The Coherence Field: A Generative Template for Coherence-Driven Evolution and Domain Flows

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Abstract

We introduce the *Coherence Field*, a functional-analytic template in which evolution is generated by the first variation of a scalar functional κ on a Hilbert space. We define a coherence functional $\kappa : \mathcal{H} \to \mathbb{R}$ as twice Fréchet differentiable, bounded below, with globally Lipschitz derivative, and we prove global well-posedness of the associated canonical (dissipative) coherence-gradient flow by standard ODE theory in Banach spaces. Along these trajectories, κ decreases monotonically and equilibria coincide with vanishing first variation.

We then place familiar regimes within the same functional-and-variation template while making explicit the role of additional geometric structure: Hamiltonian mechanics arises when the same functional generates a *symplectic* flow, imaginary-time Schrödinger dynamics arises as the gradient flow of the standard quantum energy functional (with Wick rotation connecting to unitary real-time evolution), and vacuum Einstein equations correspond to stationarity of the Einstein–Hilbert functional with the Gibbons–Hawking–York boundary term.

Finally, we introduce an operational coherence-drift observable $\|D\kappa\|$ and a spin-system steering protocol in which coherence drift separates from bare energy drift under target-pattern pressure, yielding a falsifiable signature that does not reduce to energy measurements alone.

1 Introduction

This paper proposes a common functional-and-variation template: begin with a scalar functional κ on a configuration space, consider its first variation $D\kappa$, and study the flows and stationarity conditions induced by $(\kappa, D\kappa)$ under different structural geometries. In this view, κ is not introduced as an additional physical field in spacetime, but as a generator that can be instantiated as an energy, an action, or a structured objective depending on the regime and invariances under consideration.

Scope and non-claims

The reductions in Section 5 are *not* presented as new derivations of established theories such as general relativity or quantum mechanics. They are presented as *placements* of standard variational objects within a shared functional-and-variation framework. The goal is structural clarity: (i) to establish a rigorous baseline evolution (a canonical coherence-gradient flow) under standard assumptions, (ii) to separate the *functional* from the *geometry of the induced flow*, and (iii) to propose a measurable diagnostic, the coherence drift $\|D\kappa\|$, and a protocol where it separates from bare energy drift.

Within the broader $\Delta.72$ program, κ may be interpreted as an induced functional associated with harmonic closure; here we remain agnostic and focus on the functional-analytic layer. Earlier related manuscripts introduce a conditional deterministic operator and foundational coherence mathematics [1, 2], and a companion manuscript explores coherence-based time parameterizations [3].

Related analytical background

Global well-posedness of Lipschitz vector fields in Banach spaces, and the semigroup viewpoint for evolution equations, are standard in functional analysis [4]. Lyapunov arguments and gradient-type evolutions in Hilbert spaces are likewise standard [5]. Euclidean (imaginary-time) quantum dynamics and Wick rotation are classical in functional-integral treatments [6, 7]. Variational calculus for the Einstein–Hilbert action with the Gibbons–Hawking–York boundary term is standard in general relativity [8, 9].

2 Configuration Space and Coherence Functional

Let Ω be a configuration domain (e.g., physical space, a lattice, or a manifold). We consider the Hilbert space

$$\mathcal{H} = L^2(\Omega; \mathbb{R}^n), \tag{1}$$

with inner product

$$\langle \Phi_1, \Phi_2 \rangle = \int_\Omega \Phi_1(x) \cdot \Phi_2(x)\, dx, \tag{2}$$

and induced norm $\|\Phi\| = \sqrt{\langle \Phi, \Phi \rangle}$.

Definition 1 (Coherence functional)**.** *A* coherence functional *is a map $\kappa : \mathcal{H} \to \mathbb{R}$ satisfying:*

- *κ is twice Fréchet differentiable,*
- *κ is bounded below,*
- *$D\kappa : \mathcal{H} \to \mathcal{H}$ is globally Lipschitz:*

$$\|D\kappa(\Phi_1) - D\kappa(\Phi_2)\| \le L\|\Phi_1 - \Phi_2\| \quad \forall\, \Phi_1, \Phi_2 \in \mathcal{H}$$

for some $L > 0$.

