A Symmetric Classification of Prime Numbers

Correlational, Identity, and Inversion Symmetry

part of the research cycle

"Linear Infinity $\to$ Cyclic Space"

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Abstract

Prime numbers are typically viewed as arithmetically simple yet structurally irregular objects, lacking an intrinsic organizing principle beyond divisibility. This work develops a complementary structural perspective in which every prime number $P > 2$ is assigned a well-defined *symmetric signature* determined by three mutually independent invariants.

The first invariant, the *correlational symmetry*, arises from the choice of sign in the expressions $(P \pm 1)/4$ and determines which of the two adjacent arithmetic configurations contributes to the definition of a local symmetric quantity $N(P)$. The second invariant, the *identity symmetry*, captures a uniform algebraic relation between the neighboring integers $P-1$ and $P+1$, a relation that holds for all primes and provides a structural baseline for the framework. The third invariant, the *inversion symmetry*, links local and global information by comparing $N(P)$ with the prime index $\mathbb{P}(P)$ through the integer difference

$$Z(P) = \mathbb{P}(P) - N(P).$$

Together, these invariants define a classification scheme in which primes are organized not only by numerical order but also by structural properties encoded in their symmetric signatures. The framework is established through three foundational lemmas and a unifying theorem showing that every prime $P > 2$ belongs to a unique symmetric class determined by its signature. The construction is entirely elementary and does not aim at predictive or analytic results on prime distribution; rather, it provides a structural coordinate system revealing an additional layer of organization within the prime sequence that complements classical analytic and probabilistic approaches to prime number theory.

A reproducible dataset accompanying this work provides explicit symmetric signatures for large sets of primes, enabling further computational and structural investigations.

Keywords: prime numbers, symmetric invariants, correlational symmetry, identity symmetry, inversion symmetry, symmetric signature, structural classification, number theory

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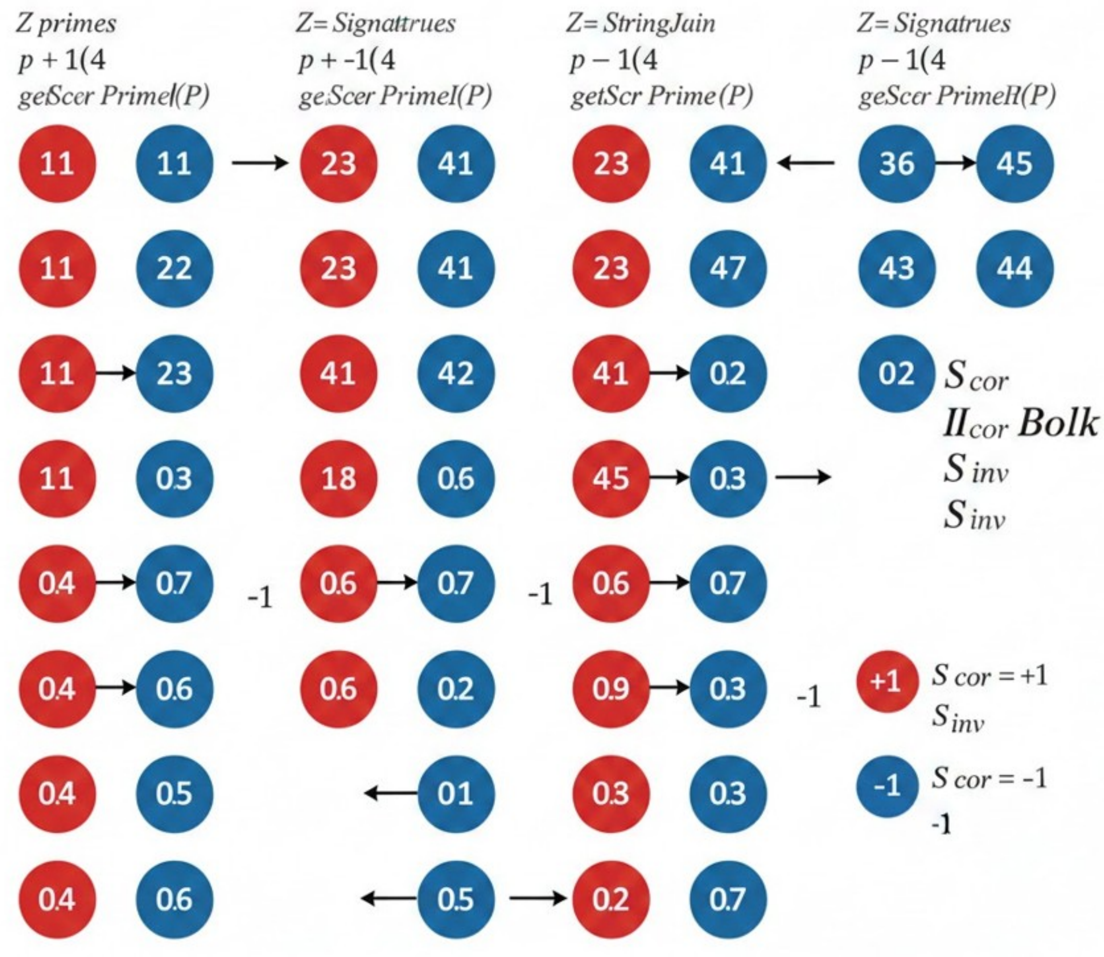
Graphical Abstract

