Hadronic Spectroscopy from the Geometry of Singular Distributions:
A First-Principles Calculation of the Mass Gap

Cláudio Silva de Melo

Department of Electrical Engineering, Federal University of Rondônia, Porto Velho, RO, Brazil

Abstract

We present a geometric derivation of the hadronic mass spectrum based on the theory of Singular Distributions on Tubular Neighborhoods. We demonstrate that the "Double Layer" distribution $\delta^{(1)}$, which arises rigorously at the Gribov Horizon, acts as an isotropic scalar stiffness. By virtue of transverse symmetry (Curie's Principle), this boundary acts as a projection operator, filtering out anisotropic modes for matter fields. By identifying the tubular thickness $\langle\lambda\rangle$ as the fundamental scale of confinement, we derive a spectral formula dependent on the zeros of Bessel functions. Calibrating the theory with Lattice QCD data for the scalar glueball (0^{++}), treated as a quadrupolar deformation of the geometry itself, we determine the tubular thickness to be $\langle\lambda\rangle \approx 0.586$ fm. Using this single parameter and the isotropic projection for vector fields (J_0), we predict the mass of the ρ meson (1^{--}) to be 809 MeV. The small deviation (4.3%) from the experimental value (775 MeV) is interpreted as the signature of the finite elasticity and surface tension of the tubular boundary. Finally, we propose that this geometric stiffness provides a deterministic origin for the Pauli Exclusion Principle via the non-orientability of the tubular manifold.

Keywords: Mass Gap, Tubular Geometry, Gribov Horizon, Geometric Spin Filter, Topological Elasticity

1. Introduction

The origin of the mass gap in Yang-Mills theory remains one of the most profound open problems in mathematical physics. While Lattice QCD provides accurate numerical estimates of the hadronic spectrum, an analytic derivation of confinement from first principles is still lacking. Standard approaches often rely on phenomenological potentials or holographic models (AdS/QCD) that introduce ad-hoc scales.

In previous works, we proposed a new framework based on the *Projective Monge Method* and the theory of *Singular Distributions* [1, 2]. We demonstrated that the configuration

Email address: `csdmelo@unir.br` (Cláudio Silva de Melo)

space of gauge theories must be treated as a tubular neighborhood of finite thickness $\langle\lambda\rangle$. This geometry naturally induces a "Double Layer" distribution $\delta^{(1)}$ at the boundary (the Gribov Horizon), which acts as a "Hard Wall" for the gauge fields.

In this Letter, we utilize the mass gap formula derived in [2] to calculate the masses of the lightest QCD states. We introduce a symmetry-based argument for mode selection and demonstrate that the mass spectrum is a direct consequence of the elastic properties of the spacetime tubular microstructure.

2. The Tubular Hard Wall and Transverse Symmetry

As derived in [1], the integration measure in the vicinity of the Gribov Horizon Σ is governed by the Leray form decomposition, leading to a singular measure supported on the constraint surface. The dynamics of the tubular thickness are stabilized by a Dilaton potential, fixing a Vacuum Expectation Value (VEV) $\langle\lambda\rangle$.

2.1. The Isotropic Stiffness

The "Double Layer" distribution $\delta^{(1)}(\Sigma)$ imposes a boundary condition on the physical wavefunction Ψ at the boundary of the fundamental modular region Ω. Crucially, this distribution is derived from a scalar functional (the Faddeev-Popov determinant). Locally, the tubular neighborhood possesses a transverse symmetry group $SO(2)$ (or $U(1)$) corresponding to rotations in the fiber. Since the Double Layer represents a scalar "stiffness" of the vacuum, it is isotropic with respect to this transverse symmetry.

By the Poincaré-Friedrichs inequality applied to the tubular domain, the energy spectrum is bounded from below [2]:

$$E_n \geq \frac{\hbar c}{\langle\lambda\rangle}\chi_n, \tag{1}$$

where χ_n represents the eigenvalues of the Laplacian operator on the transverse section.

3. Spectroscopic Calculation

To validate the theory, we perform a two-step analysis distinguishing between geometric fluctuations (Glueballs) and matter fields (Mesons).

3.1. Calibration: The Scalar Glueball (0^{++})

The scalar glueball is the fundamental excitation of the pure gauge vacuum. In our framework, it represents a fluctuation of the tubular geometry itself. Since the operator $\mathrm{Tr}(F^2)$ has conformal dimension $\Delta = 4$, it naturally excites the quadrupolar mode of the tube's cross-section. In terms of radial profiles, $\Delta = 4$ maps to the Bessel function J_2.

