Suirodoku: A Graeco-Latin Sudoku Square

CSP Formalization, Bijection Theorem, and the God Digit Problem

Jordan Maire
University of Paris 1 Panthéon-Sorbonne
`https://suirodoku.com`

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Abstract

We introduce Suirodoku, a 9×9 combinatorial puzzle combining Sudoku constraints with Graeco-Latin square orthogonality. Each cell contains an ordered pair (digit, color) where both projections satisfy Sudoku constraints, and each pair appears exactly once. We establish existence constructively, provide a complete CSP formalization within the Multi-Sorted First Order Logic (MS-FOL) framework of Berthier [14], comprising a Grid Theory (six sorts with exhaustivity axioms), a Suirodoku Theory (ten constraint axioms including a global pair uniqueness axiom), and instance-specific puzzle axioms. We prove a Bijection Theorem conferring absolute identity to each cell—a property absent from classical Sudoku. We derive solving heuristics (Rainbow and Chromatic Circle techniques), and pose the God Digit Problem: must every uniquely solvable Suirodoku puzzle contain all 9 digits among its clues? We prove a Dichotomy Theorem showing that either no symbol is critical, or all symbols are critical. The existence question remains open and calls for combinatorial and/or computational investigation.

Keywords: Constraint satisfaction, Sudoku, Graeco-Latin squares, global constraints, bijection, critical symbols

Notation: We write pairs (d, c) where d is a digit and c a color, both in $\{1, \ldots, 9\}$. Colors may also be written as Greek letters in traditional Graeco-Latin notation.

1 Introduction

Sudoku is one of the world's most popular logic puzzles: fill a 9×9 grid so that each digit from 1 to 9 appears exactly once per row, column, and 3×3 block. Graeco-Latin squares, studied by Euler [1], are grids where each cell contains a pair of symbols, with each pair appearing exactly once.

A natural question arises: can these two structures be combined? This paper answers affirmatively by introducing *Suirodoku* (from the Japanese: *sū* "digit", *shoku* "color", *doku* "unique"), a puzzle that augments Sudoku with a chromatic dimension and a global pair uniqueness constraint.

Although the underlying combinatorial object (orthogonal Sudoku squares) has been studied in the context of construction and experimental design, our contribution focuses on the puzzle/solving aspect. We make three main contributions:

(i) **CSP Formalization.** We formalize Suirodoku as a constraint satisfaction problem within Berthier's Multi-Sorted First Order Logic (MS-FOL) framework [14], comprising 10 axioms yielding 55 constraints (54 local + 1 global).

(ii) **Bijection Theorem.** We prove that valid solutions establish a bijection between cells and the space of 81 pairs, conferring absolute identity to each cell and yielding new solving heuristics.

(iii) **God Digit Problem.** We pose and partially resolve the question of whether certain symbols must necessarily appear among the clues of any uniquely solvable instance, proving a Dichotomy Theorem.

Figure 1: A complete Suirodoku grid. Each cell contains a unique (digit, color) pair. Each digit (1–9) and each color appear exactly once per row, column, and 3×3 block.

2 Background

2.1 Latin Squares

A Latin square of order n is an $n \times n$ grid where each symbol from a set of n symbols appears exactly once per row and per column. Sudoku is a Latin square of order 9 with additional 3×3 block constraints.

2.2 Graeco-Latin Squares

A Graeco-Latin square of order n (Euler [1]) places an ordered pair (a, b) in each cell such that: (1) the first symbols form a Latin square, (2) the second symbols form a Latin square, and (3) each pair appears exactly once.

Euler conjectured that Graeco-Latin squares do not exist for $n \equiv 2 \pmod 4$. Tarry [2] proved that $n = 6$ is impossible (the "36 officers problem"). Bose–Shrikhande–Parker [3] disproved Euler's conjecture for $n \geq 10$. It is now established that Graeco-Latin squares exist for all orders n except $n = 2$ and $n = 6$.

