THERMAL CONDUCTIVITY OF COPPER.

BY R. W. QUICK, C. D. CHILD, AND B. S. LANPHEAR.

PART I.

CONDUCTIVITY AT HIGH TEMPERATURES.

By C. D. Child and R. W. Quick.

I.

Method Employed.

THE subject of the conduction of heat in metals has received considerable attention during the past three quarters of a century. Many investigations have been conducted on various metals with a view of determining the absolute value of their conductivity, as well as its variation with temperature. Various methods have been employed, both in the manner of determining the value of conductivity and in the measurement of temperature, the latter determination being one of great importance.

In scanning the literature upon the subject one finds that three general methods have been followed in experiments on the determination of conductivity, viz.: —

(1) The metallic wall, whose faces are maintained at different temperatures.

(2) A long bar, one of whose ends is maintained at a constant high temperature.

(3) A long bar, one or both of whose ends may be subjected to periodic changes of temperature.

The first method was employed by Péclét, Christiansen, and Berget; the second by Despretz, Langberg, Forbes, Mitchell, Tait, and others; and the last by Ångström, Neumann, Lorenz, and H. Weber.

Temperatures have been measured by means of the mercury thermometer and the thermo-element. The heating has been accomplished by a flame directly, by hot water or steam, and by plunging the end of the bar into molten metal.

In this investigation, both in dealing with high temperatures and with low temperatures, the second general method of computing the conductivity has been employed, and the results deduced according to the well-known method first outlined and used by Forbes.

Suppose that one end of a metal bar of length a is maintained at a constant temperature either above or below that of the surrounding medium. By calculating the quantity of heat which flows per second through a cross-section of the bar, whose abscissa reckoned from the heated end as origin is x, and equating this value to the quantity of heat which is radiated (or absorbed) by the bar between x and its end a, we get an expression which gives the value of conductivity K. This is easily found to be

$$K = \frac{SD\int_x^a \frac{dv}{dt}dx}{\frac{dv}{dx}}, \tag{1}$$

where v is the temperature excess of the bar at the cross-section considered, t is time, S is specific heat, and D is density. If x is expressed in centimeters, and time in seconds, this gives K in C.G.S. units. If the length of the bar is such that a difference of temperature exists between its end a and the surrounding medium, to the numerator of (1) must be added the quantity of heat which is radiated (or absorbed) per second by the end of the bar. This is seen to be

$$\frac{ASD\frac{dv_a}{dt}}{P}$$

therms per square centimeter, where A is area of cross-section, P is the perimeter, and v_a is the temperature excess at the end.

Now in (1), $\int_x^a \frac{dv}{dt}dx$ represents the area of a curve whose ordinates are the rates of cooling (or absorption) corresponding to different temperature excesses of bar over air, and where abscissæ are distances in centimeters along the bar, reckoned from an arbitrary origin near the heated or cooled end. Thus if the bar be heated or cooled to a uniform temperature and allowed to regain its initial temperature condition while simultaneous observations of time and temperatures are recorded, the curve of cooling can be constructed, and from the tangents to this curve, together with the curve of distribution of temperature along the bar, when one of its ends is maintained at a constant temperature different from that of the surrounding air, the curve whose coördinates are x and $\frac{dv}{dt}$ can be constructed by points, and the area, $\int_x^a \frac{dv}{dt} dx$, can be computed.

To obtain $\frac{dv}{dx}$ and $\frac{dv}{dt}$, Forbes assumed v as a function of x in the case of the curve of distribution of temperature along the bar (which for brevity may be called the "statical curve"), and as a certain function of t in the case of the cooling curve. In deducing the results in this investigation, no functions were assumed, but the curve was drawn on a very large scale, utilizing in the best possible manner the points previously located by observed or calculated data; and the tangents at different points were taken by means of a straight edge, which, as determined by trial, could be set with a maximum deviation of 0.2 per cent from the mean setting.

