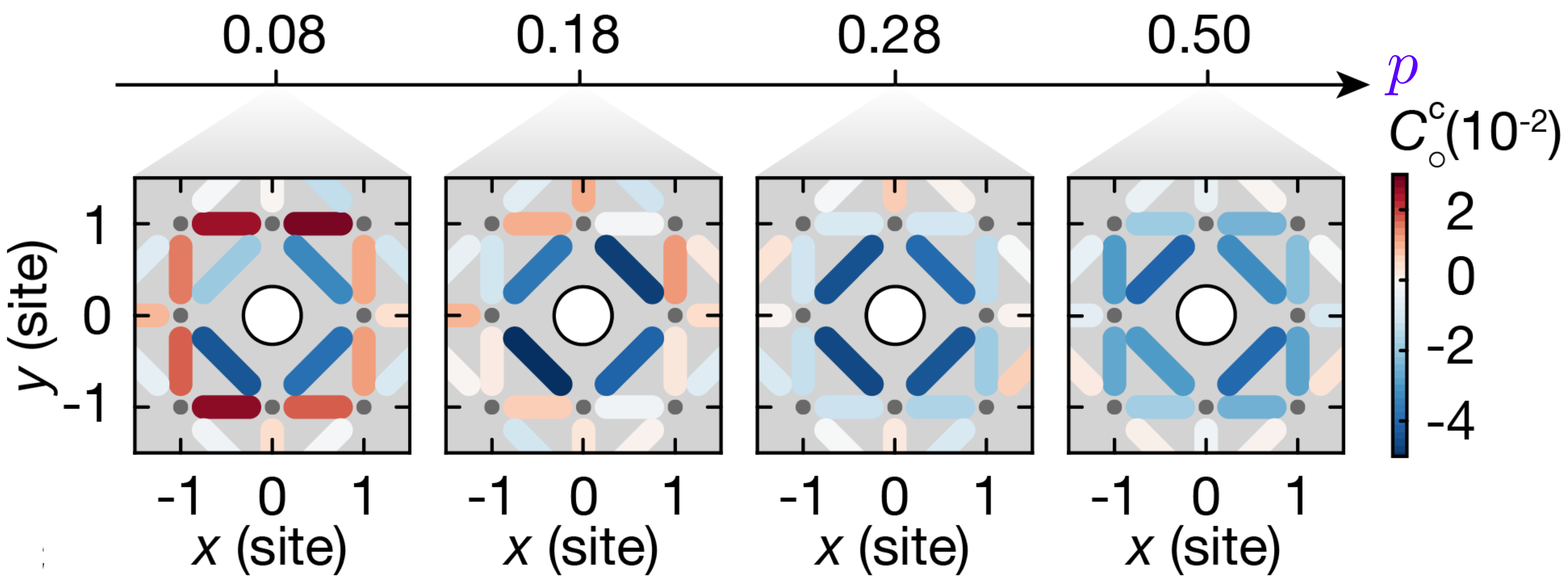
Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

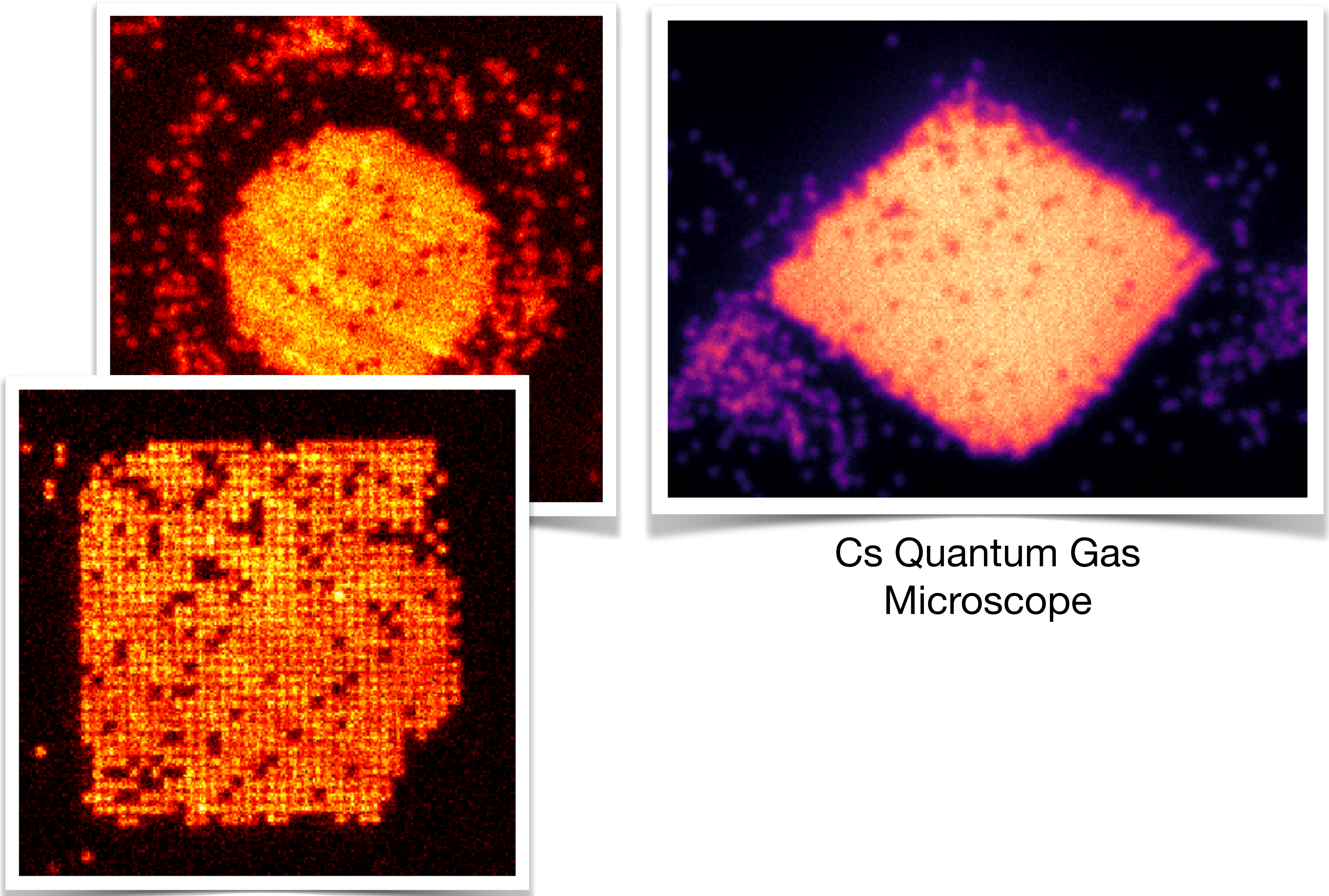
Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

Science **374** (2021) 82

Chalopin….Bloch, PNAS **123**, e2525539123 (2026)





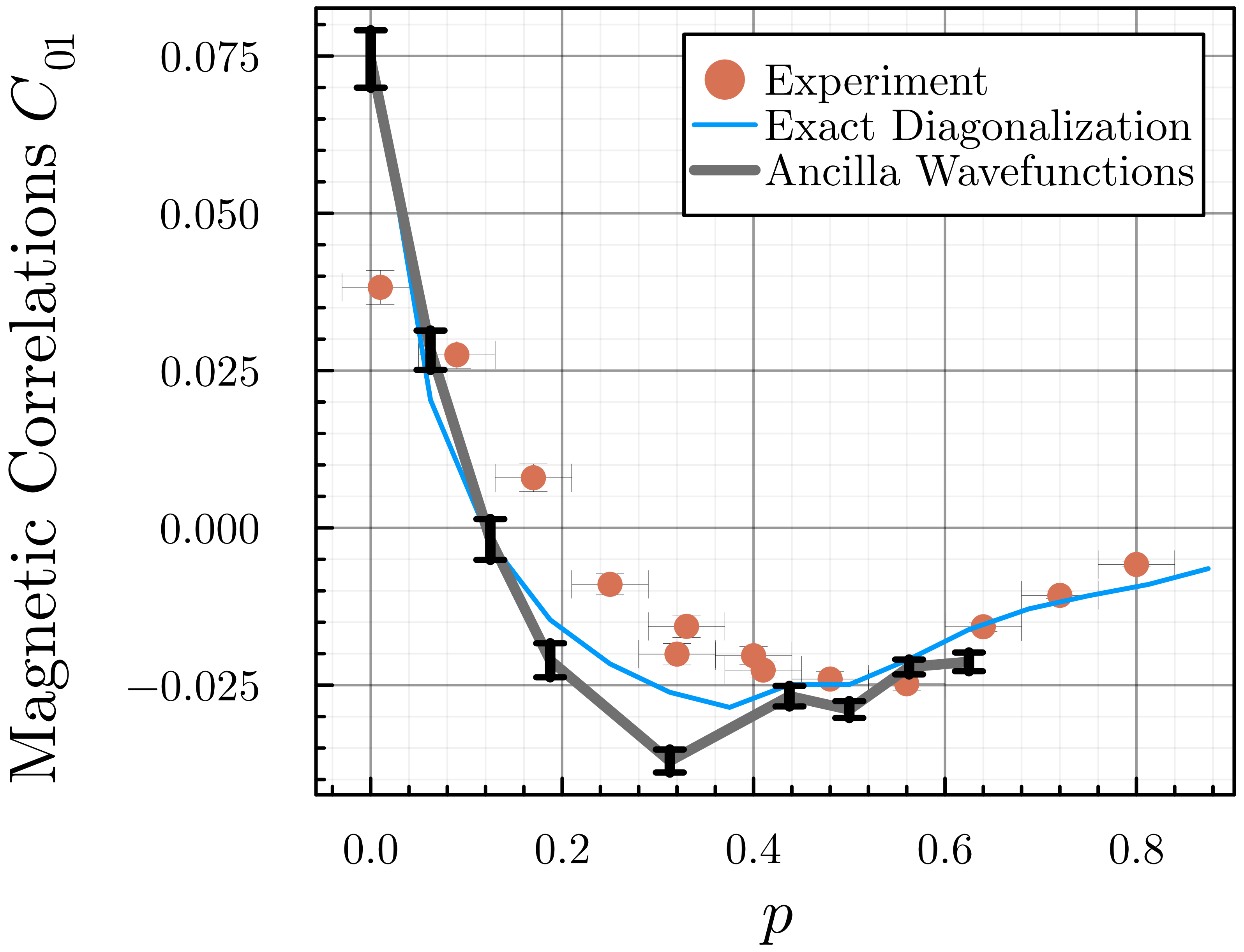


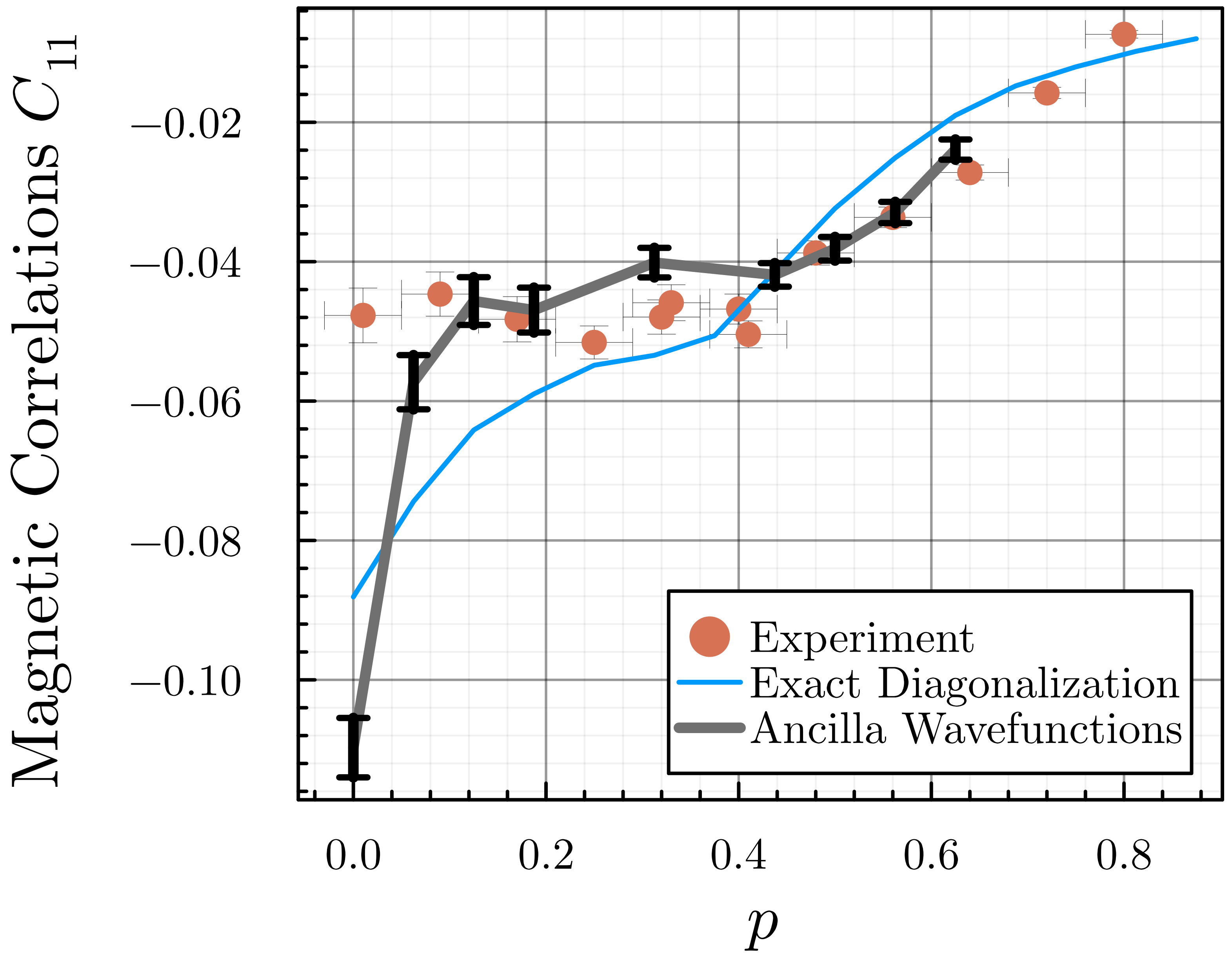
Rb Quantum Gas Microscope

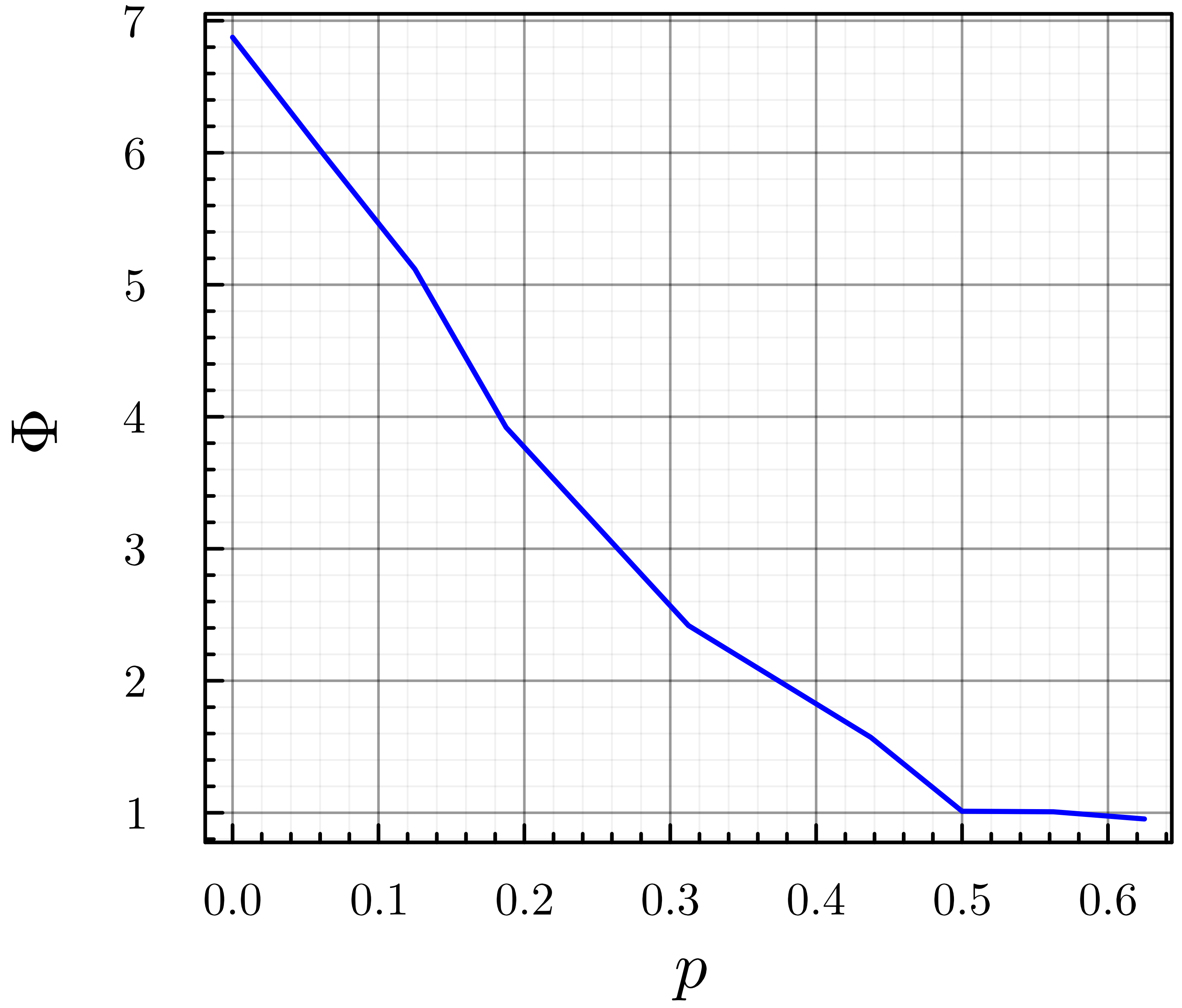
Cs Quantum Gas Microscope

see also: C. Chiu *et al.* Phys. Rev. Lett. **120**, 243201 (2018)
Idea: J.-S. Bernier *et al.* Phys. Rev. A **79**, 061601 (2009)
T.-L. Ho & Q. Zhou arXiv:0911.5506

QMC for FL* of Hubbard model







Quantum Monte Carlo on
Ancilla Layer trial wavefunction
with Φ a variational parameter
minimizing Hubbard Hamiltonian

FL*: $\Phi \neq 0$; FL: $\Phi = 0$

L. Shackleton and Shiwei Zhang, arXiv:2408.02190

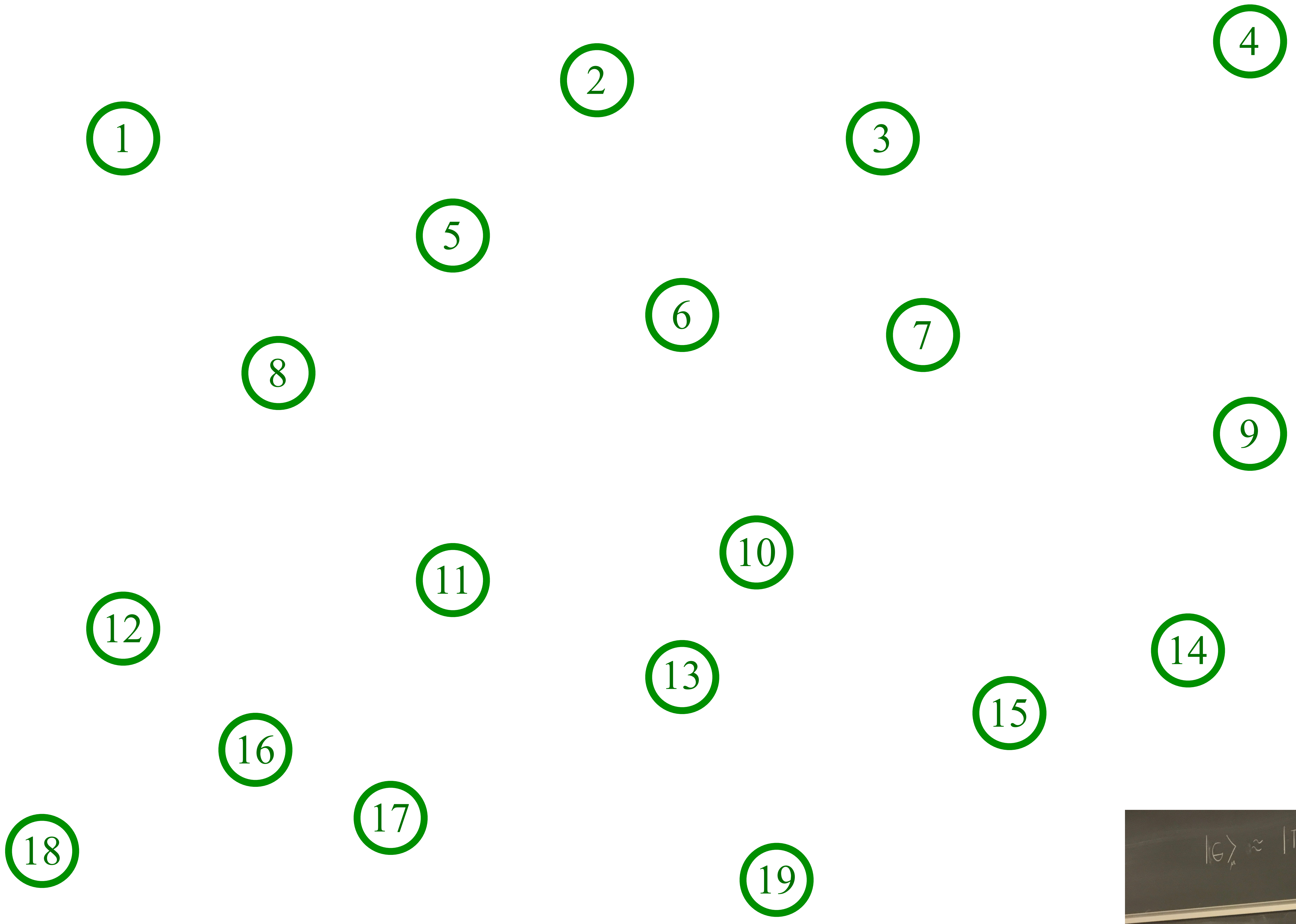
Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, PNAS **122**, e2504261122 (2025)

Many fermion entanglement II:

