**Cycling in Classical Statistical Mechanics and the Idea of Frequency**

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In physics, there are deterministic theories such as classical mechanics and statistical theories such as classical statistical mechanics and quantum mechanics. Many interpretations are applied to statistical theories and there is sometimes debate as to the application of such theories in various physical scenarios. In a previous note, we considered cycling due to exp(iEt) in quantum mechanics and tried to argue that such cycling may also appear in statistical mechanics. We made use of the example of the ground state of the quantum oscillator which has a quantum mechanical density of the same form as the classical statistical mechanical result. In this note, we wish to consider the idea of cycling in statistical mechanics using some simple examples. In addition, we wish to also examine the notion of losing information to pass from a deterministic theory to a statistical one. Finally, we wish to consider some possible aspects of the classical statistical density in a potential. It seems that even in classical statistical mechanics, the picture presented by the ‘averaging’ may actually be quite different from the physical scenario which may contain deterministic behavior which is lost in the calculation of statistical results.

**Classical Statistical Mechanics**

Many statements have been made regarding the meaning of classical statistical mechanics. For example, it has been stated in some books that statistical mechanics is a way of dealing with deterministic classical mechanical solutions of many particles in an average kind of way. In addition, classical statistical mechanics has been applied to numerous physical problems, but there have been debates as to whether ‘statistical equilibrium’ actually exists in such cases. It is known that in classical statistical mechanics, relative weights for velocity and position in a potential are given by the Maxwell-Boltzmann result:

Exp(-(.5p\*p\m + V(x))\T) ((1))

It is also known that this distribution can be obtained by maximizing Shannon’s entropy P(x,p)ln(P(x,p) subject to various constraints. The maximization of Shannon’s entropy is said to be related to a loss of information. This begs a question. In the absence of a potential V(x), could one place a number of particles, distributed according to exp(-(.5p\*p\mT)) between two walls (one dimension) and have them simply bounce (elastically) back and forth with colliding with each other and still have statistical mechanics? If statistical mechanics is simply an averaging of many deterministic classical mechanical equations, one might think that the answer would be yes. Even though at time zero, the distribution satisfies a Maxwell-Boltzmann distribution and there is no change in the distribution in time, it seems that this is not a classical statistical mechanical scenario. First of all, there is extra information in the system, namely that a particle with a certain momentum will retain the absolute value of this momentum for all time. More disturbingly, is the idea that a fast particle will bounce back and forth several times in the same interval during which a slow particle bounces back once. Thus, if one were sitting in the middle of the box, one would record that there were several fast particles, but only one slow particle during the interval of measurement. In statistical mechanics the particles are indistinguishable so one would not know that the it was the same fast particle bouncing back and forth. (In measurements of the Maxwell-Boltzmann distribution, a hole is apparently made and particles escape so such a bouncing issue does not come into play.)This, however, would change the Maxwell-Boltzmann distribution, skewing it in favour of fast particles. Thus, it seems that there is an important idea of cycling occurring. The fast particle needs to change its momentum upon colliding with the wall. The wall itself seems to be the driver of statistical mechanics as it is assumed to be at a temperature of T. This seems to mean that it will impart energy to colliding particles in a certain manner. Furthermore, a fast particle cannot simple collide and lower its momentum a little, otherwise the problem of high bounce frequencies will still persist. It seems, that in order to have equilibrium, both slow and fast particles need to create an average bouncing frequency which is the same over a certain interval of time. Then, it seems, one will have classical statistical mechanical equilibrium. This concept of a cycling frequency, in a classical gas with no potential, seems to be important and appears to be related to the idea of cycling in quantum mechanics as described in (1),(2). There, it was argued that different plane waves are created by a potential V(x). In equilibrium, one has a wavefunction W(x) which can be written as a Fourier series of these waves. Each plane wave in the series, however, would also interact with the potential leading to a cycling which overall has to preserve the form of W(x). It was argued that if each plane wave has a cycling frequency of E, the energy, one can maintain such a resonance (2). It seems that a similar physical idea may exist in the classical gas, with the average frequency of bouncing being related to the overall temperature in the gas. In general, it seems that this cycling is not discussed in classical statistical mechanics. Instead, words such as maximum disorder are used to describe the system. We try to argue that there is in fact order in the system as the bouncing particles try to maintain an overall identical average frequency. We think that this order may also be related to the fact that one can create sound waves in the medium, but that is not pursued here.