The copper on which the experiments herein described were conducted was electrolytic copper in the form of prismatic bars, having a cross-section of about 5 cm. × 2 cm. One of the bars was a little over a meter in length, and had lines ruled on its surface 5 cm. apart. This will be referred to as the "long bar," and is that which was used for obtaining the statical curve of temperatures, in the experiments both with high and low temperatures. The other bar was about 20 cm. long, and will be designated the "short bar." The original bar from which the two bars were cut was one of a set made for Professor W. A. Rogers, and loaned for the purpose of carrying on these investigations. The short bar had a hole about 1 cm. in diameter bored near the center of its length and perpendicular thereto, parallel to the longest sides of the rectangular cross-section of the bar. This hole, extending not quite through the bar, could be filled with mercury and a thermometer bulb inserted, or, when the bar was wanted as a homogeneous mass, the hole was filled with a tightly fitting copper plug.

II.

Measurement of Temperature.

As has already been stated, the methods that have been employed in measuring temperatures of the bar are the thermometer and thermo-element. The use of the former inserted in holes bored at certain intervals in the bar possesses many disadvantages. The presence of holes filled with mercury and glass destroys the homogeneity of the bar, besides necessitating many uncertain corrections relative to expansion of glass and mercury. When the thermo-junction is inserted in holes in the bar, a similar objection may be offered; and when it is used on the surface, the temperature is obtained at only one point of the perimeter of the cross-section. The method that seemed to be best adapted to the present case is the "resistance" method, and is that which was used throughout this investigation.

In dealing with ordinary and high temperatures, this method involved only a means of detecting very minute changes in the resistance of a very small wire. This wire of insulated electrolytic copper was 0.002 in. in diameter, and wound in a single layer of about twenty turns (close together as possible) upon a copper collar formed of very thin sheet copper $\frac{7}{8}$ in. in width, that fitted the bar just closely enough to admit of movement along it. After the wire was wound and the terminals joined to a coarse wire, which in turn was carefully connected to a flexible cable, the wire was given a coat of shellac to maintain it in place. This collar, bearing the copper coil of fine wire, as shown in Fig. 1, was formed from two sheets of copper. c and c' are simple extensions of the end faces of the collar turned up at right angles thereto. The cross-pieces a and b are used to stiffen c and c'.

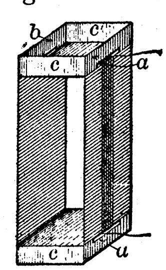

Fig. 1.

The object of these extensions c and c' was to afford a means of moving the collar on the bar. Their use will be more apparent when we come to the measurement of low temperatures.

The resistances of this copper coil were measured by a Wheatstone slide wire bridge, of which the collar formed one arm, R (Fig. 2). The wire W, which formed the ratio arms of the bridge, was an alloy of platinum and iridium, and contact was made at any point P, by means of a collar C, with vernier attachment, sliding on an insulated bar S, provided with a millimeter scale. The shaded portions are heavy copper. G is a sensitive and almost deadbeat galvanometer, and R' is a resistance compensated for temperature, against which the collar R is balanced. r and r' are resistances so chosen that, for the range of temperatures to be measured, P moves the whole length of the wire W.

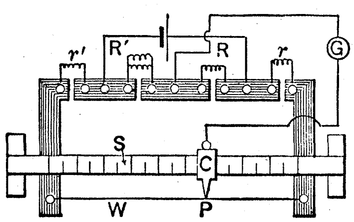

Fig. 2.

In experiments involving ordinary and high temperatures, this bridge was calibrated in the following manner, directly for temperatures of the bar: The collar in electrical connection with the bridge was placed on the short bar beside the hole already referred to, and which contained a thermometer bulb immersed in mercury. On each end of this bar, and insulated from the copper beneath by a layer of asbestos paper, were wound several turns of iron wire, through which was passed an electric current. By suitably adjusting the resistance in series with the solenoid, the bar could thus be heated to any desired temperature.[1] Two sets of simultaneous readings of the thermometer and the bridge were then taken: first, when the bar was slowly rising in temperature; and second, when its temperature was allowed to slowly fall. From these observations two curves were constructed with bridge readings as abscissæ, and temperatures of the bar as ordinates. A curve which was the mean of these two curves was then drawn, and this was used in determining temperatures of the bar from the bridge readings. The calibration was for the range 50° C. to 200° C.

[1] This method of heating has been employed quite extensively in the Cornell Physical Laboratory. See PHYSICAL REVIEW, Vol. I., p. 144.

III.

Determination of Data for the Statical Curve.