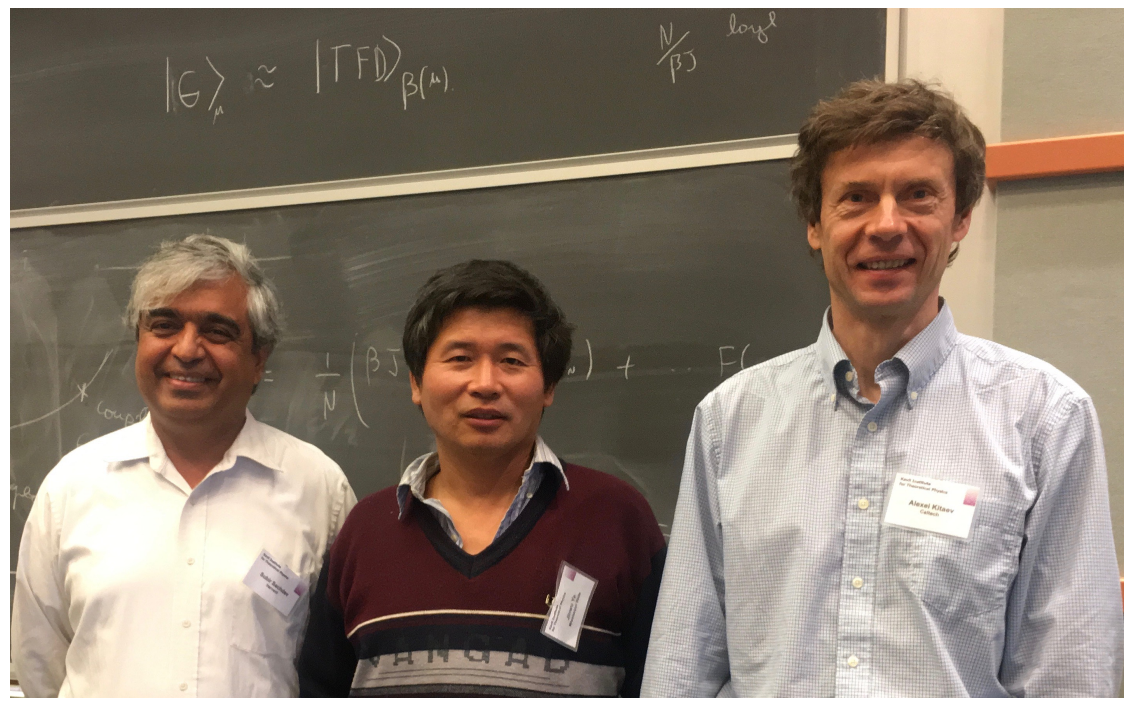
Sachdev-Ye-Kitaev model

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

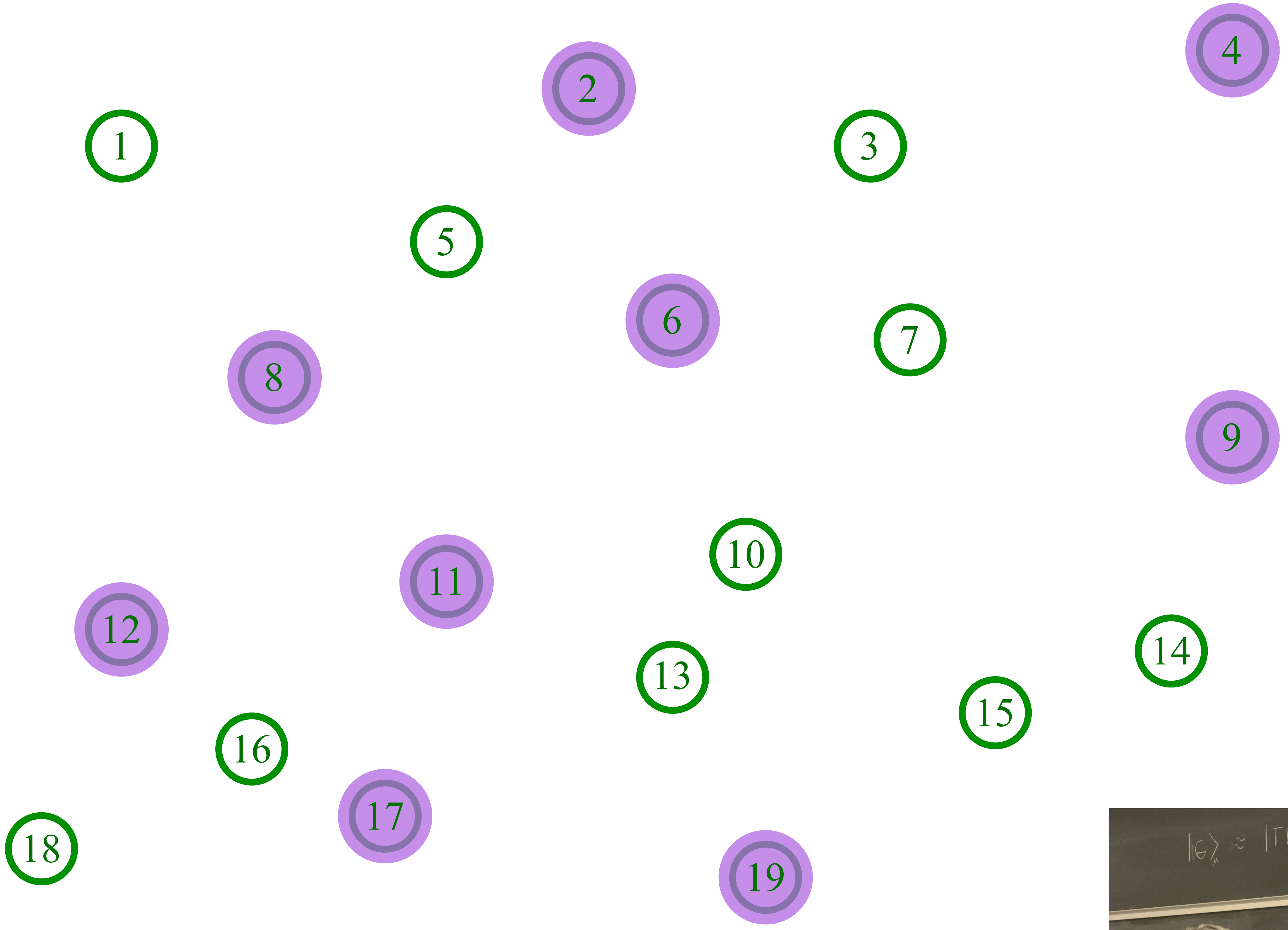


Pick a set of random positions

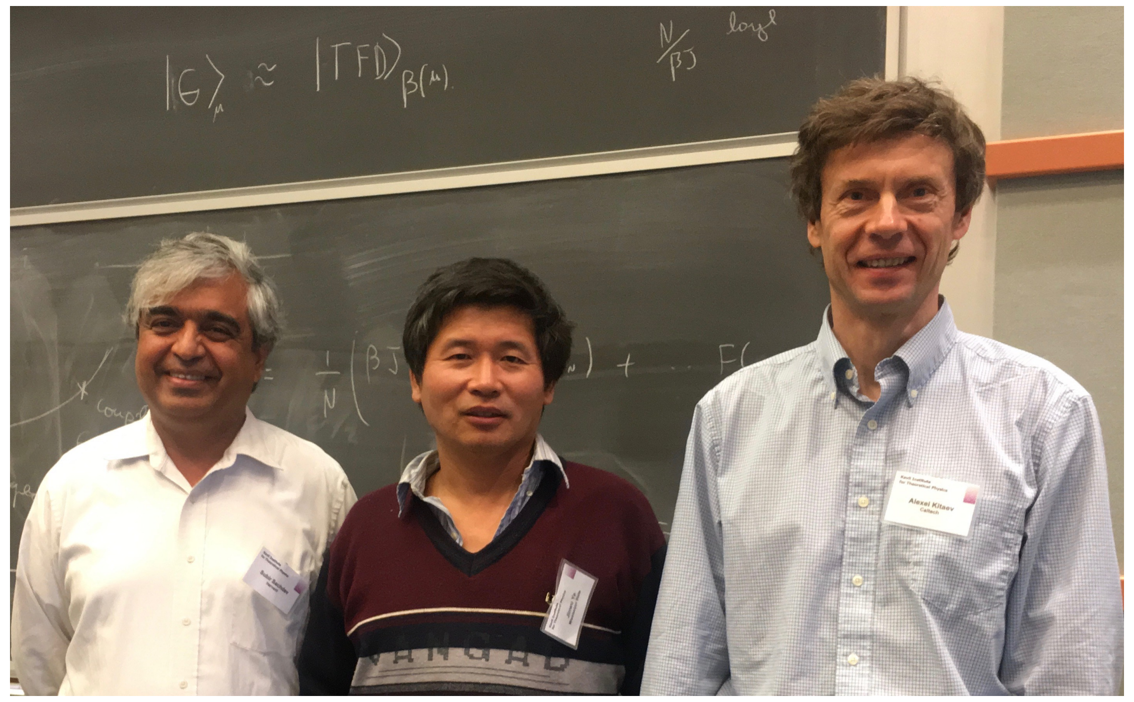


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



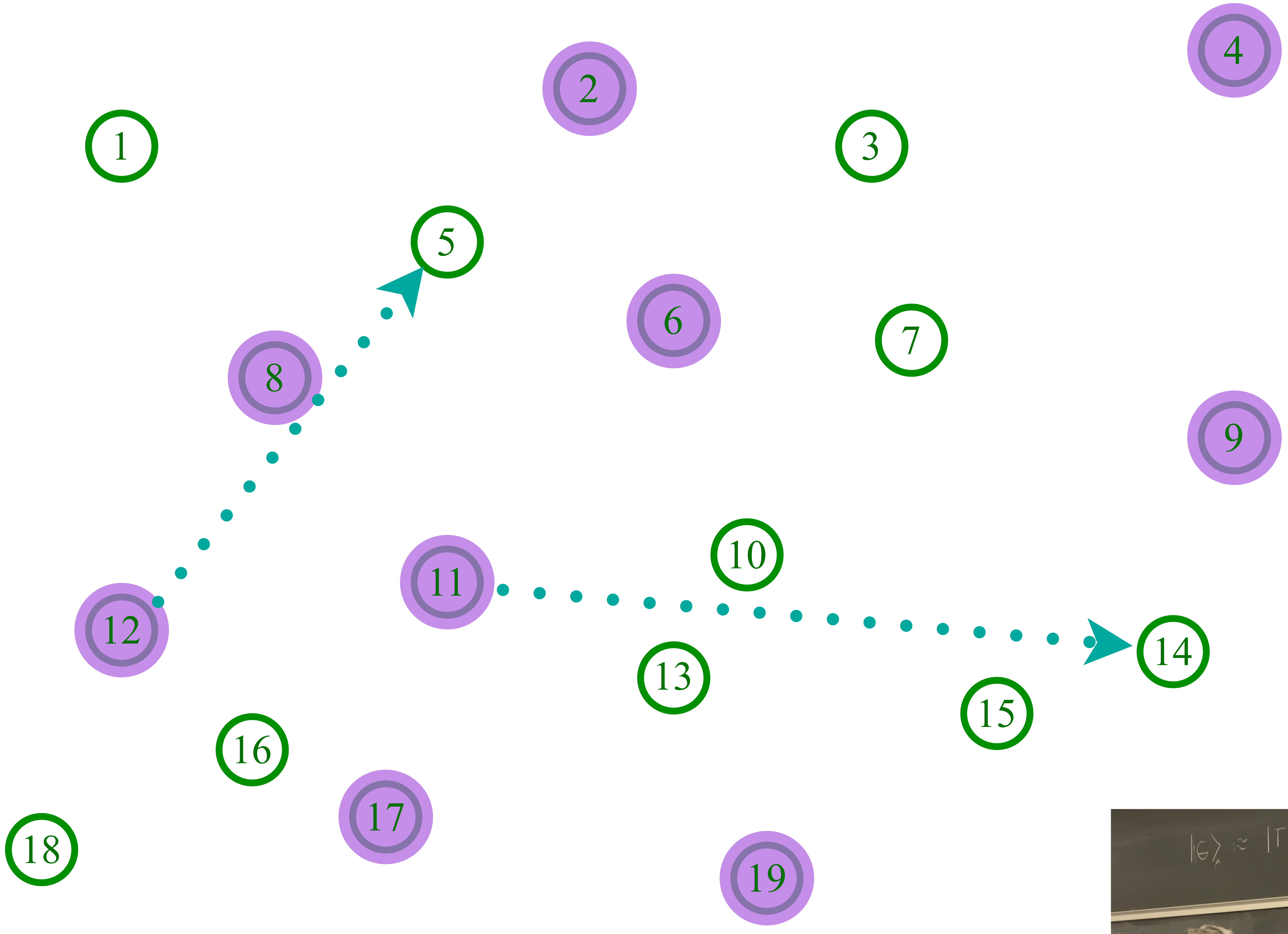
Place electrons randomly on some sites



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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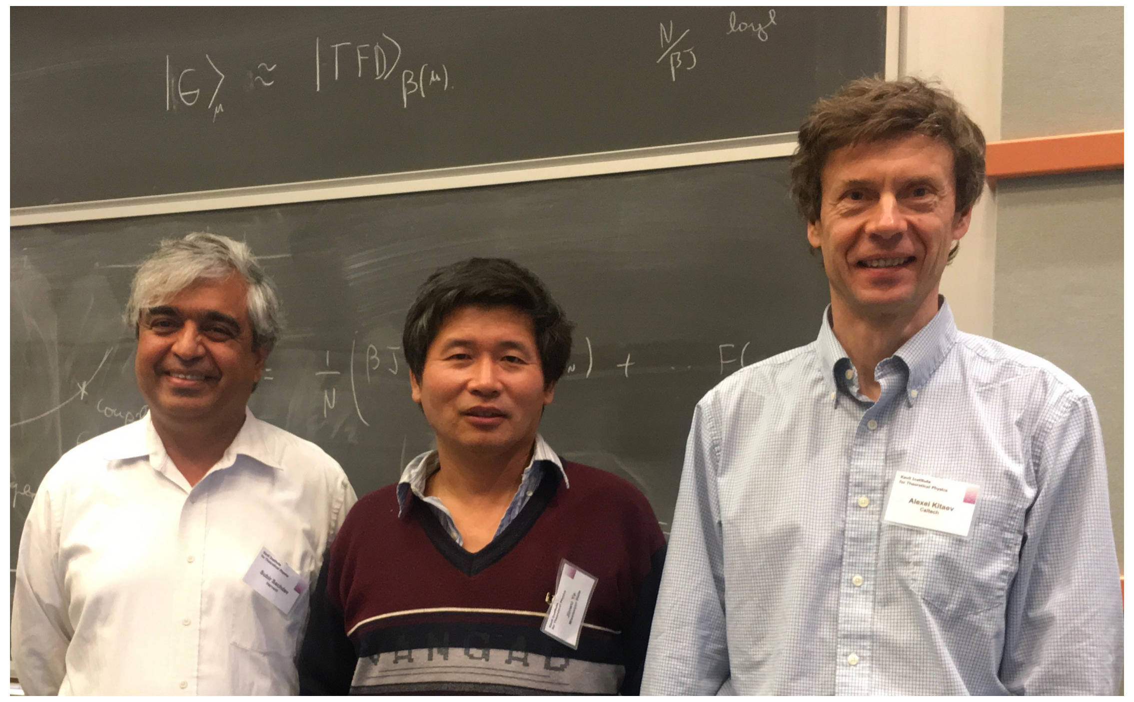

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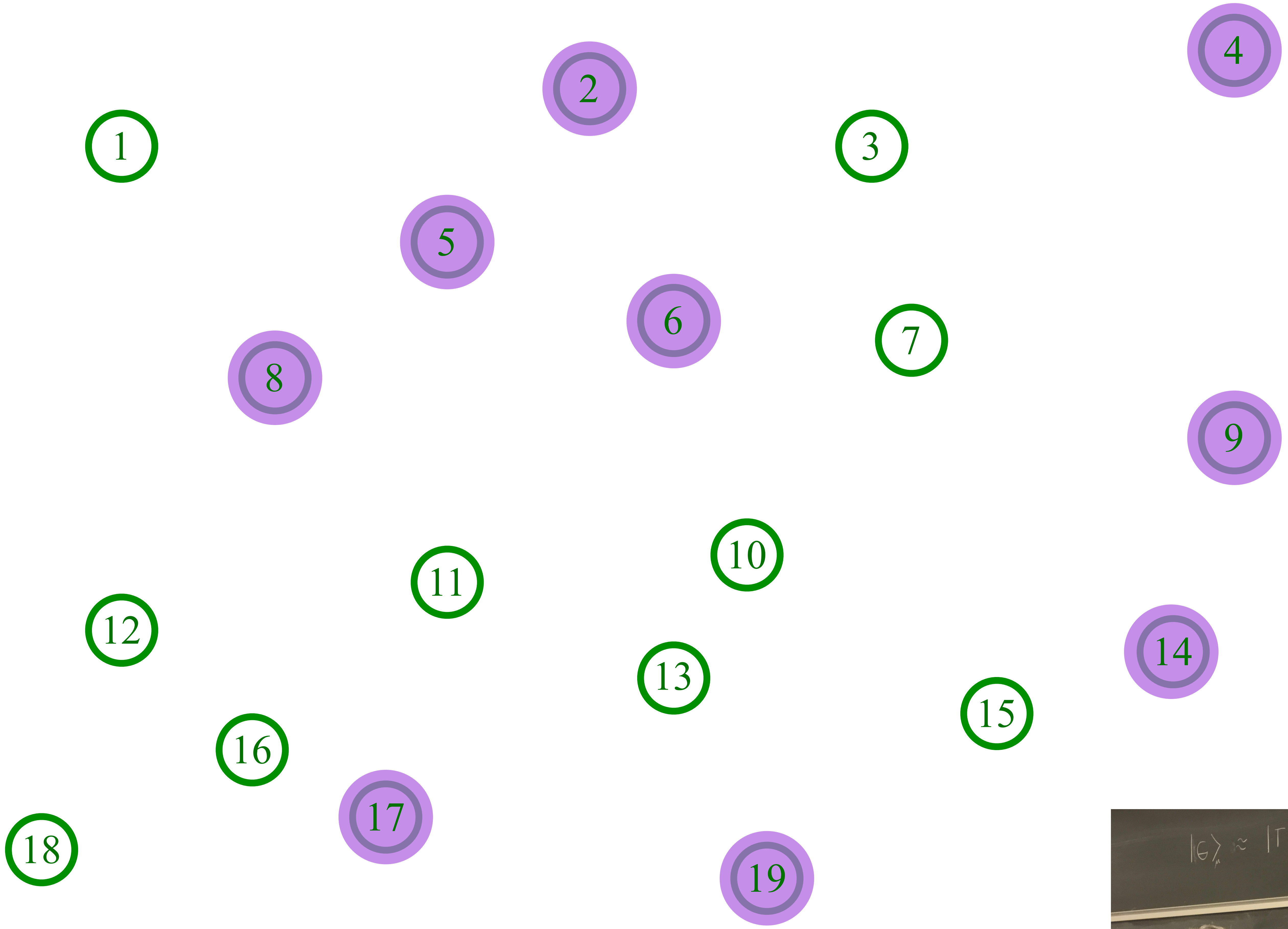
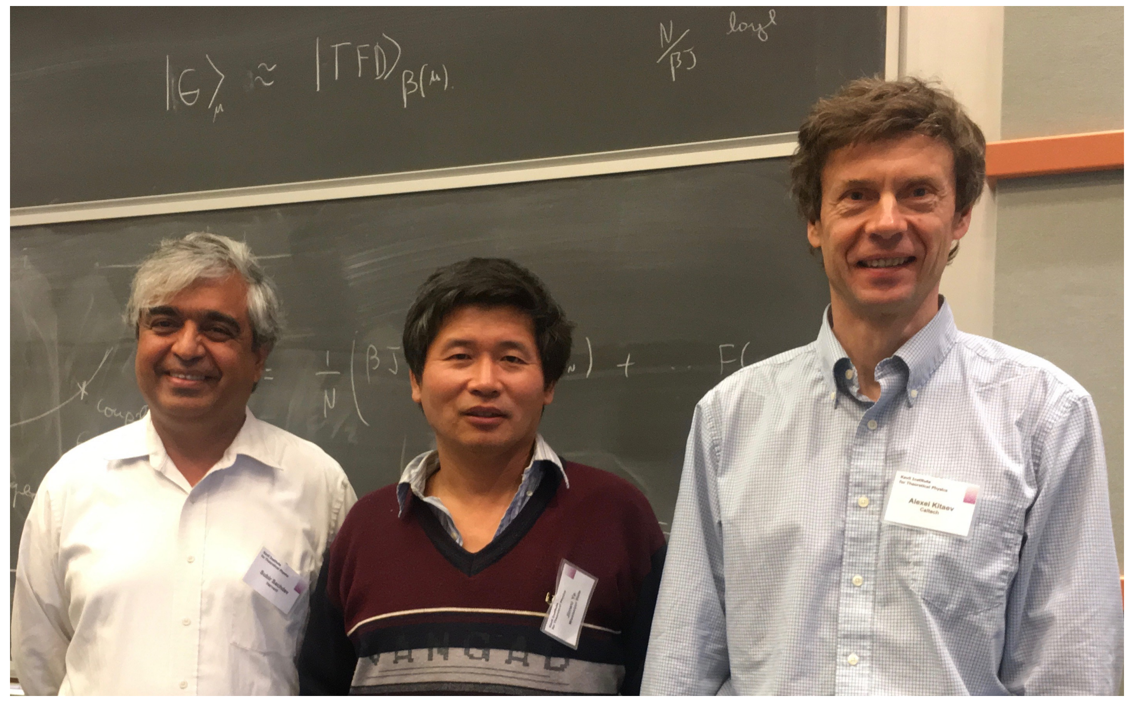


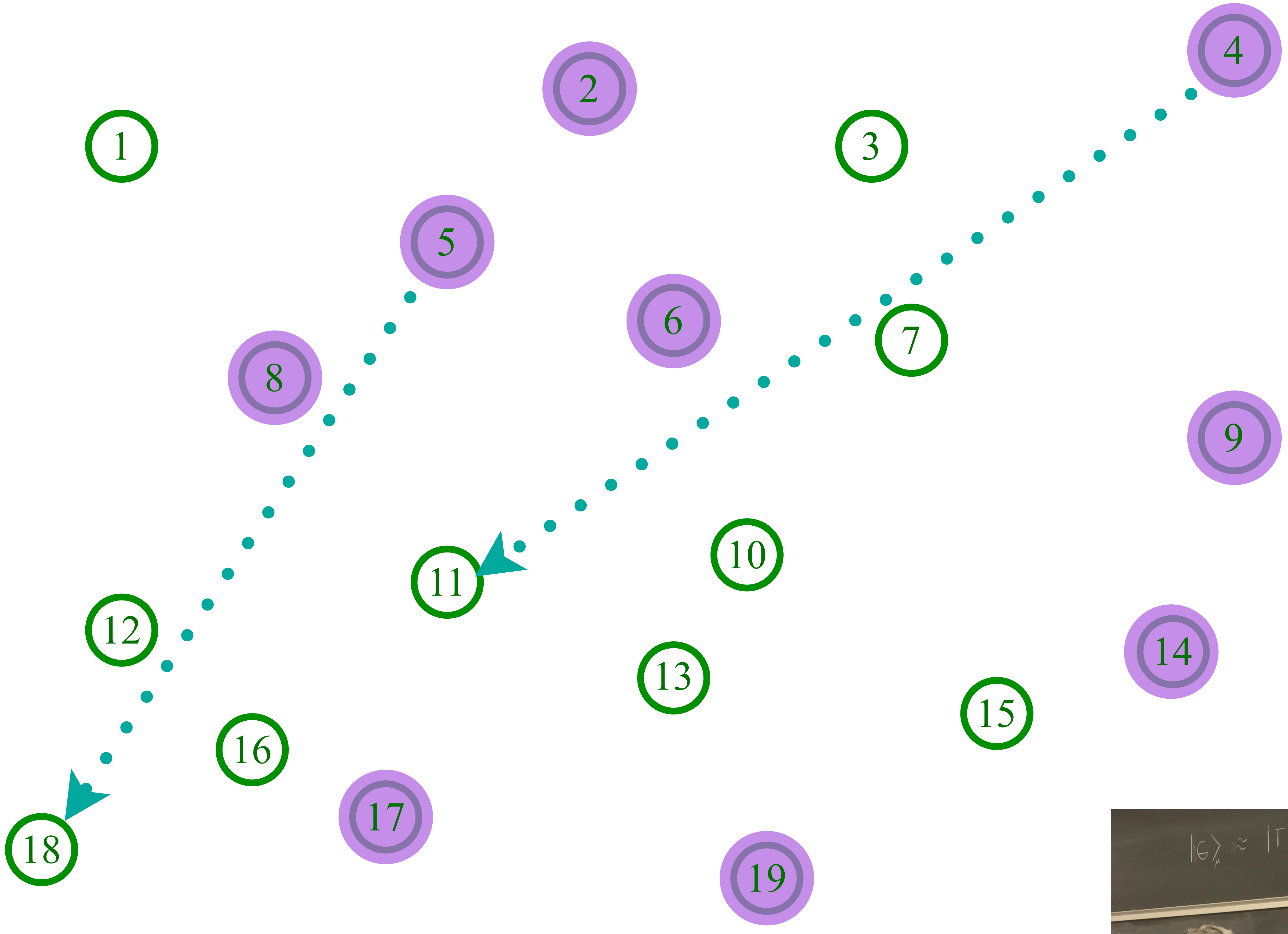
Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



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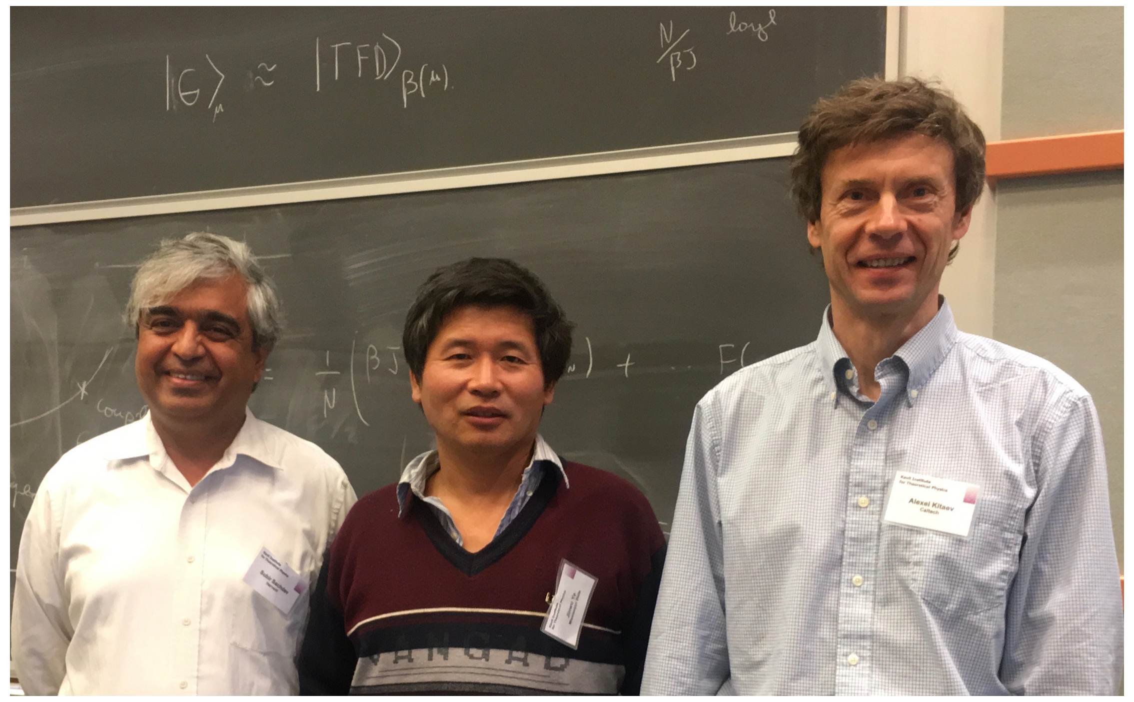