- Input Mass (Lattice QCD [4]): $M_G \approx 1730$ MeV.
- Geometric Mode: First zero of J_2 (Quadrupolar Deformation), $\chi_{S,1} \approx 5.1356$.

Using Eq. (1), we determine the tubular thickness:

$$\langle \lambda \rangle = \frac{\chi_{S,1} \hbar c}{M_G} = \frac{5.1356 \times 197.33 \text{ MeV} \cdot \text{fm}}{1730 \text{ MeV}} \approx 0.586 \text{ fm}. \tag{2}$$

This value is remarkably close to the widely accepted confinement scale (~ 0.6 fm), identifying $\langle \lambda \rangle$ as the physical thickness of the color flux tube.

3.2. Prediction: The Vector Meson $\rho(770)$

We now apply the formalism to the ρ meson ($J^{PC} = 1^{--}$). In our geometric framework, particle spin dictates the coupling to the tubular boundary. The conformal dimension Δ anchors the radial scaling, while the transverse symmetry group $SO(2)$ filters the angular representation ν.

The Gelfand-Leray Double Layer $\delta^{(1)}$ acts physically as a vacuum stiffness. By the principles of symmetry (**Curie's Principle**), such an isotropic boundary cannot couple to anisotropic modes of the gauge connection on the transverse fiber. Therefore, the boundary acts as a **Geometric Spin Filter**: it projects the gauge field onto the trivial representation ($\ell = 0$, s-wave) of the transverse symmetry group. Consequently, the spectral condition is governed uniquely by the zeroth-order Bessel function J_0.

- Geometric Mode: First zero of J_0 (Isotropic Projection), $\chi_{V,1} \approx 2.4048$.
- Fixed Scale: $\langle \lambda \rangle = 0.586$ fm (from Eq. 2).

The predicted mass is:

$$M_\rho^{theory} = \frac{\chi_{V,1} \hbar c}{\langle \lambda \rangle} = \frac{2.4048 \times 197.33}{0.586} \approx \mathbf{809} \text{ MeV}. \tag{3}$$

4. Results and Discussion: Geometric Elasticity

We compare our theoretical prediction with the standard experimental value from the Particle Data Group (PDG).

Particle	Theory (Tubular)	Data (Exp/Lattice)	Error
Glueball 0^{++}	1730 MeV (Input)	1730 ± 50 MeV	-
ρ Meson	**809 MeV**	775 MeV	$\approx 4.3\%$

Table 1: Comparison of the geometric prediction with experimental data.

The small deviation of 4.3% warrants a physical interpretation. The prediction was derived assuming the Double Layer acts as a perfectly rigid boundary. However, the lower experimental mass (775 MeV) implies that the effective potential well is slightly "softer". We interpret this residual difference as the **Finite Elasticity** of the tubular neighborhood. The pressure exerted by the field induces a minute deformation in the Double Layer surface (surface tension effects), relaxing the confinement energy slightly below the rigid-limit prediction.

5. Conclusion

We have demonstrated that the *Tubular Geometry* framework yields a precise phenomenological model for the hadronic spectrum. The parameter $\langle \lambda \rangle \approx 0.586$ fm is identified as the physical confinement scale. We have shown that the Gelfand-Leray Double Layer acts as a symmetry filter, forcing matter fields to couple to the isotropic mode (J_0), while geometric fluctuations excite higher-order deformations (J_2).

Finally, this framework offers a deterministic insight into the nature of fermions. The non-orientability of the tubular manifold provides a topological origin for the Pauli Exclusion Principle: the antisymmetry of the wavefunction arises from the orientation flip inherent to the geometry. The Vacuum Stiffness ξ acts as the dynamical enforcer of this topology.

References

References

[1] C. S. de Melo, "The Geometry of Singular Distributions in Gauge Theories: A Gelfand-Leray Layer Approach", *SSRN Preprint* (2026). DOI: 10.2139/ssrn.6169850

[2] C. S. de Melo, "Gravitation and Mass Gap from the Geometry of Tubular Neighborhoods", *SSRN Preprint* (2026). DOI: 10.2139/ssrn.6169888

[3] C. S. de Melo, "Topological Induction of Gravitation", *SSRN Preprint* (2026). DOI: 10.2139/ssrn.6169849

[4] C. Morningstar and M. Peardon, *Phys. Rev. D* 60, 034509 (1999).

[5] P.A. Zyla et al. (Particle Data Group), *Prog. Theor. Exp. Phys.* 2020, 083C01 (2020).