The long bar was placed on non-conducting supports, in a room whose temperature was maintained very constant, and one end was wound with a solenoid of iron wire as above described; in this case three layers were used, insulated from each other by asbestos paper. In series with the solenoid were placed an ammeter and a variable resistance. It was assumed that if the current were constant, the heat supply at the end of the bar would be practically constant; and so, in order to avoid the fluctuations of a dynamo current, the storage battery was used, and the current was maintained at the proper strength by adjusting the resistance. In order to prevent direct radiation from the solenoid and heated asbestos upon the copper coil of the collar, a screen was constructed in the following manner: Two sheets of tin about 50 cm. square were taken, and after the edges of one sheet were turned up at right angles on three sides of the square, the other sheet of tin was soldered to the first, leaving a space of a little over a centimeter between the tin sheets. In the center of this double thickness of tin was cut out a rectangular aperture or tunnel (whose sides were closed with tin) just large enough to admit the insertion of the long bar *without* contact with the tin. This screen was placed in a suitable standard, which supported it in a vertical plane, with one end of the bar protruding through it far enough to admit of the winding of the heating solenoid. The space between the tin sheets was then filled with water of the temperature of the room, and was constantly renewed from a tube leading out of a large jar of water placed on a level above the upper edge of the screen. After circulating through the screen, it emptied into a receptacle placed on the floor of the room. In this way direct radiation was cut off, and the screen never reached a temperature above that of the surrounding air.

After the bar had come to a statical temperature condition, as was shown by the collar in connection with the slide bridge, readings were taken on the latter, together with temperature of the surrounding air (which varied only about 2°), when the collar was placed successively at the different divisions on the bar. Table I. gives the temperature excess of the bar over air at the different divisions on the bar, counted from a point near the water screen. These excesses are obtained by subtracting the air temperature from the temperatures taken from the calibration curve of the bridge.

TABLE I.

DATA FOR STATICAL CURVE.

Distance from heated end in cm.	Temperature excess of bar over air.	Distance from heated end in cm.	Temperature excess of bar over air.
0	144.2	50	65.0
5	131.2	55	61.1
10	120.4	60	58.0
15	110.4	65	55.8
20	101.6	70	53.5
25	93.6	75	51.4
30	85.8	80	50.3
35	79.6	85	49.4
40	74.0	90	48.7
45	68.9		

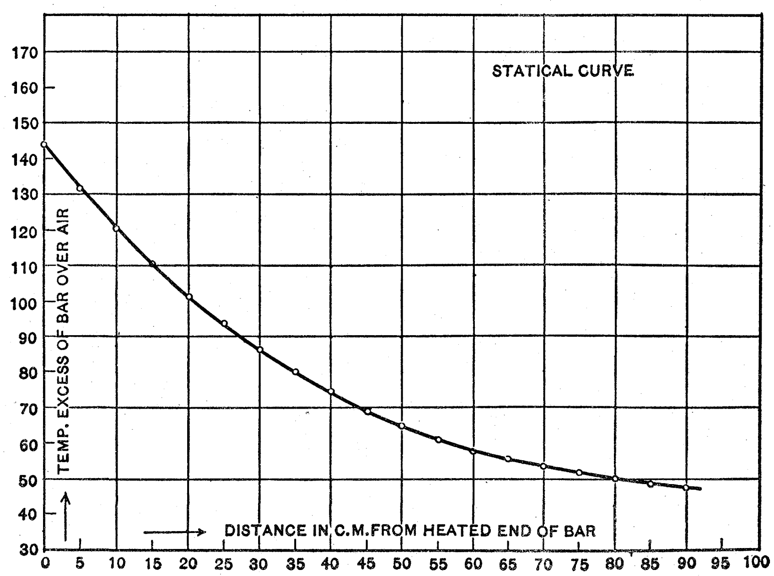

Fig. 3.

From Table I. the curve in Fig. 3 was constructed.

IV.

Determination of Data for Time Curve of Cooling of Long Bar.