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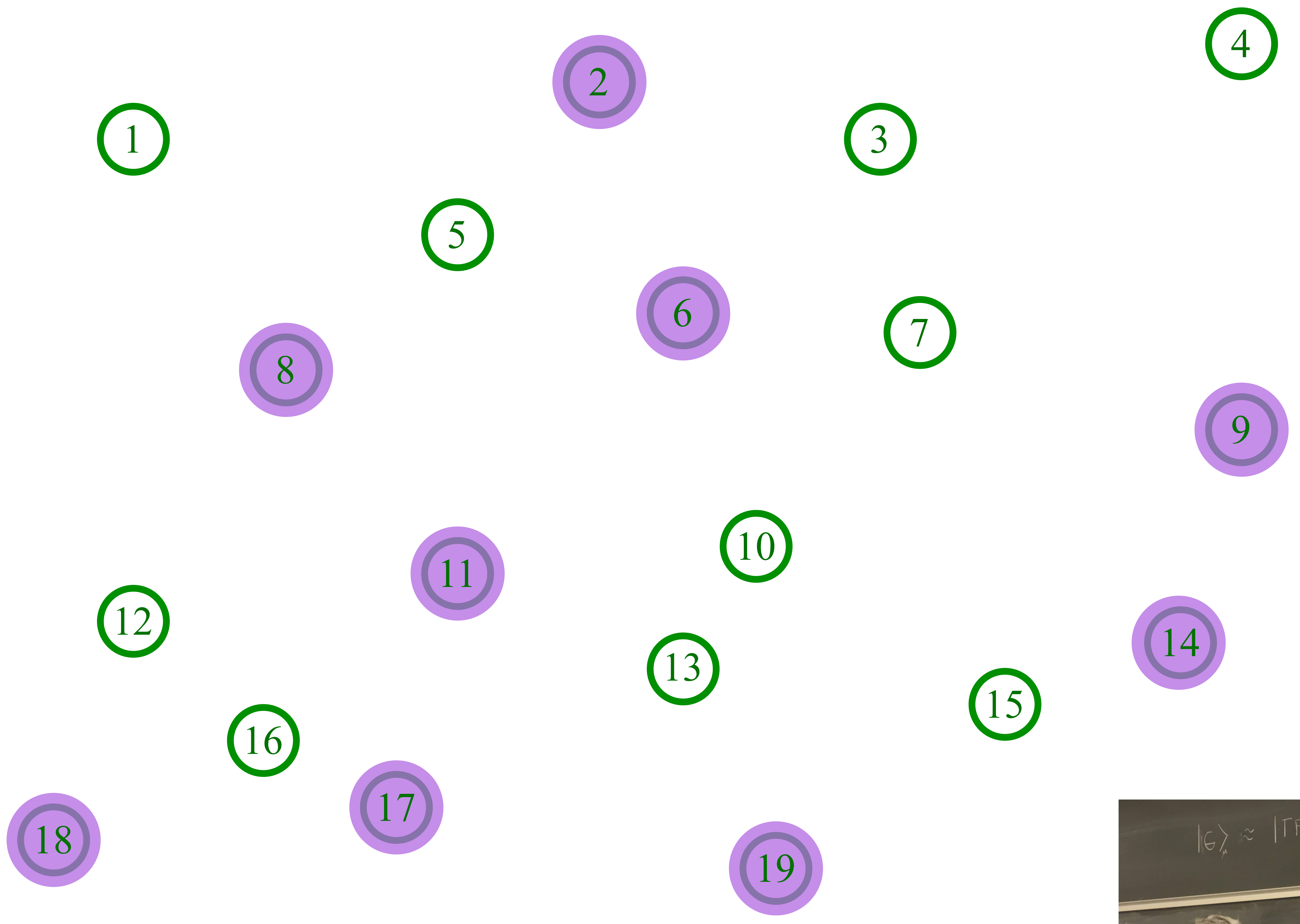

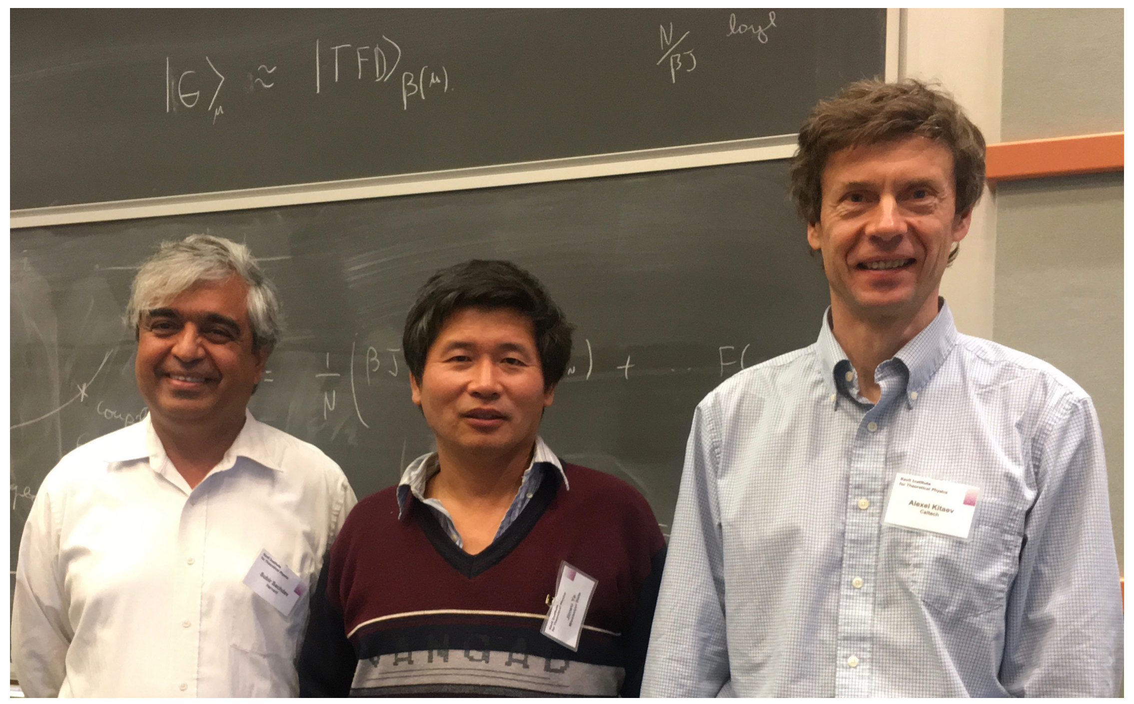

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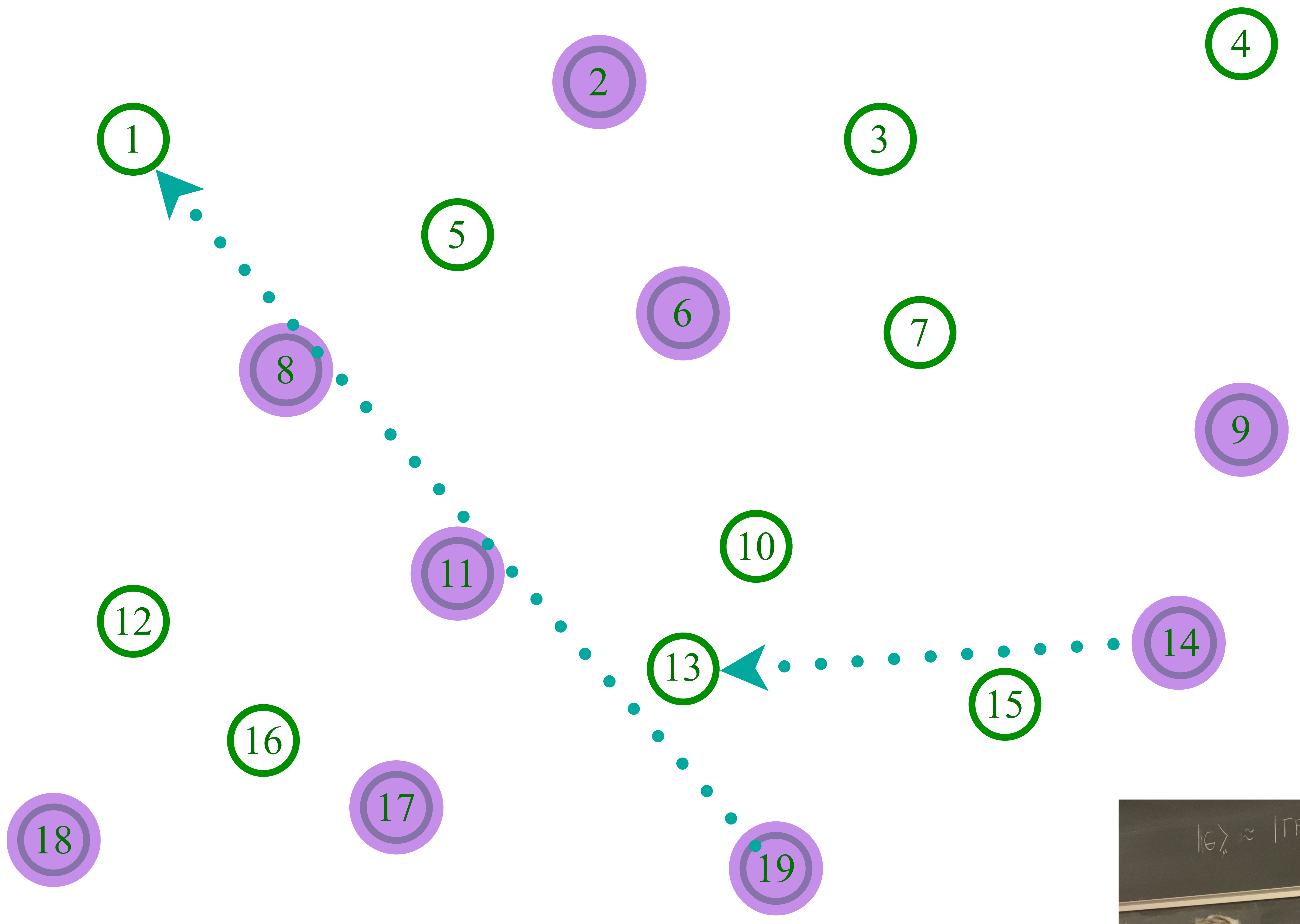

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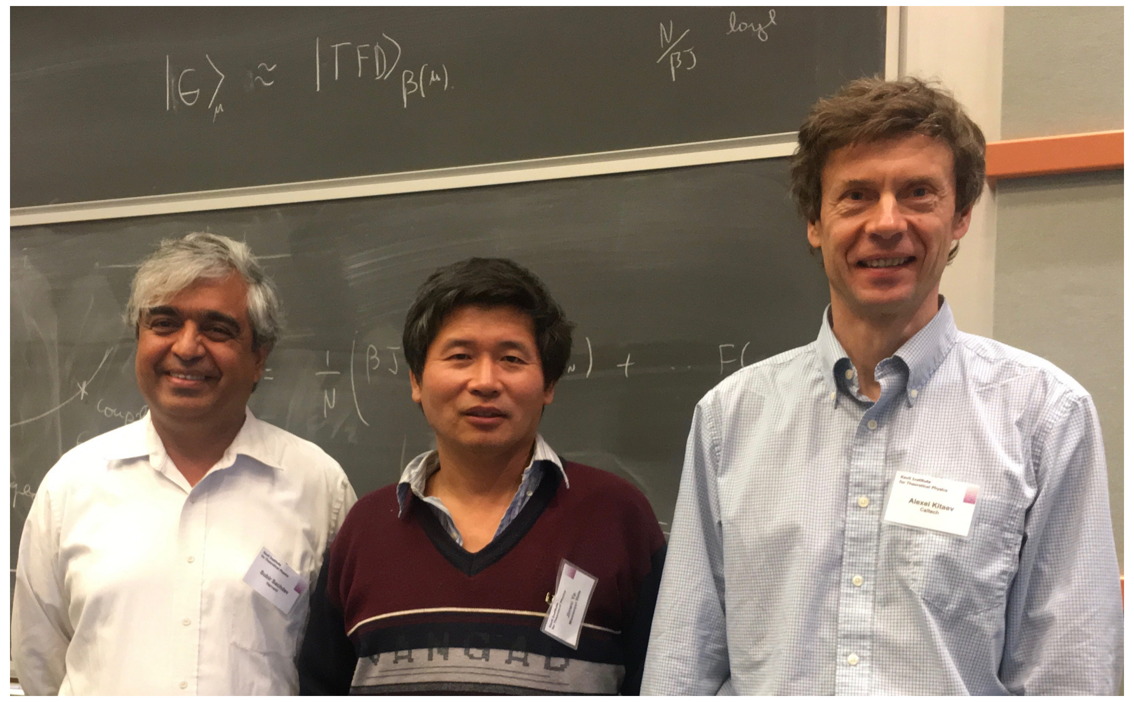

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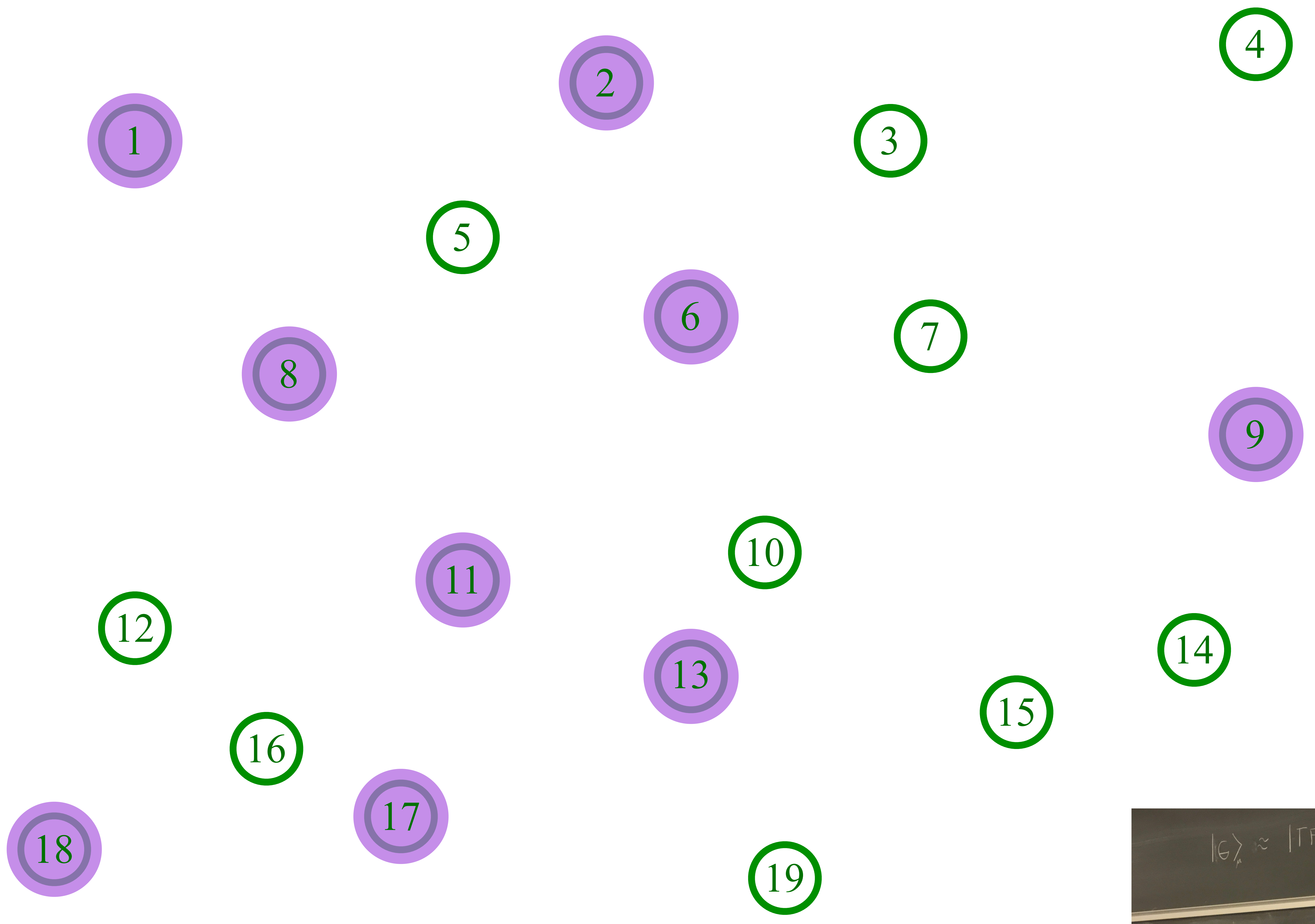

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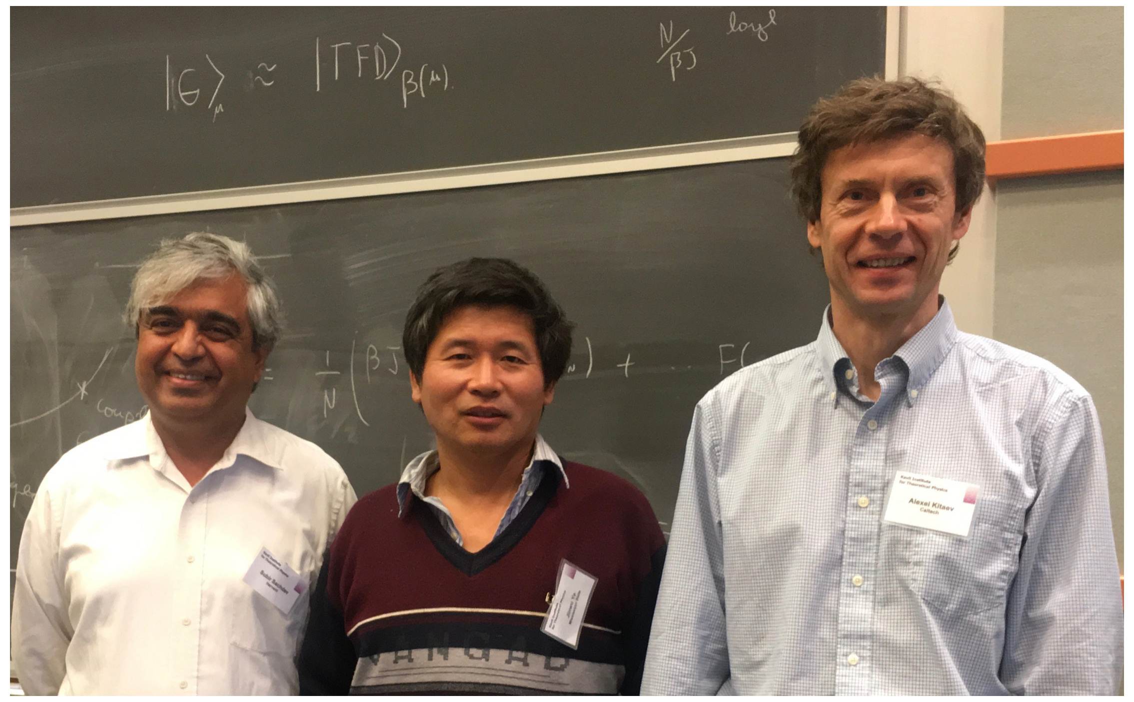

The SYK model

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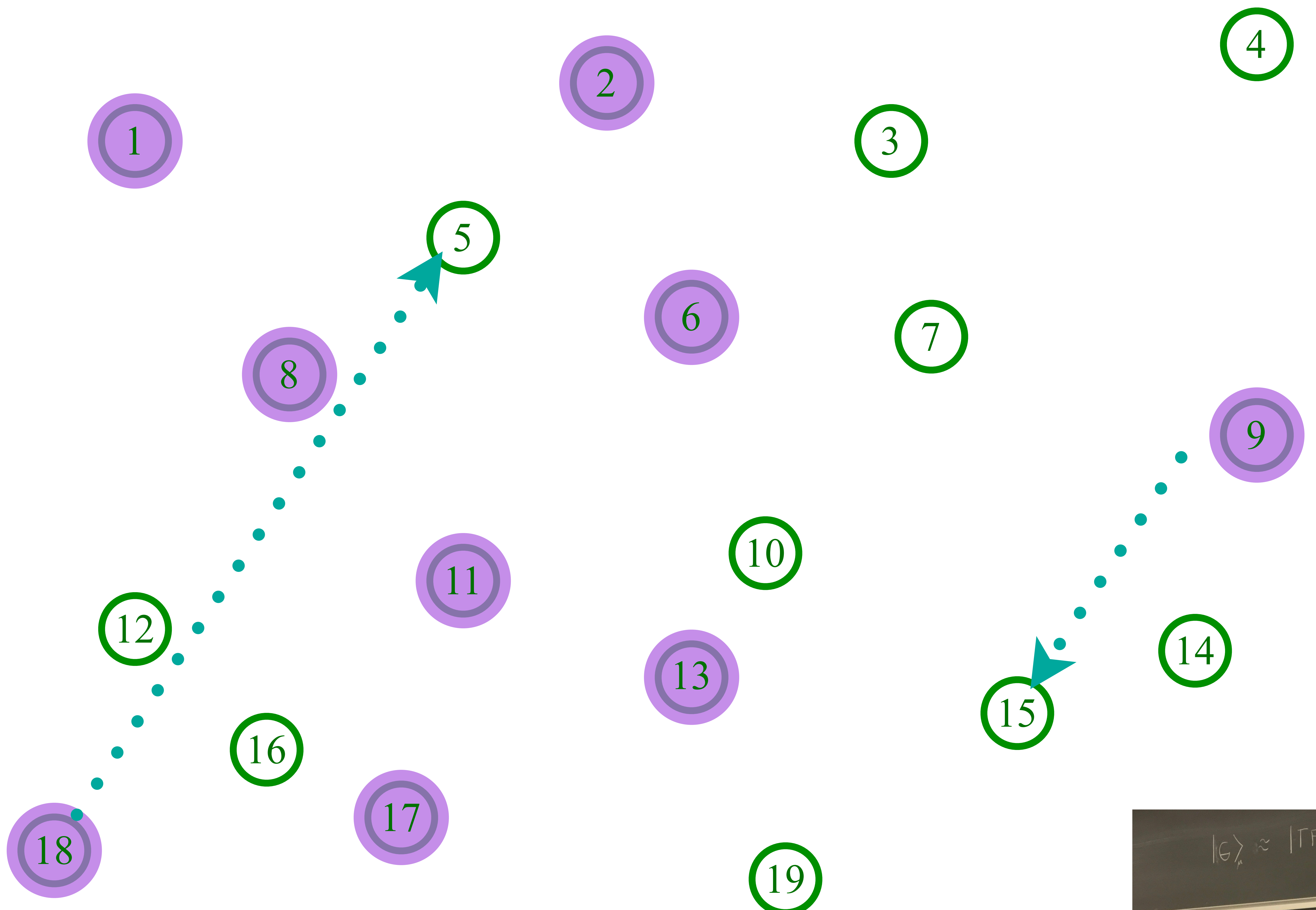
$$U_{9,18;5,15}$$



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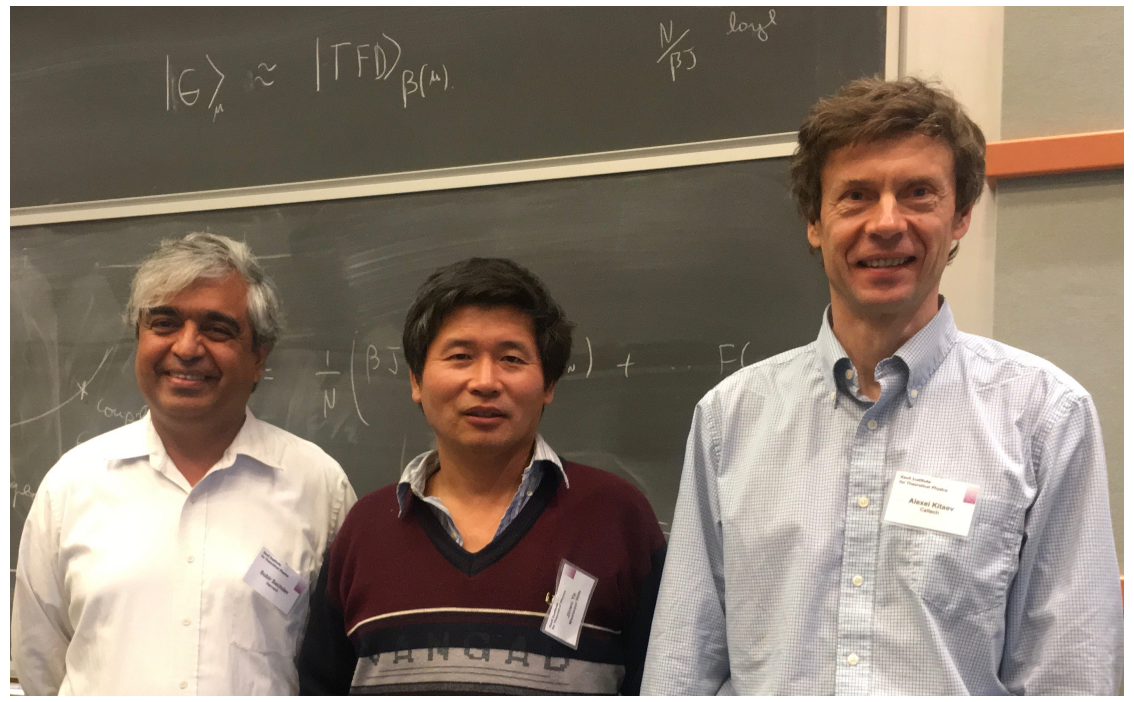