Conceptual Symmetric Invariants



Highlights

- Introduces a symmetry-based structural framework assigning to every prime number $P > 2$ a three-component symmetric signature.
- Defines the correlational symmetry, which determines whether $(P-1)/4$ or $(P+1)/4$ is a natural number and thereby fixes the local symmetric value $N(P)$.
- Establishes the identity symmetry, showing that the neighboring integers $P-1$ and $P+1$ satisfy a uniform structural relation independent of the particular prime.
- Develops the inversion symmetry linking the locally defined value $N(P)$ with the global prime index $\mathbb{P}(P)$ through the invariant $Z(P) = \mathbb{P}(P) - N(P)$.
- Demonstrates that the three invariants form a coherent classification scheme, organizing primes into symmetric classes within a structural coordinate system.

- Provides three foundational lemmas and a unifying theorem integrating the correlational, identity, and inversion symmetries into a single consistent framework.

Contents

1. Introduction

Prime numbers [1] occupy a central position in number theory [2, 3]. Despite their elementary definition, their distribution along the natural numbers exhibits a degree of irregularity that has motivated extensive analytic, algebraic, and probabilistic investigation [4]. Beyond global questions concerning density, asymptotics, and distribution laws, primes also admit local structural descriptions that can be formulated independently of analytic methods [5]. Such descriptions often arise from examining arithmetic relations in the immediate neighborhood of a prime.

The present work develops a local and structural perspective of this type. Rather than introducing new analytic estimates, we study elementary transformations involving the integers adjacent to a prime $P > 2$. These transformations give rise to three naturally defined invariants which together form what we call the *symmetric signature* of the prime. The aim is to identify structural relations that remain invariant under simple local operations and to use these invariants as the basis for a classification framework.

The first invariant, the *correlational symmetry*, determines whether $(P-1)/4$ or $(P+1)/4$ is a natural number and thereby selects a locally defined symmetric quantity $N(P)$. This invariant encodes a directional property of the prime within its immediate arithmetic neighborhood. The second invariant, the *identity symmetry*, expresses a uniform algebraic relation between the neighboring integers $P-1$ and $P+1$, a relation that holds identically for all primes and provides a structural baseline for the construction. The third invariant, the *inversion symmetry*, connects the locally defined value $N(P)$ with the global ordering of primes through the prime index function $\mathbb{P}(P)$ via the integer difference

$$Z(P) = \mathbb{P}(P) - N(P).$$

This invariant links local arithmetic structure with global positional information in the ordered sequence of primes.

The purpose of the framework is not to obtain new results concerning the analytic distribution of primes. Instead, it provides a structural classification scheme based on explicitly defined invariants, allowing primes to be organized into symmetry classes determined simultaneously by local and global properties. In this sense, the approach complements classical analytic and algebraic viewpoints by emphasizing invariant structure rather than distributional behavior.

The paper is organized as follows. Section 2 introduces the three invariants as precise definitions and establishes their basic properties in the form of three foundational lemmas. These results are then combined into a main theorem that formalizes the symmetric signature as a coherent system. Although the construction is elementary, it reveals a nontrivial structural organization of prime numbers that is not immediately visible from their usual linear ordering.

2. Preliminaries

Throughout the paper we consider prime numbers $P > 2$, unless stated otherwise. This section introduces the notation and basic definitions used in the sequel.

Modular Structure and Residue Classes

Every prime $P > 2$ satisfies the congruence

$$P \equiv \pm 1 \pmod 4.$$

This elementary fact from modular arithmetic provides the structural basis for the constructions developed below. The symmetric value $N(P)$ introduced later is determined precisely by which of the two residue classes $\pm 1 \pmod 4$ is realized by the prime.

In this sense, the present framework may be viewed as a refinement of the classical decomposition of primes into residue classes modulo 4. Rather than using congruence classes solely as arithmetic labels, we interpret them as defining invariant structural roles within a symmetry-based classification scheme. The resulting invariants combine local modular information with global ordering information through the prime index function.

The approach belongs naturally to symmetry-based classification methods in number theory, where arithmetic objects are organized according to invariant transformations rather than analytic distribution properties. No analytic assumptions are introduced; all constructions rely only on elementary modular relations and explicitly defined arithmetic quantities.

2.1. Index of a Prime Number

Let $\mathbb{P}(P)$ denote the index of a prime number P in the ordered sequence of all prime numbers. Thus,

$$\mathbb{P}(2) = 1, \qquad \mathbb{P}(3) = 2, \qquad \mathbb{P}(5) = 3, \qquad \mathbb{P}(7) = 4, \ \ldots$$

The function $\mathbb{P}(P)$ will be used to relate local symmetric quantities to the global position of P in the prime sequence.

2.2. Symmetric Value

Definition 2.1. For every prime number $P > 2$ we define the *symmetric value* $N(P)$ by

$$N(P) = \frac{P \pm 1}{4}, \tag{1}$$

where the sign is chosen so that $N(P) \in \mathbb{N}$.

The existence and uniqueness of $N(P)$ follow from the fact that $P \equiv \pm 1 \pmod 4$; see Lemma 3.2.

Symmetric Invariants

We now introduce three invariants associated with a prime number $P > 2$.

