Relativistic Shedding: A Kinematic Threshold for Emission into Weakly Coupled Eigenmodes

(Foundational formulation, Version 6)

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Abstract

We define *Relativistic Shedding* (RS) as a kinematic threshold phenomenon: a steadily moving relativistic source can radiate into any *propagating* eigenmode of the coupled field–environment system whose phase velocity (along the source direction) is below the source velocity. Cherenkov radiation is the electromagnetic special case in a dielectric. RS generalizes the same phase-velocity/synchronism criterion to *weakly coupled* eigenmodes, including hidden-sector states accessed through portal interactions.

This paper provides a self-contained foundation for RS intended to remain stable under future model, laboratory, and cosmological refinements. We present: (i) a concise, textbook-level definition; (ii) threshold criteria in both the bulk-medium language ($\beta\, n_{\mathrm{eff}} > 1$) and the guided/periodic-structure language (synchronism $\omega = k_z v$); (iii) a model-independent radiation formula in linear-response (retarded Green-function / spectral-density) form that makes the origin of the step-like turn-on explicit; and (iv) a worked example for a kinetically mixed massive vector ("dark photon"), including the in-medium effective mixing factor. We also state a minimal, falsifier-rich laboratory logic: RS predicts a reversible on/off contrast under ABAB threshold bracketing, rather than a claim about absolute rates. Implementations and cosmological applications are deferred to companion studies.

I. INTRODUCTION

The modern experimental landscape for weakly coupled new physics includes "light-shining-through-walls" (LSW) searches and cavity/haloscope approaches, spanning optical and microwave frequencies [1–7]. These experiments often exploit (i) coherent electromagnetic structures (high quality factors, narrow linewidths) and (ii) on/off or modulation strategies that reject slowly varying systematics.

Relativistic Shedding (RS) is proposed as a complementary organizing principle: a *deterministic, tunable* kinematic threshold can open (or close) a radiative channel into a weakly coupled eigenmode of the environment. In its electromagnetic limit, RS reduces to standard Cherenkov radiation [8, 9]. However, the RS framing emphasizes two points:

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1. The relevant object is not "a photon in a dielectric" but an *eigenmode* of the full coupled system (fields plus environment), which may be photon-like, hidden-like, or a mixture.

2. If a control parameter of the environment moves the eigenmode dispersion across the source's synchronism condition, the resulting *threshold turn-on* can be used as a falsifier-rich experimental discriminator.

The goal of this paper is foundational: to define RS in a manner that is independent of any specific portal, medium model, or laboratory architecture, while still yielding precise, testable implications.

II. A TEXTBOOK DEFINITION OF RELATIVISTIC SHEDDING

A. Definition

Definition (Relativistic Shedding). Let a source with constant velocity $\boldsymbol{v}$ couple (possibly very weakly) to one or more fields Φ in an environment $\mathcal{E}$ describable by linear response. Denote by $\omega_\alpha(\boldsymbol{k};\lambda)$ the dispersion relation(s) of propagating eigenmodes α of the full field–environment system, where λ is any externally controllable parameter of $\mathcal{E}$ (material, geometry, bias, temperature, etc.). *Relativistic Shedding* (RS) is the threshold turn-on of radiation into a weakly coupled eigenmode when tuning λ causes the source's kinematic synchronism condition

$$\omega = \boldsymbol{k}\cdot\boldsymbol{v}$$

to intersect a propagating branch of $\omega_\alpha(\boldsymbol{k};\lambda)$, equivalently when the source outruns the eigenmode phase velocity along its direction of motion.

The definition is intentionally *kinematic* and *mode-based*. It is stable under future refinements because it depends only on (i) existence of eigenmodes and (ii) whether a well-defined intersection exists between those modes and the source line in $(\omega,\boldsymbol{k})$ space.

B. Minimal ingredients

The definition implies three necessary ingredients:

1. **Propagating eigenmodes.** The linear response of $\mathcal{E}$ must possess propagating modes that can carry energy away (equivalently, the retarded Green function must have nonzero spectral weight on shell).

2. **A constant-velocity source.** A steadily moving source imposes a synchronism constraint $\omega = \boldsymbol{k} \cdot \boldsymbol{v}$ on its Fourier components.

3. **A tunable crossing.** A control parameter λ must move the eigenmode dispersion relative to the source line so that a propagating intersection turns on/off.

In practice, λ may be as simple as the refractive index, a geometric spacing, or a resonant frequency tuning; RS makes no demand beyond the existence of an independently characterizable threshold.