2.3 Prior Work on Orthogonal Sudoku

Equivalent structures have been studied under the names "mutually orthogonal Sudoku squares" or "Graeco-Sudoku square designs": two Sudoku grids are orthogonal if their superposition produces every pair of symbols exactly once. Subramani and Ponnuswamy introduced "Sudoku designs" in 2009 as tools for experimental design [7]. Subramani then presented the first systematic construction method for Graeco-Sudoku squares of odd orders in 2012 [8], and deepened

Latin Square

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

Graeco-Latin Square

Aα	Bβ	Cγ	Dδ
Bδ	Aγ	Dβ	Cα
Cβ	Dα	Aδ	Bγ
Dγ	Cδ	Bα	Aβ

Figure 2: Left: a Latin square of order 4. Right: a Graeco-Latin square—each (Latin, Greek) pair appears exactly once.

the analysis in 2013 [9]. Later extensions include Hyper Graeco-Latin Sudoku designs [10] and Hyper Block designs [12].

3 Suirodoku: Definition and Existence

We distinguish a *Suirodoku grid* (a complete solution satisfying all constraints) from a *Suirodoku puzzle* (a partial grid with clues, intended to have a unique solution).

Definition 1 (Suirodoku Grid). *A **Suirodoku grid** is a* 9×9 *grid where each cell contains an ordered pair* (d, c) *such that:*

(i) each digit appears exactly once per row, column, and block;

(ii) each color appears exactly once per row, column, and block;

(iii) each pair (d, c) *appears exactly once in the entire grid.*

Equivalently: a Suirodoku grid is a Graeco-Latin square of order 9 whose two projections satisfy Sudoku block constraints.

Presentation. Unlike prior work using two numerical grids, we present Suirodoku as a single grid of (digit, color) pairs, where the second component uses a visual alphabet. This facilitates readability and the formulation of color-specific solving rules.

Existence (constructive). Adding block constraints to a Graeco-Latin square is severe—order 6 admits no Graeco-Latin square at all. However, order 9 with block constraints remains satisfiable. Figure 1 provides an explicit witness; existence is therefore immediate.

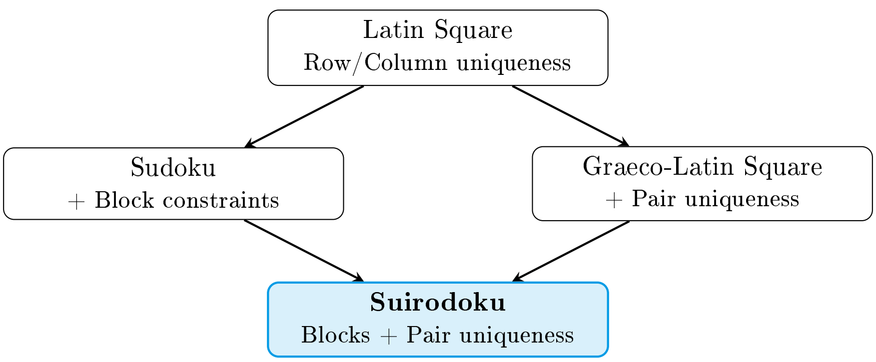

Figure 3: Structural hierarchy. Suirodoku sits at the intersection of Sudoku and Graeco-Latin square constraints.

4 CSP Formalization in Multi-Sorted First Order Logic

Following Berthier [14, 15, 16], we formalize Suirodoku as a CSP theory in Multi-Sorted First Order Logic (MS-FOL). The formalization comprises three layers: Grid Theory, Suirodoku Theory, and Puzzle Theory.

4.1 Grid Theory

4.1.1 Sorts and Exhaustivity

Definition 2 (Sort System)**.** *The Suirodoku sort system comprises six sorts, each with exactly nine elements:*

$$\begin{aligned}
\text{Digit} &= \{1, 2, \ldots, 9\} \\
\text{Row} &= \{r_1, r_2, \ldots, r_9\} \\
\text{Column} &= \{c_1, c_2, \ldots, c_9\} \\
\text{Block} &= \{b_1, b_2, \ldots, b_9\} \\
\text{Cell} &= \{s_1, s_2, \ldots, s_9\} \quad \textit{(position within a block)} \\
\text{Color} &= \{k_1, k_2, \ldots, k_9\}
\end{aligned}$$

We adopt two standard MS-FOL conventions:

Unique Name Assumption: Distinct constant symbols of the same sort denote distinct entities.

