Philosophical Magazine Series 6

ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: http://www.tandfonline.com/loi/tphm17

VIII. Investigation of the variations of magnetic hysteresis with frequency

Thomas R. Lyle M.A.

To cite this article: Thomas R. Lyle M.A. (1905) VIII. Investigation of the variations of magnetic hysteresis with frequency , Philosophical Magazine Series 6, 9:49, 102-124, DOI: 10.1080/14786440509463259

To link to this article: http://dx.doi.org/10.1080/14786440509463259

Published online: 08 Jun 2010.

of the theory is probably a distinct advantage to it when compared with other theories of climatic change; but still the theory need not be regarded as antagonistic to other theories, especially to those which have reference to changes in local conditions. There can be no doubt that variable local conditions, such as the height of the land above sea-level, the distance from the sea, and the direction of prevailing winds and ocean currents, have a very great influence on the climate of any particular locality. However, it is fairly well recognized that variations in local conditions like the above cannot be regarded as at all sufficient to account for the facts observed in connexion with the climatic changes of geological history, and a more generally operating cause must be sought for, such as we have in the variability of the carbonic acid of the atmosphere. Though I do not venture to make any definite statement about any observed connexion between volcanic activity and geological climate, I hope that geologists will state their views on the subject; and if it should be considered that there is not enough evidence to draw a definite conclusion, I hope that by and bye a sufficient amount of evidence will be obtained. In any case, as already remarked, the large amount of coal deposited in the earth is a strong argument in favour of the variability of the amount of atmospheric carbonic acid; and as it is usually considered that the periods in which large quantities of coal were deposited were warm periods, it is obvious that this also to some extent supports the view that there is a connexion between the amount of atmospheric carbonic acid and the general temperature of the atmosphere and surface of the earth.

VIII. *Investigation of the Variations of Magnetic Hysteresis with Frequency.* *By* THOMAS R. LYLE, *M.A., Professor of Natural Philosophy in the University of Melbourne* *.

[Plate II.]

IN the following paper are given some results obtained by my wave-tracer, of which a description has been published in the 'Philosophical Magazine' for November 1903.

The work described, which is of a preliminary nature, was in great part performed more than a year ago, but a severe illness has prevented me until now both from preparing it for publication and from continuing as intended the same

* Communicated by the Physical Society: read November 11, 1904.

work in a more accurate way. It is hoped that the experimental results given below will be of sufficient interest to merit publication now.

1. The experiments were made on two rings of laminated annealed iron, in one of which the radial breadth of the iron was considerable relative to its mean radius. These rings were magnetized by alternating currents of different strengths and periods; both the magnetizing-current wave and the magnetic-flux wave were quantitatively determined by the wave-tracer, using the galvanometer method described in the paper already quoted, and the wave-forms so obtained were subjected to harmonic analysis.

The experiments were divided into series in which the period and wave-form of the magnetizing current were kept as nearly constant as possible throughout any one series, while its strength was varied. The analytic expressions for the associated current and flux waves for a few series are given in tabular form, and some of their more interesting amplitude and phase relations are shown by means of curves.

From the analytic expressions for each pair of associated waves the total iron loss (I, say) per cubic centimetre per cycle was calculated, and it was found, when the magnetizing current is approximately sinusoidal, that I is given with considerable accuracy by the formula

$$I = (\cdot 00186 + \cdot 000026\ n)\, \mathfrak{B}^{1\cdot 57}$$

for ring I.,

and by

$$I = (\cdot 001684 + \cdot 0000272\ n)\, \mathfrak{B}^{1\cdot 57}$$

for ring II.,

where n is the number of periods per second and $\mathfrak{B}$ (called the effective induction) is $\sqrt{2}$ times the root of mean square of dB/dt, for all values of n, and for all values of the induction between 1000 and 12,000.

When from the total iron loss I per cm.^{3} per cycle the sum of the statical hysteresis (U, say) previously obtained by Ewing and Klaasen's method, and E the value that theory assigns to eddy-current loss, was subtracted, a considerable quantity (I − U − E) remained, which increased both when the frequency and when the flux-density increased. This quantity, called by Fleming the kinetic hysteresis, has been obtained for each experiment, and is given in the tables that are to follow, and curves are also given which show how it varies with the frequency and flux-density. That such a source of loss exists when iron is subjected to alternating magnetizing forces has already been shown by Steinmetz and by Siemens by other means.

An interesting case of transformation and what is called reflexion of energy is drawn attention to and discussed. If the E.M.F. impressed on the magnetizing circuit on the iron ring be sinusoidal, the flux-wave produced contains third, fifth, &c. harmonics. These higher flux-harmonics induce currents in the magnetizing circuit which are dissipated as heat in it. Thus we have a transformation of electric energy due to alternating currents of frequency n to energy of currents whose frequencies are $3n$, $5n$, &c., which is reflected back into the magnetizing circuit. This the author believes is a hitherto unnoticed source of transformer loss.

2. Fig. 1 (Pl. II.) shows the arrangement of the apparatus. The magnetizing current was obtained from a four-pole rotary converter T supplied with direct current from storage-cells. By means of rheostats placed both in the armature and field-circuits, the speed could be varied and adjusted. One end of the spindle of the commutator C was directly connected to the spindle of this converter, and on the other end was a screw thread which worked a tangent wheel. On this wheel an ebonite stud is fixed which momentarily breaks the circuit of an electric chronograph once every revolution, thus recording the time of every 200 periods.

The magnetizing current, drawn from the slip-rings of the rotary converter, passed through a regulating resistance, a Kelvin balance, the primary of the air-circuit transformer M, and the primary coil on the iron ring B.

One end of the secondary of M is joined to one end of that of B, and the junction connected to one of the fixed brushes of the commutator. The other ends of the secondaries are connected, as shown in fig. 1, to the three-way key K, from the lever of which connexion is made to the other fixed brush. By this means either secondary can be joined to the commutator, and thence from the movable brushes of the latter through a reversing key and a high resistance to the galvanometer.

A Clark's cell with a megohm is arranged so that it may be used at any time for the purpose of determining the reducing factor of the galvanometer. The deflexions of the latter were so nearly proportional to the currents producing them over the part of the scale used that no calibration was necessary.

3. The details of the rings, called ring I. and ring II. respectively, are as follows :—

	RING I.	RING II.
No. of laminæ	18	31
Internal diameter (r_1)	7·6 cm.	15·223
External diameter (r_2)	11·58 cm.	16·477
Mean thickness (x)	·0475 cm.	·0443
Section of magnetic circuit (a)	1·701 cm.^{2}	·861
Length of magnetic circuit (l)	30·12 cm.	49·79
Specific resistance of iron at 12° C. (ρ)	12590	13600
No. of primary turns (n_1)	164	406
No. of secondary turns (n_2)	5 or 10	10

The laminæ were well annealed, the oxide removed, and they were insulated from each other by oiled paper. The mean thickness was determined from weight, area, and specific gravity. The values obtained for the specific resistance seemed high, so the determination was checked.

4. The method by which the current and flux waves were determined was practically the same as that described in the paper already quoted, except that readings of the galvanometer which give the ordinates for a definite phase were only taken for every six degrees on the divided circle which carries the movable brushes of the commutator. This gave 15 ordinates per half wave, and from these, without plotting, the first, third, and fifth harmonics were easily obtained by an arithmetical method of analysis when the seventh and higher harmonics could be neglected. Whether or not the latter assumption was legitimate would appear during the analysis, and in all cases in which the amplitude of the seventh harmonic was greater than one per cent. of that of the first, it was determined, and though its values are not given in the tables that are to follow, its effect was allowed for.