III. THRESHOLD CRITERIA IN TWO STANDARD LANGUAGES

A. Homogeneous isotropic medium: phase velocity and effective index

In an isotropic medium, a plane-wave-like eigenmode can be characterized by a frequency-dependent wavenumber magnitude $k(\omega; \lambda)$, and we define

$$v_{\mathrm{ph}}(\omega; \lambda) \equiv \frac{\omega}{k(\omega; \lambda)}, \qquad n_{\mathrm{eff}}(\omega; \lambda) \equiv \frac{k(\omega; \lambda)}{\omega}. \tag{1}$$

With $\beta \equiv v/c$ (and $c = 1$ in natural units), a Cherenkov-like RS threshold is

$$\beta\, n_{\mathrm{eff}}(\omega; \lambda) > 1. \tag{2}$$

When Eq. (2) is satisfied for a propagating branch, radiation is emitted at an angle

$$\cos\theta(\omega) = \frac{1}{\beta\, n_{\mathrm{eff}}(\omega; \lambda)}, \tag{3}$$

recovering the standard Cherenkov relation [8, 9].

It is useful to define a detuning parameter

$$\Delta(\omega; \lambda) \equiv \beta\, n_{\mathrm{eff}}(\omega; \lambda) - 1, \tag{4}$$

so that the turn-on corresponds to $\Delta > 0$.

B. Guided or periodic structures: synchronism and detuning

Many environments of interest are not bulk homogeneous media but guided or periodic structures. Consider translational symmetry along z and label eigenmodes by (α, k_z) with dispersion $\omega = \omega_\alpha(k_z; \lambda)$. A source moving along z with speed v enforces the synchronism (phase-matching) condition

$$\omega = k_z v. \tag{5}$$

Radiation into mode α occurs when Eq. (5) intersects a propagating portion of $\omega_\alpha(k_z; \lambda)$, equivalently when the modal phase velocity ω/k_z drops below v.

This guided-mode viewpoint makes clear why engineered dispersion is powerful: by increasing k_z at fixed ω (slow-wave / large n_{eff}), one can satisfy the RS condition even when bulk material indices are close to unity.

IV. EMISSION IN LINEAR RESPONSE

A. Spectral-density formulation

Consider a field Φ coupled linearly to a (possibly conserved) source J,

$$\mathcal{L}_{\text{int}} = g\, J(x)\, \Phi(x), \tag{6}$$

where g may be a portal-suppressed effective coupling. In linear response,

$$\Phi(x) = g \int \mathrm{d}^4x'\, D_{\text{ret}}(x, x')\, J(x'), \tag{7}$$

with D_{ret} the retarded Green function of the full environment-coupled system. The power dissipated by the source into the field can be written in frequency space in terms of the *spectral density*

$$\rho(\omega, \boldsymbol{k}) \equiv -2\, \Im D_{\text{ret}}(\omega, \boldsymbol{k}). \tag{8}$$

For a translationally invariant environment, one obtains a standard quadratic form [10]

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} = \frac{g^2}{2}\, \omega \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, |J(\omega, \boldsymbol{k})|^2\, \rho(\omega, \boldsymbol{k}), \tag{9}$$

up to conventions for Fourier transforms and normalization. Equation (9) makes the RS mechanism transparent:

- A constant-velocity source yields $J(\omega, \boldsymbol{k}) \propto \delta(\omega - \boldsymbol{k} \cdot \boldsymbol{v})$ (Appendix A).

- Radiation requires $\rho(\omega, \boldsymbol{k}) \neq 0$, i.e. the presence of propagating spectral weight.

- Therefore, RS corresponds to the appearance/disappearance of support of ρ on the source line $\omega = \boldsymbol{k} \cdot \boldsymbol{v}$ as λ is tuned.

B. Mode-sum viewpoint and the origin of the step

If the environment admits a normal-mode expansion, the spectral density contains delta functions enforcing on-shell propagation. In a simple schematic form,

$$\rho(\omega, \boldsymbol{k}) \propto \sum_{\alpha} Z_{\alpha}(\boldsymbol{k}; \lambda)\, \delta\big(\omega^2 - \omega_{\alpha}^2(\boldsymbol{k}; \lambda)\big) + \cdots , \tag{10}$$

where Z_{α} are residues and the ellipsis denotes continua and/or damping contributions. Inserting the source $\delta(\omega - \boldsymbol{k} \cdot \boldsymbol{v})$ then enforces simultaneous satisfaction of the dispersion and synchronism constraints, yielding the threshold conditions in Sec. III. The "step" is simply the non-analytic change in phase space support when the intersection first appears.