Exhaustivity Axioms: Each sort contains exactly its named constants and no other elements. For each sort S with constants $s_1, \ldots, s_9$:

$$\forall x{:}S \ \ (x = s_1 \lor x = s_2 \lor \cdots \lor x = s_9)$$

These axioms guarantee that each sort has cardinality exactly 9, which is essential for the bijection argument of Section 5.

4.1.2 Coordinate Mapping

Definition 3. *The predicate* $\text{mapping}(r, c, b, s)$ *of signature* $(\text{Row}, \text{Column}, \text{Block}, \text{Cell})$ *expresses that cell* (r, c) *lies in block* b *at relative position* s*.*

The Grid Theory includes 81 ground axioms specifying this mapping for the standard Sudoku layout, along with two functionality axioms:

Axiom G1—(r, c) determines (b, s):

$$\forall r \ \forall c \ \exists!\, b \ \exists!\, s \ \ \text{mapping}(r, c, b, s)$$

Axiom G2—(b, s) determines (r, c):

$$\forall b \ \forall s \ \exists!\, r \ \exists!\, c \ \ \text{mapping}(r, c, b, s)$$

Together with the 81 ground axioms and exhaustivity, G1 and G2 establish a bijection between (Row × Column) and (Block × Cell).

4.1.3 Notational Abbreviation

Definition 4. *For any predicate* $P(\ldots, r, c, \ldots)$ *using row-column coordinates, we write* $P[\ldots, b, s, \ldots]$ *as an abbreviation for:*

$$\exists r \ \exists c \ \ [\text{mapping}(r, c, b, s) \land P(\ldots, r, c, \ldots)]$$

Square brackets [ ] indicate block-cell coordinates; parentheses ( ) indicate row-column coordinates. This is a definitional abbreviation, not a new primitive symbol.

4.2 Suirodoku Theory: Ten Axioms

Definition 5 (Central Predicate)**.** *The predicate* value(n,k,r,c) *of signature* (Digit, Color, Row, Column) *means: "the pair* (n,k) *occupies cell* (r,c)*".*

The Suirodoku Theory extends the Grid Theory with ten axioms:

Axiom A1—Cell Uniqueness. Each cell contains at most one pair:

$$\forall r\ \forall c\ \forall n_1\ \forall k_1\ \forall n_2\ \forall k_2\ \ \{\mathrm{value}(n_1,k_1,r,c)\wedge \mathrm{value}(n_2,k_2,r,c)\Rightarrow (n_1=n_2\wedge k_1=k_2)\}$$

Axiom A2—Completeness. Each cell contains at least one pair:

$$\forall r\ \forall c\ \exists n\ \exists k\ \ \mathrm{value}(n,k,r,c)$$

Axiom ST$_{rn}$—Row/Digit. Each digit appears at most once per row:

$$\forall r\ \forall n\ \forall k_1\ \forall k_2\ \forall c_1\ \forall c_2\ \ \{\mathrm{value}(n,k_1,r,c_1)\wedge \mathrm{value}(n,k_2,r,c_2)\Rightarrow c_1=c_2\}$$

Axiom ST$_{rk}$—Row/Color. Each color appears at most once per row:

$$\forall r\ \forall k\ \forall n_1\ \forall n_2\ \forall c_1\ \forall c_2\ \ \{\mathrm{value}(n_1,k,r,c_1)\wedge \mathrm{value}(n_2,k,r,c_2)\Rightarrow c_1=c_2\}$$

Axiom ST$_{cn}$—Column/Digit. Each digit appears at most once per column:

$$\forall c\ \forall n\ \forall k_1\ \forall k_2\ \forall r_1\ \forall r_2\ \ \{\mathrm{value}(n,k_1,r_1,c)\wedge \mathrm{value}(n,k_2,r_2,c)\Rightarrow r_1=r_2\}$$

