

LETTERS TO THE EDITOR.

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The Size of the Pages of Scientific Publications.

It was with much surprise that we received the circular of the Royal Society stating that it had been decided to abandon the present size of its *Proceedings* in favour of royal octavo, accompanied by a voting card on the question of a similar change in the size of the *Transactions*. At the Oxford meeting of the British Association, a Committee was appointed, by Section A, to endeavour to secure greater uniformity in the sizes of the pages of the *Transactions* and *Proceedings* of all societies which publish mathematical and physical papers. In view of the report which that Committee will present shortly at Ipswich, it is much to be hoped that the Council of the Royal Society will take no immediate steps toward carrying their recommendations into effect.

A considerable degree of uniformity already exists. The present octavo size of the *Proceedings of the Royal Society* is very nearly the same size as the *Philosophical Magazine*, the *Report of the British Association*, the *Proceedings of the London Mathematical Society*, and of the *Cambridge Philosophical Society*, and many other publications. The *Annalen der Physik und Chemie* is so very little smaller, that reprints from it can be bound up with others from the afore-mentioned sources, without paring down their margins excessively. For papers involving long mathematics or large diagrams, the quarto size of the present *Philosophical Transactions* approximates to uniformity with the *American Journal of Mathematics*, the *Comptes rendus* of the Académie des Sciences of Paris, the *Cambridge Transactions*, the *Edinburgh Transactions*, and numerous other quarto *Transactions*, such as those of the Institution of Naval Architects.

It is very important that specialists in any branch of science should be able to collect, and bind together, reprints of papers on their own particular subjects, and such volumes are of permanent value as works of reference. So long as there are only two sizes to deal with—the above-mentioned quarto and octavo—there is little difficulty about this, but occasionally one comes across a paper of intermediate size, which cannot be bound up with either, and the collection is thus necessarily incomplete. It is hoped that the report, so shortly to be presented, will be a guide to authors of papers in indicating which publications to select, and which to avoid, if they desire to conform to the average standard sizes. Although the work of the Committee is at present confined to mathematical and physical papers, it might perhaps be of advantage that the matter should be discussed in, and representatives on the Committee appointed from the other Sections of the British Association as well. The question of changing the size of the *Proceedings* was recently discussed by the London Mathematical Society, but it was decided to retain the existing form, at any rate for the present, mainly on account of its uniformity with other publications. It will be most unfortunate if the Royal Society takes any retrograde step which may prevent the sizes of its *Proceedings* and *Transactions* from being adopted as the standards.

G. H. BRYAN,
SYLVANUS P. THOMPSON.

On the Minimum Theorem in the Theory of Gases.

YOU would oblige me by inserting the following lines in NATURE. The last remark made by Mr. Burbury points out, indeed, the weakest point of the demonstration of the H-theorem. If condition (A) is fulfilled at $t=0$, it is not a mechanical necessity that it should be fulfilled at all subsequent times. But let the mean path of a molecule be very long in comparison with the average distance of two neighbouring molecules; then the absolute position in space of the place where one impact of a given molecule occurs, will be far removed from the place where the next impact of the same molecule occurs. For this reason, the distribution of the molecules surrounding the place of the second impact will be independent of the conditions in the neighbourhood of the place where the first impact occurred, and therefore independent of the motion of the molecule itself. Then the probability that a second molecule moving with given velocity should fall within the space traversed by the first

molecule, can be found by multiplying the volume of this space by the function f . This is condition (A).

Only under the condition, that all the molecules were arranged intentionally in a particular manner, would it be possible that the frequency (number in unit volume) of molecules with a given velocity, should depend on whether these molecules were about to encounter other molecules or not. Condition (A) is simply this, that the laws of probability are applicable for finding the number of collisions.

Therefore, I think that the assumption of external disturbances is not necessary, provided that the given system is a very large one, and that the mean path is great in comparison with the mean distance of two neighbouring molecules.

LUDWIG BOLTZMANN.

9 Tuerkenstrasse, Vienna, June 20.

Argon and the Kinetic Theory.

THE spectrum exhibited by argon undoubtedly shows that, under the conditions of the experiment, the molecules composing the gas are set into an intense state of vibration, while the ratio of the specific heats ($5/3$, about) shows, according to the equation

$$\beta = \frac{\frac{5}{3}}{\gamma - 1}, \text{ that } \beta = 1, \text{ and therefore the gas is, as pointed out by}$$

Lord Rayleigh, monatomic, and cannot therefore be capable of vibrating. But there is, I think, a very simple explanation of this apparent contradiction, and that is, that the above equation is not true, and that it should be, as will be proved hereafter, $\beta = 3k(\gamma - 1)$, where k is very nearly 1 for argon and other so-called permanent gases. This latter equation gives 2 for the value of β in argon, a value easily understood.