C. Recovering the Frank–Tamm spectrum

In bulk electromagnetism, the above machinery reproduces the Frank–Tamm spectrum for Cherenkov radiation [8, 9]. For orientation, the photon yield per unit length per unit frequency can be written as

$$\frac{\mathrm{d}^2 N_{\gamma}}{\mathrm{d}x\, \mathrm{d}\omega} = \frac{\alpha}{c} \left(1 - \frac{1}{\beta^2 n^2(\omega)}\right) \Theta(\beta n - 1), \tag{11}$$

with $\alpha = e^2/(4\pi\hbar c)$ and refractive index $n(\omega)$. RS can be viewed as the statement that an analogous threshold factor appears whenever a propagating eigenmode is accessible, with the overall magnitude controlled by the effective coupling of the source to that eigenmode.

V. WORKED EXAMPLE: KINETICALLY MIXED MASSIVE VECTOR

A. Minimal vector portal

A canonical hidden-sector example is a massive vector V_μ kinetically mixed with the photon [11, 12]:

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu + e\, J^\mu_{\mathrm{em}} A_\mu. \tag{12}$$

In a medium, photon propagation is modified by the polarization tensor $\Pi_{\mu\nu}(\omega, \boldsymbol{k})$, and the propagating eigenmodes are mixtures of A and V.

B. In-medium effective mixing

For small vacuum mixing $\kappa \ll 1$, the probability to produce a predominantly-V eigenmode from an electromagnetic source is controlled by an *in-medium effective mixing factor*. For transverse (T) and longitudinal (L) polarizations one may write [13, 14]

$$\kappa^2_{T,L}(\omega, \boldsymbol{k}) = \kappa^2 \, \frac{m_V^4}{\left(m_V^2 - \Re\Pi_{T,L}(\omega,\boldsymbol{k})\right)^2 + \left(\Im\Pi_{T,L}(\omega,\boldsymbol{k})\right)^2}. \tag{13}$$

Equation (13) compactly encodes resonant enhancement when $m_V^2 \simeq \Re\Pi$ and damping through $\Im\Pi$. It also makes clear when a bulk medium cannot substantially "slow" the heavy hidden mode: if $m_V^2 \gg |\Pi|$, then $\kappa_{T,L} \simeq \kappa$ and the eigenmode remains vacuum-like.

C. A simple closure: heavy on-shell mediators in ordinary matter

For an on-shell massive vector in vacuum,

$$\omega^2 = k^2 + m_V^2 \quad \Rightarrow \quad n_{\mathrm{eff}}(\omega) = \frac{k}{\omega} = \sqrt{1 - \frac{m_V^2}{\omega^2}} < 1, \tag{14}$$

so the phase velocity satisfies $v_{\mathrm{ph}} = \omega/k > 1$. A charge with $v < 1$ cannot outrun the phase velocity of a *vacuum-like* massive mode, and thus Eq. (2) cannot be satisfied for that branch.

To realize RS for a heavy on-shell mediator, the environment must reshape the relevant eigenmode dispersion so that an accessible branch becomes subluminal at $\omega \gtrsim m_V$. In ordinary matter, the scale entering Π (and thus any refractivity) is typically set by plasma/atomic response energies (eV–keV), not by MeV–GeV mediator masses, so $|\Pi| \ll m_V^2$

in that regime and the medium does not generate a subluminal V-like branch. This motivates engineered structures (slow-wave guides, periodic media, resonators) as the natural laboratory space for RS in the sub-eV regime, where microwave eigenmodes can have very large effective indices and tunable dispersion.

D. Near-threshold scaling and the RS signature

When a weakly mixed eigenmode does admit a subluminal branch, the hidden-channel spectrum inherits a Cherenkov-like threshold factor multiplied by the effective coupling. Schematically, in a bulk-like setting one expects

$$\frac{\mathrm{d}^2 N_V}{\mathrm{d}x\,\mathrm{d}\omega} \sim \kappa_T^2(\omega)\,\frac{\mathrm{d}^2 N_\gamma}{\mathrm{d}x\,\mathrm{d}\omega}, \tag{15}$$

with $\mathrm{d}^2 N_\gamma/(\mathrm{d}x\,\mathrm{d}\omega)$ given by Eq. (11). The precise prefactor and polarization structure depend on geometry and losses, but the non-analytic turn-on in $\Delta(\omega;\lambda)$ is robust and is the defining experimental handle.