5. In order to explain the procedure by which the results arrived at were deduced, the galvanometer readings for a pair of associated waves will be given, and the treatment to which these readings were subjected will be indicated.

The following table gives four times the galvanometer deflexions for a pair of associated current and flux waves, with the corresponding divided-circle readings. The latter, when doubled, give the corresponding phase-angles, as one complete revolution of the divided circle corresponds to two full waves, the rotary converter and the commutator both being four-pole. The readings for the current-wave are represented by γ, and those for the flux-wave by β. The other necessary details of the experiment are also given.

Experiment with Ring I.

Resistance in γ circuit $= r_1 = 5030\ \omega$.
" in β circuit $= r_2 = 15030\ \omega$.
Period by chronograph $= \mathrm{T} = \cdot 0307$ sec.
Kelvin-balance reading of current C at start $= \cdot 1873$ amp.
" " " at finish $= \cdot 1868$ amp.
Reducing factor of galvanometer $= \lambda = 1\cdot 287 \times 10^{-8}$ amp.
M of air-circuit transformer $= 0\cdot 00061$ henry.
Secondary turns on ring $= n_2 = 5$.

Divided Circle.	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
4γ	103	338	583	795	1003	1160	1270	1323	1314	1239	1069	872	638	395	147
4β	−636	−561	−450	−296	−82	183	449	661	774	804	795	785	760	729	690

Each of these numbers is the sum of four separate readings of γ or of β, two of these being at corresponding points on the positive and negative halves of each wave, and the other two being a similar pair got after switching the galvanometer.

Analysing the above, we obtain

$$4\gamma = 1293 \sin \omega t - 41 \sin 3(\omega t - 7) + 5 \sin 5(\omega t - 23\cdot 4)$$

$$4\beta = 868 \sin(\omega t - 51\cdot 76) + 120 \sin 3(\omega t - 63\cdot 1) + 23 \sin 5(\omega t - 69\cdot 2).$$

6. The factors (c and f say) to be applied to the different values of γ and of β in order to obtain the corresponding values of current (C) and of flux (F) are (see Phil. Mag. *loc. cit.*)

$$c = \frac{\lambda r_1 \mathrm{T}}{4\mathrm{M}}, \quad f = \frac{\lambda r_2 \mathrm{T}}{4 n_2}, \quad . \quad . \quad . \quad . \quad (1)$$

which become

$$c = \cdot 00008145, \qquad f = 29\cdot 7$$

for the experiment being discussed when the figures given in § 5 are substituted.

These factors can also be obtained from the Kelvin-balance reading C of the magnetizing current; for if $\bar{\gamma}$ be the R.M.S. of the γ readings

$$\mathrm{C} = c\bar{\gamma}$$

and

$$\bar{\gamma}^2 = \tfrac{1}{2}\{\gamma_1^2 + \gamma_3^2 + \gamma_5^2 + \ldots\}$$

where γ_1, γ_3, γ_5, &c. are the amplitudes of the first, third, fifth, &c. harmonics of γ, and these have already been obtained by analysis (§ 5).

The factor f can then be deduced from c, as equations (1) above give us the relation

$$f=\frac{r_2}{r_1}\frac{M}{n_2}c.$$

By this method we obtain for the experiment in hand

$$c=\cdot00008178, \qquad f=29\cdot8,$$

which agree satisfactorily with the values obtained above by the other method.

The latter method of obtaining c and f is the one usually adopted.

The factors (h and b say) to be applied to γ and β in order to obtain H and B, where H is the M.M.F. round the ring divided by its mean circumference l, and B is the total flux F divided by the iron cross-section α, are

$$h=\frac{4\pi n_1 c}{l}, \qquad b=\frac{f}{\alpha}.$$

Substituting the values of n_1, l, and α given in the details of ring I. in § 3, and those of c and f just obtained, we find that

$$h=\cdot0056, \qquad b=17\cdot52\,;$$

so that

$$H=\cdot0056\gamma, \quad B=17\cdot52\beta.$$

Hence the final expressions for the pair of associated waves investigated are

$$H=1\cdot81[\sin\omega t-\cdot0317\sin 3(\omega t-7)+\cdot004\sin 5(\omega t-23\cdot4)]$$

$$B=3802[\sin(\omega t-51\cdot76)+\cdot1382\sin 3(\omega t-63\cdot1)+\cdot0265\sin 5(\omega t-69\cdot2)].$$

7. It is easy to show that as the secondary (galvanometer) current is inappreciable, the energy (D say) dissipated per cycle in the iron of the ring is given by

$$D=n_1\int_0^T C\frac{dF}{dt}\,dt,$$

which becomes (see § 6)

$$D=n_1 cf\int_0^T \gamma d\beta.$$

Hence when the galvanometer readings for β are plotted as ordinates against the corresponding ones for γ as abscissæ for a complete period a closed curve is obtained, whose area when multiplied by n_1cf gives D the total energy dissipated per cycle in the iron of the ring.

Dividing D by the volume $l\alpha$ of iron we get for quotient the space average throughout the ring of the iron loss per cubic centimetre per cycle. In the sequel this quantity will be represented by I.

I can also be obtained from the harmonic expressions for H and B in § 6.

For as

$$Hl=4\pi n_1 C, \qquad B\alpha=F,$$

$$D=\frac{l\alpha}{4\pi}\int_0^T H dB\,;$$

hence

$$I=\frac{1}{4\pi}\int H dB,$$

and it is easy to show that if

$$H=H_1[\sin\omega t+h_3\sin 3(\omega t-\phi_3)+h_5\sin 5(\omega t-\phi_5)+\&c.]$$
$$B=B_1[\sin(\omega t-\theta_1)+b_3\sin 3(\omega t-\theta_3)+b_5\sin 5(\omega t-\theta_5)+\&c.],$$

then

$$\frac{1}{4\pi}\int H dB=\frac{H_1 B_1}{4}\left\{\sin\theta_1+3h_3 b_3\sin 3(\theta_3-\phi_3)+5h_5 b_5\sin 5(\theta_5-\phi_5)+\&c.\right\}.$$

Applying this formula to the expressions for H and B in § 6 we find, for the experiment being discussed, that

$$I=1346.$$

8. For the two rings used what has been called the static hysteresis (U, say) was determined for various induction densities by Ewing and Klaasen's method. It was found that the Steinmetz coefficients σ, where

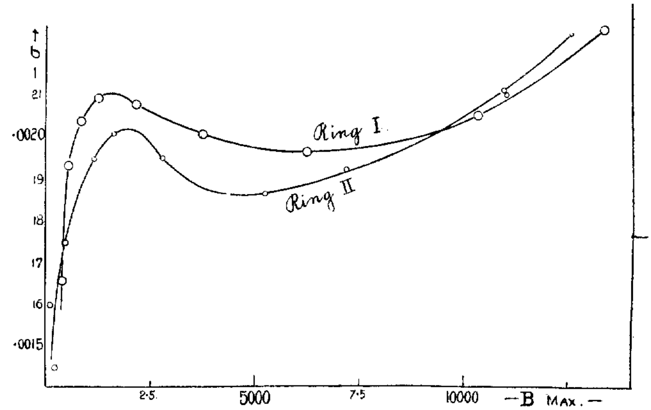
$$U=\sigma B_{\text{Max.}}^{1\cdot 6},$$

for them were not constant but varied with the induction, and that the variation was quite as great for the narrow ring as for the broad one.—

In fig. 2 the Steinmetz coefficients σ for these two rings are plotted against maximum induction (B_0 say). It will be seen that σ tends to be very small in weak fields, possibly vanishing with B_0, that it increases rapidly to a maximum, then diminishes to a minimum, and then steadily increases. It is possible that the maximum σ point, which is very marked, may give the induction at which some definite physical change due to magnetization begins to take place in the iron, perhaps that at which magnetization begins to produce lengthening.