Definition 2 (Coherence Field (first-variation operator))**.** *The* Coherence Field *is the first-variation operator associated with κ:*

$$\mathcal{C}(\Phi) = D\kappa(\Phi) \in \mathcal{H}. \tag{3}$$

Here "field" is used in the variational sense (an operator-valued map on configuration space), not as an additional spacetime field.

Remark 1. *The global Lipschitz condition ensures global well-posedness of the canonical flow defined in Section 4 by standard Banach-space ODE theory [4]. The bounded-below condition supports monotone descent and Lyapunov arguments along dissipative flows [5].*

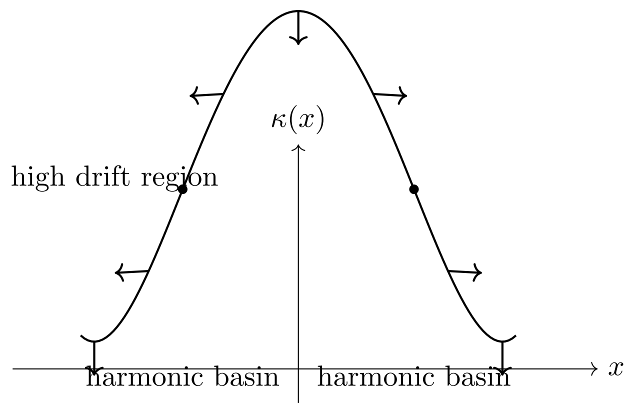

Figure 1: Schematic coherence landscape: under a dissipative flow, trajectories descend along $\nabla\kappa$ into basins where the first variation vanishes.

3 Coherence landscape intuition

The functional κ induces a scalar landscape over $\mathcal{H}$. In dissipative settings, the canonical flow descends along the gradient of κ and converges toward critical regions where the first variation vanishes. In one dimension, this is analogous to sliding down a potential surface into a basin.

4 Canonical coherence-gradient dynamics and harmonic closure

Definition 3 (Canonical (dissipative) coherence-gradient flow)**.** *Given κ, define the canonical evolution*

$$\frac{d\Phi}{dt} = -\lambda\, D\kappa(\Phi), \qquad \lambda > 0. \tag{4}$$

Theorem 1 (Global well-posedness and monotone descent)**.** *Let κ satisfy Definition 1. For any $\Phi_0 \in \mathcal{H}$, the initial value problem (4), $\Phi(0) = \Phi_0$, admits a unique solution $\Phi : [0, \infty) \to \mathcal{H}$. Moreover, $t \mapsto \kappa(\Phi(t))$ is nonincreasing and*

$$\frac{d}{dt}\kappa(\Phi(t)) = -\lambda\, \|D\kappa(\Phi(t))\|^2 \le 0.$$

Proof. Define $F(\Phi) = -\lambda D\kappa(\Phi)$. Since $D\kappa$ is globally Lipschitz, so is F. By the Picard–Lindelöf theorem for Banach-space ODEs, there exists a unique maximal solution. Global Lipschitz implies the solution extends for all $t \ge 0$ [4].

For descent, differentiate along the flow:

$$\frac{d}{dt}\kappa(\Phi(t)) = \langle D\kappa(\Phi(t)), \dot{\Phi}(t)\rangle = -\lambda\, \|D\kappa(\Phi(t))\|^2 \le 0.$$

□

Definition 4 (Harmonic closure)**.** *A configuration $\Phi^* \in \mathcal{H}$ is in* harmonic closure *if $D\kappa(\Phi^*) = 0$ and $D^2\kappa(\Phi^*)$ is positive definite as a bounded operator:*

$$D\kappa(\Phi^*) = 0, \qquad \langle D^2\kappa(\Phi^*)[\delta\Phi], \delta\Phi\rangle > 0\ \forall\, \delta\Phi \ne 0. \tag{5}$$