One of the great difficulties that have hitherto been encountered in obtaining these observations was that of heating the bar to a uniform temperature. A method employed in this case, and which proved satisfactory, was that of electric heating by means of a solenoid. The collar was first placed on the long bar near its center, and the latter was wound throughout its entire length with iron wire, whose turns were quite close together and insulated from the bar by asbestos paper. After the collar indicated a temperature of about 25° above the highest temperature recorded in the statical experiment, the solenoid was quickly removed from the bar, and simultaneous observations of bridge readings, time, and temperature of the room were taken. Table II. contains the data obtained from these observations, and from which the curve in Fig. 4 was constructed.

Table II.

DATA FOR TIME CURVE OF COOLING OF LONG BAR.

Time.	Temperature excess of bar over air.	Time.	Temperature excess of bar over air.
min. sec.		min. sec.	
0 52	155.6	20 2	92.8
2 10	149.7	22 29	87.3
3 35	143.7	25 3	81.8
5 5	137.9	27 52	76.4
6 35	132.1	30 51	71.1
8 12	126.5	34 5	65.8
9 57	120.9	37 44	60.5
11 43	115.3	41 38	55.2
13 37	109.6	46 7	50.0
15 38	104.0	50 0	45.9
17 46	98.3		

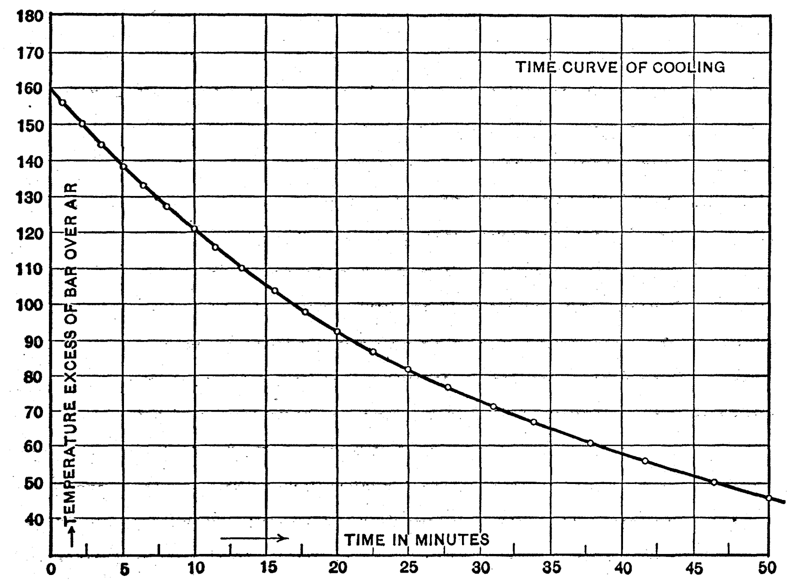

Fig. 4.

V.

Deduction of Results.

It now remains to construct, by aid of the two curves already drawn, the curve whose abscissæ are distances from heated end,

TABLE III.

DATA FOR INTEGRATION CURVE.

Distance in cm. (x) from heated end of bar.	Tangents to cooling curve $-\frac{dv}{dt}$	Distance in cm. (x) from heated end of bar.	Tangents to cooling curve $-\frac{dv}{dt}$
0	0.0690	50	0.0260
5	0.0598	55	0.0240
10	0.0540	60	0.0220
15	0.0490	65	0.0207
20	0.0450	70	0.0200
25	0.0396	75	0.0190
30	0.0362	80	0.0187
35	0.0325	85	0.0183
40	0.0298	90	0.0178
45	0.0278	92.12	0.0177

and whose ordinates are $-\frac{dv}{dt}$. This curve, for convenience, will be designated the "integration curve"; for the area, $\int_x^a \frac{dv}{dt}dx$, is proportional to the quantity of heat radiated per second by the bar between the neutral end and x. The numbers in the second column of Table III. are obtained by taking tangents to the cooling curve at temperature excesses that are the same as those of the bar at the different distances in column 1 when it was in the statical temperature condition. The statical curve shows this. The curve constructed from Table III., and shown in Fig. 5, was integrated in