Definition 2.2 (Correlational symmetry)**.** The *correlational symmetry* of P is defined by

$$\mathbb{S}_{cor}(P) = \begin{cases} +1, & \text{if } \dfrac{P+1}{4} \in \mathbb{N}, \\ -1, & \text{if } \dfrac{P-1}{4} \in \mathbb{N}. \end{cases} \tag{2}$$

This invariant determines which of the two possible local configurations contributes to the definition of $N(P)$.

Definition 2.3 (Identity symmetry)**.** For a prime number $P > 2$, consider the quantities

$$\frac{(P-1) \pm 1}{4}, \qquad \frac{(P+1) \pm 1}{4}.$$

Lemma 3.6 shows that none of these values belongs to $\mathbb{N}$. Accordingly, the *identity symmetry* $\mathbb{S}_{ind}$ is defined as a universal invariant, identical for all primes $P > 2$.

Definition 2.4 (Inversion symmetry)**.** The *inversion symmetry* of a prime number $P > 2$ is defined by

$$\mathbb{S}_{inv}(P) = Z(P) = \mathbb{P}(P) - N(P). \tag{3}$$

Lemma 3.10 establishes that $Z(P) \in \mathbb{Z}$ for all primes $P > 2$.

Symmetric Signature

Definition 2.5. The *symmetric signature* of a prime number $P > 2$ is the quadruple

$$\Sigma(P) = \big(\mathbb{S}_{cor}(P),\, \mathbb{S}_{ind},\, \mathbb{S}_{inv}(P),\, \mathbb{P}(P)\big). \tag{4}$$

For notational convenience we also employ the symbolic representation

$$^{\mathbb{S}_{inv}(P)}_{\mathbb{S}_{cor}(P)}\mathbf{P}^{\mathbb{P}(P)}_{\equiv}, \tag{5}$$

where the symbol $\equiv$ denotes the universal identity symmetry.

Remark 2.6 (Relation to residue class symmetry). The correlational symmetry $\mathbb{S}_{cor}(P)$ is equivalent to the classical decomposition of odd primes into the residue classes

$$P \equiv \pm 1 \pmod 4.$$

In this sense, the symmetric value $N(P)$ may be viewed as a normalized representative of the corresponding residue class.

The present framework, however, extends beyond modular classification. While residue classes describe purely local arithmetic structure, the inversion symmetry

$$\mathbb{S}_{inv}(P) = \mathbb{P}(P) - N(P)$$

incorporates global ordering information through the prime index. The symmetric signature therefore combines local modular symmetry with global positional structure, yielding a classification that is not visible at the level of residue classes alone.

3. Symmetries of Prime Numbers

The symmetric signature may be interpreted as a refinement of the classical residue-class decomposition, lifting local modular information to a mixed local–global invariant. The symmetries introduced below do not represent new arithmetic properties of primes; rather, they reorganize existing arithmetic information into a coherent structural framework.

3.1. Correlational Symmetry

The correlational symmetry is the first and most elementary invariant entering the symmetric signature of a prime number. It arises from the observation that for every prime $P > 2$ exactly one of the quantities $(P-1)/4$ and $(P+1)/4$ is a natural number. This induces a canonical binary partition of the primes, determined by the side on which the symmetric value appears.

From the present viewpoint, this invariant identifies the direction in which a prime is locally aligned with its symmetric representative $N(P)$ and therefore provides the local component of the symmetric classification.

Definition 3.1. For every prime number $P > 2$ we define the *correlational symmetry* by

$$\mathbb{S}_{cor}(P) = \begin{cases} +1, & \text{if } \dfrac{P+1}{4} \in \mathbb{N}, \\ -1, & \text{if } \dfrac{P-1}{4} \in \mathbb{N}. \end{cases} \tag{6}$$

This invariant determines the direction in which P correlates with its symmetric value $N(P)$ defined in (1).

Lemma 3.2. *For every prime number $P > 2$ exactly one of the numbers $\dfrac{P-1}{4}$ and $\dfrac{P+1}{4}$ is a natural number. Hence the correlational symmetry $\mathbb{S}_{cor}(P)$ defined in* (6) *is well defined and takes values only in* $\{+1, -1\}$.

Proof. Every prime number $P > 2$ is odd and therefore can be written in one of the two forms

$$P = 4k + 1 \quad \text{or} \quad P = 4k + 3,$$

where $k \in \mathbb{Z}$.

- If $P = 4k + 1$, then

$$\frac{P-1}{4} = k \in \mathbb{N}, \qquad \frac{P+1}{4} = k + \frac{1}{2} \notin \mathbb{N}.$$

- If $P = 4k + 3$, then

$$\frac{P+1}{4} = k + 1 \in \mathbb{N}, \qquad \frac{P-1}{4} = k + \frac{1}{2} \notin \mathbb{N}.$$

In both cases exactly one of the two values is a natural number, and the claim follows. □

Remark 3.3 (Arithmetic reformulation). Since every prime $P > 2$ satisfies $P \equiv 1$ or $3 \pmod 4$, it follows that

$$P \pm 1 \equiv 0 \text{ or } 2 \pmod 4.$$

Hence exactly one of the two neighbors $P-1$ and $P+1$ is divisible by 4, while the other is congruent to $2 \pmod 4$. Consequently, exactly one of the values $(P-1)/4$ and $(P+1)/4$ is a natural number.