Proposition 1 (Local asymptotic stability of harmonic closure)**.** *If Φ^* satisfies (5), then Φ^* is a locally asymptotically stable equilibrium of (4).*

Proof. Positive definiteness of the second variation implies Φ^* is a strict local minimum of κ. Along (4), κ is a Lyapunov function and decreases strictly unless $D\kappa = 0$. Standard gradient-flow stability arguments in Hilbert spaces apply [5]. □

Remark 2 (Structured flows are not all dissipative)**.** *The canonical flow* (4) *is dissipative. Other regimes are generated by the same functional* under different geometric structures*, for example symplectic structure (Hamiltonian flow) or unitary structure (Schrödinger evolution). We make this explicit next.*

4.1 A coherence-based reparameterization (optional)

Along a trajectory $t \mapsto \Phi(t)$ solving (4)*, one may define a derived path parameterization*

$$\tau(t) = \int_0^t \frac{\|\dot{\Phi}(s)\|}{\|D\kappa(\Phi(s))\|}\, ds, \tag{6}$$

whenever $D\kappa(\Phi(s)) \neq 0$. Conceptual interpretation is developed in [3].

5 Domain representations: classical, quantum, and geometric

5.1 Classical mechanics: symplectic flow generated by a functional

Let $\Phi = (q,p) \in \mathbb{R}^n \times \mathbb{R}^n$ and take $\kappa(q,p) = H(q,p)$. Define

$$D\kappa(q,p) = (\nabla_q H, \nabla_p H), \qquad J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

The symplectic (Hamiltonian) flow generated by κ is

$$\dot{\Phi} = J\, D\kappa(\Phi), \tag{7}$$

which yields Hamilton's equations

$$\dot{q} = \nabla_p H, \qquad \dot{p} = -\nabla_q H.$$

Remark 3. *Equation* (7) *is not a dissipative gradient flow; it is a structured flow induced by the same generator under symplectic geometry. This isolates the role of geometry in determining the dynamical form.*

5.2 Quantum mechanics: imaginary-time flow and Wick rotation

Let $\psi \in L^2(\mathbb{R}^d)$. Consider the standard quantum energy functional

$$\kappa(\psi) = \frac{\hbar^2}{2m} \int_{\mathbb{R}^d} |\nabla\psi(x)|^2\, dx + \int_{\mathbb{R}^d} V(x)\, |\psi(x)|^2\, dx. \tag{8}$$

Its L^2-gradient is

$$D\kappa(\psi) = -\frac{\hbar^2}{2m}\Delta\psi + V\psi,$$

so the canonical gradient flow yields

$$\frac{\partial\psi}{\partial t} = -D\kappa(\psi) = \frac{\hbar^2}{2m}\Delta\psi - V\psi, \tag{9}$$

the Euclidean (imaginary-time) Schrödinger equation. Wick rotation and the functional-integral viewpoint connecting Euclidean and real-time evolution are standard [6, 7].

Remark 4 (Decoherence and drift diagnostics). *In open quantum systems, additional terms can be added to κ to encode coupling to effective environment degrees of freedom. In such settings, $\|D\kappa\|$ provides a diagnostic of drift away from stationary coherent structure even when energy changes are small; see [12] for standard decoherence context.*

5.3 General relativity: stationarity of the Einstein–Hilbert functional

Let M be a four-dimensional spacetime manifold with metric $g_{\mu\nu}$, scalar curvature R, and induced metric h_{ij} on ∂M. Define

$$\kappa(g) = -\int_M R(g)\sqrt{-g}\, d^4x - \int_{\partial M} K\sqrt{|h|}\, d^3x, \tag{10}$$

the Einstein–Hilbert functional with the Gibbons–Hawking–York boundary term. Stationarity under compactly supported metric variations yields vacuum Einstein equations, a standard result [8, 9]. We emphasize again that this subsection records a standard variational result to illustrate template placement, not a proposed modification of general relativity.