VI. MINIMAL LABORATORY VALIDATION LOGIC

Although RS is defined without committing to a particular apparatus, it implies experimentally checkable consequences. A minimal validation strategy requires:

1. **A tunable control parameter** λ that moves $\Delta(\omega_{\mathrm{sys}};\lambda)$ across zero at a known system frequency ω_{sys} (or known synchronism line).

2. **A source** whose velocity (or effective phase velocity) is known and stable, so that the threshold prediction is objective.

3. **A receiver channel** that is sensitive to the weak eigenmode, ideally narrowband around ω_{sys} (e.g. a resonator).

4. **ABAB bracketing:** alternate between below-threshold (A) and above-threshold (B) states while holding all other parameters fixed.

The RS hypothesis predicts a reversible on/off contrast in the receiver (power, events, etc.) correlated with the sign of Δ:

$$\Delta P \equiv P_B(\omega_{\mathrm{sys}}) - P_A(\omega_{\mathrm{sys}}) > 0 \quad \text{only if} \quad \Delta(\omega_{\mathrm{sys}};\lambda_B) > 0 > \Delta(\omega_{\mathrm{sys}};\lambda_A). \tag{16}$$

Further falsifiers are naturally available: receiver detuning (line disappears and reappears on retune), geometry/polarization swaps, and shielding/leakage discriminants. Existing cavity-based LSW searches provide clear demonstrations of narrowband regeneration techniques and attainable noise floors in the sub-eV regime [3, 5–7], and thin-wall/evanescent extensions exist for $m_V > \omega$ [15, 16].

VII. DISCUSSION AND OUTLOOK

The RS definition isolates a common physical core shared by Cherenkov radiation, slow-wave excitation, and resonant regeneration: a moving (or effectively moving) source can excite an eigenmode only when a synchronism condition intersects a propagating dispersion branch. When that intersection is controlled by a tunable parameter, the resulting threshold provides a powerful systematic canceller.

The novel claim of RS is not a new portal or a new mixing formalism; these are standard [1, 11, 12]. Rather, RS advocates treating *threshold modulation* as a first-class search strategy for hidden-sector emission in laboratory structures where dispersion engineering provides exquisite control of $n_{\text{eff}}(\omega; \lambda)$.

Companion papers can specialize this foundation to concrete radiator designs (slow-wave, photonic crystal, metasurface), receiver architectures (resonant regeneration, interferometric pickup, quantum-limited readout), and astrophysical/cosmological environments where $\Pi_{\mu\nu}$ is plasma-driven and evolves with temperature.

VIII. CONCLUSIONS

We have provided a stand-alone definition and formal foundation for Relativistic Shedding (RS) as a kinematic threshold phenomenon: a relativistic source emits into a weakly coupled eigenmode when it outruns that mode's phase velocity (equivalently, when the synchronism condition intersects a propagating eigenmode branch). Expressing emission in linear-response / spectral-density language makes the threshold origin explicit and environment-agnostic.

A worked example for a kinetically mixed massive vector illustrates the in-medium effective mixing factor and clarifies why ordinary dielectrics cannot realize heavy on-shell RS without engineered dispersion. Finally, we stated a minimal laboratory validation logic emphasizing ABAB threshold bracketing and falsifiers, consistent with the broader LSW/cavity experimental ethos.

Appendix A: Synchronism δ-function for a constant-velocity source

For a point charge q moving along z with constant speed v,

$$\rho(\boldsymbol{x}, t) = q\, \delta(x)\, \delta(y)\, \delta(z - vt), \qquad \boldsymbol{J}(\boldsymbol{x}, t) = q\, v\, \hat{\boldsymbol{z}}\, \delta(x)\, \delta(y)\, \delta(z - vt). \tag{A1}$$

Fourier transforming yields

$$\int \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{e}^{-\mathrm{i}k_z v t} = 2\pi\, \delta(\omega - k_z v), \tag{A2}$$

which is Eq. (5). This is the origin of phase matching for all constant-velocity radiation processes.

Appendix B: A convenient near-threshold linearization

For $\beta \simeq 1 - 1/(2\gamma^2)$ and $n_{\mathrm{eff}}(\omega; \lambda) = 1 + \delta n$ with $|\delta n| \ll 1$, the threshold $\beta n_{\mathrm{eff}} > 1$ implies

$$\delta n > \frac{1}{2\gamma^2}. \tag{B1}$$

Ultra-relativistic sources can therefore access extremely small index offsets, motivating engineered structures where effective indices are large and tunable.

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