Remark 3.4 (Structural role). The correlational symmetry partitions the primes into two classes,

$$\mathbb{S}_{cor}(P) = +1 \quad \Longleftrightarrow \quad P \equiv 3 \pmod 4,$$

$$\mathbb{S}_{cor}(P) = -1 \quad \Longleftrightarrow \quad P \equiv 1 \pmod 4.$$

Within the symmetric signature (4), this invariant provides the local modular component of the classification. The remaining invariants introduced below incorporate global positional information, so that the full symmetric signature combines residue-class symmetry with global ordering structure.

3.2. Identity Symmetry

The second symmetry in the classification framework concerns the structural behavior of the immediate neighbors of a prime number $P > 2$. While the correlational symmetry depends on the side of P on which the natural value $N(P)$ (defined in (1)) arises, the identity symmetry expresses a complementary phenomenon: the neighboring integers $P-1$ and $P+1$ exhibit completely uniform behavior with respect to the symmetric construction.

More precisely, all expressions that could potentially generate additional symmetric candidates in the immediate neighborhood of a prime are necessarily fractional. As a consequence, the symmetric configuration around every prime is rigidly fixed. This rigidity is independent of the particular prime and therefore defines a universal structural invariant.

Definition 3.5. For every prime number $P > 2$ we define the *identity symmetry* as the structural condition that none of the numbers

$$\frac{(P-1)\pm 1}{4}, \qquad \frac{(P+1)\pm 1}{4}$$

is a natural number. This property is independent of the particular prime and reflects a universal restriction on immediate prime neighbors. Accordingly, we represent this symmetry by the fixed value

$$\mathbb{S}_{ind} \equiv 0. \tag{7}$$

Lemma 3.6. *For every prime number $P > 2$ the numbers $\frac{(P-1)\pm 1}{4}$ and $\frac{(P+1)\pm 1}{4}$ are fractional. Hence the identity symmetry $\mathbb{S}_{ind}$ is a universal invariant, independent of P.*

Proof. Let $P > 2$ be an arbitrary prime number. Then P is odd and can be written as $P = 4k+1$ or $P = 4k+3$.

- **Case $P = 4k+1$.**

$$P-1 = 4k, \qquad P+1 = 4k+2.$$

Thus

$$\frac{(P-1)\pm 1}{4} = k \pm \frac{1}{4}, \qquad \frac{(P+1)\pm 1}{4} = k + \frac{1}{2} \pm \frac{1}{4}.$$

All values are fractional.

- **Case $P = 4k+3$.**

$$P-1 = 4k+2, \qquad P+1 = 4k+4.$$

Thus

$$\frac{(P-1)\pm 1}{4} = k + \frac{1}{2} \pm \frac{1}{4}, \qquad \frac{(P+1)\pm 1}{4} = k + 1 \pm \frac{1}{4}.$$

Again, all values are fractional.

In both cases none of the four expressions is a natural number. Therefore $\mathbb{S}_{ind}$ is a universal invariant. □

Remark 3.7 (Structural role of the identity symmetry). The universality of $\mathbb{S}_{ind}$ does not imply triviality. Although the underlying arithmetic argument applies to all odd integers, its function within the present framework is structural rather than classificatory. The identity symmetry fixes the admissible configuration of symmetric candidates around a prime and thereby establishes the background constraint against which the remaining invariants operate. In this sense, $\mathbb{S}_{ind}$ acts as a normalization condition that ensures the uniqueness of the locally defined symmetric value $N(P)$.

Remark 3.8. The identity symmetry shows that the immediate neighbors of every prime number $P > 2$ exhibit identical symmetric behavior. Consequently, $\mathbb{S}_{ind}$ is the only invariant in the symmetric signature that does not depend on the particular prime. Within the signature (4) it plays the role of a fixed structural axis with respect to which the correlational and inversion symmetries are arranged.

3.3. Inversion Symmetry

The third symmetry in the classification framework establishes a link between the locally defined symmetric value $N(P)$ and the global position of the prime number P in the ordered sequence of primes. While the correlational symmetry determines the direction of the local symmetric construction and the identity symmetry fixes the universal structure of admissible local configurations, the inversion symmetry compares local symmetric information with global ordering.

In contrast to the previous invariants, which are purely local in nature, the inversion symmetry introduces a mixed local–global quantity measuring the discrepancy between these two levels of description.

Definition 3.9. For every prime number $P > 2$ we define the *inversion symmetry* by the invariant

$$\mathbb{S}_{inv}(P) = Z(P) = \mathbb{P}(P) - N(P), \tag{8}$$

where $\mathbb{P}(P)$ denotes the index of P in the sequence of prime numbers and $N(P)$ is the symmetric value defined in (1).

The invariant $Z(P)$ therefore measures the offset between the local symmetric position determined by arithmetic structure and the global position determined by prime ordering.

Lemma 3.10. *For every prime number $P > 2$ the quantity*

$$Z(P) = \mathbb{P}(P) - N(P)$$

is an integer-valued invariant.

Proof. By Lemma 1 the symmetric value $N(P)$ is uniquely defined and belongs to $\mathbb{N}$. By definition, the index $\mathbb{P}(P)$ is also a natural number. Hence their difference $Z(P) = \mathbb{P}(P) - N(P)$ is an integer for every prime $P > 2$.