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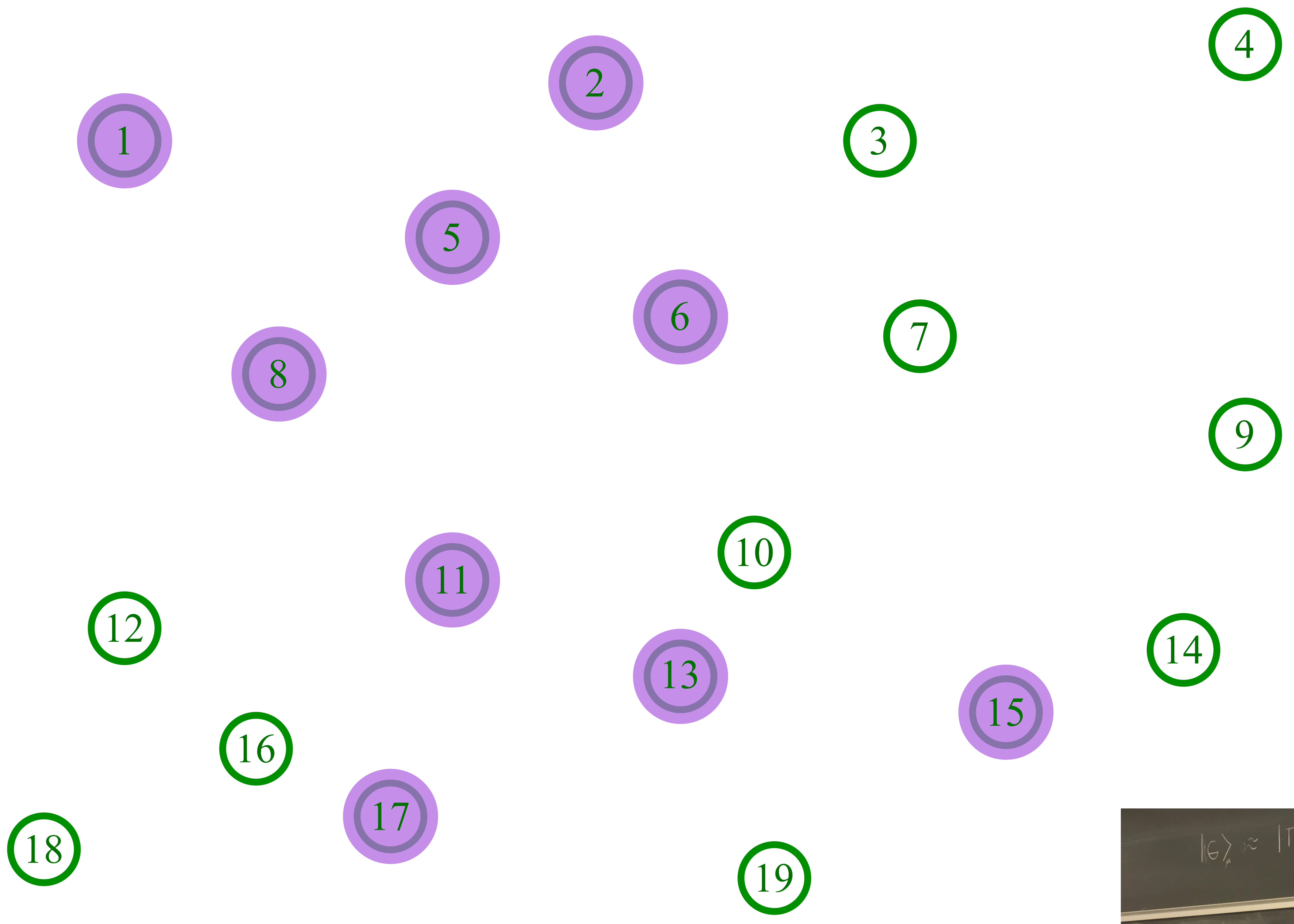

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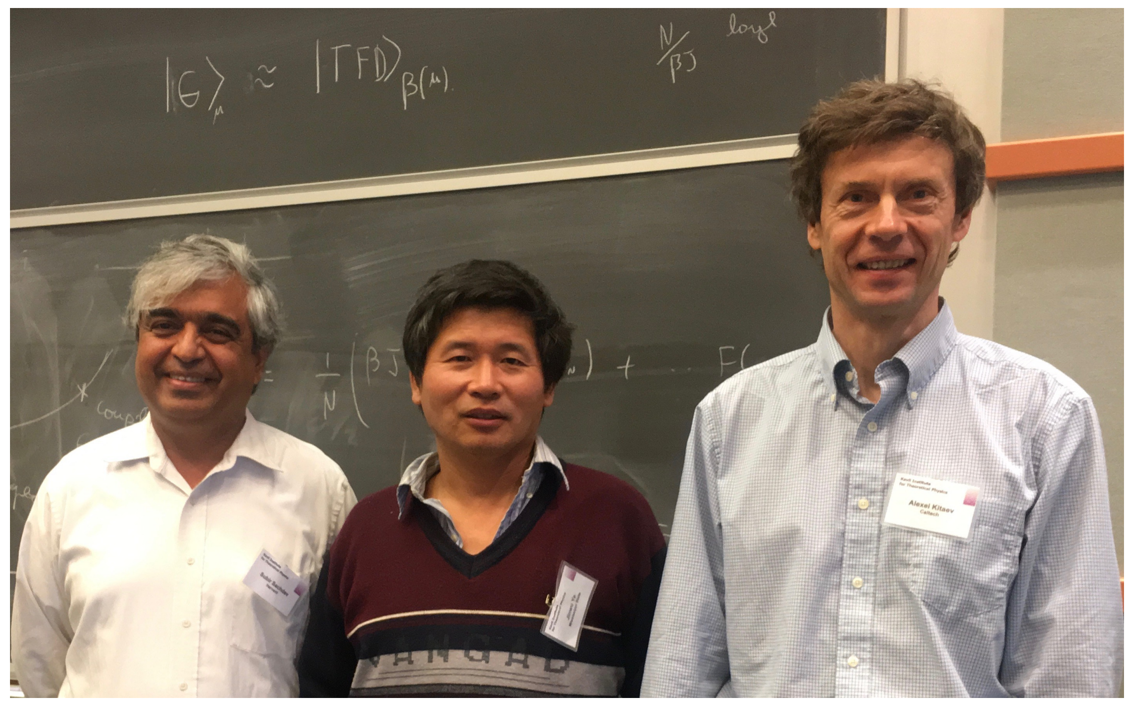
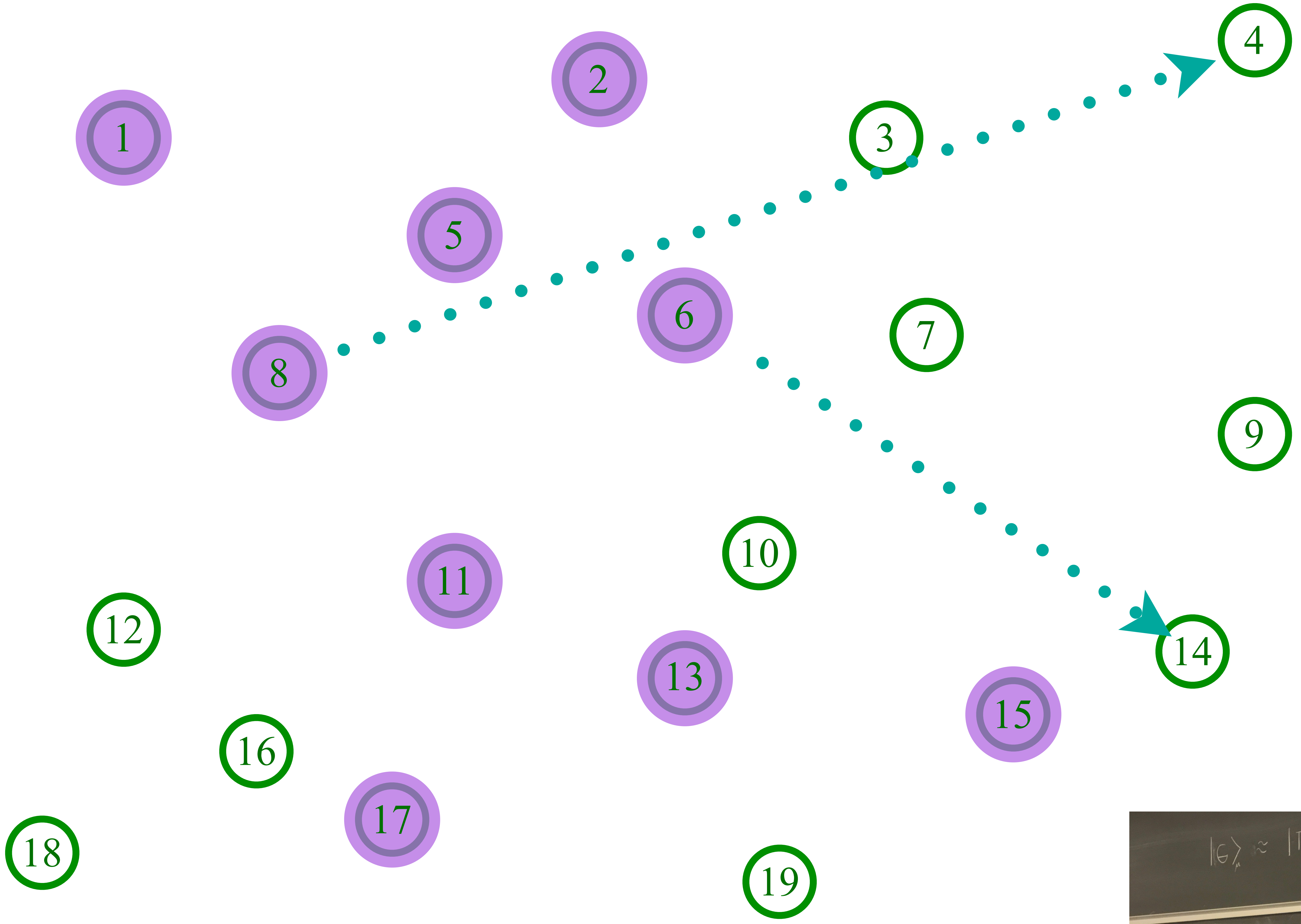


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{6,8;4,14}$



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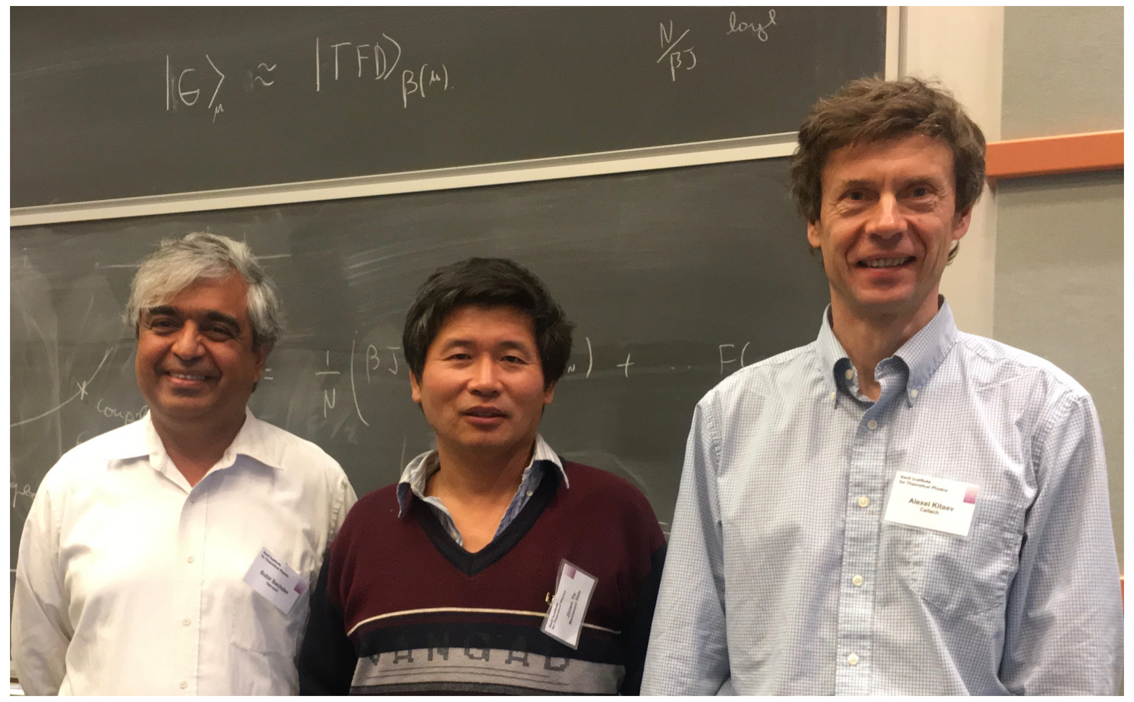

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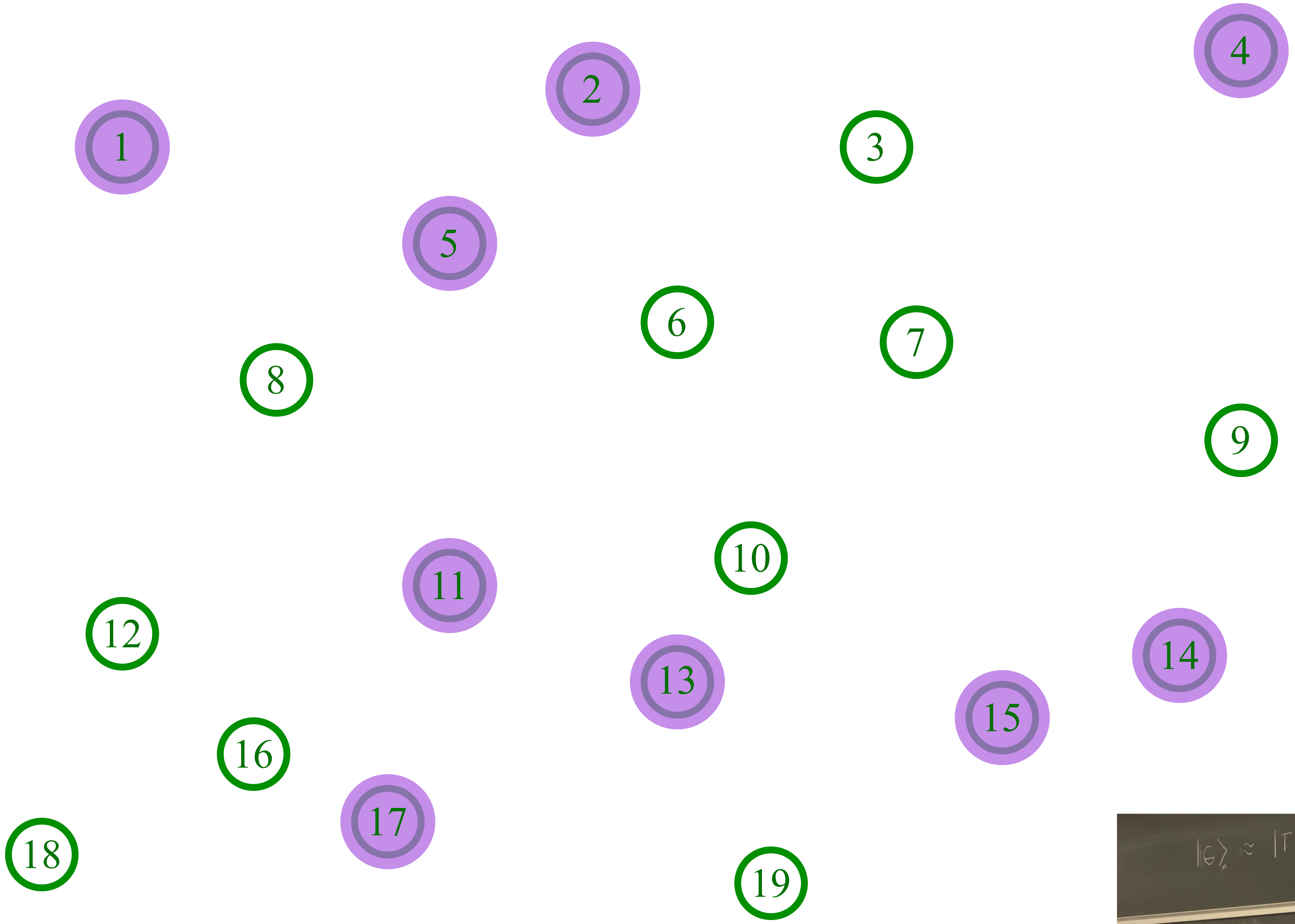

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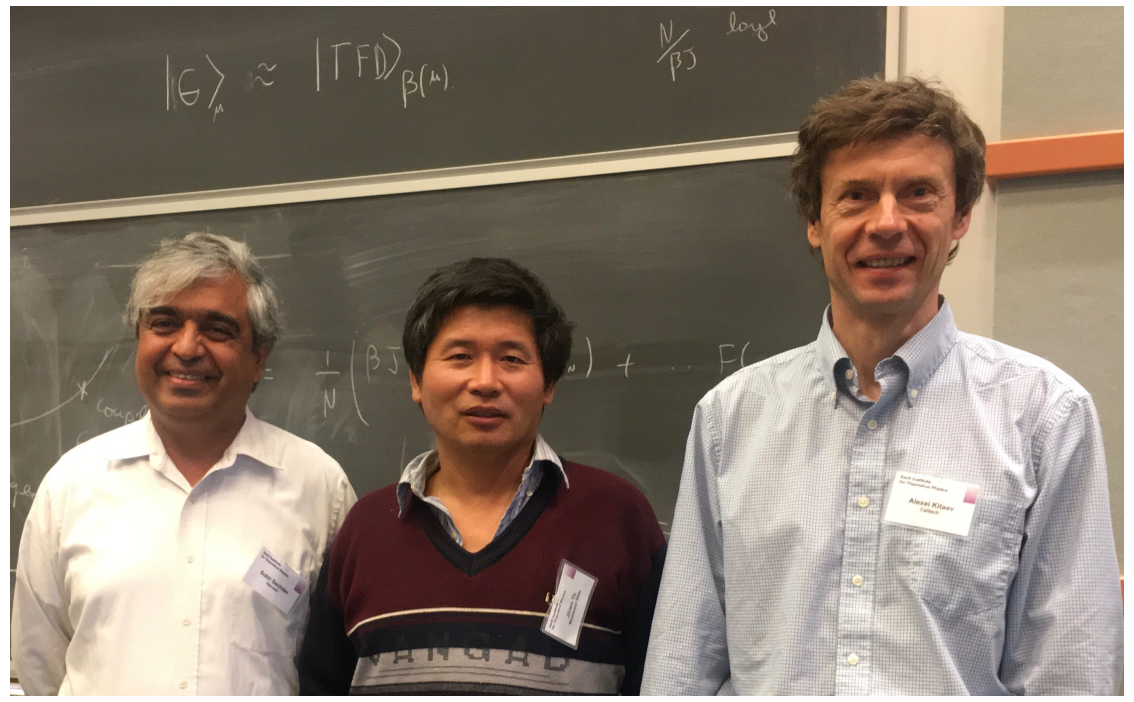
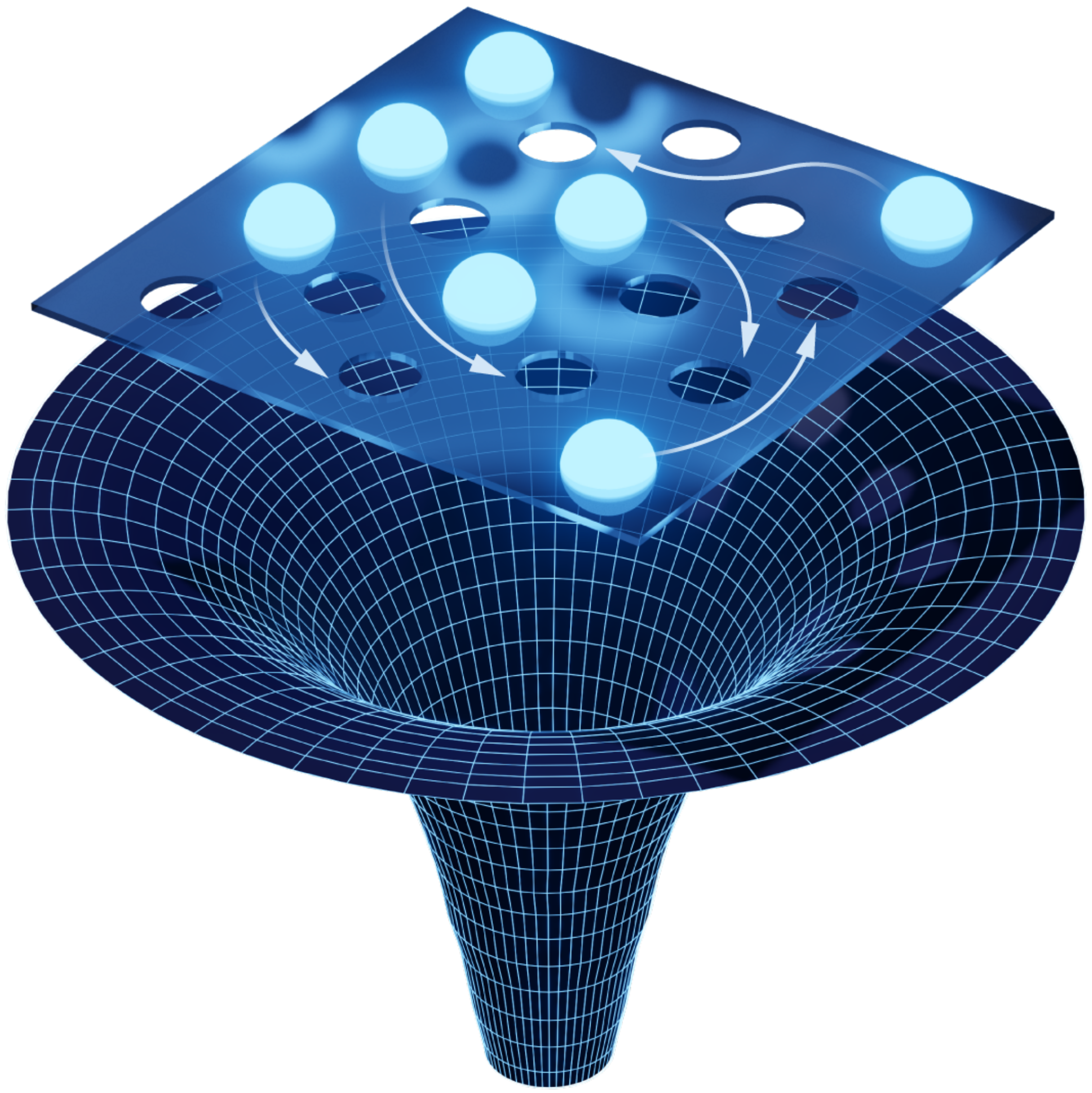


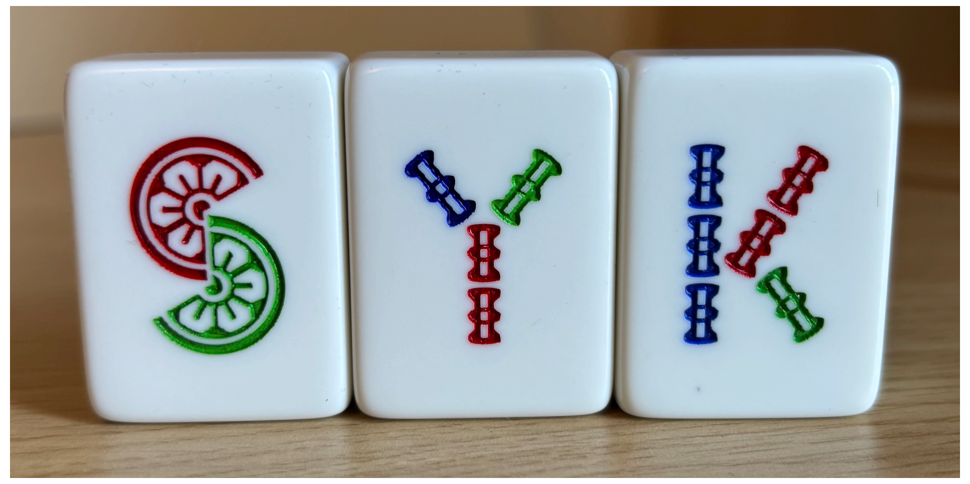
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

Solvable models of multi-particle quantum entanglement with mobile fermions.



Yields a metal whose excitations are not particle-like
i.e. no bosons, fermions, anyons….



Current is carried by an "entangled quantum soup"

The SYK model

At $T > 0$, solutions are fully characterized by a universal, frequency-dependent, 'Planckian', relaxation time,

$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$

where Φ_τ is a known universal function.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev (**GPS**), PRB **63**, 134406 (2001)

arXiv:2503.15646

Planckian dissipation, anomalous high temperature THz non-linear response and energy relaxation in the strange metal state of the cuprate superconductors

Dipanjan Chaudhuri#,[1] David Barbalas#,[1] Fahad Mahmood,[1,2,3] Jiahao Liang,[1] Ralph Romero III,[1] Anaëlle Legros,[1] Xi He,[4] Hélène Raffy,[5] Ivan Božović,[4,6] and N.P. Armitage[1,7]

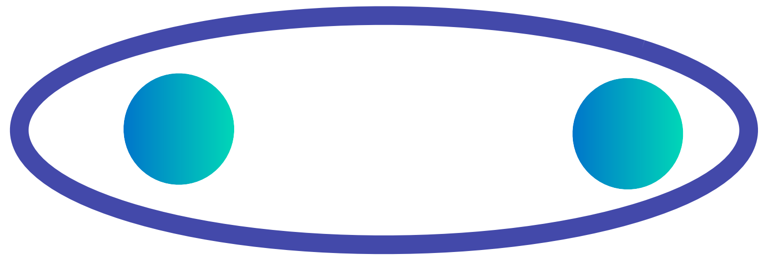
…the momentum relaxation rate is linear in T and close to its "Planckian" form ($\Gamma_M \approx 2k_BT/\hbar$). We find (the energy relaxation rate) Γ_E to be 10-40 times smaller than the momentum relaxation. This shows that the scattering that causes momentum loss (and T-linear) resistivity do not remove appreciable energy from the electrons.