Remark 5. *This subsection is a placement of standard GR variational structure in the template; it is not a new derivation.*

6 Pipeline diagram: domain emergence

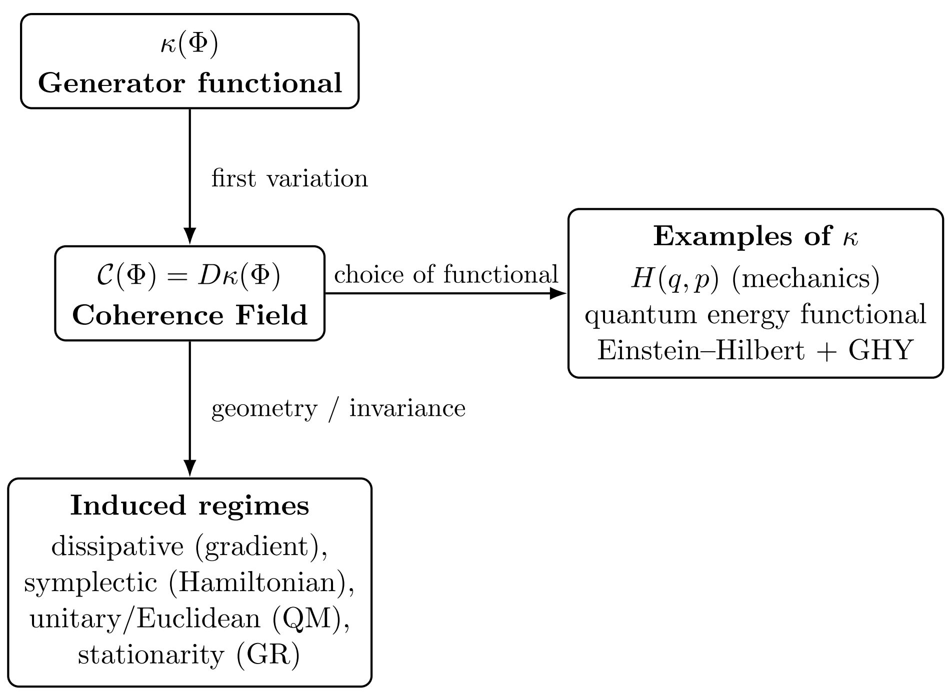

Figure 2: A single generator κ defines a Coherence Field $D\kappa$. The observed regime depends on the geometry/invariance used to induce dynamics (gradient-type, symplectic-type, unitary/Euclidean-type, or stationarity conditions).

7 Spin-system steering: coherence drift vs energy drift

We now define a concrete functional where a protocol-level separation between energy drift and coherence drift can be tested.

Consider an Ising-like system on a graph with spins $s_i \in \{-1, 1\}$ *and couplings* J_{ij}*. Let* $\bar{s}$ *be a target (reference) configuration. With* $\Phi = (s_1, \ldots, s_N)$*, define*

$$\kappa(\Phi) = -\sum_{i<j} J_{ij} s_i s_j + \beta \sum_{i=1}^{N} (s_i - \bar{s}_i)^2, \tag{11}$$

where $\beta > 0$ *controls target-pattern pressure. The first term is standard Ising energy; the second is a steering penalty analogous in spirit to biasing used in associative-memory / Hopfield-type models [10].*

Relax $s_i \in [-1, 1]$ *and consider the dissipative evolution*

$$\dot{s}_i = -\frac{\partial \kappa}{\partial s_i}. \tag{12}$$

Then

$$\frac{\partial \kappa}{\partial s_i} = -\sum_{j \neq i} J_{ij} s_j + 2\beta (s_i - \bar{s}_i).$$

Definition 5 (Coherence drift)**.** *Define the coherence drift observable*

$$D_\kappa(\Phi) = \|D\kappa(\Phi)\|.$$

In empirical or numerical instantiations, Φ *is taken to be a finite-dimensional embedding of measured state variables (or a discretization of a field), and* $\|\cdot\|$ *is the induced Euclidean/*L^2 *norm after a stated normalization, making* D_κ *a well-defined, protocol-dependent observable.*