The virial equation for smooth elastic spheres of finite magnitude is $\frac{2}{3}PV = \frac{1}{2}mv^2 - \frac{1}{2}\Sigma Rr$; and since the resilience is unity and r finite, the term $-\frac{1}{2}\Sigma Rr$ cannot vanish. Now the term $\frac{2}{3}PV$ represents work or its equivalent of energy; hence the right-hand member of the equation must represent the same, and since the term $\frac{1}{2}mv^2$ is obviously kinetic energy, or its equivalent of work, the term $-\frac{1}{2}\Sigma Rr$ must also represent work or energy. Now we can find the value of $\frac{2}{3}PV$ in terms of $\frac{1}{2}mv^2$, as follows. Imagine a cube box so constructed that one side of each pair can be used as a spring to discharge any mass in contact with a velocity v . And suppose three smooth elastic spheres each of mass $\frac{M}{3}$ to be discharged by the three spring

sides with the above velocity into the interior of the box. Then the work done on each mass will be $\frac{1}{2} \cdot \frac{M}{3}v^2$. Put this equal to

PV and take V equal to the volume of the box. The total work done is evidently $3PV = \frac{1}{2}Mv^2$. If, instead of three elastic spheres, we imagine a very great number of very minute ones of the same total mass to be discharged by the spring sides with the same velocity, the energy will be the same as before, and the above equation will still be applicable; and the state of affairs now represented would be that of an ideal gas. But owing to collisions after first starting the velocities of the particles will vary, and therefore we must write the equation

$$3PV = \frac{1}{2}M\bar{v}^2; \dots \dots \dots (1)$$

where \bar{v}^2 is the mean square velocity of the particles. By hypothesis V has the same value in the above equation as in the virial equation; and P can be proved, if necessary, to have the same value in the two equations as follows.

If \bar{f} = the mean acceleration or retardation, as the case may be, of the cr. of gr. of an elastic sphere impinging directly against a plane; then $\bar{f}t = v$. Also $\bar{f} = \frac{v^2}{2s}$, $\therefore t = \frac{2s}{v}$. Here t is half the time of impact, and v the velocity normal to the plane before and after impact. Now if it can be shown that the time taken by the spring side of our imaginary box to give the same velocity is the same as the above, then it is obvious that the mean pressures in the two cases must be identical.

Assume s^3 to be the volume of the cube box, then s^2 is the area of each side. Now let the spring side be drawn back so as to act through a distance s on the mass $\frac{M}{3}$ with a constant pressure P per unit of surface; then $P s^2 \times s = PV$ represents the work done. The velocity given to the mass is v , and the acceleration constant. Hence the mean velocity of the spring

side in passing through the distance s is $v/2$, and the time is $s \div v/2 = 2s/v$, the same as in the first case. Which proves the proposition.

Since from (1) we have $3PV = \frac{1}{2} M\bar{v}^2$ or $\frac{3}{2} PV = \frac{1}{4} M\bar{v}^2$, we may substitute this value in the virial equation, and remembering that $\Sigma \frac{1}{2} m\bar{v}^2 = \frac{1}{4} M\bar{v}^2$, we get $-\frac{1}{2} \Sigma Rr = -\frac{1}{4} M\bar{v}^2$. Hence also

$$P = \frac{1}{3} \rho \bar{v}^2 \dots \dots \dots (2)$$

where $\rho = \frac{M}{V}$ the density. The above equation is easily obtainable without the use of the virial equation *when the time of impact* is taken into consideration. A phenomenon which cannot be assumed to be instantaneous without upsetting the dynamical definition of the measurement of a force; which expressed algebraically is $Ft = Mv$. From which it is evident that when t the time is 0, v the velocity, is also 0.

When the virial equation is made applicable to the case of a gas composed of molecules capable of vibrating, it seems obvious that the term $\Sigma \frac{1}{2} m\bar{v}^2$ should be written $\Sigma \frac{1}{2} \beta m\bar{v}^2$; because, as shown by Clausius, the internal energy of the molecules bears a constant ratio to the energy of agitation. We must look to the *mechanical structure of the molecule* for the reason of this. Here the fact is simply accepted, not explained; but it is obvious that the same forces which impart translatory energy to a molecule will impart vibratory energy also. The same reasoning applies to the term $-\frac{1}{2} \Sigma Rr$, which now becomes $-\Sigma \beta(Rr)$. The volume of the gas is unaltered by the vibrations, and the pressure is dependent on the two other terms. Hence the equation may be written

$$\frac{3}{2} PV = \Sigma \frac{1}{2} \beta m\bar{v}^2 - \frac{1}{2} \Sigma \beta(Rr) \dots \dots \dots (3)$$

And from this we get

$$P = \frac{1}{3} \beta \rho \bar{v}^2 \dots \dots \dots (4)$$

The above equation may be written

$$P_2 = \frac{1}{3} \rho \bar{v}_i^2; \dots \dots \dots (5)$$

Where $\bar{v}_i = \beta \bar{v}$. Again equation (2) may be written

$$P_i = \frac{1}{3} \rho \bar{v}_i^2; \dots \dots \dots (6)$$

the suffix i denoting that the pressure, density, and mean square velocity are those of an ideal gas composed of smooth elastic spheres.