Axiom ST$_{ck}$—Column/Color. Each color appears at most once per column:

$$\forall c\ \forall k\ \forall n_1\ \forall n_2\ \forall r_1\ \forall r_2\ \ \{\mathrm{value}(n_1,k,r_1,c)\wedge \mathrm{value}(n_2,k,r_2,c)\Rightarrow r_1=r_2\}$$

Axiom ST$_{bn}$—Block/Digit. Each digit appears at most once per block:

$$\forall b\ \forall n\ \forall k_1\ \forall k_2\ \forall s_1\ \forall s_2\ \ \{\mathrm{value}[n,k_1,b,s_1]\wedge \mathrm{value}[n,k_2,b,s_2]\Rightarrow s_1=s_2\}$$

Axiom ST$_{bk}$—Block/Color. Each color appears at most once per block:

$$\forall b\ \forall k\ \forall n_1\ \forall n_2\ \forall s_1\ \forall s_2\ \ \{\mathrm{value}[n_1,k,b,s_1]\wedge \mathrm{value}[n_2,k,b,s_2]\Rightarrow s_1=s_2\}$$

Axiom ST$_{\mathrm{global}}$—Global Pair Uniqueness. Each digit-color pair appears at most once in the entire grid:

$$\forall n\ \forall k\ \forall r_1\ \forall c_1\ \forall r_2\ \forall c_2\ \ \{\mathrm{value}(n,k,r_1,c_1)\wedge \mathrm{value}(n,k,r_2,c_2)\Rightarrow (r_1=r_2\wedge c_1=c_2)\}$$

This axiom distinguishes Suirodoku from classical Sudoku. It creates a global dependency linking all 81 cells.

Constraint Count. The ten axioms yield 55 constraints: 27 local digit constraints (ST$_{rn}$, ST$_{cn}$, ST$_{bn}$ for each of the 9 rows, 9 columns, 9 blocks), 27 local color constraints (ST$_{rk}$, ST$_{ck}$, ST$_{bk}$), and 1 global pair constraint (ST$_{\mathrm{global}}$).

Remark 1 ("Exactly Once")**.** *Axioms ST$_{rn}$ through ST$_{bk}$ are stated as "at most once" constraints. Combined with A2 (completeness) and sort exhaustivity, they imply "exactly once": each row contains 9 cells, each cell contains exactly one pair (by A1 + A2), and no digit can repeat (by ST$_{rn}$), so each digit appears exactly once per row. The same reasoning applies to colors and to columns/blocks.*

55 Constraints

27 local digit	27 local color	1 global pair
9 rows × ST_{rn} 9 cols × ST_{cn} 9 blocks × ST_{bn}	9 rows × ST_{rk} 9 cols × ST_{ck} 9 blocks × ST_{bk}	$\mathrm{ST}_{\text{global}}$ AllDifferent on all 81 pairs

Figure 4: Constraint decomposition: 54 local + 1 global = 55 total constraints.

4.3 Puzzle Theory

A specific puzzle P is defined by ground axioms (clues) of the form value(n_i, k_j, r_p, c_q).

Definition 6 (Clue)**.** *A clue is a constraint specifying that cell (r, c) contains the pair (n, k).*

Definition 7 (Puzzle)**.** *A puzzle is a finite set of clues. It is* uniquely solvable *if exactly one complete solution satisfies all constraints and contains all specified clues.*

A solution of P is a model of Grid Theory ∪ Suirodoku Theory ∪ Puzzle(P).

Following Berthier [14]: "a resolution theory can only solve instances of the CSP that have a unique solution". Uniqueness is neither an axiom nor a theorem; it is an assumption imposed by the puzzle generator.

5 Structural Properties

5.1 Bijection Theorem

Definition 8. *The* pair space *is $\mathcal{P} = \text{Digit} \times \text{Color}$. By exhaustivity, $|\mathcal{P}| = 9 \times 9 = 81$.*

Theorem 1 (Bijection)**.** *In any model of* Grid Theory ∪ Suirodoku Theory*, the map*

$$\varphi : \text{Row} \times \text{Column} \to \mathcal{P}$$

defined by $\varphi(r, c) = (n, k)$ where value(n, k, r, c) *holds, is a bijection.*

Proof. Well-definedness: By A2, $\exists n\ \exists k\ \ \text{value}(n, k, r, c)$. By A1, this pair is unique. So $\varphi(r, c)$ is uniquely determined.