In the case of magnetization by means of alternating currents, it is fair to assume that the static hysteresis will depend on the maximum value of B, (B_0 say), during the

Fig. 2.



Steinmetz Coefficient σ v. Max. Induction ($U=\sigma B^{1 \cdot 6}$).

alternation. B_0 is obtained from the maximum β reading by multiplying the latter by b (see § 6), and

$$U=\sigma B_0^{1 \cdot 6},$$

in which for σ we take from the curve in fig. 2 for the ring used the value corresponding to B_0.

For the experiment being discussed,

$$\text{Max. } \beta=\beta_0=201 \text{ (see § 5)}$$

$$B_0=17 \cdot 52\beta_0=3522 \text{ (see § 6)},$$

for which induction for ring I.

$$\sigma = \cdot 002012,$$

hence

$$U=952.$$

9. No formula is available by means of which the eddy-current loss in an annular lamina can be exactly determined, but a fair approximation to it can be obtained in the following way.

Searle and Bedford* have shown that if X be the space average of the heat dissipated in a thin strip by eddy-currents,

$$\frac{dX}{dt}=\frac{x^2}{\rho}\left(\frac{db}{dt}\right)^2\left\{\frac{1}{12}-\cdot 0525\frac{x}{y}\right\},$$

* Phil. Trans. vol, cxcviii,

where x is the thickness, y the width of the strip, ρ the specific resistance, and b the induction, provided that db/dt has the same value at all points of a cross section.

In ring laminæ when magnetized in the usual way neither b nor db/dt is constant across their section, but an upper limit will be given to the rate of dissipation per cm.[3] at any point where the induction is b by the equation

$$\frac{dX}{dt}=\frac{x^2}{12\rho}\left(\frac{db}{dt}\right)^2,$$

and the rate of dissipation of energy by eddy-currents in the whole ring will be

$$=\frac{x^2}{12\rho}\times\text{vol. of ring}\times\text{space average of }\left(\frac{db}{dt}\right)^2\text{ throughout the ring.}$$

Now it can be shown that this average will not differ much from $(dB/dt)^2$ where B has the meaning already assigned to it, namely, the average induction across any section. The latter statement is roughly indicated by the fact that though the amplitude of b and hence of db/bt may vary considerably from the inner to the outer radius of the ring, being greatest at the inner radius as the amplitude of the magnetizing force (which varies inversely as the distance from the centre of the ring) is greatest there, still, since the inner circumference is less than the outer one, there will, in making up the space average of $(db/dt)^2$, be a smaller relative volume of the iron at the high induction than at the low induction. Hence, finally, if E be the average eddy-current loss throughout the ring per cm.[3] per cycle, we have, approximately, that

$$E=\frac{x^2}{12\rho}\int_0^T\left(\frac{dB}{dt}\right)^2 dt.$$

If

$$B=B_1[\sin\omega t+b_3\sin 3(\omega t-\theta_3)+b_5\sin 5(\omega t-\theta_5)+\&c.]$$

then

$$\int_0^T\left(\frac{dB}{dt}\right)^2 dt=\frac{2\pi^2}{T}\mathfrak{B}^2,$$

where

$$\mathfrak{B}^2=B_1^2\{1+9b_3^2+25b_5^2+\&c.\}$$

and

$$E=\frac{\pi^2x^2}{6\rho T}\mathfrak{B}^2.$$

It will be seen that $\mathfrak{B}$ is a quantity of considerable importance in this theory, and it will be called the *effective induction*.

It is the amplitude of the sinusoidal induction-wave that would by its variation generate in any circuit looped on it the same virtual E.M.F. as would be produced in the same circuit by the actual induction B ; or otherwise stated,

$$\mathfrak{B}=\sqrt{2}\ \text{R.M.S.}\left(\frac{dB}{dt}\right).$$

When we write the equation for E in the form

$$E=e\frac{\mathfrak{B}^2}{T},$$

e will be a constant for a particular ring and we find, using the values for x and ρ given in § 3, that,

$$\text{for Ring I.}\ e=2{\cdot}95\ \times 10^{-7},$$

$$\text{and for Ring II.}\ e=2{\cdot}373\times 10^{-7}.$$

In the experiment with Ring I. that is being discussed,

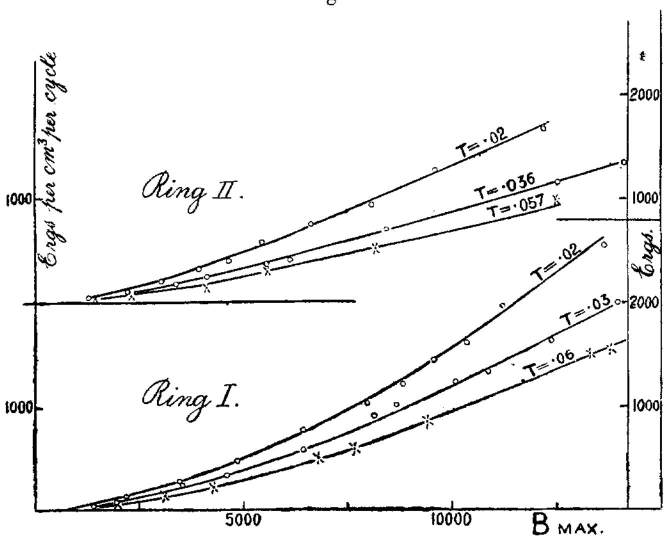
$$\mathfrak{B}^2=17{\cdot}2\times 10^6$$

$$T={\cdot}0307$$

hence $$E=165.$$

10. In every case with either ring it was found that the

Fig. 3.



Kinetic Hysteresis v. Max. Induction.

sum of the statical hysteresis (U) and the theoretical eddy-current loss (E) was less than the total iron loss (I) as determined by the wave-tracer. The difference I−(U+E), which Fleming has called the kinetic hysteresis, was determined for each experiment, and is given in the tables that are to follow; and its values for different induction densities and frequencies for both rings are plotted in fig. 3 (p. 111) against B_0 the maximum induction in each case. The results expressed by these curves seem to thoroughly verify the observations of Steinmetz and of Siemens both as to the existence of such a quantity as kinetic hysteresis and as to the general character of its variation with induction density and period. As this question has been much disputed the verification, by a new method, of the results obtained by previous investigators is not without significance.

11. In Table I. (p. 113) are given the analytical results deduced as indicated in the preceding paragraphs from a series of experiments with Ring I., in which the period was approximately ·019 sec. The wave-forms of the magnetizing currents in experiments 1 to 13 were approximately sinusoidal, and the E.M.F. impressed on the circuit was practically of the sine form in all experiments with the exception of No. 15, Table I.

In fig. 4 (p. 114) the more important characteristics of the induction-waves given in Table I. Nos. 1 to 13, *i. e.* produced by q.p. sine currents, are plotted against the amplitudes B_1 of the first harmonic of these waves. These curves are typical of any series of induction-waves of constant period produced by currents of similar wave-forms. No such regularity, however, would be obtained if the wave-forms of the magnetizing currents were allowed to vary, as will be seen by marking on fig. 4 the points for the characteristics of the induction-wave of Experiment 15, in which the H wave was greatly distorted (made saddle-shaped) by artificially distorting the applied E.M.F. wave.