If P_i , ρ_i , and \bar{v}_i in (6) are taken respectively equal to P , ρ , and \bar{v}_i in (5); then it is evident that \bar{v}_i in (5) is the velocity of mean square of an ideal gas which, having the same density, would give the same pressure as a natural gas. Hence \bar{v}_i can be found from (6). Now the total energy in unit mass of a gas is given by the equation

$$K_v T = \frac{1}{2} \beta \bar{v}^2; \dots \dots \dots (7)$$

where K_v is the specific heat at constant volume, and T is the absolute temperature. From which equation $\bar{v} \sqrt{\beta}$ can be found. We have also from above

$$\bar{v} \beta = \bar{v}_i \dots \sqrt{\beta} = \frac{\bar{v}_i}{\bar{v} \sqrt{\beta}} \dots \dots \dots (8)$$

from which equation the value of $\sqrt{\beta}$ and consequently β can be found.

The equation $\beta = 3k(\gamma - 1)$ can now be proved as follows. Multiplying both sides of (4) by V , the volume of unit mass, and combining with (7), we get

$$K_v T = 3PV \dots \dots \dots (9)$$

Now from (5) and (6), taking $\rho = \rho_i$, we get $P = P_i/\beta$, and substituting in (9) $K_v = 3P_i V/\beta T$. But $P_i V/T = K_{vp} - K_{tv}$; or the difference between the specific heats at constant pressure and constant volume; the suffix i indicating, as before, that the symbols refer to an ideal gas. Hence

$$\beta = \frac{3(K_{vp} - K_{tv})}{K_v} = \frac{3k(K_p - K_v)}{K_v} = 3k(\gamma - 1) \dots (10)$$

Here k is some factor which for so-called permanent gases is very nearly unity. For such gases we may write (10)

$$\beta = 3(\gamma - 1); \text{ or } \gamma = \frac{1}{3}(\beta + 3) \dots \dots \dots (11)$$

In the following table the values of β , except in the case of argon, are calculated from equation (8); and \bar{v}_i , the velocity of ideal gases having the same pressure and density as their cor-

responding natural gases, at standard temperature and pressure, from (6). The velocities are given in feet per second, and the value of gravity is taken at 32.2 . Column (4) gives the values of γ for the different gases calculated from equation (11); and column (5) gives the experimental values of γ . The close agreement between these values is a significant fact.

(1)	(2)	(3)	(4)	(5)	(6)
β	\bar{v}_i	\bar{v}	$\gamma = \frac{1}{3}(\beta + 3)$	γ Ex-periment	k
Hydrogen ... 1'234 ... 8551 ... 6925 ... 1'4115 ... 1'412 ... 1'00035					
Oxygen ... 1'197 ... 2140 ... 1787 ... 1'399 ... 1'402 ... 1'0021					
Nitrogen ... 1'227 ... 2282 ... 1860 ... 1'409 ... 1'411 ... 1'0014					
Dry air ... 1'222 ... 2250 ... 1841 ... 1'407 ... 1'409 ... 1'0014					
Argon ... 2 (about) ... 1940 ... 970 ... 1'7 ... 1'7 ...					

8, Norfolk Square, W., June 13.

C. E. BASEVI.

Romano-British Land Surface.—Flint Flakes Replaced.

IN the early spring of the present year, whilst passing a newly-opened excavation near Caddington Church, three miles south-east of Dunstable, I noticed a very thin horizontal line of sharp flint flakes, embedded a foot deep from the surface-line of an old pasture. I could see at once that the line represented an old living surface, so I took a few of the flints away. In removing the stones from the soil, one or two little fragments of Romano-British pottery came away with them. The flakes were lustrous,

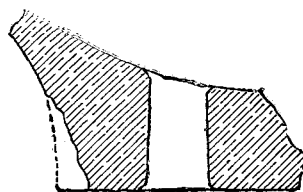


FIG. 1.—Fragment of perforated Romano-British pottery (half actual size).

chiefly black and brown-grey, and as sharp as when first struck. On looking over the flints in the evening, I was able to replace five on to each other. This fact, and the occurrence of the pottery fragments, proved the old surface to have remained intact from Romano-British times.

A little later in the spring, about six square yards of the superincumbent soil were carefully removed for me, when other flakes were found *in situ* to the exact number of four hundred; with these were eighteen fragments of Romano-British pottery, one piece—somewhat like the bottom of a pot—perforated, as here



FIG. 2.—Four conjoined flint-flakes (half actual size).

illustrated. Amongst the flints were two cores, two hammer-stones, three scrapers, part of one edge of a chipped celt, and several neatly chipped but apparently unfinished little implements. A middle-brass Roman coin, too corroded for identification, was found on the same surface in a second excavation close by; with this was a small piece of wood carved to represent a horse's fore-leg, and a well-finished and perfect unpolished flint celt.

In sorting the flints I was able to replace thirty-eight on to each other in groups of from two to five. Two of these groups