The quantities entering the definition of $Z(P)$ arise from distinct structures: $N(P)$ depends only on the local arithmetic symmetry determined by the residue class of P modulo

4, whereas $\mathbb{P}(P)$ reflects the global ordering of primes. Consequently, $Z(P)$ measures the discrete mismatch between local symmetric positioning and global placement. □

Remark 3.11 (Local–global coupling). Unlike the previous two symmetries, the inversion symmetry combines local and global information. The invariants $\mathbb{S}_{cor}(P)$ and $\mathbb{S}_{ind}$ depend only on the immediate arithmetic neighborhood of P, whereas $Z(P)$ compares this local structure with the global ordering of primes. As a result, primes sharing identical local symmetry may belong to different inversion classes, reflecting differences in their global distribution.

Remark 3.12. Within the symmetric signature (4), the inversion symmetry plays the role of an inversion coordinate. It distinguishes primes that are locally equivalent but globally separated in the prime sequence, thereby completing the transition from purely local symmetry to a mixed local–global classification framework.

4. Main Theorem

The three symmetries introduced above describe complementary aspects of the structure associated with every prime number $P > 2$. The following theorem shows that they combine into a single well-defined classification framework.

Theorem 4.1. *For every prime number $P > 2$ there exists a unique symmetric signature*

$$\Sigma(P),$$

defined by (4)*, characterized by the following conditions:*

(i) exactly one of the numbers $\dfrac{P-1}{4}$ *and* $\dfrac{P+1}{4}$ *is a natural number;*

(ii) none of the numbers $\dfrac{(P-1)\pm 1}{4}$ *and* $\dfrac{(P+1)\pm 1}{4}$ *is a natural number;*

(iii) the quantity $Z(P) = \mathbb{P}(P) - N(P)$ is an integer.

Consequently, every prime number $P > 2$ belongs to exactly one symmetric class determined by its signature $\Sigma(P)$.

Proof. Property (i) follows from Lemma 3.2, property (ii) from Lemma 3.6, and property (iii) from Lemma 3.10. Together these conditions uniquely determine the invariants $\mathbb{S}_{cor}(P)$, $\mathbb{S}_{ind}$, and $\mathbb{S}_{inv}(P)$. Since the index $\mathbb{P}(P)$ is uniquely defined, the signature $\Sigma(P)$ is therefore unique. □

Remark 4.2. The theorem shows that the three symmetries form a coherent classification system. Each prime number is assigned a unique symmetric signature, allowing the set of primes to be viewed not only as a sequence, but also as a collection of well-defined symmetric classes.

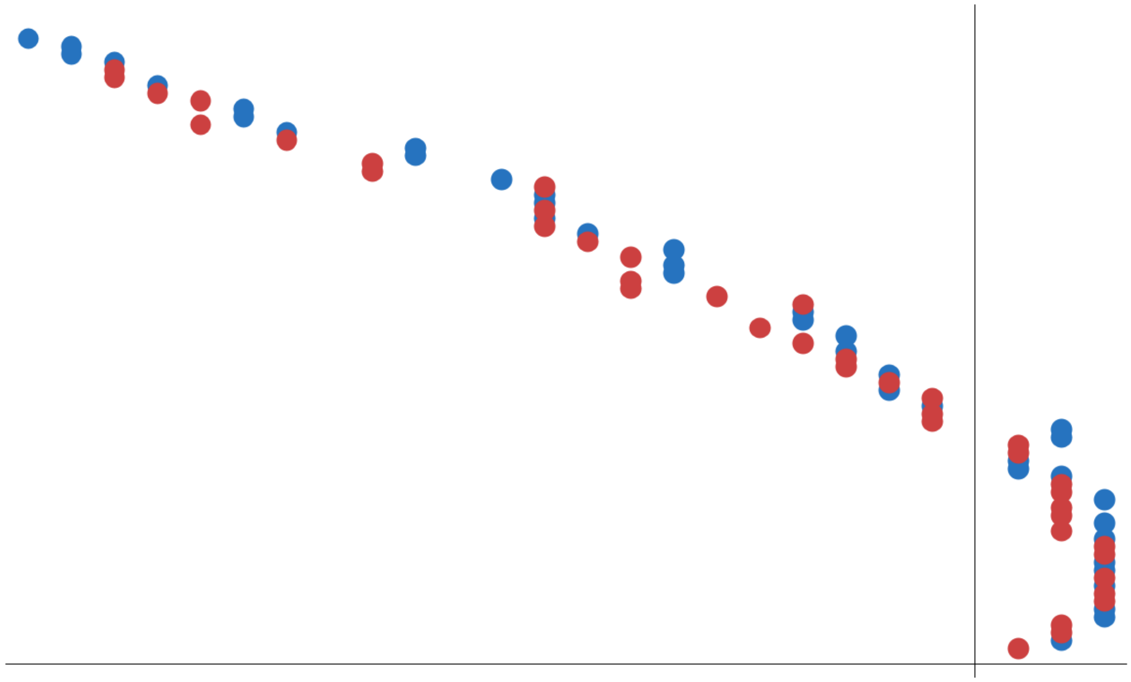


Figure 1: A minimalist visualization of the symmetric classification of prime numbers. The horizontal axis represents the inversion symmetry $Z(P)$, the vertical axis the index $\mathbb{P}(P)$, and the color encodes the correlational symmetry $\mathbb{S}_{cor}(P) \in \{-1, +1\}$.