The characteristics μ_0 and θ, which are the connecting links between the H wave and the B wave it produces, both fall to small values for small values of B_1, θ probably vanishing with B_1. Fig. 4 shows clearly how both rise to maxima and then diminish as B_1 increases.

The curve for b_3 the ratio of the third (B_3) to the first harmonic B_1 of B is striking, apparently issuing from the origin (see series 2, Table II. p. 115, & fig. 5, p. 118), it rises quickly between $B_1=0$ and $B_1=1000$ (q.p.), from which it continues for larger values of B_1 as a straight line. Hence for all values of B_1 greater than 1000 the amplitude B_3 of the third harmonic of B is of the form

$$\alpha B_1+\beta B_1^2.$$

Table I.—Ring I.

$T = 2\pi/\omega = \cdot 019$ (q.p.).

$\mu_0 = B_1/H_1$.

$\mathfrak{B} = \sqrt{2}\,.\,\text{R.M.S.}\left(\frac{dB}{dt}\right)$.

Type forms $\begin{cases} H = \dfrac{\text{M.M.F.}}{\text{length}} = H_1[\sin \omega t + h_3 \sin 3(\omega t - \phi_3) + h_5 \sin 5(\omega t - \phi_5) + \ldots]. \\ B = \dfrac{\text{Flux}}{\text{area}} = B_1[\sin(\omega t - \theta) + b_3 \sin 3(\omega t - \theta - \psi_3) + b_5 \sin 5(\omega t - \theta - \psi_5) + \ldots]. \end{cases}$

$I = \dfrac{\text{Iron loss per cycle in ring}}{\text{vol. of iron in ring}}$. $U = \dfrac{\text{Statical hysteresis per cycle in ring}}{\text{vol. of iron in ring}}$. $E = \left\{ \begin{array}{l}\text{Theoretical eddy-current} \\ \text{loss per cm.}^3 \text{ per cycle.}\end{array}\right.$

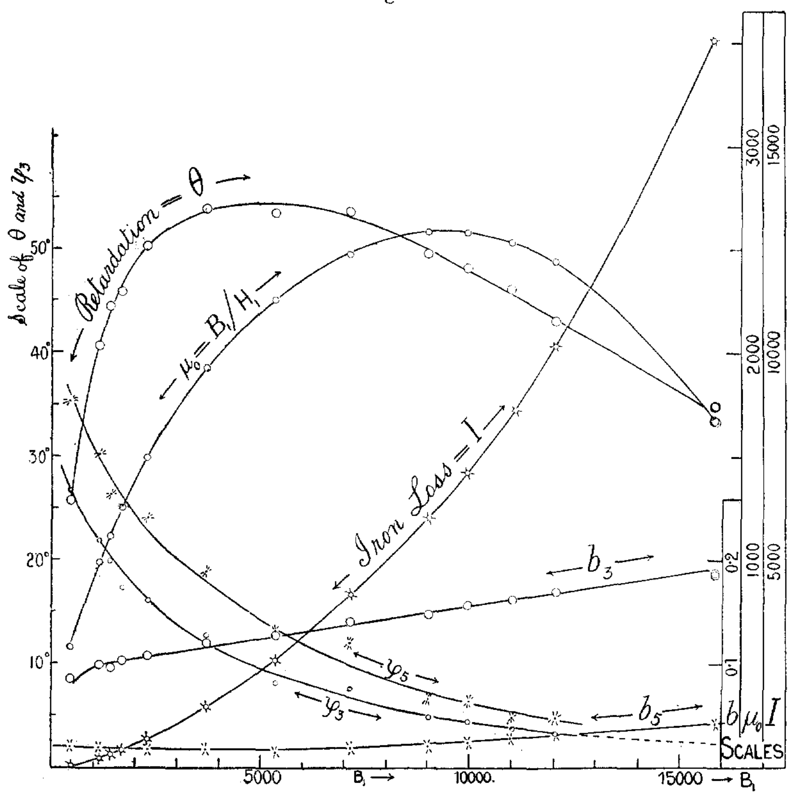
No.	T.	H_1.	$-h_3$.	ϕ_3.	h_5.	ϕ_5.	B_1.	θ.	b_3.	ψ_3.	b_5.	ψ_5.	μ_0.	Max. B.	$\mathfrak{B}$.	I.	U.	E.	I-U-E.
1 …	·01877	0·773	·0463	−0·12	…	…	448	25·75	·085	26·76	·021	35·25	580	450	465	37·2	31	3·4	2·8
2 …	·01887	1·156	·042	1·9	…	…	1147	40·65	·099	21·89	·020	30·14	992	1124	1204	216	158	22·6	35·4
3 …	·01849	1·261	·033	4·17	…	…	1407	44·55	·096	19·84	·016	26·32	1116	1370	1468	311	219	34·4	57·6
4 …	·01894	1·35	·037	2·67	…	…	1689	45·85	·1025	17·32	·017	25·15	1251	1643	1772	409	293	49	67
5 …	·01860	1·536	·034	1·14	…	…	2291	50·34	·108	16·05	·016	24	1492	2199	2414	680	463	92·5	124·5
6 …	·0190	1·93	·025	0·0	…	…	3708	53·93	·120	12·73	·017	18·85	1921	3482	3956	1452	936	243	273
7 …	·01875	2·377	·012	17·4	…	…	5360	53·54	·128	8·10	·015	13	2255	4884	5753	2573	1588	521	464
8 …	·01892	2·892	0	…	…	…	7160	53·66	·140	7·60	·019	12	2476	6466	7800	4170	2457	948	765
9 …	·01878	3·497	·020	57	·005	21·0	9040	49·61	·148	4·82	·021	6·76	2585	8032	9934	6027	3443	1550	1034
10 …	·01905	3·897	·014	41·4	·007	14·5	10050	48·18	·157	4·56	·025	6·5	2580	8855	11170	7260	4115	1935	1210
11 …	·01868	4·347	·017	50·36	·008	16·0	11010	46·06	·162	3·84	·030	4·92	2534	9604	12350	8614	4762	2408	1444
12 …	·01835	4·943	·022	42·85	·008	8·8	12070	43·10	·170	3·28	·033	4·90	2442	10440	13690	10170	5540	3015	1615
13 …	·01840	5·67	·040	36·53	·014	7·4	13020	40·35	·175	2·65	·034	3·30	2296	11250	14500	11820	6380	3560	1880
End of approximately sinusoidal magnetizing waves.																			
14 …	·01869	9·42	·1635	8·72	−·056	29·78	15860	34·94	·186	4·77	·043	9·18	1677	13670	18542	17500	9530	5427	2543
15 …	·01844	2·886	−·218	−·45	·042	26·09	6488	40·84	·134	−2·9	·022	−1·5	2248	5780	7037	3451	2059	792	600

Within the limits of experimental error b_5 may be taken as a straight line, so that B_5 is also of the form

$$\gamma B_1 + \delta B_1^2.$$

The curves for ψ_3 and ψ_5 show the gradual change of position as regards phase of the third and fifth harmonics with respect to B_1 the first harmonic of B as B_1 increases.

Fig. 4.



Both ψ_3 and ψ_5 seem to attain small limiting values for very high values of B_1, which means that all the harmonics of B pass through their zero values in the same direction at nearly the same instant when B_1 is very large.

In Tables II. and III. are given the analytical results for two other series of experiments with Ring I. for periods ·03 sec. and ·058 sec. approximately, and in Tables IV., V., and VI. are given the results of three series of experiments with Ring II.

TABLE II.—Ring I.