Injectivity: Suppose $\varphi(r_1, c_1) = \varphi(r_2, c_2) = (n, k)$. Then value$(n, k, r_1, c_1)$ and value(n, k, r_2, c_2) both hold. By $\mathrm{ST}_{\text{global}}$, $r_1 = r_2$ and $c_1 = c_2$.

Surjectivity: By exhaustivity, $|\text{Row} \times \text{Column}| = 81$ and $|\mathcal{P}| = 81$. An injective function between finite sets of equal cardinality is surjective. □

Corollary 2. *Each pair $(n, k) \in \mathcal{P}$ appears exactly once in any valid Suirodoku grid.*

Corollary 3 (Absolute Cell Identity)**.** *The inverse φ^{-1} maps each pair to a unique cell. In classical Sudoku, knowing a digit gives 9 possible locations; in Suirodoku, knowing a pair (d, c) determines exactly one cell.*

Propagation Rule: Once $(4, \text{green})$ is placed in cell $(3, 5)$, this pair is eliminated from all 80 other cells globally—not just from the same row/column/block.

5.2 Solving Heuristics

The bijection (Theorem 1) implies that for any fixed digit n, the nine pairs $(n, k_1), \ldots, (n, k_9)$ occupy nine distinct cells. Similarly, for any fixed color k, the nine pairs $(1, k), \ldots, (9, k)$ occupy nine distinct cells.

Lemma 4 (Pair Existence)**.** *For every $n \in \text{Digit}$ and $k \in \text{Color}$, there exist unique r, c such that* $\text{value}(n, k, r, c)$.

Proof. Immediate from the Bijection Theorem (Theorem 1). □

This lemma yields two solving heuristics with no analogue in classical Sudoku:

Rainbow Technique. For any fixed digit n, the nine pairs $(n, k_1), \ldots, (n, k_9)$ occupy nine distinct cells. Tracking digit n across colors: if n appears with 8 colors, the 9th pair (n, k_{missing}) must exist; its location is constrained by ST_{rn}, ST_{cn}, ST_{bn}.

Chromatic Circle Technique. For any fixed color k, the nine pairs $(1, k), \ldots, (9, k)$ occupy nine distinct cells. Tracking color k across digits: if k appears with 8 digits, the 9th pair (n_{missing}, k) must exist; its location is similarly constrained.

These heuristics are direct consequences of $\text{ST}_{\text{global}}$.

6 Graeco-Latin Representation

A Suirodoku grid can be written in traditional Graeco-Latin notation, using Latin letters A–I for digits and Greek letters for colors:

Cβ	Eη	Bγ	Fδ	Dε	Gζ	Aη	Iθ	Hι
Aι	Gγ	Iε	Bη	Cβ	Hζ	Dι	Fθ	Eδ
Fθ	Hδ	Dη	Eβ	Iζ	Aγ	Bι	Cε	Gθ
Bζ	Iη	Hα	Cγ	Gδ	Fε	Eθ	Aι	Dβ
Dβ	Fθ	Gη	Aζ	Eγ	Iε	Cδ	Hι	Bα
Eε	Cβ	Aδ	Dθ	Hη	Bι	Fζ	Gγ	Iα
Hγ	Bι	Cζ	Iη	Fθ	Dδ	Gε	Eβ	Aγ
Gη	Dε	Fθ	Hδ	Aα	Eζ	Iβ	Bγ	Cι
Iδ	Aζ	Eι	Gγ	Bβ	Cθ	Hα	Dη	Fε

Figure 5: Suirodoku in Graeco-Latin notation. Each (Latin, Greek) pair appears exactly once.

Both layers satisfy Sudoku constraints (row, column, block), and each pair appears exactly once—the defining property of a Graeco-Latin square.

7 Comparison of Structures

8 The God Digit Problem

The minimum number of clues problem for classical Sudoku was solved by McGuire, Tugemann, and Civario [6]: the answer is 17. We study a different question for Suirodoku.