Remark 4.3 (Synthesis of the three symmetries)*.* The theorem shows that the three symmetries play complementary roles and together form a complete structural description of the symmetric signature.

The correlational symmetry $\mathbb{S}_{cor}(P)$ arises from residue-class structure modulo 4 and determines the direction of the local symmetric construction. The identity symmetry $\mathbb{S}_{ind}$ provides a universal structural constraint, fixing the admissible configuration of symmetric candidates around every prime and thereby establishing a common background for the classification. The inversion symmetry $\mathbb{S}_{inv}(P)$ connects this local structure with the global ordering of primes through the index $\mathbb{P}(P)$.

Taken together, these invariants realize a transition from purely local arithmetic symmetry to a mixed local–global classification framework. The symmetric signature $\Sigma(P)$ may therefore be interpreted as a minimal invariant that simultaneously encodes residue-class information, structural constraints, and global positional information. In this sense, the symmetric classification complements the traditional linear ordering of the primes by introducing an additional symmetry-based organization.

4.1. Symbolic Notation

For notational convenience, we introduce a compact symbolic representation of the symmetric signature associated with a prime number $P > 2$. The collection of invariants together with the index $\mathbb{P}(P)$ is denoted by

$$ {}^{\mathbb{S}_{inv}(P)}_{\mathbb{S}_{cor}(P)}\mathbf{P}^{\mathbb{P}(P)}_{\equiv}, \tag{9} $$

where the symbol $\equiv$ represents the universal identity symmetry $\mathbb{S}_{ind} \equiv 0$ defined in (7).

This notation may be viewed as a symbolic lift of the symmetric signature $\Sigma(P)$ into a compact algebraic form that simultaneously records the local orientation of the symmetry, the universal structural constraint, and the global position of the prime.

In this notation:

- the upper left index $\mathbb{S}_{inv}(P)$ denotes the inversion symmetry;
- the lower left index $\mathbb{S}_{cor}(P)$ denotes the correlational symmetry;
- the upper right index $\mathbb{P}(P)$ denotes the index of P in the ordered sequence of primes;
- the lower right symbol $\equiv$ encodes the universal identity symmetry.

The expression (9) therefore provides a concise encoding of the symmetric signature $\Sigma(P)$ defined in (4), allowing primes to be represented directly by their structural invariants.

4.2. Example

We illustrate the notation with three non-adjacent prime numbers: $P = 11$, $P = 23$, and $P = 41$. These examples demonstrate how identical or comparable inversion values may arise for primes belonging to different local symmetry orientations.

1. *The prime* $P = 11$..

$$N(11) = \frac{11-1}{4} = 3, \qquad \mathbb{P}(11) = 5, \qquad Z(11) = 5 - 3 = 2.$$

Since $(11-1)/4 \in \mathbb{N}$, we have $\mathbb{S}_{cor}(11) = -1$. Hence,

$${}^{2}_{-1}\mathbf{11}^{5}_{\equiv}.$$

2. *The prime* $P = 23$..

$$N(23) = \frac{23+1}{4} = 6, \qquad \mathbb{P}(23) = 9, \qquad Z(23) = 9 - 6 = 3.$$

Since $(23+1)/4 \in \mathbb{N}$, we have $\mathbb{S}_{cor}(23) = +1$. Hence,

$${}^{3}_{+1}\mathbf{23}^{9}_{\equiv}.$$

3. *The prime* $P = 41$..

$$N(41) = \frac{41-1}{4} = 10, \qquad \mathbb{P}(41) = 13, \qquad Z(41) = 13 - 10 = 3.$$

Since $(41-1)/4 \in \mathbb{N}$, we have $\mathbb{S}_{cor}(41) = -1$. Hence,

$${}^{3}_{-1}\mathbf{41}^{13}_{\equiv}.$$

These examples show that different primes may share the same inversion symmetry while differing in correlational symmetry or index. The notation (9) makes such structural relations explicit without reference to numerical proximity.

4.3. Symmetric Classes of Prime Numbers

The symmetric signature $\Sigma(P)$ defined in (4) allows the prime numbers to be grouped into classes that share the same values of the symmetric invariants. Fixing the correlational symmetry $\mathbb{S}_{cor}(P)$ and the inversion symmetry $\mathbb{S}_{inv}(P)$ determines such a class, while the index $\mathbb{P}(P)$ varies within it.

In this sense, the classification induced by $\Sigma(P)$ is structural rather than metric: primes may be arbitrarily far apart in the natural ordering while remaining close in the symmetry-based organization.

Prime P	$\mathbb{S}_{cor}(P)$	$N(P)$	$\mathbb{P}(P)$	$\mathbb{S}_{inv}(P)$
11	-1	3	5	2
41	-1	10	13	3
71	-1	17	20	3

Table 1: Three non-adjacent primes belonging to the correlational class $\mathbb{S}_{cor} = -1$ and having close inversion values.

Table 1 illustrates this situation using three non-adjacent primes that belong to the same correlational class and exhibit similar inversion values. The primes themselves are separated by large gaps, emphasizing that the classification is not based on numerical proximity.