No.	T.	H.	$-h_3$.	ϕ_3.	h_5.	ϕ_5.	B_1.	θ.	b_3.	ψ_3.	b_5.	ψ_5.	μ_0.	Max. B.	𝔅.	I.	U.	E.	I–U–E.
1...	·0302	·4	·054	5·1	...	...	148	13·9	·053	37·8	·0153	45·3	370	145	150	3·2			
2...	·0299	·577	·048	2	...	...	258	19·4	·070	32·0	·014	40·1	447	259	264	12·2			
3...	·03	·783	·047	4·2	...	...	483	25·11	·081	26·8	·019	38	617	479	500	39·5	35·3	2·5	1·7
4...	·0303	·955	·043	2·12	...	...	776	32·41	·103	23·1	·024	32·7	813	768	818	98·4	83·2	6·5	8·7
5...	·0305	1·202	·038	2·46	...	...	1432	41·18	·113	18·5	·024	27·5	1192	1388	1520	282	224	22·3	36
6...	·0303	1·366	·043	4	·005	12·25	2025	47	·1253	16·4	·0284	24·4	1481	1944	2182	505	381	46	78
7...	·0307	1·807	·031	7	·004	23·5	3802	51·76	·1382	11·4	·0265	17·4	2104	3522	4147	1344	952	165	227
8...	·0297	2·114	·021	13·9	·008	25·3	5050	51·7	·147	9·43	·028	14·9	2389	4613	5568	2078	1445	308	325
9...	·0295	2·708	·014	25	·008	11·7	7284	49·13	·155	6·65	·030	10·2	2690	6462	8105	3692	2455	657	580
10...	·0296	3·366	·018	35·6	·009	11·8	9318	46·07	·167	4·77	·035	6·9	2769	8142	10550	5590	3580	1110	900
11...	·0295	4·53	·044	29·1	·012	7·4	11720	39·89	·181	2·34	·042	3·9	2587	10120	13580	8310	5234	1844	1232
								End of approximately sinusoidal magnetizing waves.											
12...	·0295	4·737	·115	13·9	·036	3·0	12400	43·88	·163	6·17	·034	12·0	2618	10900	13964	9308	6026	1950	1332
13...	·0295	6·243	·148	11·6	−·046	34·3	14210	39·28	·178	5·46	·038	10·7	2276	12420	16340	12115	7817	2665	1633
14...	·0301	9·83	·1895	6·5	−·058	26·5	16310	31·1	·196	4·4	·052	8·2	1660	13970	19410	15630	9930	3704	2000

TABLE III.—Ring I.

No.	T.	H_1.	$-h_3$.	ϕ_3.	h_5.	ϕ_5.	B_1.	θ.	b_3.	ψ_3.	b_5.	ψ_5.	μ_0.	Max. B.	$\mathfrak{B}$.	I.	U.	E.	I-U-E.
1 ...	·0574	·774	·038	1·6	...	...	483	23·62	·089	24·08	·021	34·9	624	477	500	36·8	...	...	
2 ...	·057	·971	·04	0·6	...	...	847	31·34	·123	19·2	·026	28·7	873	825	911	105	94	4·3	6·7
3 ...	·0572	1·211	·032	7·4	...	...	1625	39·8	·131	16·5	·031	22·6	1342	1550	1763	311	267	16	28
4 ...	·0576	1·348	·045	7·5	·016	8	2102	42·5	·143	14·2	·038	27·7	1560	1965	2322	470	386	27	57
5 ...	·0578	1·653	·026	9·9	·008	16·3	3398	46·0	·155	11·2	·042	15·5	2055	3118	3814	997	790	74	133
6 ...	·0582	1·94	·024	15·1	·009	15·7	4765	46·35	·165	8·7	·043	12·4	2456	4275	5413	1645	1284	148	213
End of approximately sinusoidal magnetizing waves.																			
7 ...	·0593	2·583	·074	14·6	·027	17·9	7685	48·46	·170	7·6	·043	12·0	2975	6800	8785	3545	2668	384	493
8 ...	·0585	2·91	·082	14·5	·026	2·8	8690	46·84	·174	7·1	·042	11·4	2987	7686	9975	4337	3255	501	581
9 ...	·0602	3·737	·108	13·9	·036	0·5	10780	43·17	·179	5·7	·038	10·7	2885	9440	12450	6255	4620	760	875
10 ...	·0590	6·41	·293	15·6	·061	10·7	14710	37·71	·123	8·6	·018	19·9	2295	13330	15750	11730	8982	1241	1507
11 ...	·0643	9·328	·189	3·45	·074	22·6	16165	28·93	·203	4·0	·053	7·0	1733	13800	19500	13090	9800	1748	1542

TABLE IV.—Ring II.

No.	T.	H_1.	$-h_3$.	ϕ_3.	h_5.	ϕ_5.	B_1.	θ.	b_3.	ψ_3.	b_5.	ψ_5.	μ_0.	Max. B.	$\mathfrak{B}$.	I.	U.	E.	I-U-E.
1 ...	·0210	·582	·034	−1·4	...	...	291	21·2	·073	25·9	·017	35·9	500	291	300	15			
2 ...	·0216	·894	·036	1·8	...	...	712	32·4	·082	22·6	·015	34·4	796	703	736	85	69	6	10
3 ...	·0204	1·161	·029	6·2	...	...	1357	41·45	·089	19·2	·016	26	1169	1316	1410	260	193	23	44
4 ...	·0205	1·46	·0306	7·2	...	...	2325	47·2	·101	16	·016	24·5	1592	2230	2430	621	453	68	100
5 ...	·0212	1·737	·03	6·8	...	...	3225	48·97	·116	15·1	·022	22·5	1857	3047	3435	1055	724	132	199
6 ...	·0209	2·046	·032	17·5	...	...	4173	49·45	·118	12·5	·023	19·3	2040	3916	4450	1604	1059	225	320
7 ...	·0204	2·276	·036	21·4	·008	15	5011	48·75	·1195	11·5	·020	19·25	2202	4657	5350	2110	1383	333	394
8 ...	·0204	2·647	·042	18·6	·008	18·7	5910	47·45	·127	10·6	·022	17·2	2233	5443	6356	2824	1775	470	579
9 ...	·0208	3·18	·052	19·8	·013	13·8	7300	44·6	·136	8·8	·029	15	2295	6638	7956	3948	2477	722	749
10 ...	·0206	4·234	·068	19·1	·015	5·3	9074	38·78	·151	7·32	·031	12·7	2143	8094	10050	5707	3575	1163	969
11 ...	·0203	5·65	·076	16·6	·016	0	10900	32·51	·155	6·09	·035	9·19	1930	9612	12165	7760	4763	1730	1267
12 ...	·01994	10·57	·076	11·5	−·0256	25·8	14260	21·4	·184	2·8	·052	4·4	1350	12250	16700	12710	7716	3318	1676

TABLE V.—Ring II.