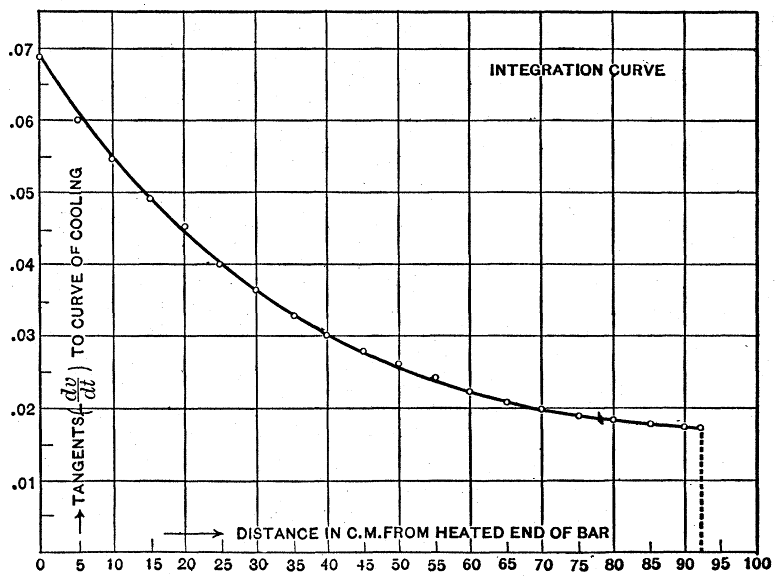

Fig. 5.

parts by drawing it on a sufficiently large scale, so that consecutive small lengths of the curve deviated very little from a straight line. The area included between two ordinates, the x-axis, and a small length of the curve, could then be easily obtained, and by adding together such consecutive areas, the area between any ordinate and the end of the bar can be easily obtained. In Table IV. the lower limit of integration x in column 2 is the same x as in column

1, and a represents the total length of the bar from the assumed origin near the heated end. To each of the areas in column 2 has been added the area that corresponds to the heat radiated per second from the face end of the bar. The numbers of column 3 are

TABLE IV.

DATA FOR CURVE OF CONDUCTIVITY (K).

Distance on bar x.	$\int_x^a \frac{dv}{dt} dx$	Tang. to statical curve $-\frac{dv}{dx}$	Temperature.	Conductivity K.
80	0.232	0.211	74.0	0.914
70	0.425	0.385	77.2	0.920
60	0.634	0.550	81.8	0.963
50	0.871	0.768	88.2	0.952
40	1.149	1.06	97.3	0.915
35	1.306	1.21	103.1	0.915
30	1.478	1.35	109.5	0.932
25	1.668	1.51	116.9	0.945
20	1.878	1.71	124.9	0.944
15	2.111	1.88	133.8	0.971
10	2.369	2.05	143.6	1.006
5	2.656	2.32	154.6	1.004
0	2.979	2.57	166.8	1.024

the tangents to the statical curve corresponding respectively to distances in column 1. The temperatures in the fourth column are obtained by adding to the temperature excess of the bar the temperature of the room at the time of the statical experiment. The values of K are then computed from equation (1); that is, dividing column 2 by column 3 and multiplying by SD. But since both specific heat and density are functions of temperature, cognizance must be taken of this change. Most writers give the average value of S for copper between 0° and 100° as 0.093, and this coincided with a determination made by Shepard[1] on the same specimen of copper that was used in this investigation. Moreover, as this corresponds to the value given by Bède's formula

$$S_t = 0.0892(1 + 0.00073\, t),$$

[1] Thesis, Physical Properties of Copper, Library of Cornell University.

the value of S was obtained from the above in computing absolute values of K. Density also varies with temperature, but as the 5 cm. divisions were marked on the bar itself, a small error in measurement would be introduced, owing to linear expansion of the bar. Therefore in order to correct for this, instead of using D_t, we use $D_t^{\frac{1}{3}} \cdot D_{20}^{\frac{2}{3}}$, where D_{20} is the density at the temperature at which the bar was marked. Now the writers obtained 8.86 for the value of density of the bar at a temperature of 4° C., which, combined with the temperature factor $(1-0.000056\,t)$ as determined by R. W. Stewart,[1] gives

$$D_t = 8.862(1-0.000056\,t).$$

Using the values of specific heat and density as given above, the values of K were computed at the different temperatures, showing an increase in conductivity as temperature increases through the range 70° to 170°. This result will be shown graphically in Part II., which gives the result of the investigation on conductivity at low temperatures.

[1] Proc. Roy. Soc. London, V. 53, p. 151, 1893.