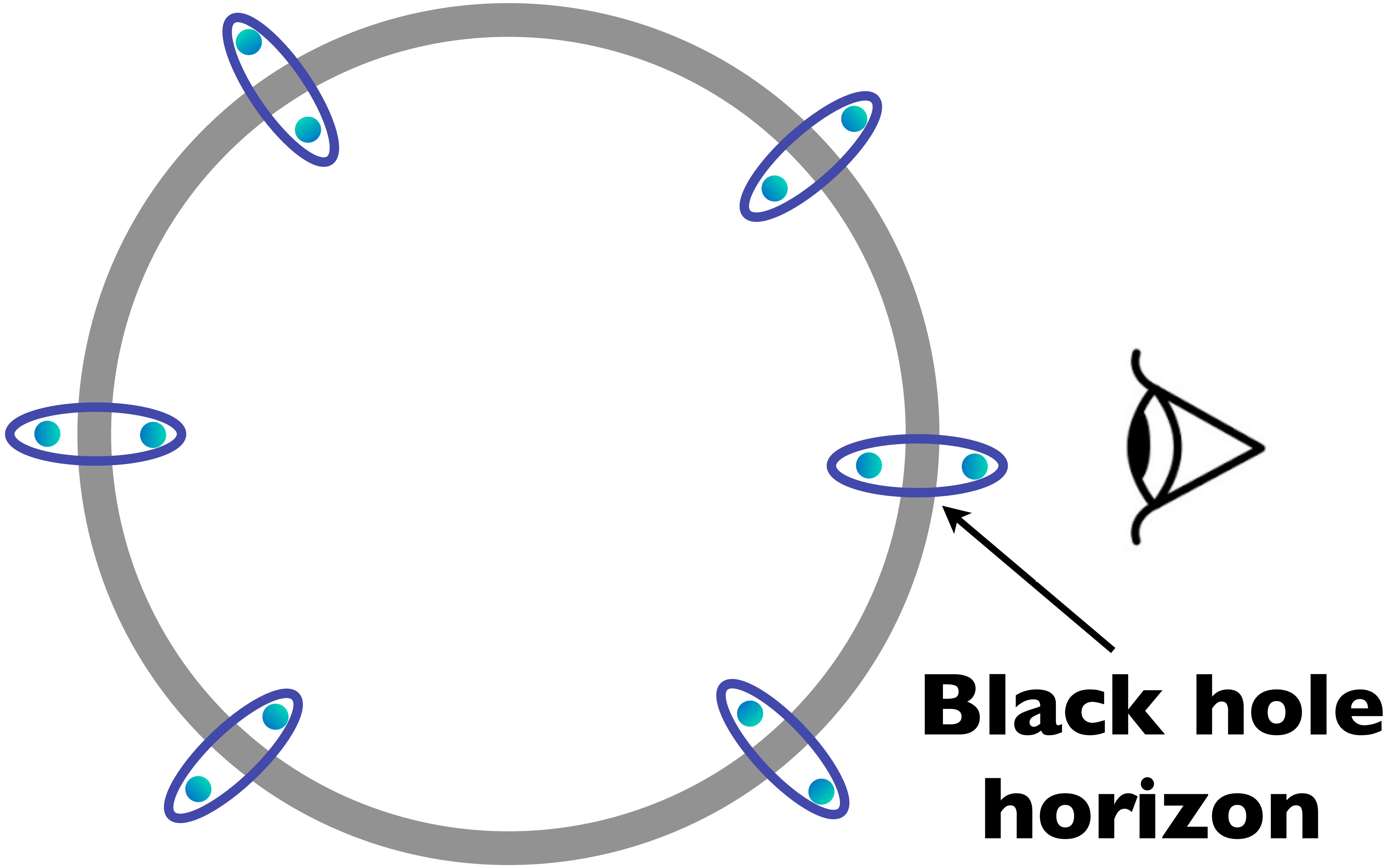
Many fermion entanglement II:

The SYK model and black holes

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Hawking obtained the black hole entropy

$$S = \frac{Ac^3}{4G\hbar}$$

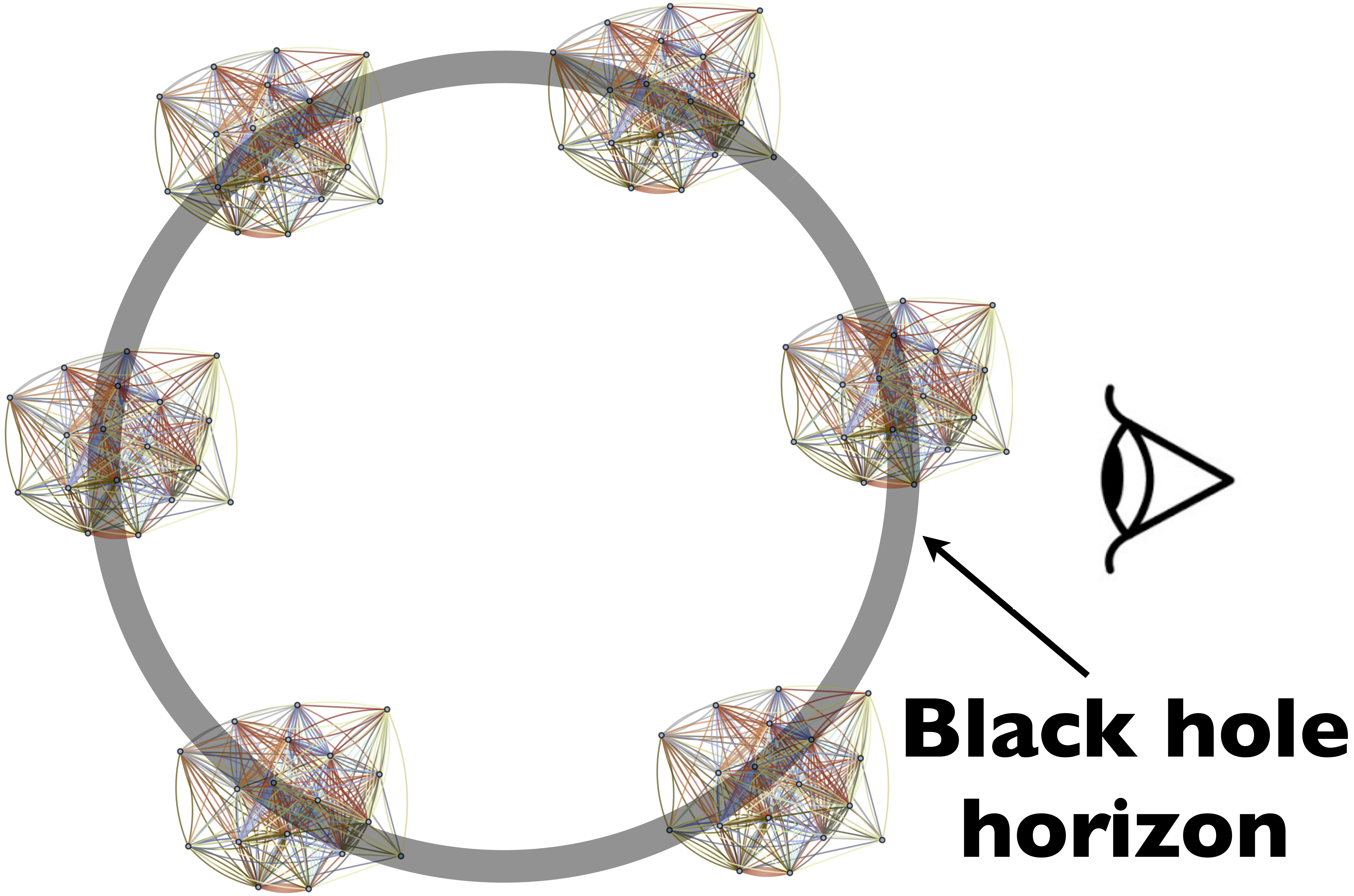
where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

S. Sachdev, PRL **105**, 151602 (2010)



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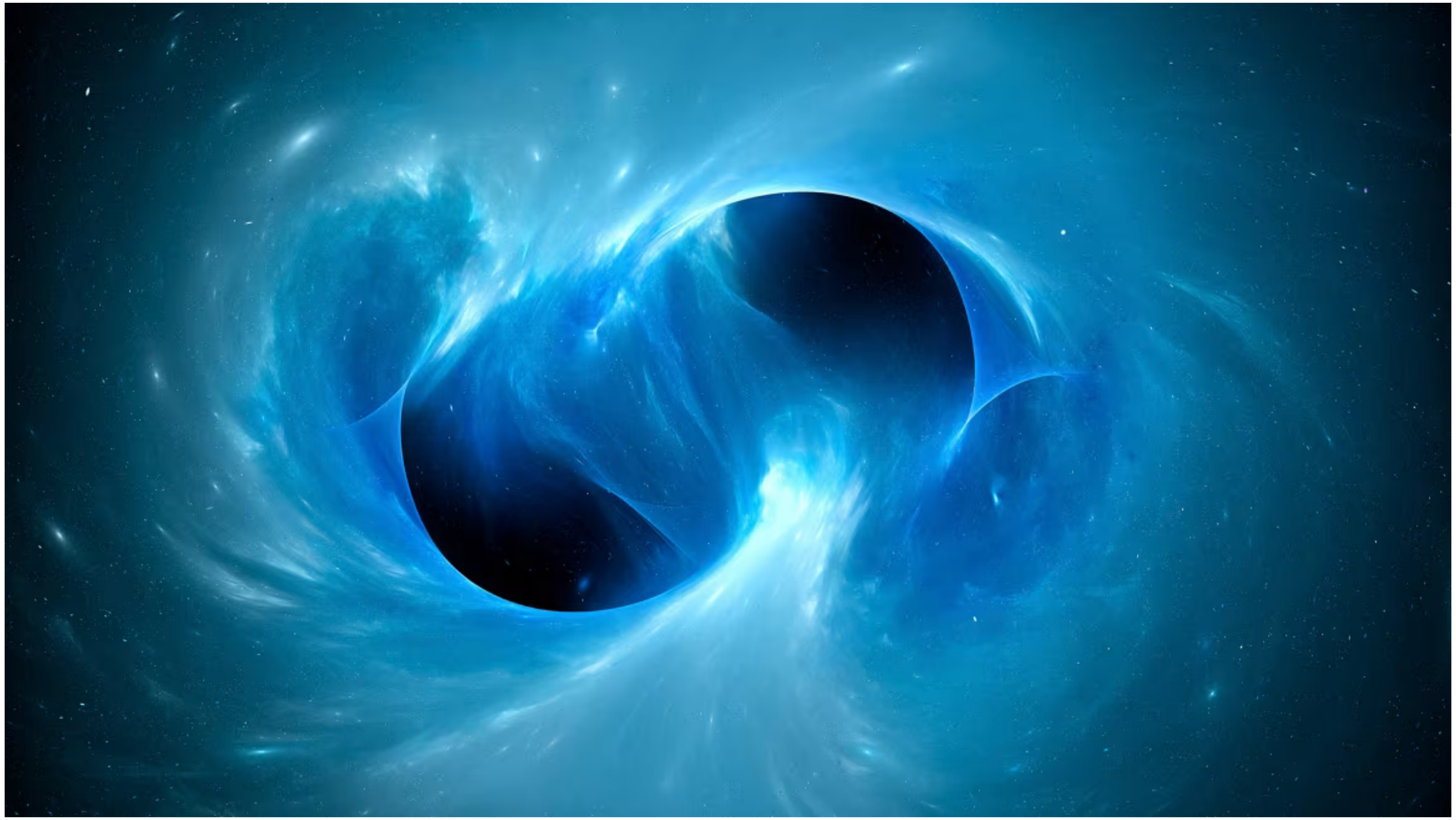


Sakkmesterke/Science Photo Library RF/Getty Images

$$\tau_{\rm ring-down} \sim \frac{\hbar}{k_B T}$$

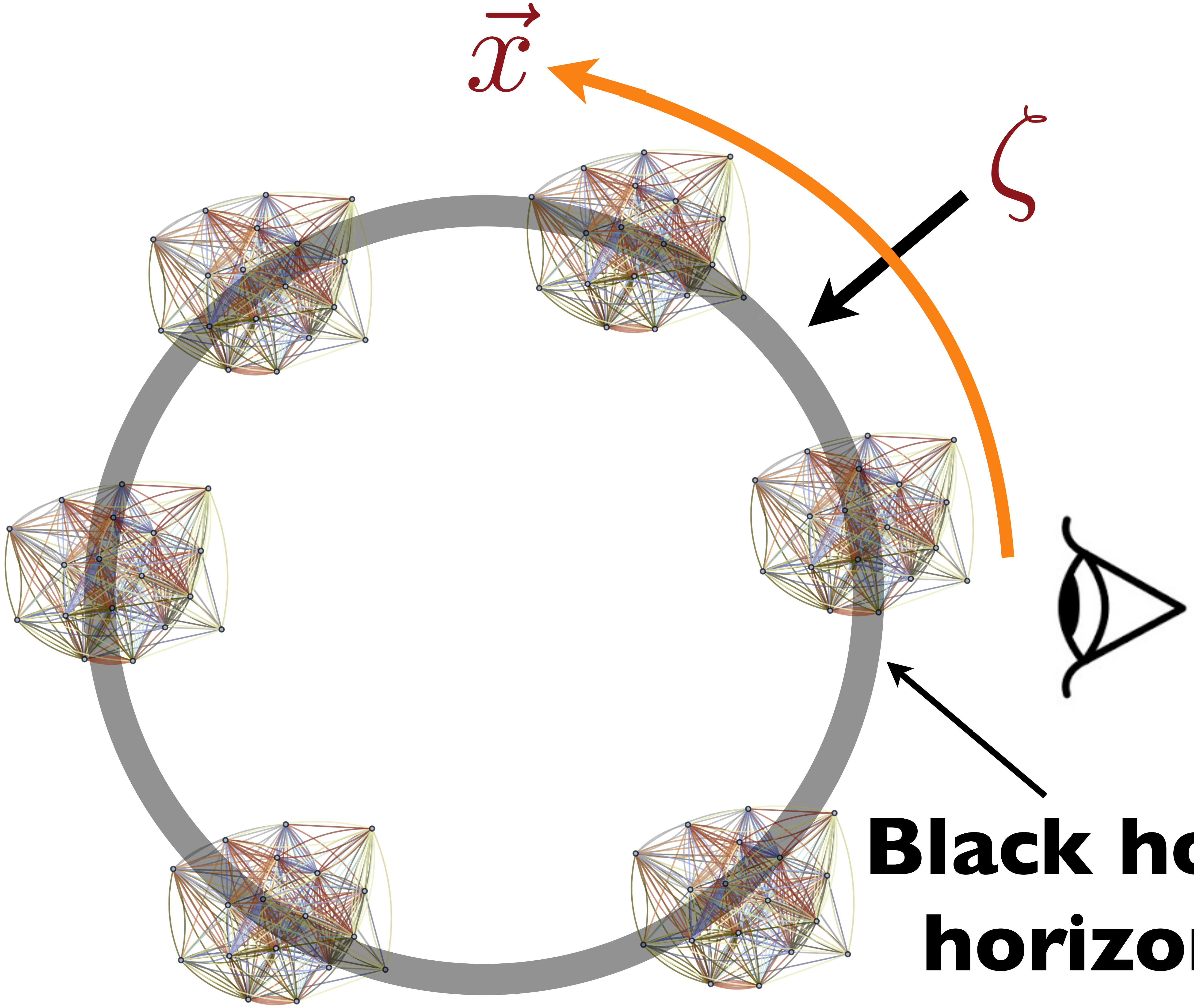
Planckian dynamics of quasi-normal modes!

C.V. Vishveshwara
Nature **227**, 936 (1970)

T is the Hawking temperature of the black hole

S. Sachdev, PRL **105**, 151602 (2010)

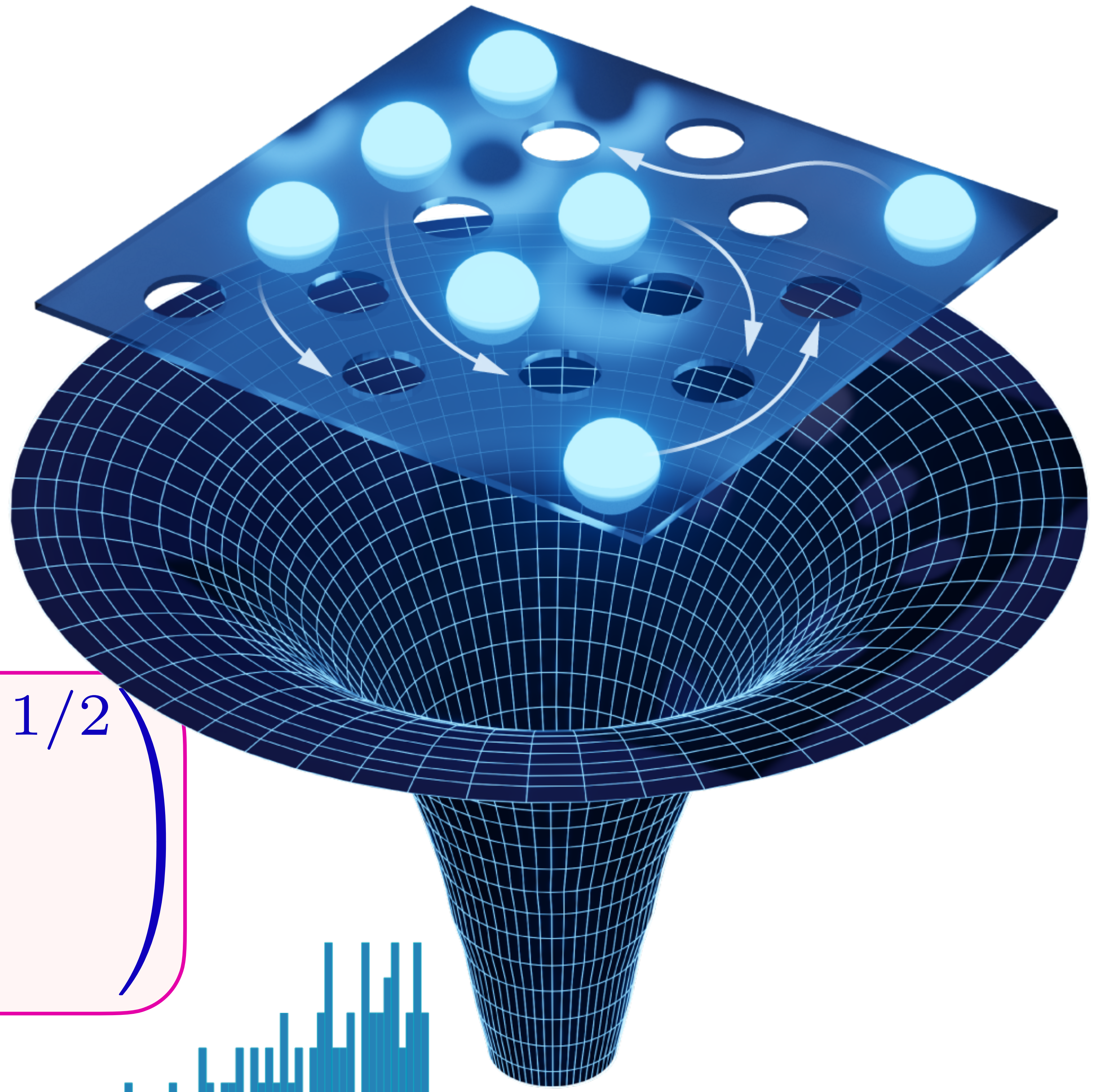
Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The quantum versions of
Maxwell's and Einstein's
equations in
ζ space and time are
also the equations describing
electron entanglement
in the SYK model!

D. Chowdhury, A. Georges, O. Parcollet, and S. S.,
Rev. Mod. Phys. **94**, 035004 (2022)

D(E) of charged black holes from the SYK model



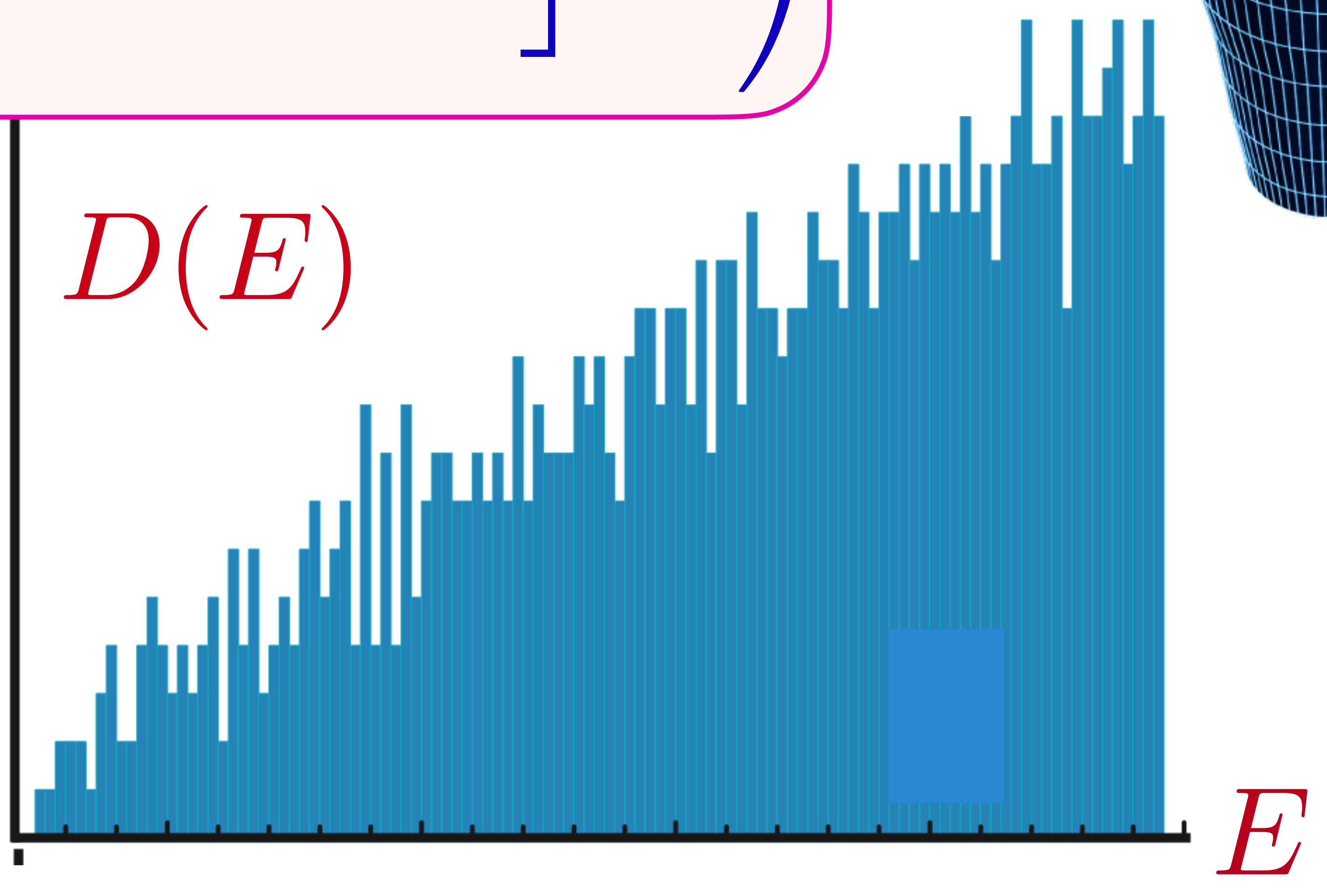
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}{A_0}^{3/2}c^2}{\hbar^2 G}E\right]^{1/2}\right)$$

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



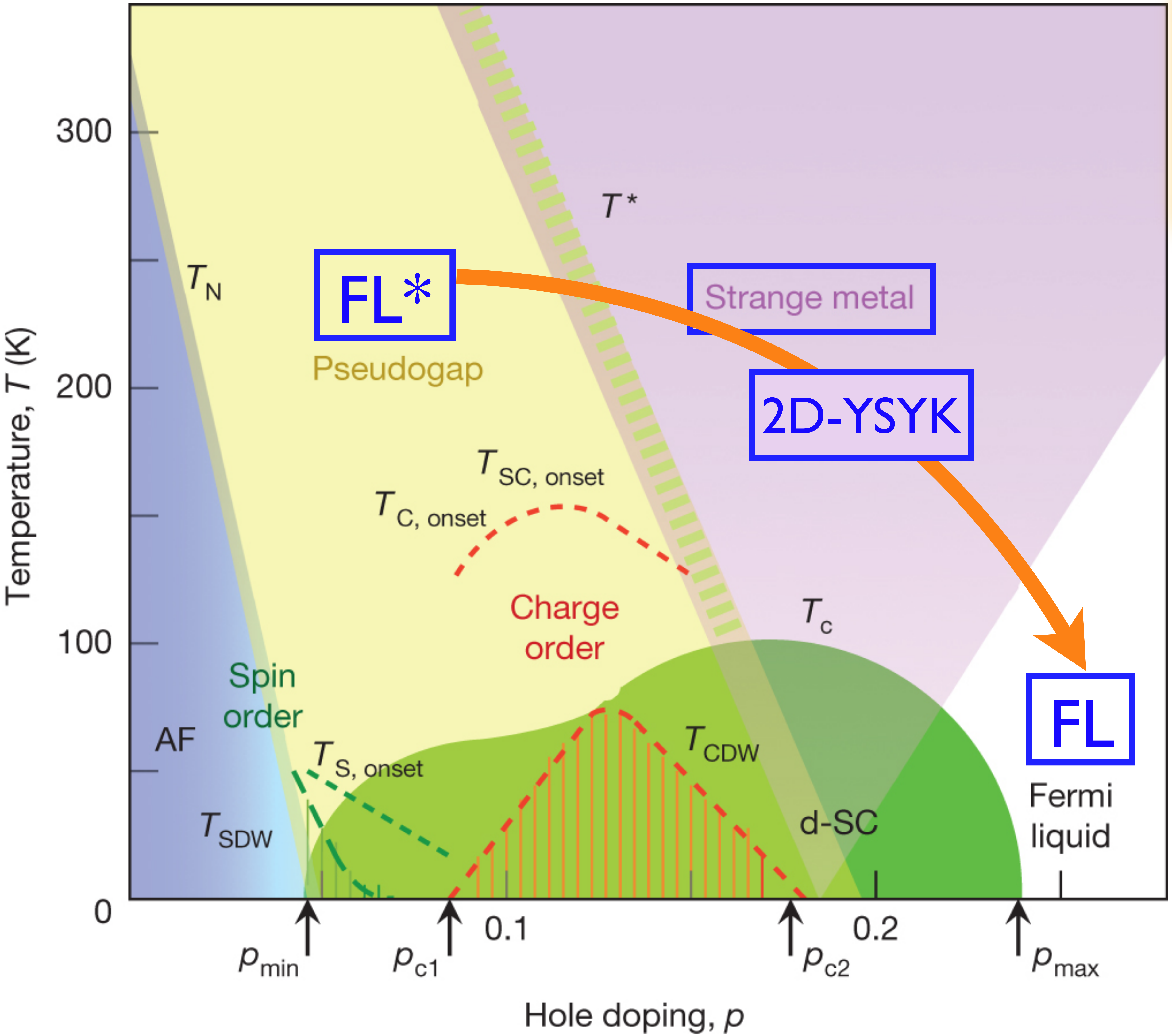
Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Many fermion entanglement I & II:

Theory of the strange metal

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Quantum entanglement of mobile fermions without an energy gap

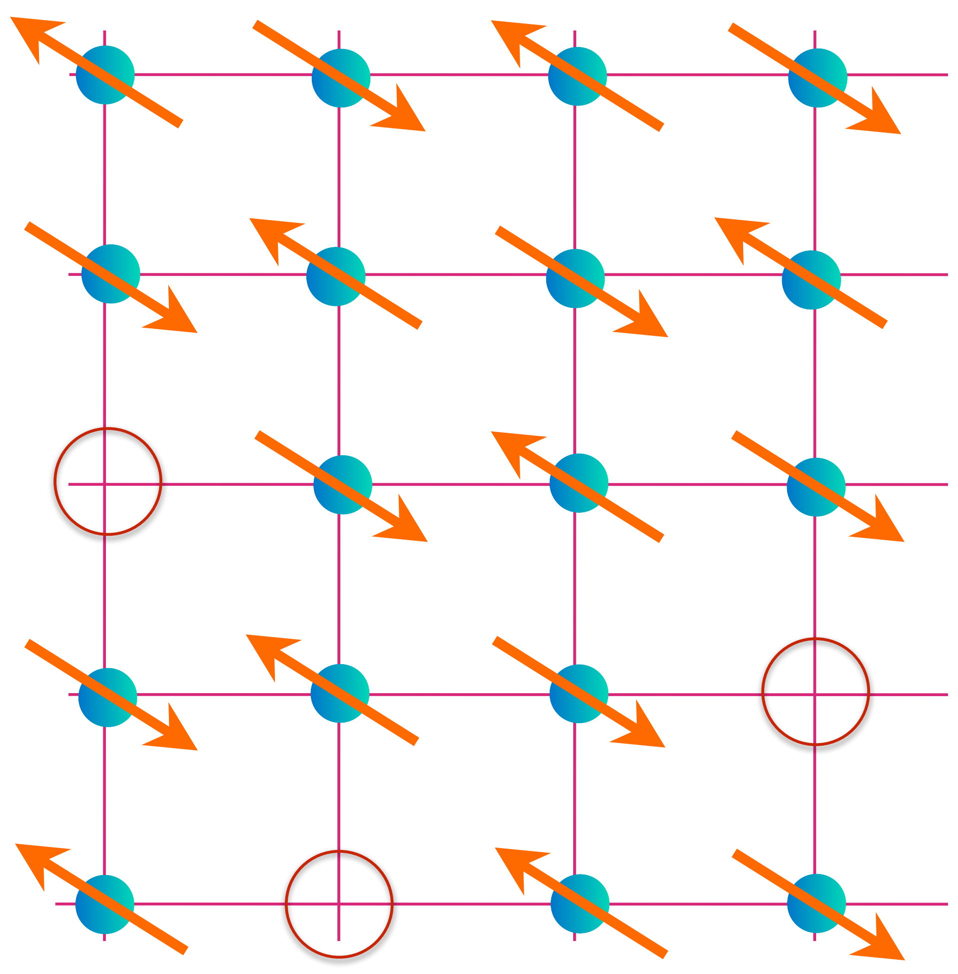
II. Sachdev-Ye-Kitaev (SYK) liquid

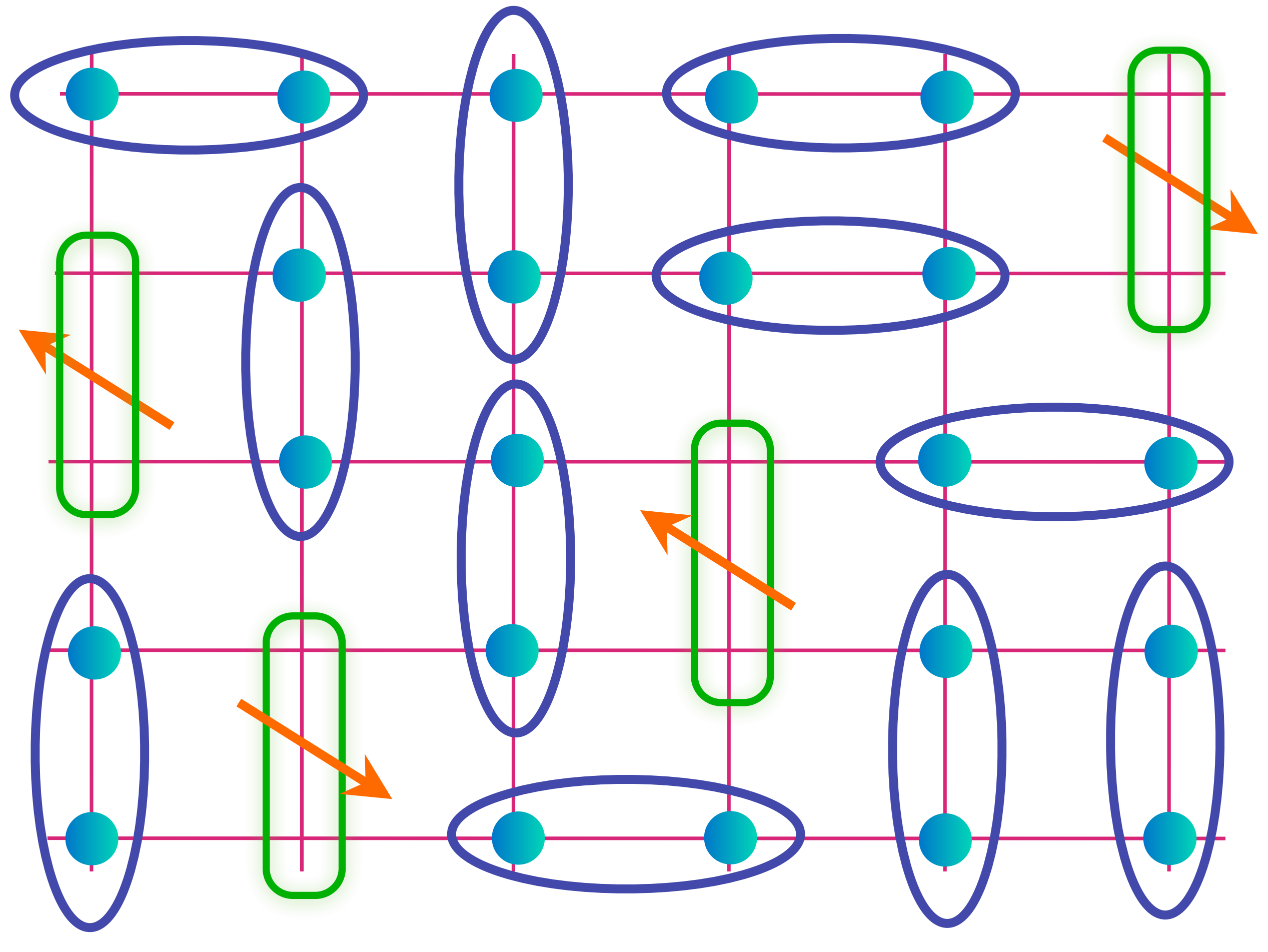
- Compressible state with no quasiparticles.
- SYK: low energy theory of generic charged black holes in asymptotically flat 3+1 dimensional space.
- 2D-YSYK: universal theory of strange metals: FL*-FL transition in cuprates.

AF Metal

FL*

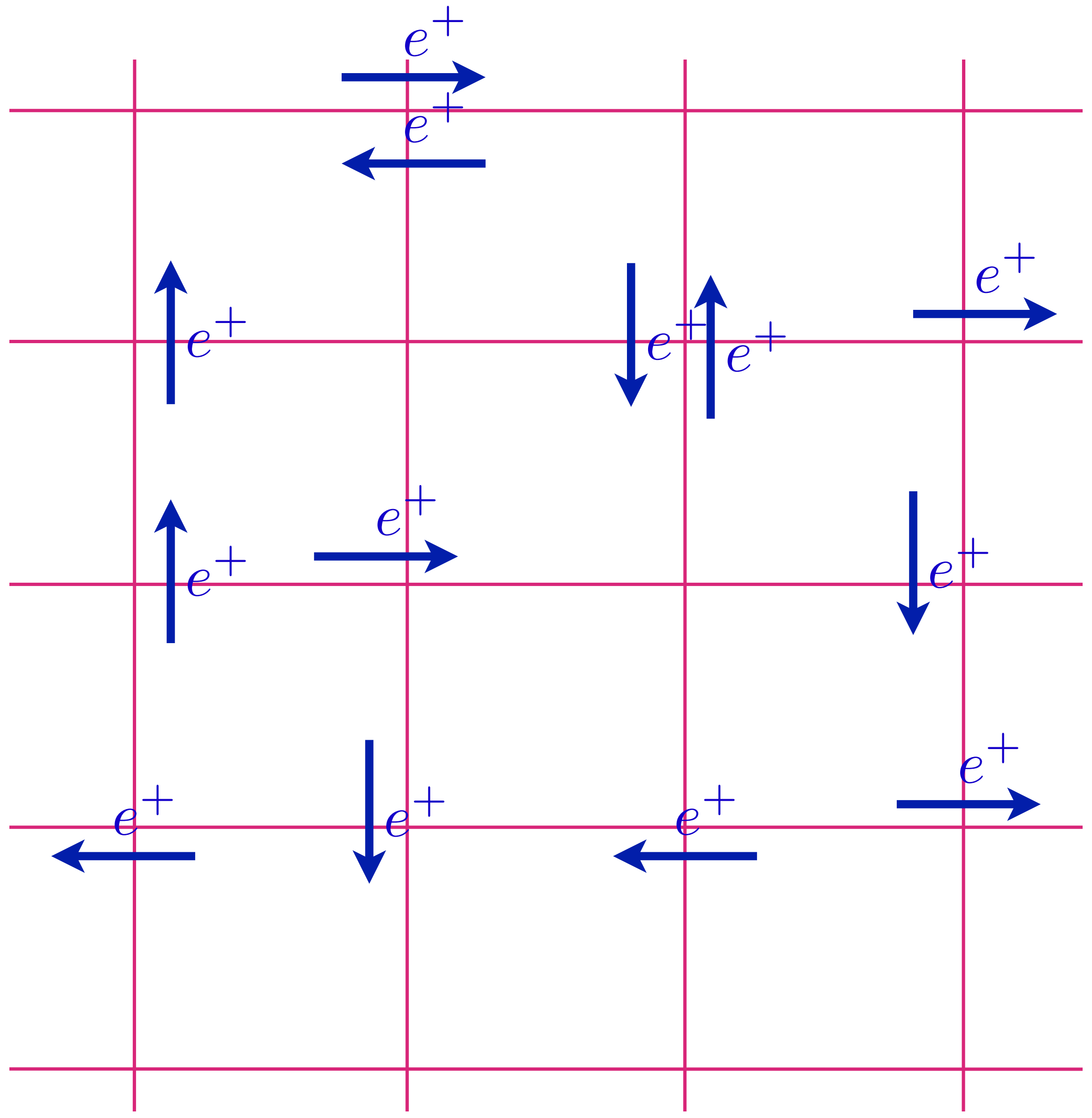
FL





$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



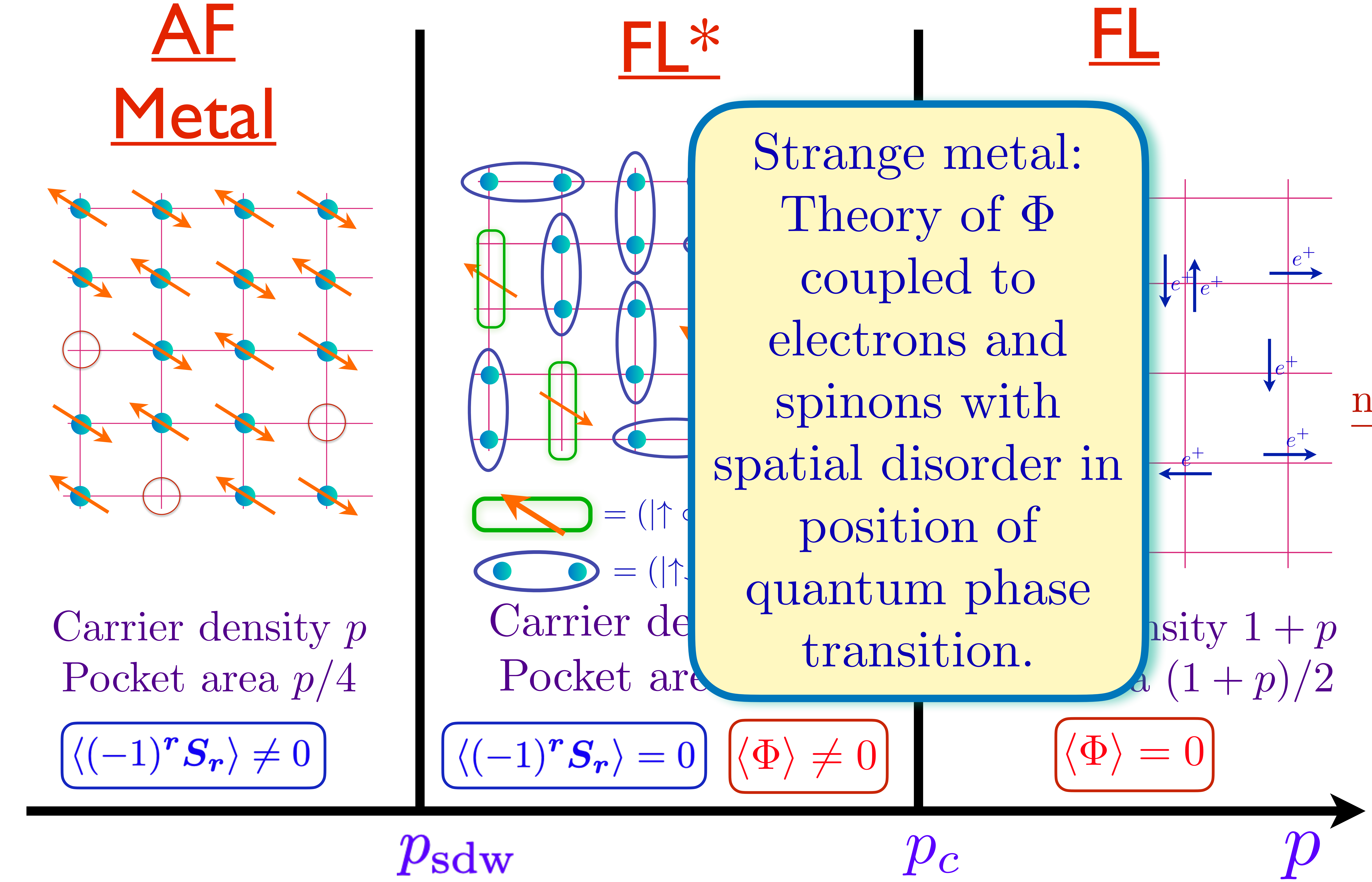
FL*-FL: Quantum phase transition between two metals at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

AF Metal	FL*	FL
Carrier density p	Carrier density p	Carrier density $1 + p$
Pocket area $p/4$	Pocket area $p/8$	Fermi area $(1 + p)/2$
$\boxed{\langle (-1)^{\boldsymbol{r}} \boldsymbol{S_r} \rangle \neq 0}$	$\boxed{\langle (-1)^{\boldsymbol{r}} \boldsymbol{S_r} \rangle = 0}$ $\boxed{\langle \Phi \rangle \neq 0}$	$\boxed{\langle \Phi \rangle = 0}$

$p_{\rm sdw}$ $\qquad$ p_c $\qquad$ p

S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



FL*-FL:
Quantum
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no symmetry breaking.

Described by the
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Higgs field Φ.

Spatial disorder
leads to
momentum
relaxation

S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Yukawa-Sachdev-Ye-Kitaev model

$$\mathcal{H} = -\mu \sum_i c_i^\dagger c_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \Phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} c_i^\dagger c_j \Phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-Sachdev-Ye-Kitaev model

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with $g_{ij\ell}$ independent random numbers with zero mean.

Properties very similar to the SYK model, including Planckian dynamics:

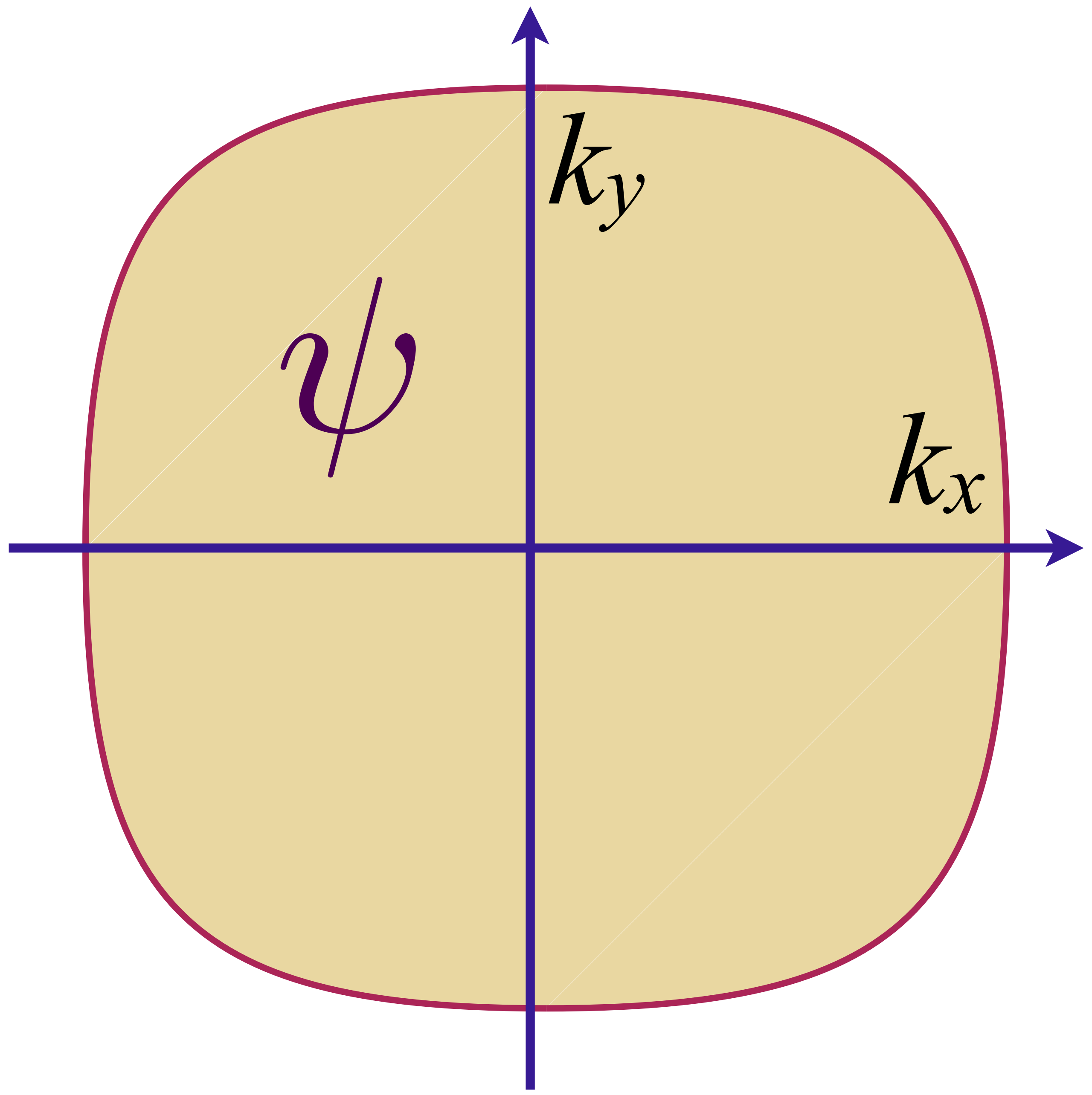
$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T}\right)$$

where Φ_τ is a known universal function.

I. Esterlis and J. Schmalian,
PRB **100**, 115132 (2019)
See also Yuxuan Wang,
PRL **124**, 017002 (2020)

2D-YSYK model: Fermi surface + Higgs boson with interaction disorder

$$\mathcal{L} = c^\dagger_{\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\alpha} + f^\dagger_{1\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \widetilde{\varepsilon}(\boldsymbol{k})\right)f_{1\boldsymbol{k}\alpha}$$



$$+ [\nabla\Phi(\boldsymbol{r})]^2 + s\,[\Phi(\boldsymbol{r})]^2 + u\,[\Phi(\boldsymbol{r})]^4$$

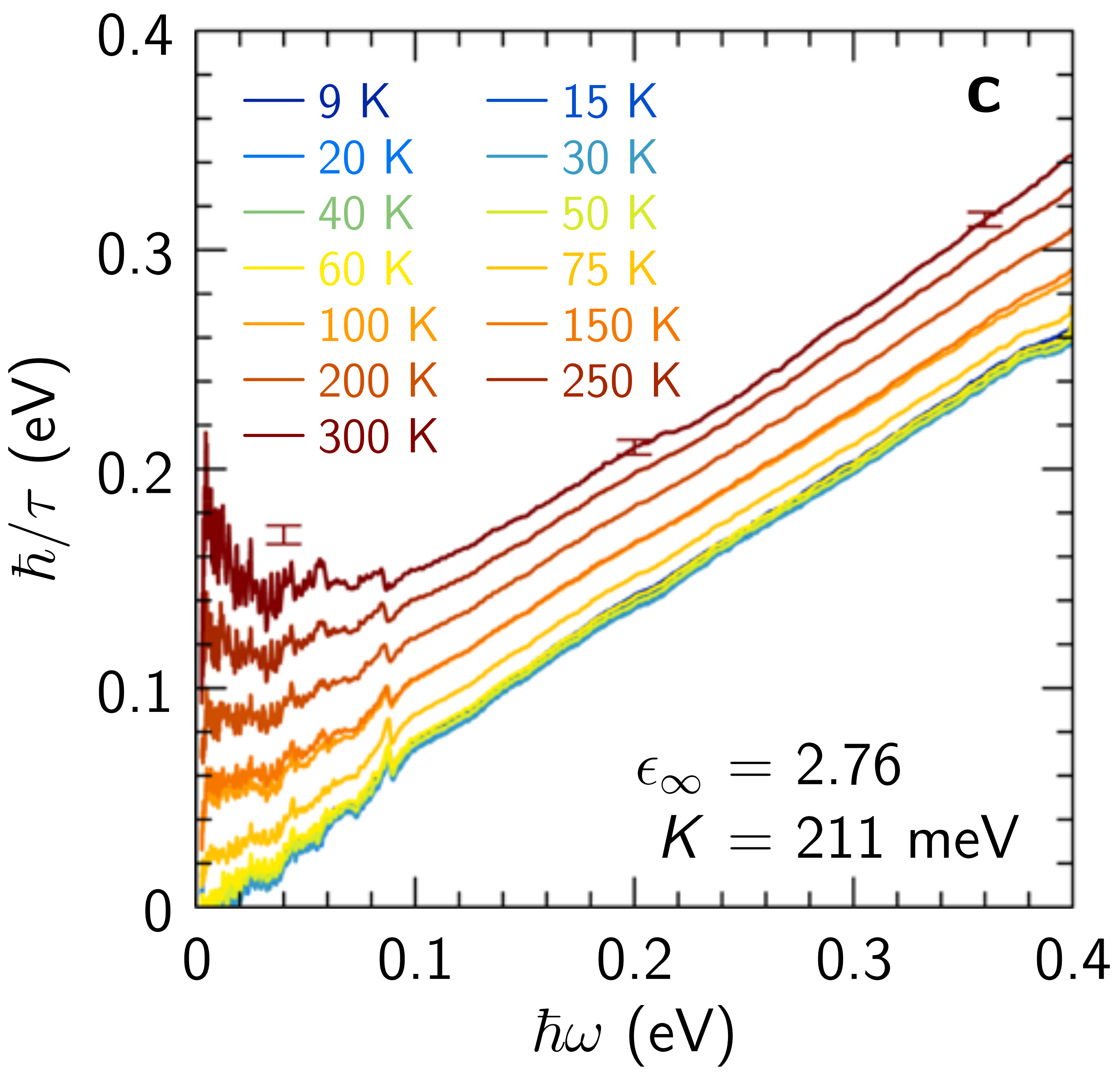
$$+ [g + g'(\boldsymbol{r})]\,c^\dagger_\alpha(\boldsymbol{r})f_{1\alpha}(\boldsymbol{r})\,\Phi(\boldsymbol{r}) + \text{H.c.}$$

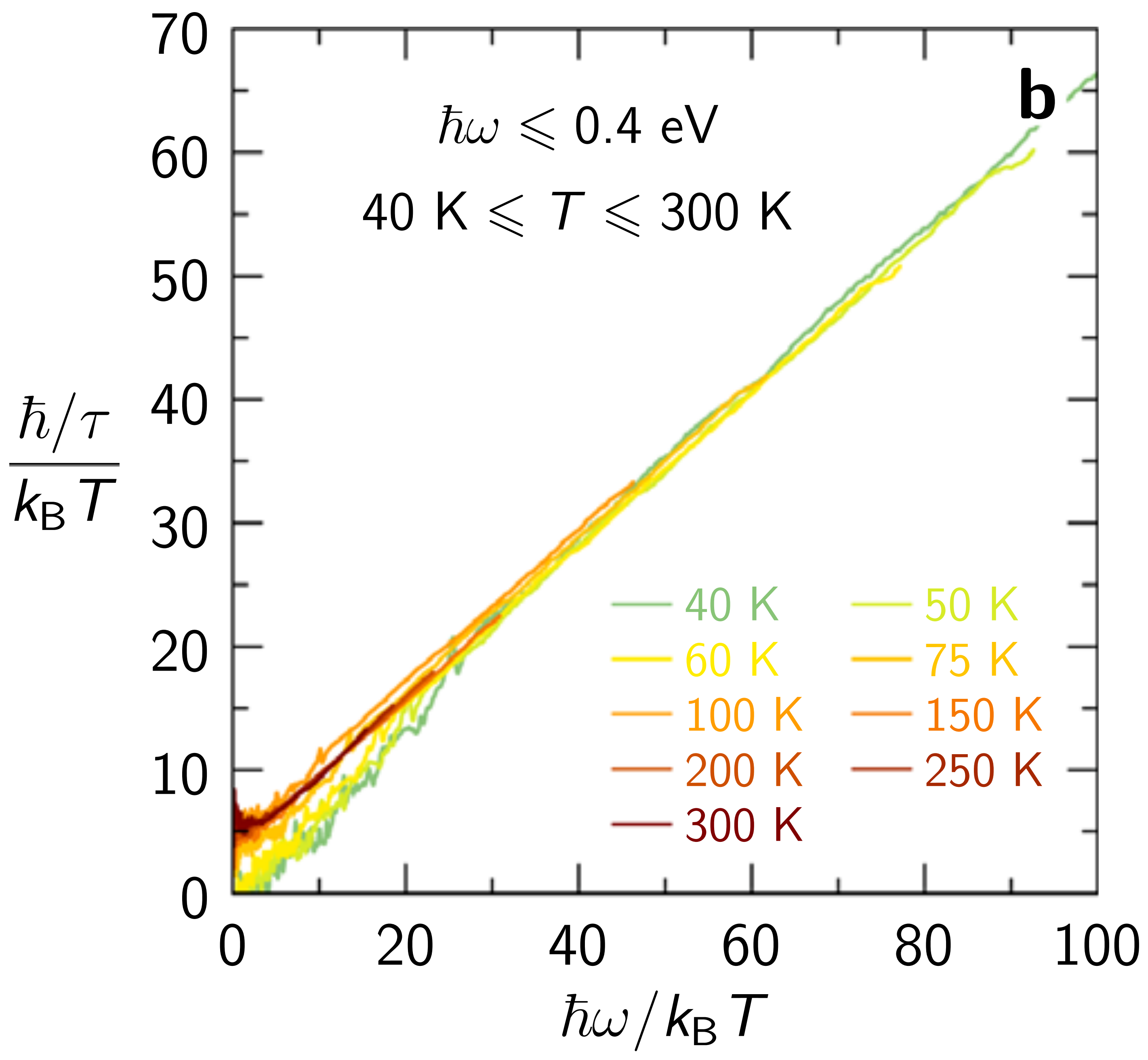
$$+v(\boldsymbol{r})c^\dagger_\alpha(\boldsymbol{r})c_\alpha(\boldsymbol{r})$$

Φ^2 "mass" disorder $s \to s + \delta s(\boldsymbol{r})$ is strongly relevant; rescale Φ to move disorder to the Yukawa coupling.