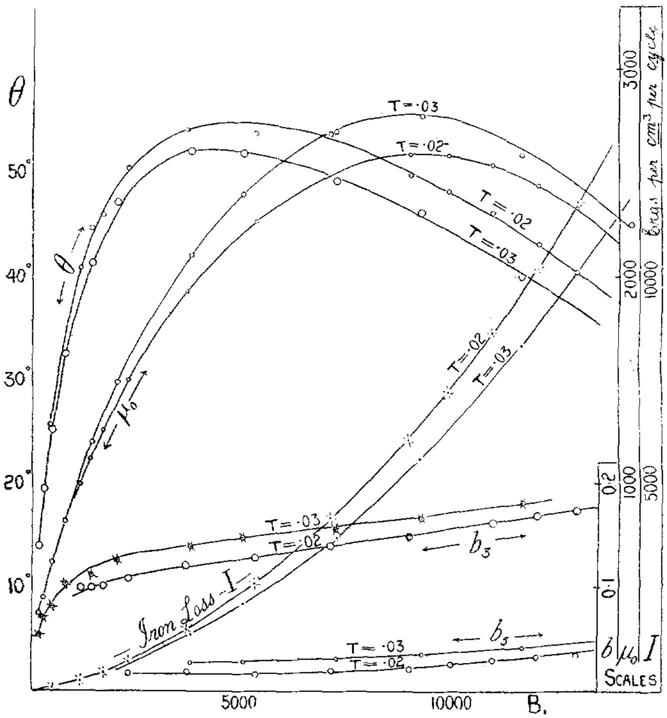
No.	T.	H_1.	$-h_3$.	ϕ_3.	h_5.	ϕ_5.	B_1.	θ.	b_3.	ψ_3.	b_5.	ψ_5.	μ_0.	Max. B.	𝔅.	I.	U.	E.	I-U-E.
1 ...	·0352	0·881	·051	4·6	...	...	802	34·17	·10	22·6	·025	30·8	938	788	836	97	78	4·7	14·3
2 ...	·0358	1·445	·047	7	·005	5·0	2610	45·82	·131	14·7	·038	19·8	1861	2448	2850	670	521	54	95
3 ...	·0360	1·734	·048	11·6	·005	12·5	3636	46·36	·140	13·0	·032	18·8	2097	3372	3990	1120	839	105	176
4 ...	·0359	1·997	·050	16·2	·011	13	4503	45·32	·146	11·1	·032	17·0	2255	4128	4970	1555	1147	163	245
5 ...	·0360	2·573	·065	17·6	·017	10·2	6110	42·07	·149	9·2	·032	14·6	2375	5560	6770	2513	1841	302	370
6 ...	·0358	2·843	·073	19·4	·019	6·8	6770	40·43	·15	8·7	·032	13·3	2381	6133	7500	2948	2172	373	403
7 ...	·0359	4·425	·088	16·7	·023	2·1	9570	32·22	·158	5·6	·037	8·5	2163	8420	10740	5247	3780	763	704
8 ...	·0350	9·97	·210	3·23	−·07	25	14030	24·65	·148	8·5	·034	14·6	1407	12540	15550	10900	8100	1641	1160
9 ...	·0359	14·1	·171	2·57	−·068	17·1	16180	20·96	·176	6·33	·053	11·5	1147	14120	18860	14140	10470	2354	1316

TABLE VI.—Ring II.

No.	T.	H_1.	$-h_3$.	ϕ_3.	h_5.	ϕ_5.	B_1.	θ.	b_3.	ψ_3.	b_5.	ψ_5.	μ_0.	Max. B.	𝔅.	I.	U.	E.	I-U-E.
1 ...	·0572	1·13	·0363	0·9	...	...	1525	38·7	·133	15·4	·0355	22·1	1350	1437	1670	265	224	10·5	30·5
2 ...	·0567	1·392	·0268	11·4	·01	1·9	2525	41·54	·143	12·7	·0374	17·7	1814	2330	2795	573	486	32·5	54·5
3 ...	·0573	1·959	·0581	15·3	·02	3·3	4557	41·2	·163	9·6	·0437	14·6	2326	4138	5170	1400	1151	111	138
4 ...	·0572	2·56	·065	13·65	·015	1·87	6206	38·88	·171	8·1	·0465	11·6	2425	5544	7120	2350	1832	210	308
5 ...	·0568	4·185	·090	13·0	−·023	29·1	9340	30·93	·173	5·4	·049	7·7	2232	8168	10770	4550	3543	484	523
6 ...	·0565	10·38	·1583	1·38	−·062	11·8	14370	22	·176	6·1	·05	10·65	1384	12490	16710	10230	8067	1174	990

In fig. 5 some of the characteristics of the induction-waves produced in Ring I. for two different speeds are plotted against B_1 for the purpose of showing the general effect of change of frequency on these characteristics.

Fig. 5.



12. For the two rings experimented with I find that I, the total iron loss per cm.[3] per cycle, is given very approximately in terms of $\mathfrak{B}$, the effective induction, and n the number of periods per second, when $\mathfrak{B}$ lies between 1000 and 12,000, by a simple formula analogous to that of Steinmetz for statical hysteresis.

Thus for Ring I.,

$$I = (\cdot 00186 + \cdot 000026\, n)\, \mathfrak{B}^{1\cdot 57},$$

and for Ring II.,

$$I = (\cdot 001684 + \cdot 0000272\, n)\, \mathfrak{B}^{1\cdot 57}.$$

That the same power of the effective induction should appear in the formula for each ring is striking, and suggests the probability that for any given frequency when the wave-form of H is approximately sinusoidal the total iron loss per cm.^{3} per cycle is proportional to

$$\mathfrak{B}^{1\cdot57},$$

or to some power of $\mathfrak{B}$ differing little from 1·57 independently of the kind of iron or the thickness of the laminæ.

In order to show the degree of accuracy with which the above formulæ give the iron losses, Table VII. has been compiled. In it are given in parallel columns the values of I obtained experimentally by means of the wave-tracer and those determined from the above formula for Ring II. for different values of n and $\mathfrak{B}$.

Table VII.

Comparing observed values of I for Ring II. with those given by formula $I=(\cdot001684+\cdot00002717\,n)\,\mathfrak{B}^{1\cdot57}$.

n.	$\mathfrak{B}$.	I. Obs.	I. Calc.	n.	$\mathfrak{B}$.	I. Obs.	I. Calc.
49·07	1410	260	265	27·77	6770	2513	2517
48·71	2430	621	621	27·90	7500	2948	2962
47·24	3435	1055	1056	27·86	10740	5247	5200
47·91	4450	1604	1596	34·03	3061	793	775
49·02	5350	2110	2153	34·22	3780	1080	1083
49·02	6356	2824	2819	34·66	4770	1569	1568
48·03	7956	3948	3975	34·91	5818	2139	2145
48·54	10050	5707	5760	33·49	7536	3165	3170
49·21	12165	7760	7710	17·62	2795	573	557
27·90	2854	670	649	17·45	5170	1400	1459
27·76	3990	1120	1098	17·48	7120	2350	2409
27·81	4970	1550	1551	17·60	10770	4550	4625

13. In the tables giving the analytical results of the different experiments, it will be seen that as H_1 gets large the wave-form of H differs more and more from the sine form, and that in the slower speeds the range over which it was possible to obtain magnetizing currents of approximately sine wave-form was less. This was due to the reaction of the iron in the ring and may be explained as follows:—

If we assume that the applied E.M.F. was sinusoidal [it was nearly so], a current would flow which would produce a flux wave that contained 1st, 3rd, 5th, &c. harmonics. The 3rd, 5th, &c. harmonics produced would induce in the magnetizing circuit currents of periods $3n$, $5n$, &c., which would appear as 3rd, 5th, &c., harmonics in the magnetizing current, and would be greater as the impedances of the primary circuit for currents of these different periods were less, if we assume that the amplitudes of the upper harmonics of the flux are fixed.