Using the symbolic notation introduced in (9), these primes are represented as

$$ {}^{2}_{-1}\mathbf{11}^{5}_{\equiv}, \qquad {}^{3}_{-1}\mathbf{41}^{13}_{\equiv}, \qquad {}^{3}_{-1}\mathbf{71}^{20}_{\equiv}. $$

Remark 4.4. Table 1 illustrates that primes distant in the natural ordering may nevertheless belong to the same symmetric class. Agreement in $\mathbb{S}_{cor}(P)$ together with proximity of the inversion invariant $\mathbb{S}_{inv}(P)$ indicates that the symmetric signature captures structural properties independent of local spacing. The resulting organization complements the classical linear ordering of the primes by introducing a symmetry-based perspective.

Remark 4.5 (Interpretational scope). The symmetric signature introduced in this work is not intended as a prediction tool for prime numbers, nor does it aim to generate new primes or provide estimates for their distribution. Instead, it should be understood as a structural coordinate system on the set of primes.

Within this perspective, each prime number is assigned a position determined by explicitly defined invariants that combine local arithmetic symmetry with global ordering information. The resulting classification complements the classical description of primes through modular arithmetic and residue classes by lifting modular symmetry into a mixed local–global framework.

Accordingly, the symmetric signature organizes prime numbers geometrically in terms of symmetry and structure rather than in terms of numerical prediction or asymptotic behavior.

4.3.1. Second Symmetric Class

To complement the previous example, we now consider primes belonging to the opposite correlational class,

$$ \mathbb{S}_{cor}(P) = +1. $$

As before, the symmetric value $N(P)$ is determined by the appropriate choice of $(P\pm1)/4$, while the inversion symmetry $\mathbb{S}_{inv}(P)$ reflects the difference between the local symmetric structure and the global index.

Prime P	$\mathbb{S}_{cor}(P)$	$N(P)$	$\mathbb{P}(P)$	$\mathbb{S}_{inv}(P)$
23	$+1$	6	9	3
47	$+1$	12	15	3
83	$+1$	21	23	2

Table 2: Primes from the correlational class $\mathbb{S}_{cor} = +1$ with close inversion values.

Remark 4.6. Table 2 provides the symmetric counterpart of Table 1, exhibiting primes with opposite correlational orientation. Together, the two tables illustrate that the symmetric signature $\Sigma(P)$ induces a partition of the primes determined by structural symmetries rather than by arithmetic proximity. In particular, primes that are widely separated along the number line may occupy neighboring positions in the symmetric coordinate system defined by $(\mathbb{S}_{cor}, \mathbb{S}_{inv})$.

$$ {}^{3}_{+1}\mathbf{23}^{9}_{\equiv}, \qquad {}^{3}_{+1}\mathbf{47}^{15}_{\equiv}, \qquad {}^{2}_{+1}\mathbf{83}^{23}_{\equiv}. $$

4.3.2. Third Symmetric Class

For completeness we present a third example of a symmetric class of primes sharing the same correlational symmetry and exhibiting nearby inversion values. In this case,

$$ \mathbb{S}_{cor}(P) = -1, $$

while the corresponding inversion values $\mathbb{S}_{inv}(P)$ remain within a narrow range despite the primes being far apart in the sequence of primes.

Prime P	$\mathbb{S}_{cor}(P)$	$N(P)$	$\mathbb{P}(P)$	$\mathbb{S}_{inv}(P)$
19	-1	5	8	3
59	-1	15	17	2
79	-1	20	22	2

Table 3: A third example of a symmetric class: primes with $\mathbb{S}_{cor} = -1$ and nearby inversion values.

Remark 4.7. This third example complements the previous two and illustrates that symmetric signatures do not occur in isolation but form families connected through the invariants defining the signature. Although the primes involved are widely separated in both value and index, their proximity in the symmetric space determined by $(\mathbb{S}_{cor}, \mathbb{S}_{ind}, \mathbb{S}_{inv})$ reveals structural relations that are invisible in the usual linear ordering of primes.

$$ {}^{3}_{-1}\mathbf{19}^{8}_{\equiv}, \qquad {}^{2}_{-1}\mathbf{59}^{17}_{\equiv}, \qquad {}^{2}_{-1}\mathbf{79}^{22}_{\equiv}. $$

4.3.3. Non-Adjacent Primes with the Same Inversion Symmetry

While correlational symmetry partitions the primes into two broad classes, the inversion symmetry

$$\mathbb{S}_{inv}(P) = Z(P) = \mathbb{P}(P) - N(P)$$

defined in (8) determines natural classes of primes for which the value of $Z(P)$ coincides. These classes often contain primes that are far apart both in numerical value and in index, yet are "close" in the inversion space.

As an example, consider the inversion class $Z(P) = 2$. For several primes we obtain:

$$\begin{aligned}
&P = 5, && N(5) = 1, && \mathbb{P}(5) = 3, && Z(5) = 2,\\
&P = 7, && N(7) = 2, && \mathbb{P}(7) = 4, && Z(7) = 2,\\
&P = 11, && N(11) = 3, && \mathbb{P}(11) = 5, && Z(11) = 2,\\
&P = 59, && N(59) = 15, && \mathbb{P}(59) = 17, && Z(59) = 2,\\
&P = 71, && N(71) = 18, && \mathbb{P}(71) = 20, && Z(71) = 2.
\end{aligned}$$

$$ {}^{2}_{-1}\mathbf{5}^{3}_{\equiv}, \quad {}^{2}_{+1}\mathbf{7}^{4}_{\equiv}, \quad {}^{2}_{-1}\mathbf{11}^{5}_{\equiv}, \quad {}^{2}_{+1}\mathbf{59}^{17}_{\equiv}, \quad {}^{2}_{+1}\mathbf{71}^{20}_{\equiv}.$$

Remark 4.8. This example shows that the inversion symmetry $\mathbb{S}_{inv}(P)$ groups together primes that are distant both in value and in index, yet share the same difference between the local symmetric value $N(P)$ and the global index $\mathbb{P}(P)$. Inversion classes therefore provide an additional structural layer that is independent of local numerical adjacency.