Structure	Row/Col	Block	Two symbols	Unique pair
Latin square	Yes	No	No	—
Sudoku	Yes	Yes	No	—
Graeco-Latin square	Yes	No	Yes	Yes
Suirodoku	**Yes**	**Yes**	**Yes**	**Yes**

Table 1: Comparison of combinatorial structures.

8.1 Symmetries

The Suirodoku constraint system admits:

- **Digit permutations** $\sigma \in \mathrm{Sym}(\mathcal{D})$, acting on clues by transforming each pair (n, k) to $(\sigma(n), k)$.
- **Color permutations** $\tau \in \mathrm{Sym}(\mathcal{K})$, acting on clues by transforming each pair (n, k) to $(n, \tau(k))$.
- **Standard Sudoku geometric symmetries**, acting on cell coordinates (r, c).

Digit permutations act transitively on $\mathcal{D}$; color permutations act transitively on $\mathcal{K}$.

8.2 Critical Symbols

Definition 9. *A digit $n \in \mathcal{D}$ is* critical *if every uniquely solvable Suirodoku puzzle contains at least one clue whose digit component is n.*

Definition 10. *A color $k \in \mathcal{K}$ is* critical *if every uniquely solvable Suirodoku puzzle contains at least one clue whose color component is k.*

8.3 Dichotomy Theorem

Theorem 5 (Dichotomy)**.** *If one digit is critical, then all digits are critical. If one color is critical, then all colors are critical.*

Proof. Suppose digit n_1 is critical but digit n_2 is not. Then there exists a uniquely solvable puzzle P omitting n_2. Let $\sigma = (n_1\ n_2)$ be the transposition swapping n_1 and n_2. Applying σ to P by transforming each clue $(n, k, (r, c))$ to $(\sigma(n), k, (r, c))$ yields the puzzle $P' = \sigma(P)$. Since σ is a symmetry, P' is uniquely solvable. But P' omits n_1 (since P omitted n_2 and σ swaps them), contradicting the criticality of n_1. The argument for colors is identical. □

Corollary 6 (Digit Dichotomy)**.** *Exactly one of the following holds:*

(a) For every digit n, there exists a uniquely solvable puzzle whose clues omit n.

(b) Every uniquely solvable puzzle contains all 9 digits in its clues.

Corollary 7 (Color Dichotomy)**.** *Exactly one of the following holds:*

(a) For every color k, there exists a uniquely solvable puzzle whose clues omit k.

(b) Every uniquely solvable puzzle contains all 9 colors in its clues.

Remark 2. *Theorem 5 establishes these dichotomies independently for digits and for colors. Whether digit criticality implies color criticality (or vice versa) is not proven here and remains an additional open question.*

8.4 Formal Statement

Open Question 1 (God Digit Problem[1]). *Does there exist a uniquely solvable Suirodoku puzzle whose clues omit a digit $n \in \mathcal{D}$ entirely?*

Difference from Sudoku. In classical Sudoku, the question "can a puzzle omit the digit 7?" trivially has answer yes by relabeling. In Suirodoku, omitting digit n means omitting all nine pairs $(n, k_1), \ldots, (n, k_9)$. This interacts non-trivially with the global bijection constraint.

8.5 Unavoidable Sets

Computational proofs about Sudoku rely on unavoidable sets and hitting set enumeration [6]. We adopt the "swap" formulation:

Definition 11. *Let G be a complete Suirodoku solution. A set U of cells is* unavoidable *with respect to G if there exists another solution $G' \neq G$ such that G and G' agree outside U.*

Any puzzle having G as its unique solution must *hit* U—i.e., contain at least one clue from U.

Classical example. In classical Sudoku, for any two digits d_1, d_2, the 18 cells containing those digits form an unavoidable set: swapping $d_1 \leftrightarrow d_2$ everywhere produces another valid grid. This immediately implies that 7 clues never suffice: two digits must be absent, leaving this set unhit [6].

Important remark. This argument gives no direct bound on digit omission in Suirodoku. The global pair uniqueness constraint changes which swaps are admissible: some classical swaps become invalid, while new coupled digit-color swaps may emerge.