Now in order to obtain the larger magnetizing currents it was necessary to reduce the impedance, and hence in these cases the higher harmonics produced as above by the reaction of the ring became greater and the magnetizing current more distorted. Also in order to get the slower speeds fewer cells were used to supply current to the rotary converter, thus reducing the applied E.M.F., and in order to get the same currents as before the impedances used had to be proportionally reduced. Hence, in the slower speeds the upper harmonics of the current due to the reaction of the ring would be relatively greater and the distortion of its wave-form more marked.

This phenomenon is an interesting case of transformation and what may be called reflexion of energy. Thus the iron receives energy from the first harmonic (frequency n) of the exciting current, some of which it transforms to vibratory energy of frequencies $3n$, $5n$, &c., and sends back energy to the primary circuit by means of currents of these higher frequencies, where in general this reflected energy will be dissipated as heat.

This reflexion of energy occurred in nearly all the experiments recorded in this paper, and the amount in each case for each harmonic is obtained when calculating the iron loss by the analytical method given in § 7.

Thus the energy D received per cycle by the iron and dissipated as heat in it is given by

$$\begin{aligned} \mathrm{D} &= n_1 \int_0^{\mathrm{T}} \mathrm{C} \frac{d\mathrm{F}}{dt} dt \\ &= n_1 \int_0^{\mathrm{T}} \left\{ \mathrm{C}_1 \frac{d\mathrm{F}_1}{dt} + \mathrm{C}_3 \frac{d\mathrm{F}_3}{dt} + \mathrm{C}_5 \frac{d\mathrm{F}_5}{dt} + \text{\&c.} \right\} dt \\ &= \mathrm{D}_1 + \mathrm{D}_3 + \mathrm{D}_5 + \text{\&c. (say)}, \end{aligned}$$

where C_1, C_3, C_5, F_1, F_3, F_5 are the 1st, 3rd, and 5th harmonics of current and flux respectively.

Now if, for instance,

$$\mathrm{D}_3 \text{ or } n_1 \int_0^{\mathrm{T}} \mathrm{C}_3 \frac{d\mathrm{F}_3}{dt} dt$$

is negative, the iron by means of the third harmonics is sending back per cycle to the magnetizing circuit energy to the amount $-D_3$, and similarly if D_5 &c. are negative.

We have (see § 7)

$$\frac{D}{v}=I=\frac{H_1B_1}{4}\{\sin\theta_1+3h_3b_3\sin 3(\theta_3-\phi_3)+5h_5b_5\sin 5(\theta_5-\phi_5)+\&c.\}$$

$$=I_1+I_3+I_5+\&c.\ \text{(say)},$$

where v is the volume of iron in the ring:

hence $$D_1=I_1v=v\frac{H_1B_1}{4}\sin\theta_1$$

$$D_3=I_3v=v\frac{H_1B_1}{4}\times 3h_3b_3\sin 3(\theta_3-\phi_3)$$

$$D_5=I_5v=v\frac{H_1B_1}{4}\times 5h_5b_5\sin 5(\theta_5-\phi_5).$$

For example, in calculating I by means of the above formula for experiment 14 (Table I.) in which

$$H=\ 9{\cdot}42\{\sin\omega t-{\cdot}1635\sin 3(\omega t-8{\cdot}72)$$
$$-{\cdot}0565\sin 5(\omega t-29{\cdot}78)+{\cdot}0159\sin 7(\omega t-15{\cdot}86)\}$$
$$B=15860\{\sin(\omega t-34{\cdot}94)+{\cdot}1865\sin 3(\omega t-39{\cdot}71)$$
$$+{\cdot}0434\sin 5(\omega t-44{\cdot}12)+{\cdot}0117\sin 7(\omega t-47{\cdot}9)\}$$

we find that

$$I=37350\{{\cdot}5727-{\cdot}0914-{\cdot}0116-{\cdot}0009\}$$
$$=21390-3414-434-34$$
$$=17508,$$

or $$I_1=21390,\quad I_3=-3414,$$
$$I_5=-434,\quad I_7=-34,$$

which shows that per cm.^{3} of iron per cycle 21390 ergs entered the ring by means of the first harmonics, and of this there was sent back or reflected to the primary circuit 3414 ergs by means of the third, 434 ergs by means of the fifth, and 34 ergs by means of the seventh harmonic, while the remainder, $=I=17508$ ergs, was dissipated as heat in the iron of the ring.

These reflected current harmonics will obviously, if of sufficient magnitude, greatly modify the wave-form of C, and they will also by their reaction reduce the amplitudes and change the phases of the corresponding harmonics of F from which they arise, thus modifying the wave-form of F (the flux).

Now I find that when the upper harmonics of the flux are damped as above, μ_0 or B_1/H_1 becomes greater, and the iron loss I for the same value of $\mathfrak{B}$, or even of B_1, becomes greater. To make up for this greater iron loss as well as to supply the energy that is reflected, more energy must enter the iron by means of the first harmonic of C, and this is effected by a considerable increase in θ_1, the angle of lag of F_1 behind C_1.

If the applied E.M.F. (supposed sinusoidal) and the circuit be so controlled that while F_1 or αB_1 is kept constant the impedance is regularly diminished, the reflected energy due to any of the upper harmonics (C_3, F_3, say) will not go on increasing as the impedance diminishes, but will at first increase to a maximum and then diminish, in the limit vanishing when the impedance becomes zero. This would follow if we assumed that on account of some property of the iron there is always associated with a given primary sinusoidal flux oscillation F_1 or αB_1 of frequency n a series of magneto-motive forces M_3, M_5, &c., of frequencies $3n$, $5n$, &c. given in amplitude and fixed in phase relative to F_1.

F_3 would then, for a given impedance, be the flux produced by the vector resultant of M_3 and $4\pi n_1 C_3$, where C_3 is the reflected current produced in the circuit by variation of F_3.

In the simple case in which the circuit outside the ring is non-inductive, and when magnetic lag is neglected, the relations between M_3, C_3, F_3, and the reflected energy can be shown by means of a vector right-angled triangle of which the hypotenuse is proportional to M_3 and is fixed. One of its sides is proportional to C_3 and the other to F_3, while the area of the triangle is proportional to the reflected energy. The locus of the right-angled vertex is thus a semicircle, and as C_3 increases from zero until the triangle becomes isosceles, the reflected energy increases from zero to a maximum, after which any further increase of C_3 causes a diminution in the reflected energy, which finally vanishes with F_3.

If we took account of magnetic lag and of the inductance of the circuit the vector diagram would be more complicated, but the general result would be similar.

The following results of three experiments with Ring I. will illustrate the preceding.

The ring was rewound with two layers of 207 turns each, one or both of which could be used, thus enabling me to obtain for the same values of F_1 two series of values, one double the other, for the reaction E.M.F.'s due to variation of the upper harmonics of F.

Experiment 1.

$T = \cdot 0295$ sec. Magnetizing current sent through 207 turns. Full pressure of secondary battery on to rotary converter, and current in magnetizing circuit cut down by iron-less inductances to keep the reflected currents due to higher harmonics of F as small as possible.

$$H = 4\cdot 53\{\sin \omega t - \cdot 0443 \sin 3(\omega t - 29\cdot 11) + \cdot 0124 \sin 5(\omega t - 7\cdot 41)\}$$

$$B = 11720\{\sin(\omega t - 39\cdot 89) + \cdot 1814 \sin 3(\omega t - 42\cdot 23) + \cdot 042 \sin 5(\omega t - 43\cdot 8)\},$$

$\mu_0 = B_1/H_1 = 2587.$ $\mathfrak{B} = 13580.$ Max. $B = 10120.$

$I_1 = 8541,$ $I_3 = -203,$ $I_5 = -1,$

$I = I_1 + I_2 + I_3 = 8310,$

$I/\mathfrak{B}^{1\cdot 57} = \cdot 0027.$

Experiment 2.