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

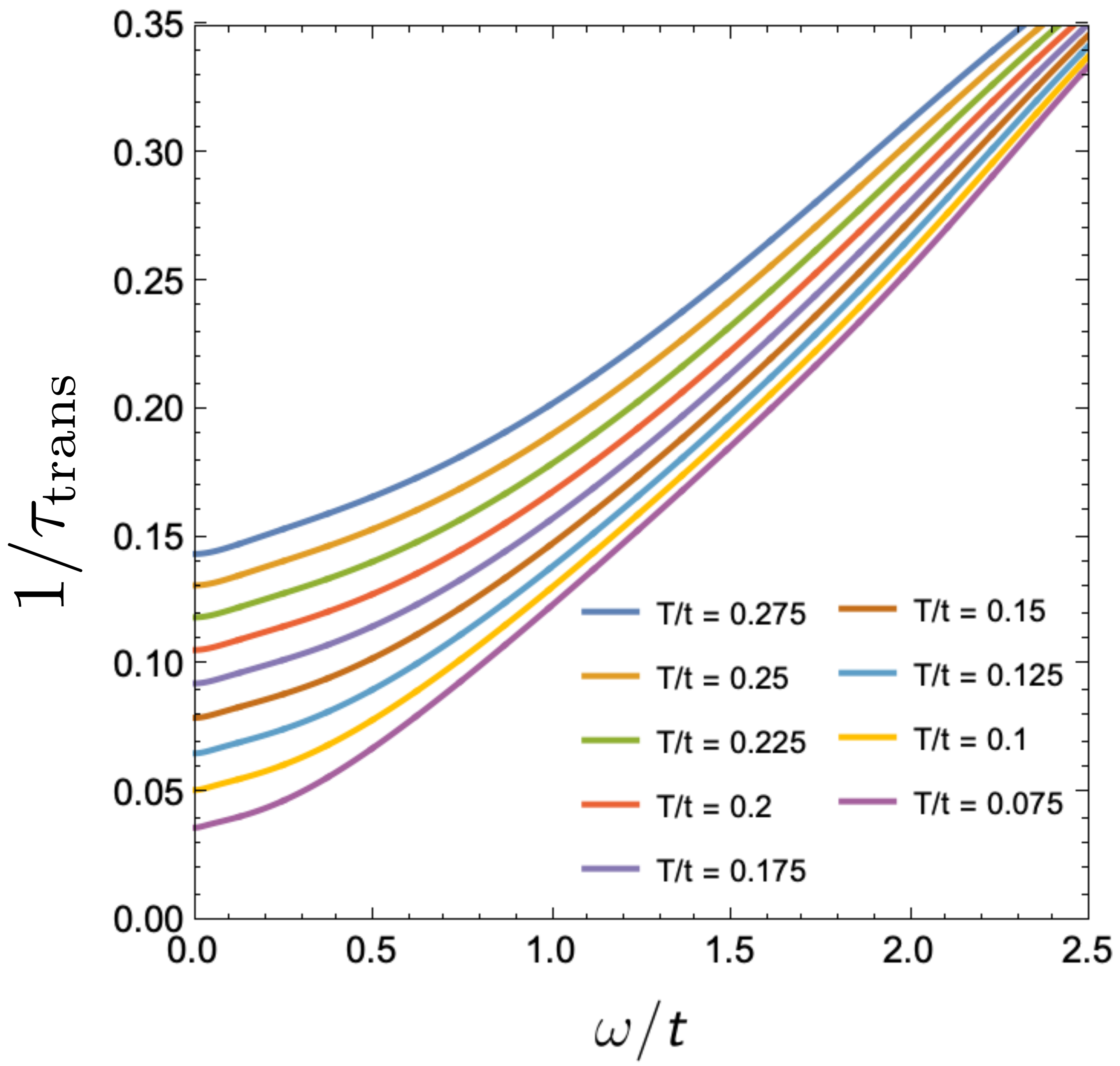
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

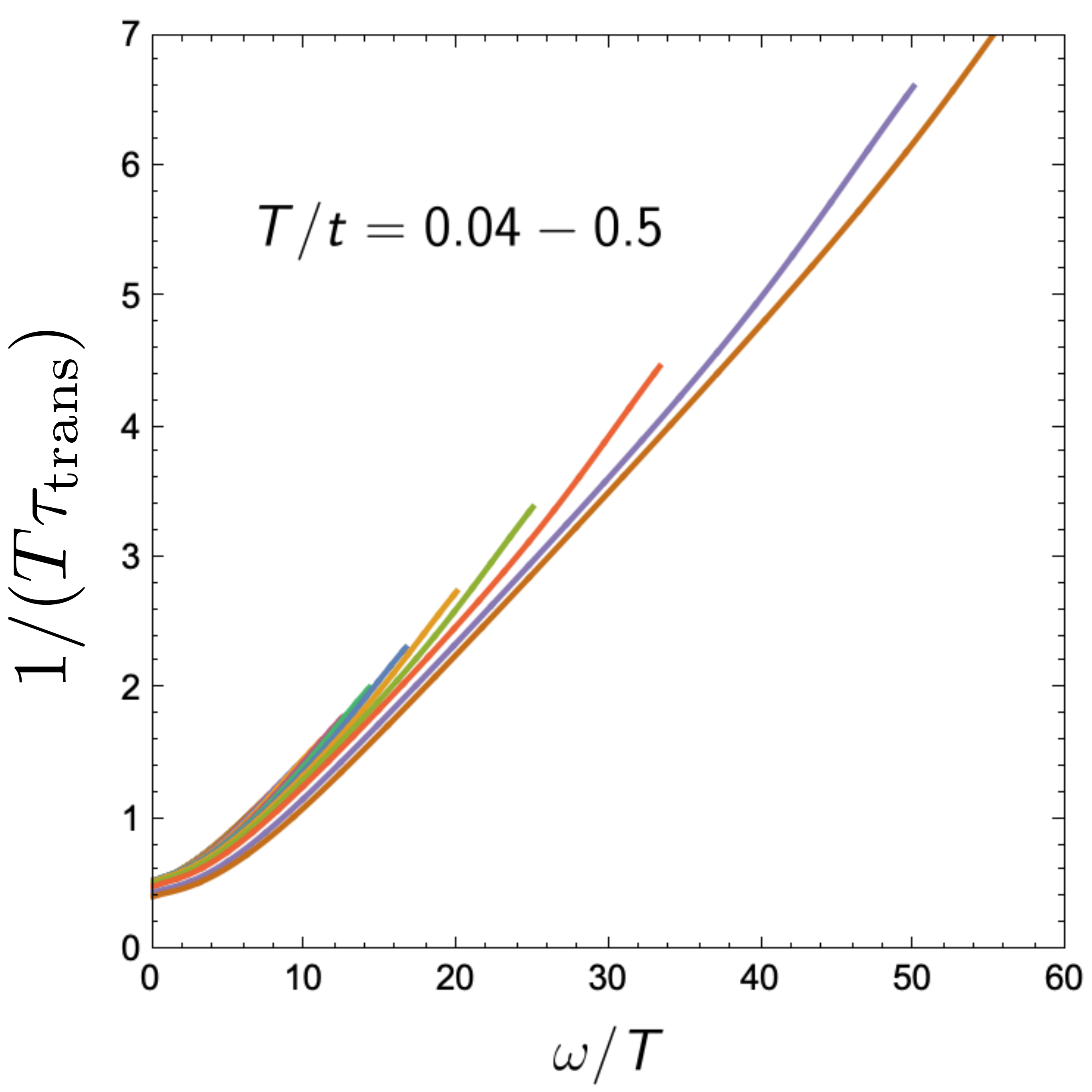




$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$

From optical conductivity data of Michon et al. (2023)





$$\frac{\hbar}{\tau(\omega)} = k_B T\, \Phi_\tau\left(\frac{\hbar\omega}{k_B T}\right)$$

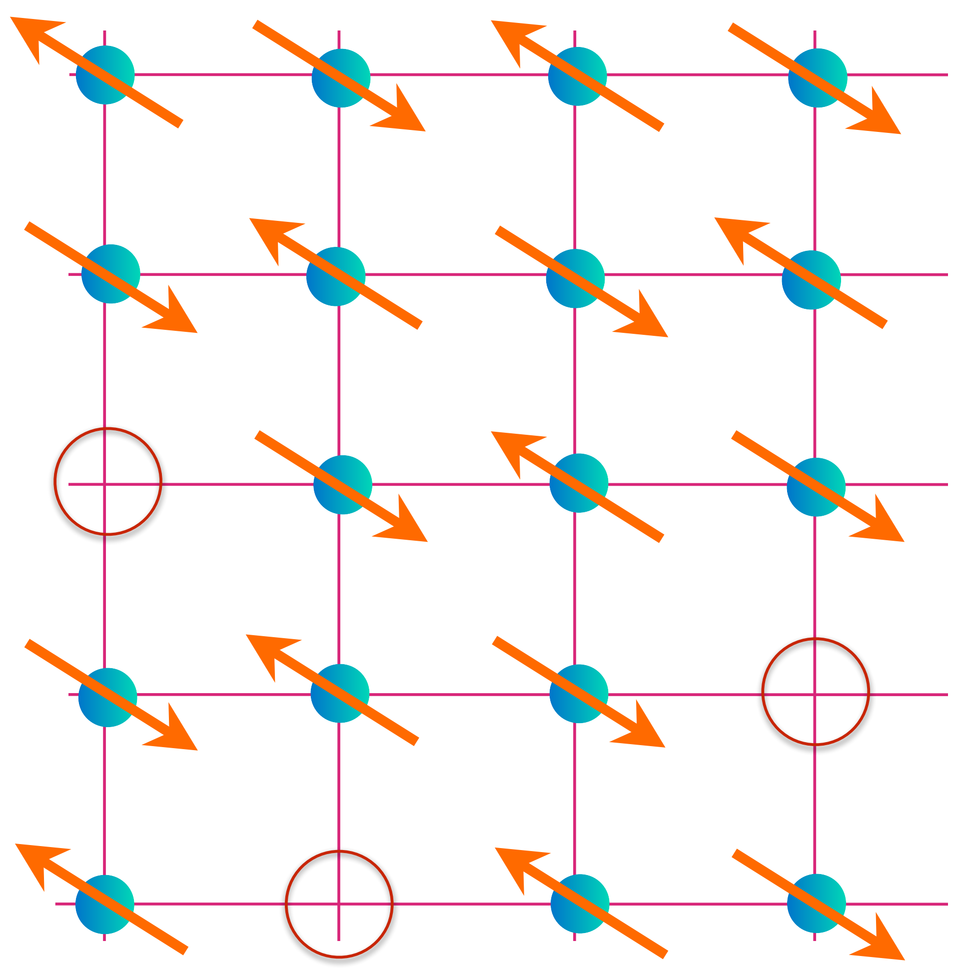
2d-YSYK theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

Summary

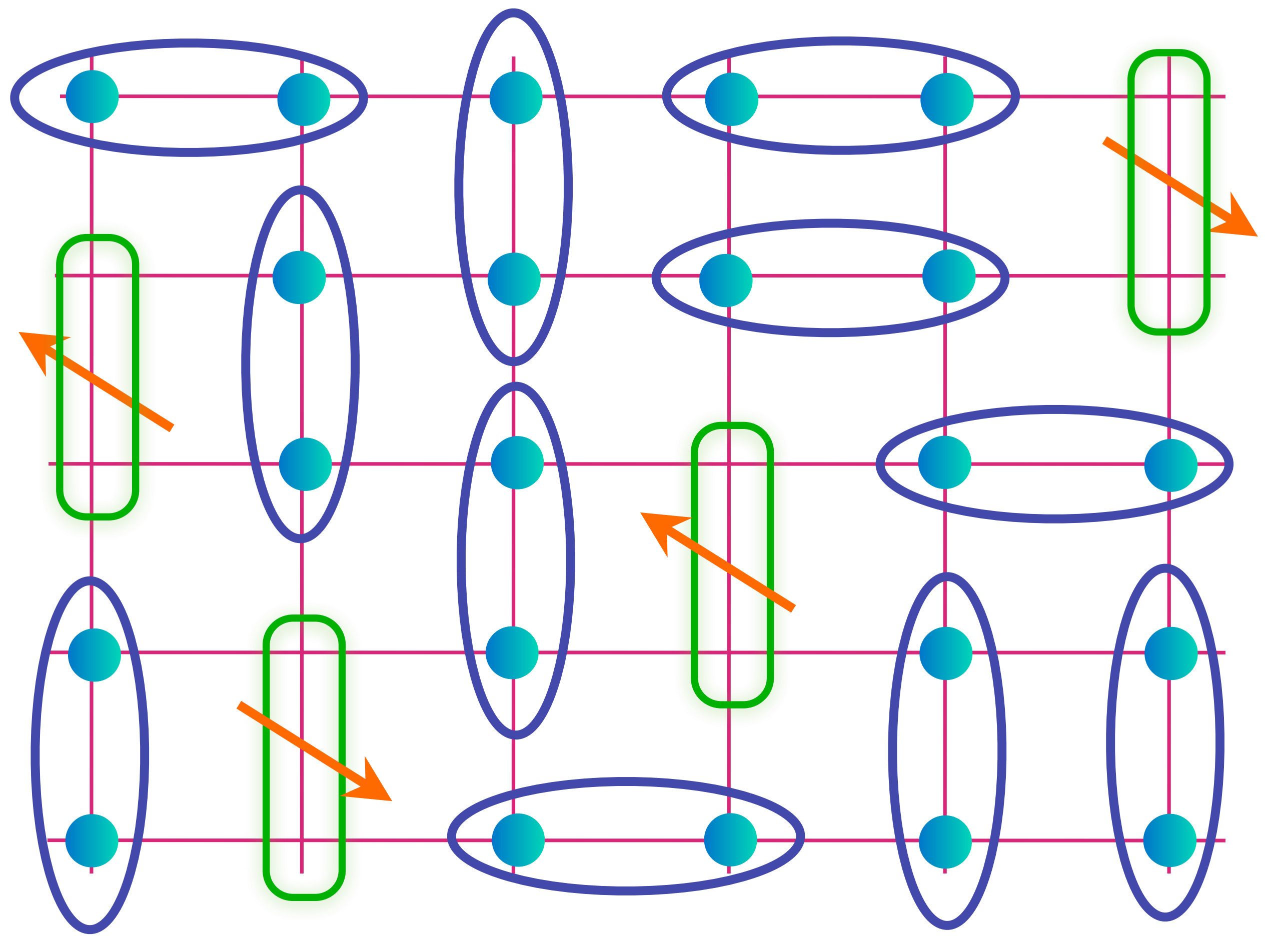
AF Metal

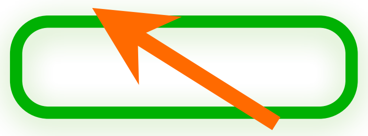


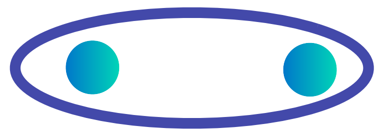
Carrier density p
Pocket area $p/4$

$\boxed{\langle (-1)^{\boldsymbol{r}} \boldsymbol{S}_{\boldsymbol{r}} \rangle \neq 0}$

FL*



 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

Carrier density p
Pocket area $p/8$

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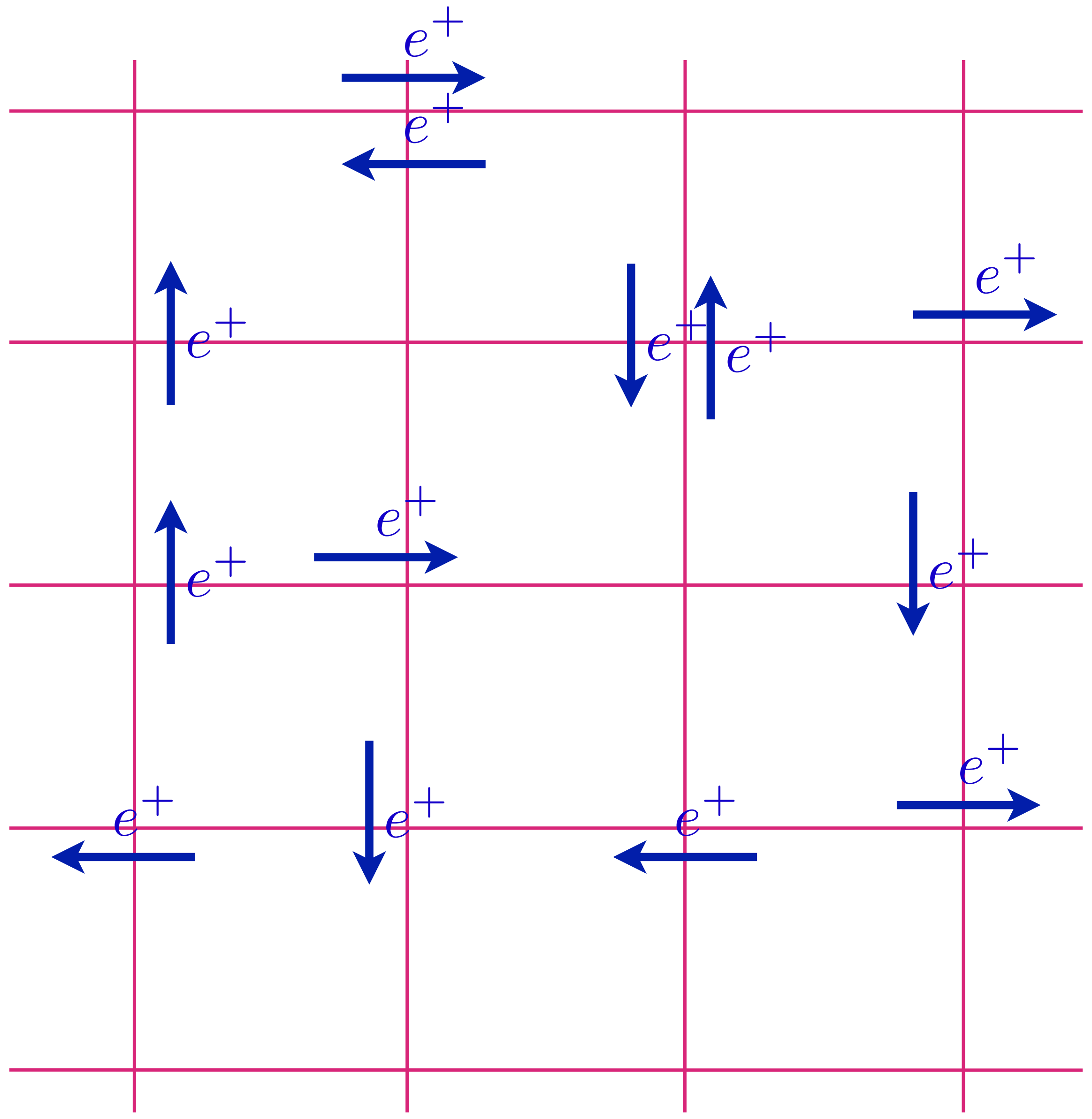
FL



Carrier density $1 + p$
Fermi area $(1 + p)/2$

$\boxed{\langle \Phi \rangle = 0}$

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between two metals
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condensation of a
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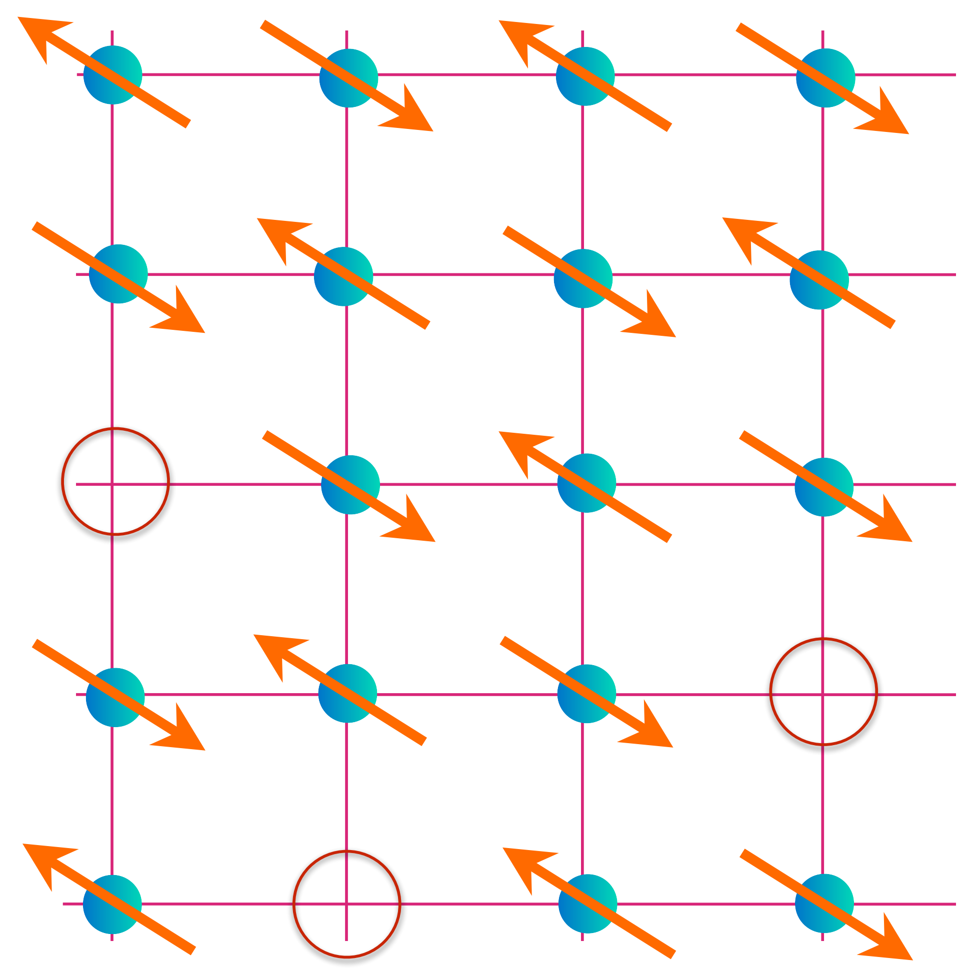
S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