**Deterministic Classical Mechanics**

In classical mechanics, a particle is treated as a point (center of mass) and one knows the velocity and position of this point at each time. (One can also have rotations about the center of mass but that is not considered here.) This is the deterministic picture. In (3), however, the authors have argued that a classical particle can be described in terms of a probability density given by dt\v. Thus, information about time is being lost. They apply this result to the classical oscillator, using conservation of energy, to obtain:

probability density proportional to dx\sqrt(E - .5kx\*x) ((2))

One may try to interpret this equation statistically as there is no information about time included here. Given that there is a particle moving in a potential V(x)= .5kx\*x one would assume that there are different velocities at different points, but there can be velocities in both directions. Considering ((2)) to be a steady state solution, one would not really consider an overall collective flow. In addition, one would try to assign a type of pressure balance. This is of course, not what is happening physically if one brings in the full time dependent deterministic oscillator solution, but rather seems to be a kind of statistical averaging scenario. The point seems to be that one needs to be careful with the interpretation of statistical results. It should be noted that the density of ((2)) has been compared to that of high energy solutions of the quantum mechanical oscillator and it has been shown that on average the two tend to agree, although quantum mechanics shows signs of an underlying wavelength. Thus, a deterministic theory can be described in terms of statistical mechanics if one loses information, but one has to be careful because some of the ‘average concepts’ may correspond to physical reality in an unusual way. For example, the idea of pressure from motion to the right or left in the statistical picture is somewhat different from the deterministic view that the particle moves to the right and then returns at a much later time. Furthermore, overall statistical observables such as the probability density ((2)) may mask physical dynamics in the system. For example, on average at each point x, there is no velocity flow because the velocity to the right cancel that to the left, but in reality there are two velocity currents. It seems that similar physical ideas occur in quantum mechanics where the wavefunction contains information about dynamics which are lost when one calculates the density, which is the physical observable.

**Classical Statistical Mechanics with a Potential V(x)**

The Maxwell-Boltzmann factor ((1)) shows that in the presence of a potential V(x), the spatial density changes. It can be shown that this variation in spatial density represents a pressure balance, at least in the ideal gas case, between the pressure at x, -dV\dx and the pressure at x+ delta where delta is a small number. This approach is often used in statistical mechanical textbooks to calculate the spatial density in the presence of gravity. In the previous section, however, it has been seen that a statistical picture of a deterministic model can yield similar considerations even though physically different things are happening. Consider again the case of particles bouncing back and forth between two walls. The bouncing cannot be elastic as argued in order to maintain statistical mechanics. In the case of an oscillator potential, each particle will follow conservation of energy at each point x dictated by the oscillator. It will also follow the oscillator frequency. When considering classical statistical mechanics, however, there is no overall conservation of energy at each point (i.e. average kinetic energy + average potential energy = E) and there is no oscillator frequency in the result ((1)). Nevertheless, both of these are physically present. In addition, the pressure balance given by the spatial part of ((1)) seems here to be a consequence of conservation of energy for each particle at each point x. Consider one of the bouncing particles moving according to deterministic classical mechanics in the oscillator potential. Its physical density is given by ((2)). Then:

Density(x+b) = density(x) + b.5 dV\dx (density(x) cubed) ((3))

According to the Maxwell-Boltzmann case one has:

Density(x+b)= density(x) + density(x)\T b dV\dx ((4))

Both of these are of a similar form with the change of density being related to dV\dx which is related to force. In the case of ((1)), there is a temperature in the problem which is not part of ((2)). Furthermore ((3)) applies to one particle, while ((4)) is an overall average. Nevertheless, the point we wish to make is that physically one may have a very clear picture of a set of particles each moving according to classical mechanics with statistical mechanics only coming in during the collision at the wall, yet the overall description given by statistical mechanics loses this deterministic information.

**Conclusion**

In conclusion, we have tried to argue that cycling is an important physical part of statistical mechanics. We argue that it is not sufficient to have a Maxwell-Boltzmann distribution, but that particles must cycle through different values of this distribution. We have presented a simple picture in which this cycling can occur if each particle has the same average ‘bounce’ frequency, for the case of particles bouncing back and forth between walls at a temperature T. We think that this frequency may imply that statistical mechanics (for the case of no potential) is a type of resonance. We have argued in other notes that quantum mechanics may be a similar type of resonance with cycling occurring according to exp(iEt).

References

1. Ruggeri, Francesco R. The Hamiltonian and Time Evolution for Classical Statistical Systems and Quantum Systems and a Possible Origin of exp(iEt) in the Wavefunction (preprint, zenodo, 2018)

2. Ruggeri, Francesco R. Cycling in Quantum and Classical Statistical Mechanics (preprint, zenodo, 2018)

3. Majernik, V. and Opartny, T. Entropic Uncertainty Relations for a Harmonic Oscillator (J. Phys. A. Math 1996, 2187-2197)

.upol.cz/tom/clanky/jpa96-ent.pdf