$T = \cdot 02957$ sec. $n_1 = 414$ turns.

E.M.F. the same as in Exp. 1, but current cut down by non-inductive resistances.

$$H = 4\cdot 4\{\sin \omega t - \cdot 1758 \sin 3(\omega t - 22\cdot 56) + \cdot 039 \sin 5(\omega t - 1488)\}$$

$$B = 11850\{\sin(\omega t - 48\cdot 04) + \cdot 1246 \sin 3(\omega t - 56\cdot 82) + \cdot 0164 \sin 5(\omega t - 66\cdot 73)\}.$$

$\mu_0 = 2693.$ $\mathfrak{B} = 12700.$ Max. $B = 10840.$

$I_1 = 9692.$ $I_3 = -830.$ $I_5 = -42.$

$I = 8820.$ $I/\mathfrak{B}^{1\cdot 57} = \cdot 00318.$

Experiment 3.

$T = \cdot 0294$ sec. $n_1 = 414$ turns.

Impressed E.M.F. reduced and non-inductive resistances proportionally reduced.

$$H = 4\cdot 23\{\sin \omega t - \cdot 25 \sin 3(\omega t - 30\cdot 25) + \cdot 0572 \sin 5(\omega t - 32\cdot 24)\}$$

$$B = 11560\{\sin(\omega t - 50\cdot 77) + \cdot 0718 \sin 3(\omega t - 62\cdot 19) + \cdot 0096 \sin 5(\omega t - 82\cdot 7)\}.$$

$\mu_0 = 2734.$ $\mathfrak{B} = 11840.$ Max. $B = 10900.$

$I_1 = 9465.$ $I_3 = -654.$ $I_5 = -32.$

$I = 8780.$ $I/\mathfrak{B}^{1\cdot57} = \cdot00353.$

Though the applied E.M.F.'s in these experiments were only approximately sinusoidal, they illustrate the main features of the phenomenon being discussed. Thus we have with diminution of the impedance of the circuit while B_1 is kept approximately constant, the increase in the reflected upper harmonics of C and the diminution of the corresponding harmonics of F, the increase of μ_0 and of I and the increase and subsequent decrease of the reflected energies I_3 and I_5.

Attention is drawn to the great variation in the ratio of I to $\mathfrak{B}^{1\cdot57}$ with change of wave-form of H, and therefore of B, thus showing that the formula given in § 12 for I can only apply to a series in which the wave-form of H is nearly sinusoidal.

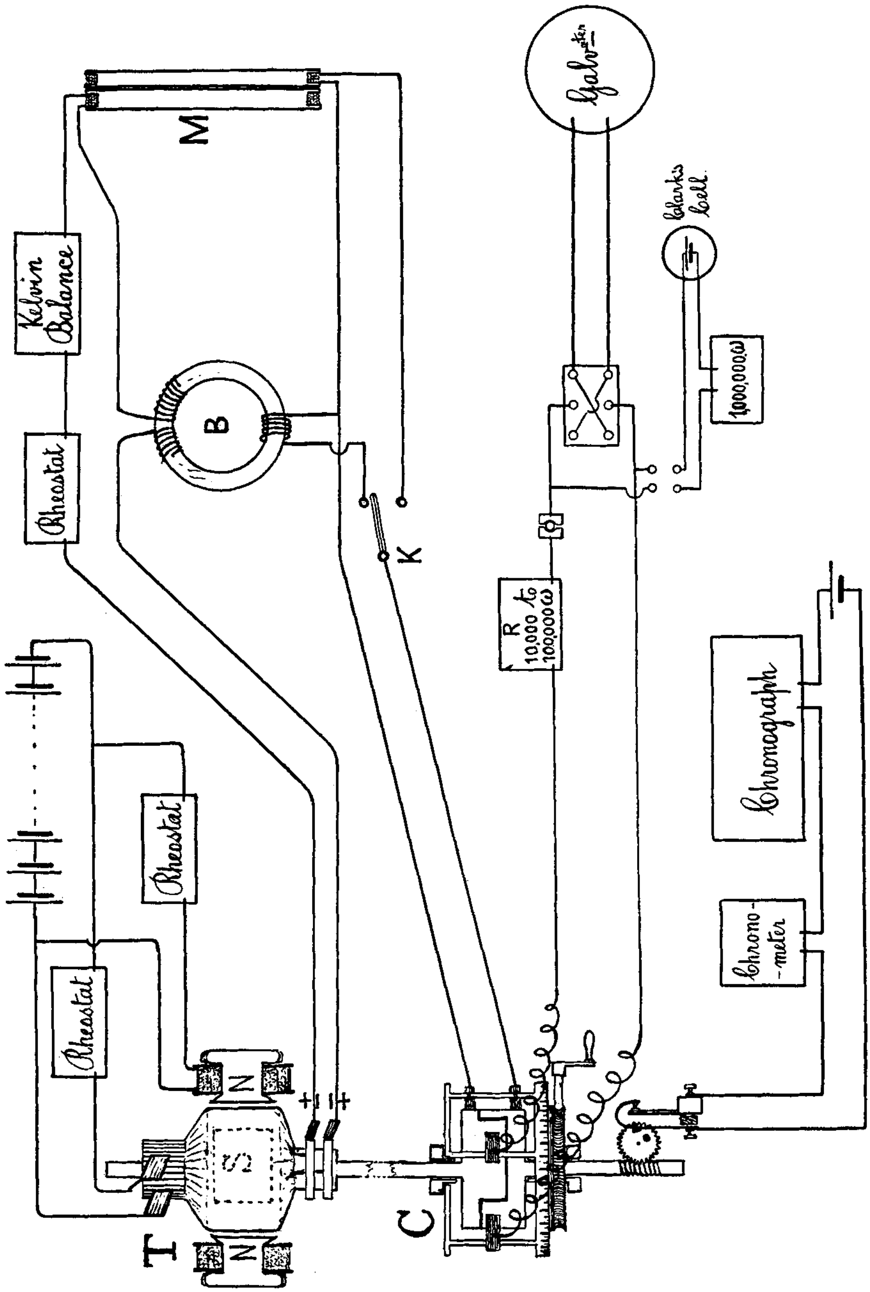
In order to fully investigate the phenomenon drawn attention to in this paragraph, it would be necessary to determine the impressed E.M.F. wave as well as that of H and of B. This is being done, and will form the subject of a future communication.

Finally, the matter dealt with in this section seems to me to have an important practical significance in the case of transformers. In calculating their losses all that has hitherto been counted as iron loss is the energy that is dissipated as heat in the iron. But now we see that it is possible for the iron to send back a considerable amount of energy to the primary circuit, where it also is dissipated as heat. This reflected energy should be counted against the iron.

Thus Experiment 2 above may, for the sake of illustration, be supposed to refer to a transformer on open secondary. The true dissipation of energy associated with its operation per cm.^{3} of iron per cycle is I_1 or 9692 ergs, of which $I = 8820$ ergs, hitherto called the total iron loss, is dissipated as heat in the iron, and $-(I_3 + I_5) = 872$ ergs is reflected by the iron to the primary circuit, where it also is dissipated as heat.

I desire to thank Mr. K. S. Cross and Mr. E. Machin for valuable assistance in carrying out the experiments discussed in this paper.

Fig. 1.


Kelvin Balance
Rheostat
Rheostat
Rheostat
M
B
K
Clarks Cell
1000,000ω
R 10,000 to 100,000ω
Chronograph
Chronometer
C
T
N
N
S