5. Conclusion

In this work we introduced a symmetric framework that assigns to every prime number $P > 2$ a natural collection of invariants forming a symmetric signature. The correlational symmetry determines the direction of the local symmetric value $N(P)$ defined in (1), the identity symmetry captures the uniform structural behavior of the neighboring integers $P-1$ and $P+1$, and the inversion symmetry links local and global structure through the integer difference $Z(P) = \mathbb{P}(P) - N(P)$ defined in (8). Together, these invariants define the symmetric signature (4), which assigns each prime to a well-defined symmetric class.

The purpose of the framework is not to derive new analytic results concerning the distribution of prime numbers. Rather, it provides an alternative organizational viewpoint in which primes are regarded not only as elements of a linear sequence but also as points in a discrete symmetry space. Within this space, primes that are distant in numerical value or index may nevertheless share common structural characteristics encoded by their symmetric signatures.

The examples presented illustrate that correlational and inversion symmetries naturally generate global classes that are not visible from purely numerical ordering. From this perspective, the symmetric signature acts as a structural coordinate system: it organizes existing arithmetic information without introducing predictive assumptions about prime occurrence. Its role is descriptive rather than predictive, providing a language in which relations between local modular structure and global ordering can be expressed in a unified manner.

Several directions for further investigation arise naturally from this point of view. These include the statistical behavior of inversion classes, possible connections between symmetric invariants and classical arithmetic properties, and the use of symmetric signatures as descriptive tools in computational or exploratory studies of prime structure.

Although the construction relies only on elementary arithmetic observations, the resulting framework reveals a nontrivial structural organization of the primes. The symmetric framework introduced here should therefore be understood not as a closed theory but as an open coordinate system: a formal setting in which new structural questions about prime numbers may be formulated precisely.

The accompanying Zenodo dataset supports reproducible exploration of these ideas by providing explicit symmetry vectors and classification data. No hidden laws governing primes are claimed; rather, the framework establishes a consistent structural language through which existing arithmetic phenomena may be organized, compared, and potentially reinterpreted. It is hoped that this perspective will serve as a starting point for future work in which prime numbers are studied not only as a sequence, but as a collection of symmetric forms within a discrete structural space.

Within the broader research program, the geometric foundations of the linear–cyclic transition are developed in the companion article *Global Affine Time and Metric Uniqueness: A Geometric Characterization of Linear and Cyclic Temporal Structure*, which provides the affine and metric framework underlying the structural interpretation adopted across the cycle.

6. Author's Note

The symmetric framework presented in this work originated from earlier considerations related to structural encoding and pattern recognition in non-random numerical sequences, including questions arising in the context of technosignature analysis and SETI-oriented signal interpretation. In that setting, the guiding idea was that structurally meaningful patterns, if present in numerical data, should admit a representation in terms of stable invariants rather than isolated numerical coincidences.

The present article does not make claims concerning extraterrestrial signals or detection methodologies. Instead, the SETI context served as an initial conceptual motivation for seeking coordinate-like descriptions of numerical structures that remain stable under changes of scale or representation. The symmetric signature introduced here is therefore presented purely as a mathematical and structural framework, independent of any specific application domain.

Possible applications to information encoding, signal classification, or pattern detection in large numerical datasets remain speculative and are mentioned only as directions for future interdisciplinary exploration.

7. Series Note

More generally, the transition from linear to cyclic structure is investigated from complementary viewpoints: topological, informational, arithmetic, and geometric. The present work constitutes the arithmetic component of this program. In parallel, the geometric foundations of the linear–cyclic transition are developed in a companion study establishing the role of affine structure and metric uniqueness in the interpretation of temporal and asymptotic limits.

The full cycle consists of four interconnected parts:

- **Part I:** Hybrid Linear–Cyclic Topological Structures for Digital Sequence Encoding and Technosignature Analysis
  DOI: 10.5281/zenodo.18473473
- **Part II:** Informational Geometry of the Positive Half-Line. A World Without Negative Numbers
  DOI: 10.5281/zenodo.18474513
- **Part III:** A Symmetric Classification of Prime Numbers. Correlational, Identity, and Inversion Symmetry
- **Part IV:** Global Affine Time and Metric Uniqueness: A Geometric Characterization of Linear and Cyclic Temporal Structure
  DOI: 10.5281/zenodo.18505857

The cycle is supported by a FAIR-compliant dataset and computational framework available via Zenodo, ensuring reproducible access to the symmetry vectors and invariants discussed across the individual parts. Collectively, these works provide a modular foundation for further exploration into the geometric nature of asymptotic structure, prime organization, and the emergence of cyclic behavior from linear regimes under appropriate metric interpretation.

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