AF Metal



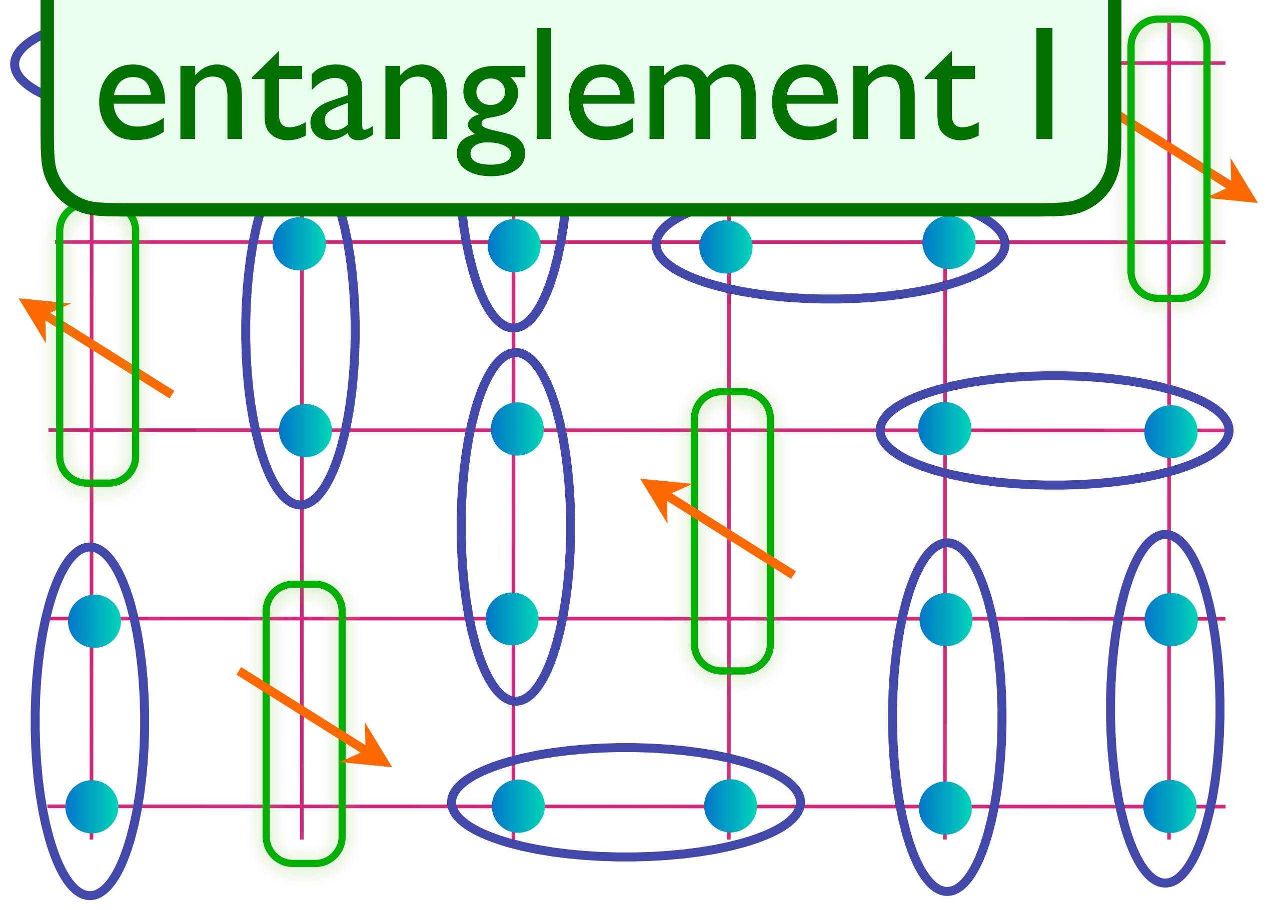
Carrier density p

Pocket area $p/4$

$\langle (-1)^{\boldsymbol{r}} \boldsymbol{S_r} \rangle \neq 0$

FL*

Many fermion entanglement I



$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

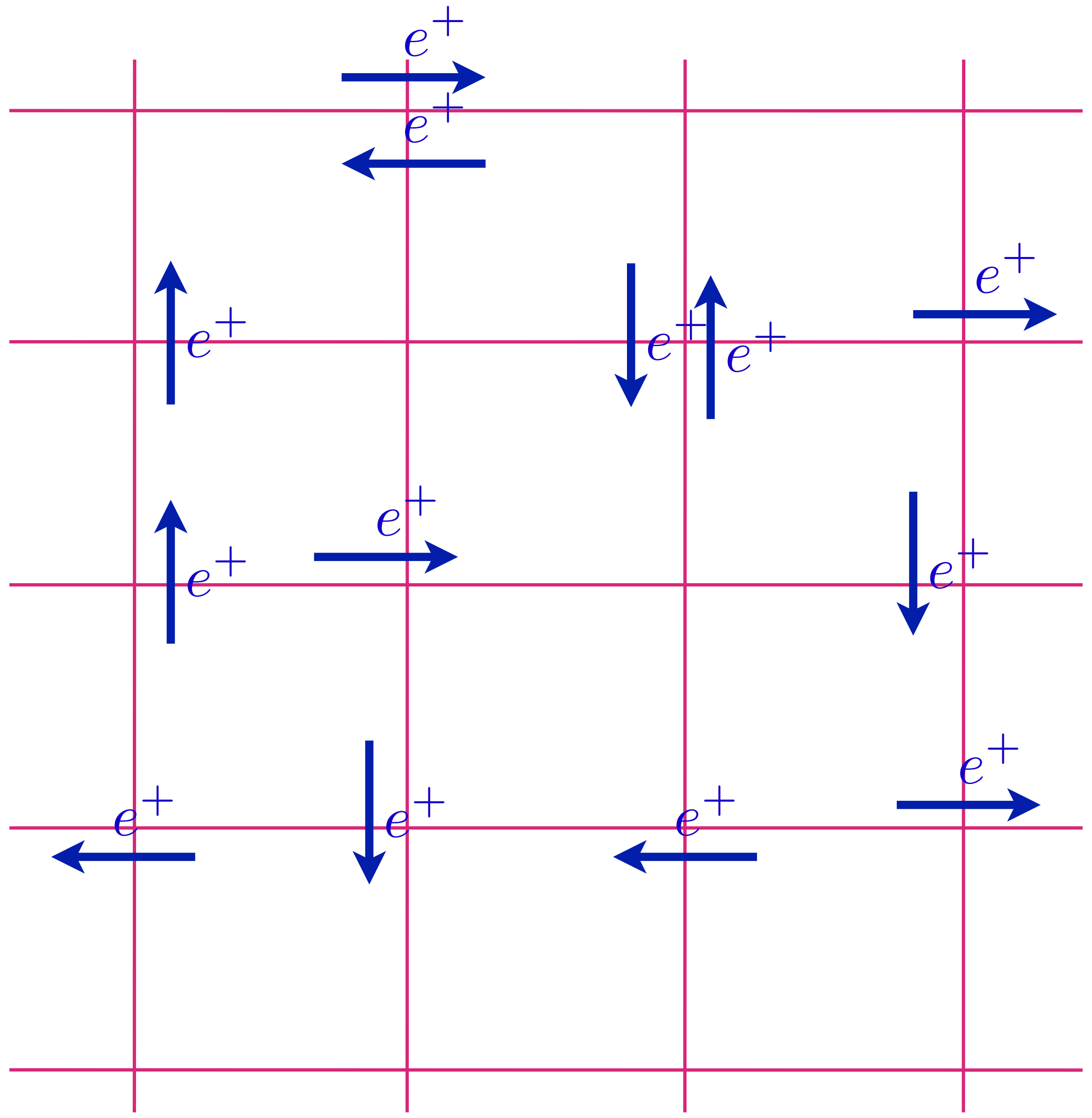
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

Carrier density p

Pocket area $p/8$

$\langle (-1)^{\boldsymbol{r}} \boldsymbol{S_r} \rangle = 0$ $\quad$ $\langle \Phi \rangle \neq 0$

FL



Carrier density $1 + p$

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$\langle \Phi \rangle = 0$

p_{sdw} $\qquad$ p_c $\qquad$ p

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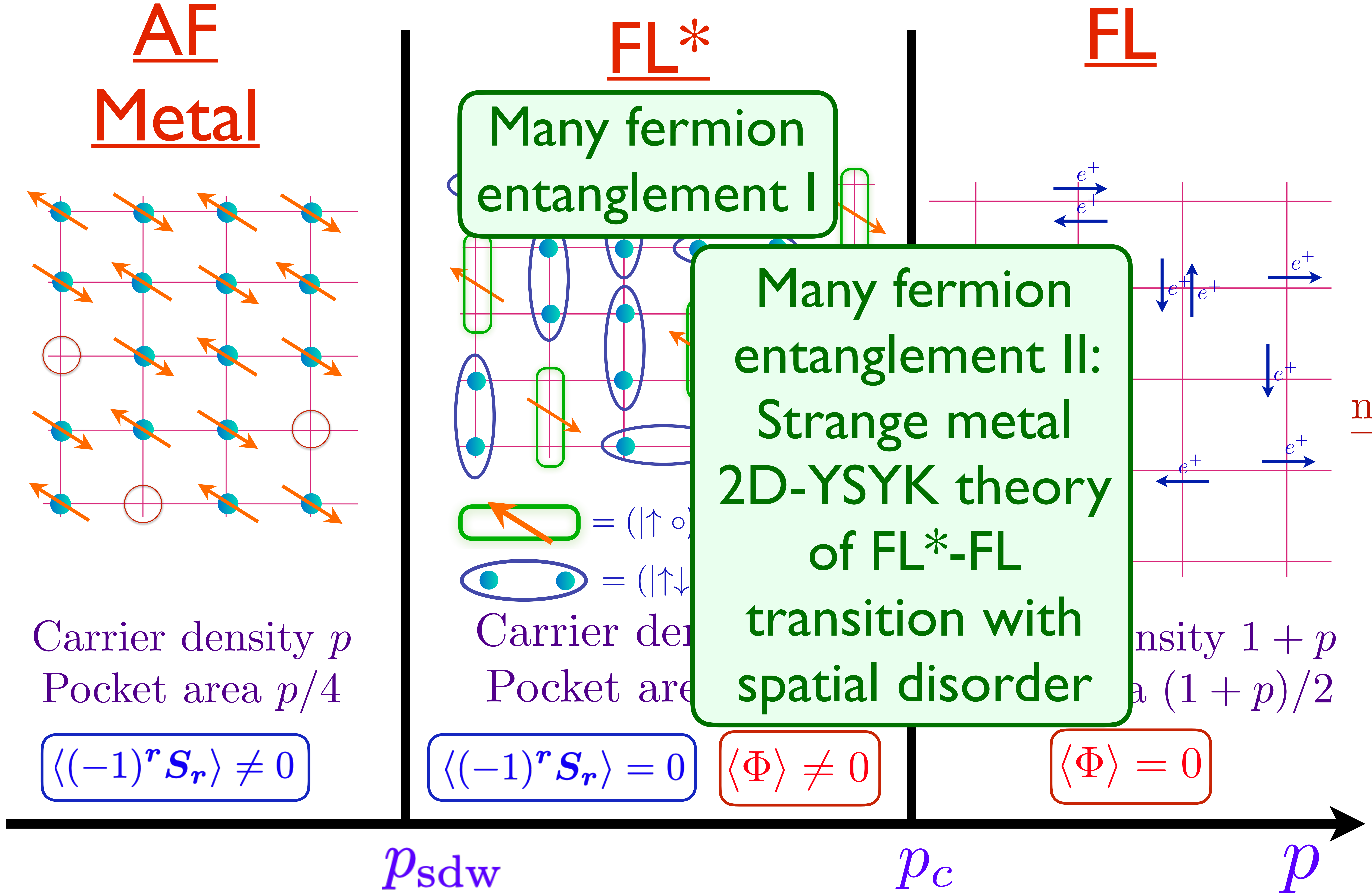

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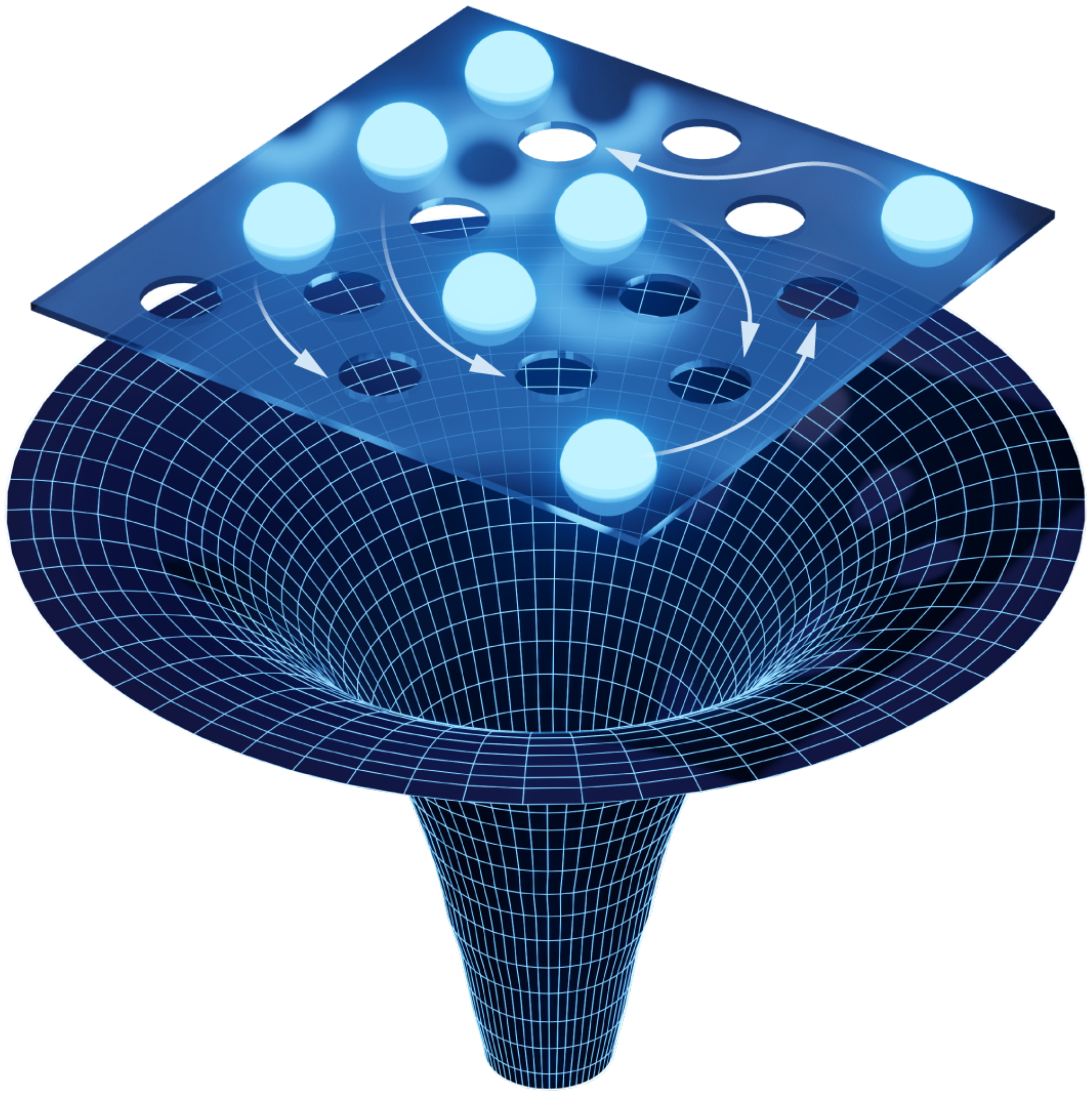
The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

Many fermion entanglement II



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Many fermion entanglement II

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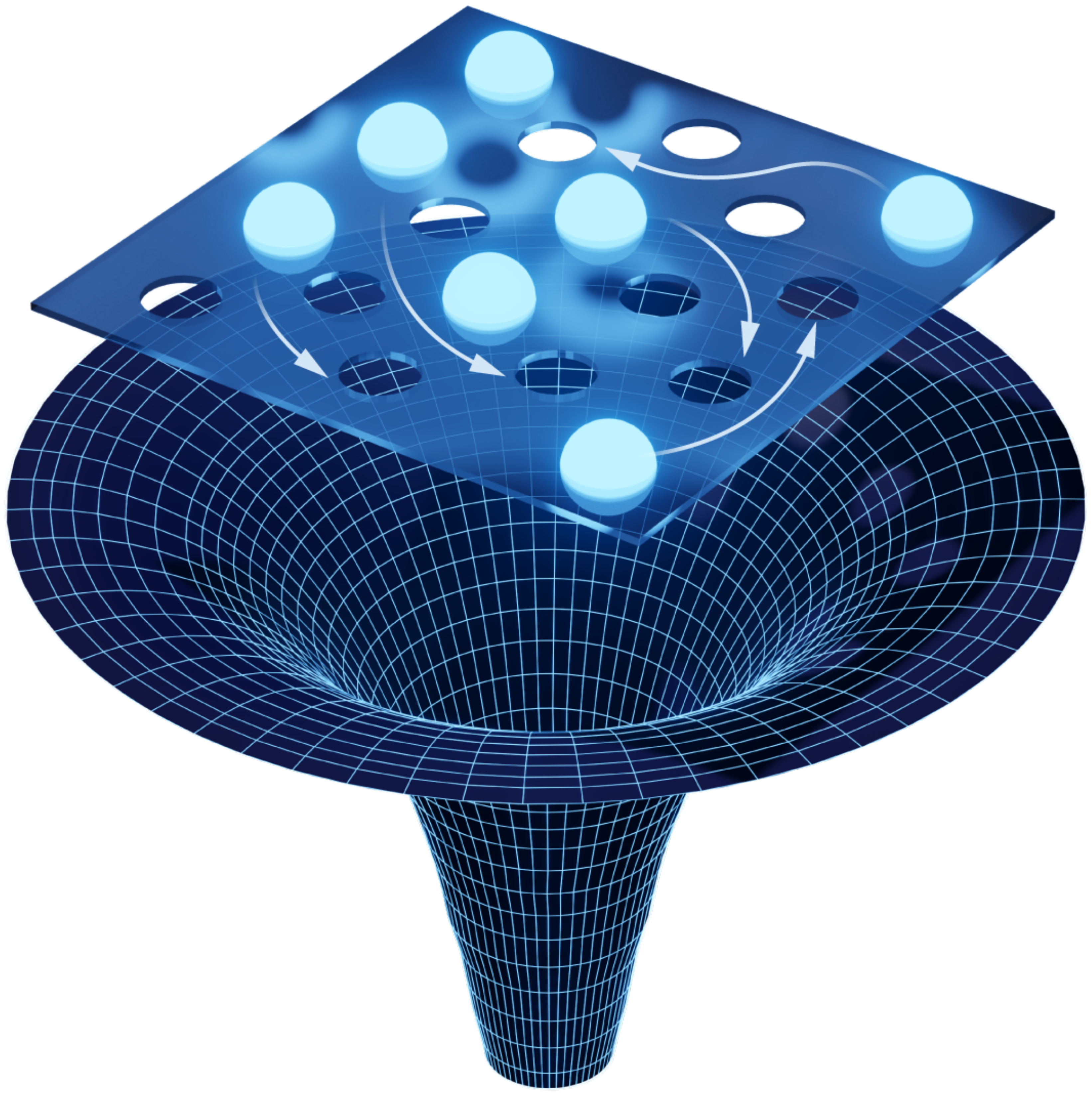

Extending to 2D-YSYK theory of FL*-FL transition, it helps describe the ***strange*** electrical properties of YBCO

Patel, Guo, Esterlis, Sachdev (2023)

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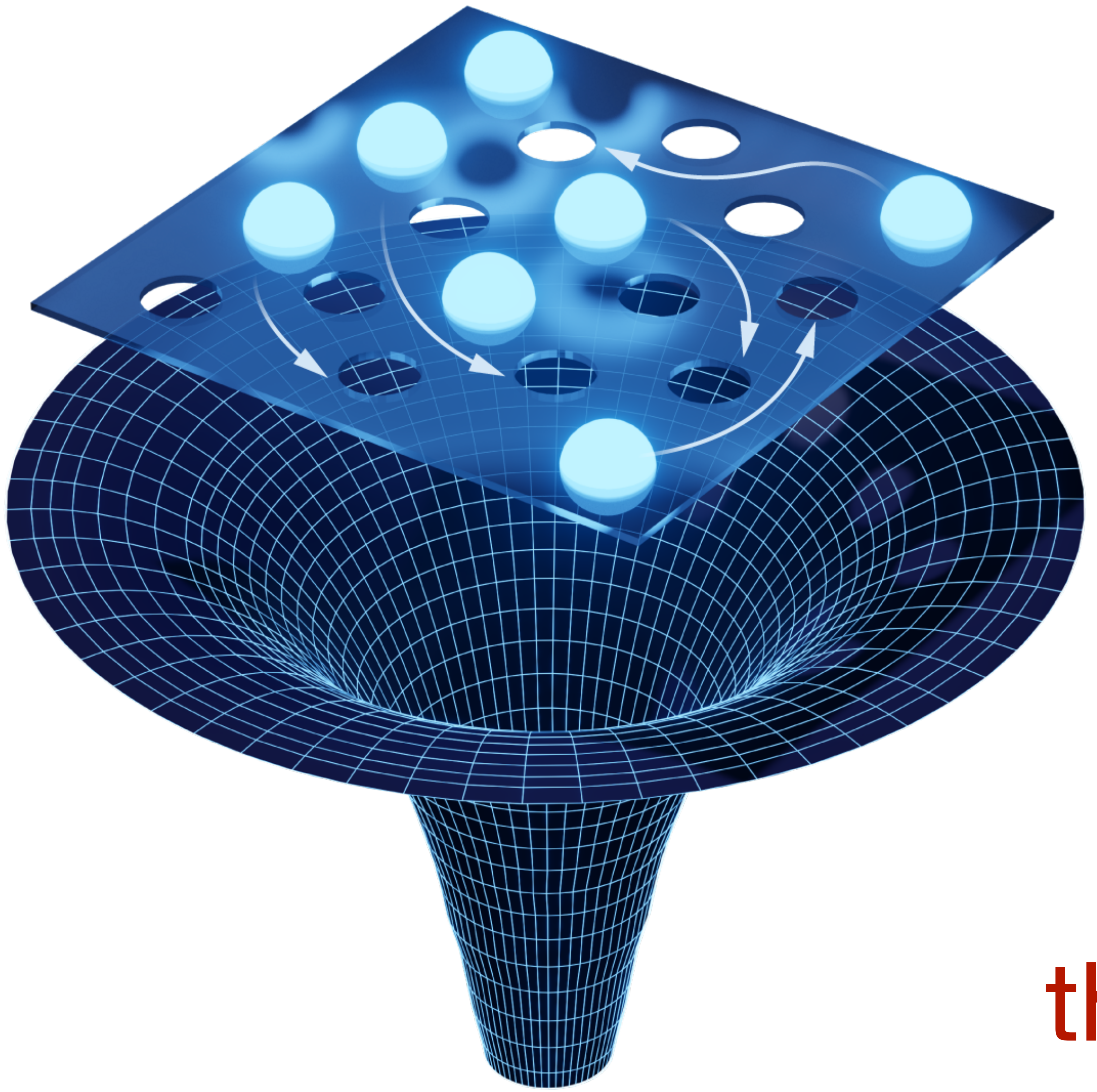

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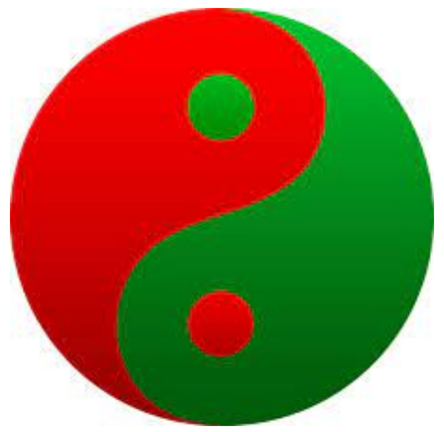
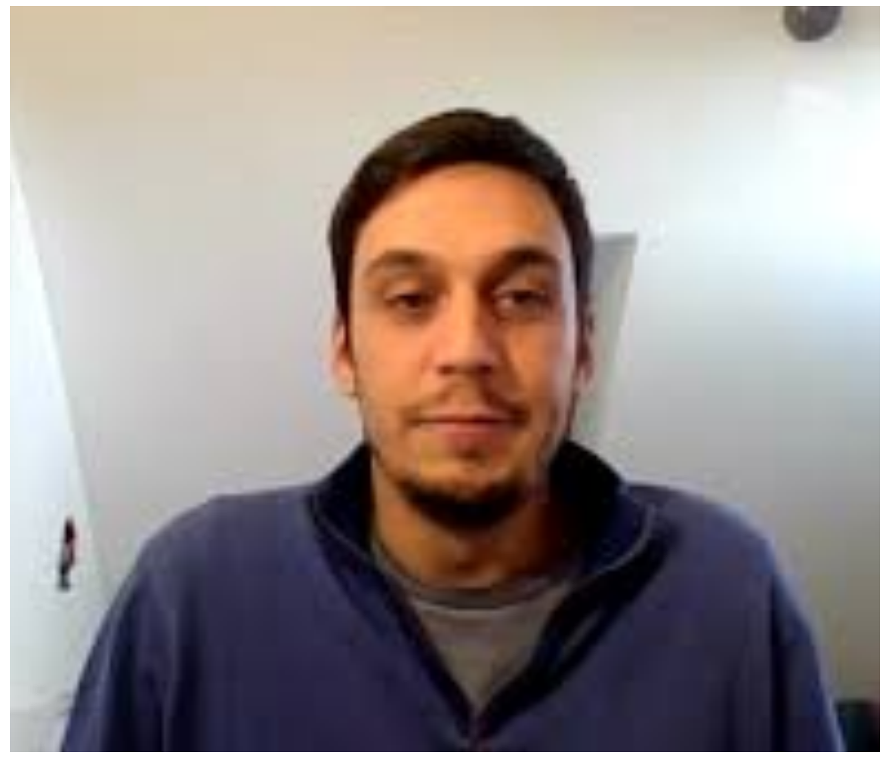
Patel, Guo, Esterlis, Sachdev (2023)



In a *dual* set of variables it describes the quantum horizon of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

Maine Christos
Caltech



Pietro Bonetti
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Sayantan Sharma

- *Lectures on insulating and conducting quantum spin liquids*, S. Sachdev, arXiv:2512.23962
- *Fractionalized Fermi liquids and the cuprate phase diagram*, P. M. Bonetti, M. Christos, A. Nikolaenko, A.A. Patel, and S. Sachdev, arXiv:2508.20164
- *Thermal SU(2) lattice gauge theory of the cuprate pseudogap: reconciling Fermi arcs and hole pockets*, H. Pandey, M. Christos, P. M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, and S. Sachdev, arXiv:2507.05336