We identify three plausible swap types under $\mathrm{ST}_{\mathrm{global}}$:

- **Type 1 (inherited):** Classical digit swaps that preserve color constraints.
- **Type 2 (chromatic):** Color swaps analogous to digit swaps.
- **Type 3 (coupled):** Swaps involving simultaneous digit and color changes, enabled or constrained by the global bijection.

A systematic classification of minimal unavoidable sets in Suirodoku is an open problem.

8.6 Toward a Computational Approach

Symmetry reduction. It suffices to study the omission of digit 1: any result transfers to other digits by permutation.

Hitting set strategy. Following McGuire et al. [6], a computational approach would proceed as follows:

1. Fix a complete Suirodoku solution G.
2. Generate a family of unavoidable sets for G.
3. Enumerate hitting sets (candidate clue sets) that intersect all unavoidable sets, subject to the constraint: no clue contains digit 1.
4. Test each candidate for solution uniqueness.

[1] The name alludes to "God's Number" in Rubik's Cube theory (the minimum number of moves to solve any configuration).

Standard optimizations apply: hit vectors, dead clue vectors, effective size ordering, and higher-degree unavoidable sets for early pruning [6].

Complexity. The number of essentially distinct Suirodoku grids (equivalence classes under the standard symmetry group) is unknown. For comparison, classical Sudoku has 5 472 730 538 essentially distinct grids [5]. A classification of Suirodoku-specific unavoidable sets would be needed for efficient enumeration.

9 Open Problems

Enumeration. How many Suirodoku grids exist? The total number of complete Sudoku grids is approximately 6.7×10^{21} [4]. The number for Suirodoku is unknown.

Symmetries. Natural symmetries include: digit permutation (9!), color permutation (9!), row permutations within bands ($3!^3$), column permutations within stacks ($3!^3$), band/stack permutations ($3!^2$), and transposition. A complete analysis of the symmetry group would aid enumeration.

Minimum number of clues. What is the minimum number of clues for a uniquely solvable Suirodoku puzzle?

God Digit Problem. See Section 8. Whether digit criticality implies color criticality also remains open.

Pair omission. Can a puzzle omit a specific pair (n, k)? (Weaker than omitting a digit.)

Unavoidable set classification. What is the structure of minimal unavoidable sets in Suirodoku?

Other orders. Does Suirodoku exist for 4×4 grids (2×2 blocks) or 16×16 grids (4×4 blocks)?

10 Conclusion

We have introduced Suirodoku, a puzzle combining Sudoku and Graeco-Latin square structures. The CSP model, formalized within Berthier's MS-FOL framework, comprises six sorts, a Grid Theory with coordinate mapping, and a Suirodoku Theory with 10 axioms yielding 55 constraints (54 local AllDifferent + 1 global AllDifferent). The Bijection Theorem establishes absolute cell identity—a unique property absent from classical Sudoku. The Rainbow and Chromatic Circle techniques exploit this bijection for solving. The Dichotomy Theorem shows that either no symbol is critical, or all symbols are critical, partially resolving the God Digit Problem. Questions of enumeration, minimum number of clues, unavoidable set classification, and full resolution of the God Digit Problem remain open.

Play online: `https://suirodoku.com`

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A SAT Encoding

For solver implementation, Suirodoku admits a direct Boolean encoding:

Variables: $x_{r,c,d,k} \in \{0,1\}$ meaning “cell (r,c) contains the pair (d,k)”. Total: $9^4 = 6561$ Boolean variables.

Clause families:

1. **Cell ALO:** each cell has at least one pair (81 clauses of length 81)
2. **Cell AMO:** each cell has at most one pair ($81 \times \binom{81}{2}$ binary clauses)
3. **House-digit:** AMO for each digit in each house (27×9 constraints)
4. **House-color:** AMO for each color in each house (27×9 constraints)
5. **Global-pair:** AMO for each pair (d,k) across all cells (81 constraints)

Alternative encodings: CP-SAT with table constraints, or ILP with binary variables. The global AllDifferent admits specialized propagators based on matching theory.