Proposition 2 (Falsifiable separation protocol)**.** *Fix* J_{ij} *and vary the steering strength* β *in* (11)*. There exist regimes in which increasing* β *produces a sharp reduction in* $D_\kappa(\Phi)$ *(rapid approach toward stationarity of the full functional) while the bare Ising energy term* $-\sum_{i<j} J_{ij} s_i s_j$ *changes slowly or exhibits a delayed response. This predicts a measurable separation between coherence drift and energy drift under target-pattern steering.*

Remark 6. *The novelty claim here is operational: the protocol-level separation between drift diagnostics, not the existence of an Ising-plus-penalty functional form. Standard spin-glass baselines and diagnostics can be used for comparison [11].*

8 Discussion and outlook

The Coherence Field (first-variation operator) contributes a rigorous template and a measurable diagnostic.

1. ***A canonical well-posed evolution.*** *Under the assumptions of Definition 1 on* κ*, the canonical dissipative coherence-gradient flow is globally well-posed, and* κ *decreases monotonically along trajectories (Theorem 1).*

2. ***A structural separation between generator and geometry.*** *Classical, quantum, and geometric regimes can be expressed in a single functional-and-variation language once the relevant geometric structure is made explicit: symplectic structure for Hamiltonian flow, Euclidean/unitary structure for quantum evolution, and stationarity conditions for GR variational calculus.*

3. ***A drift observable and a falsifiable protocol.*** *The coherence drift* $\|D\kappa(\Phi)\|$ *provides a measurable indicator of distance from stationarity in the induced landscape. In steering settings, drift signatures may separate from changes in an energy-like component, yielding a testable prediction (Proposition 2). In empirical or numerical instantiations,* Φ *is taken to be a finite-dimensional embedding of measured state variables (or a discretization of a field), and* $\|\cdot\|$ *is the induced Euclidean/*L^2 *norm after a stated normalization; the choice of embedding is part of the model specification.*

For readers less familiar with the distinction between energy drift and coherence drift, it is useful to summarize their roles side by side.

Quantity	**Interpretation**
Energy drift	Change in an energy/action component along evolution. Encodes energetic constraints, but may not distinguish structural alignment under steering or invariance constraints.
Coherence drift	The norm $\|D\kappa(\Phi)\|$, measuring distance from stationarity of the chosen generator functional, even when energy-like terms change slowly.

Table 1: Schematic comparison of energy drift and coherence drift.

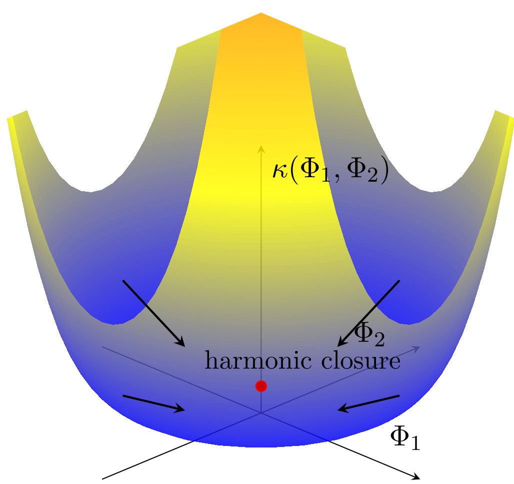

Figure 3: Illustrative coherence landscape slice. The surface represents $\kappa(\Phi_1, \Phi_2)$ on a two-dimensional slice of configuration space. Arrows indicate dissipative drift toward a stationary basin where $D\kappa$ vanishes.

9 Conclusion

We defined the Coherence Field as the first variation $D\kappa$ of a coherence functional on a Hilbert space, proved global well-posedness and monotone descent for a canonical dissipative coherence-gradient flow, and showed how standard domain objects can be placed within a single functional-and-variation template once the geometry of the induced dynamics is made explicit. We introduced a coherence drift observable $\|D\kappa\|$ and a steering protocol in a spin-system setting where coherence drift is predicted to separate from bare energy drift, yielding a falsifiable signature.

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