# The Schrödinger Equation as a Resolution Geometry Theorem

## *Deriving Quantum Mechanics as Phase Tension Preservation on a Complex Scaffold*

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## Abstract

This companion paper demonstrates that the Schrödinger equation is not an arbitrary postulate of quantum mechanics, but a geometric inevitability: it is the unique dynamics permitted for a complex-valued ledger that must preserve probability amplitude while minimizing tension.

While the heat equation describes the relaxation of a scalar magnitude (dissipation), the Schrödinger equation describes the evolution of a magnitude-plus-phase ledger (rotation). In Resolution Geometry terms, the imaginary unit i acts as the Layer 2 rotation operator, converting spatial gradient tension not into decay, but into temporal frequency.

We derive the Schrödinger equation as the Euler-Lagrange condition for maintaining a smooth phase field on a 2D scaffold under the constraint of norm conservation. The resulting dynamics preserve the total 'phase tension' (energy) by converting it into unitary propagation. This completes the 'Resolution Geometry Trilogy,' demonstrating that Black-Scholes (finance), the Heat Equation (thermodynamics), and Schrödinger (quantum mechanics) are three expressions of the same constraint geometry—distinguished only by whether their ledger is scalar or complex, and whether their constraint is dissipative or unitary.

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## 1. The Structural Correspondence

Resolution Geometry (Connerty, 2026a) posits that all physical laws emerge from the requirements of identity management on a constrained scaffold.

The formal relationship between the Heat Equation and the Schrödinger Equation is well known (Wick rotation: $t \to it$). However, the geometric interpretation of this relationship is often obscured. Resolution Geometry clarifies it:

**Heat Equation:** A system smoothing out inequality (magnitude differences) via redistribution.

**Schrödinger Equation:** A system smoothing out twist (phase differences) via rotation.

The correspondence is structural:

| Component | Heat Equation | Schrödinger Equation |
|---|---|---|
| Scaffold | Spatial manifold x | Spatial manifold x |
| Ledger Field | Scalar $u(x,t)$ | Complex $\psi(x,t)$ |
| Ledger Content | Magnitude (Mass/Heat) | Mag + Phase (Probability) |
| Geometric Cost | Gradient Energy $(\nabla u)^2$ | Total Tension $\lvert\nabla \psi\rvert^2$ |
| Constraint | Conservation of Mass | Conservation of Norm |
| Dynamics | Gradient Flow (Dissipative) | Hamiltonian Flow (Unitary) |
| Operator | Identity (1) | Rotation (i) |
| Result | Relaxation (Smoothing) | Propagation (Wave Motion) |

## 2. The Geometric Interpretation

### 2.1 The Scalar Ledger (Heat)

In a scalar ledger, gradients represent a 'tension of inequality.' If slot A has value 100 and slot B has value 0, the system minimizes tension by moving value from A to B until both are 50.

**Result:** Dissipation. The gradients vanish. The tension is 'spent.'

### 2.2 The Complex Ledger (Schrödinger)

In a complex ledger (Layer 2 of the framework), the field $\psi$ encodes orientation (phase) as well as magnitude. Gradients represent a 'tension of twist'—slot A is rotated relative to slot B. The term $|\nabla\psi|^2$ captures the total structural cost, including both magnitude gradients (concentration) and phase gradients (twist).

**Constraint:** The system cannot 'spend' this tension by collapsing the amplitude, because it must preserve the total probability norm ($\int|\psi|^2 = 1$).

**Solution:** The system resolves the spatial twist by rotating the local clock.

**Result:** Propagation. The gradients do not vanish; they drive the phase rotation. High spatial curvature induces high temporal frequency.

The 'wave function' is simply a ledger that tracks orientation.

## 3. The Variational Derivation

We now derive the general Schrödinger equation (including potential V) as the condition that minimizes the 'Action of Phase'—the trade-off between spatial twist and temporal spin.

### 3.1 The Action Functional

We define the Schrödinger Action on the scaffold. This action represents the cost of maintaining a dynamic phase field. Unlike the dissipative Dirichlet energy, this is a Lagrangian density $\mathscr{L}$ that includes potential energy V(x):

$$S[\psi] = \iint [\ i\hbar\psi^*(\partial\psi/\partial t) - (\hbar^2/2m)\nabla\psi^*\cdot\nabla\psi - V(x)\psi^*\psi\ ]\ dx\ dt$$

**Spatial Tension:** $\nabla\psi^*\cdot\nabla\psi = |\nabla\psi|^2$. This measures the energetic cost of 'twisting' the phase field across the scaffold.

**Potential:** $V(x)|\psi|^2$ represents the local cost of occupancy (the 'terrain' of the scaffold).

**Temporal Spin:** The term involving $\partial\psi/\partial t$ measures the cost of rotating the phase in time.

### 3.2 Euler-Lagrange Optimization

The system evolves along the path that makes this action stationary ($\delta S = 0$). We vary the action with respect to the conjugate field $\psi^*$ (treating $\psi$ and $\psi^*$ as independent variables in the variation):

$$\partial\mathscr{L}/\partial\psi^* - \nabla\cdot(\partial\mathscr{L}/\partial(\nabla\psi^*)) - (\partial/\partial t)(\partial\mathscr{L}/\partial\dot{\psi}^*) = 0$$

Computing the terms:

**Density Variation:** $\partial\mathscr{L}/\partial\psi^* = i\hbar(\partial\psi/\partial t) - V\psi$

**Gradient Variation:** $\partial\mathscr{L}/\partial(\nabla\psi^*) = -(\hbar^2/2m)\nabla\psi$

**Time Derivative Variation:** $\partial\mathscr{L}/\partial\dot{\psi}^* = 0$ (since $\dot{\psi}^*$ does not appear in $\mathscr{L}$)

Substituting into the Euler-Lagrange equation:

$$(\ i\hbar(\partial\psi/\partial t) - V\psi\ ) - \nabla\cdot(\ -(\hbar^2/2m)\nabla\psi\ ) = 0$$

### 3.3 The Result

Rearranging terms yields the time-dependent Schrödinger equation:

$$\mathbf{i\hbar(\partial\psi/\partial t) = [\ -(\hbar^2/2m)\nabla^2 + V(x)\ ]\ \psi}$$

**Conclusion:** The Schrödinger equation is the Euler-Lagrange condition where Total Energy (Spatial Tension + Potential) is perfectly balanced by Temporal Rotation (Frequency).

## 4. Probability Conservation (Exchange Consistency)

For the Resolution Geometry framework to hold, the complex ledger must be exchange-consistent—identity cannot be created or destroyed, only moved. We prove this by deriving the continuity equation from the result in Section 3.3.

Define the probability density $\rho = \psi^*\psi$. The time evolution of the density is:

$$\partial\rho/\partial t = (\partial\psi^*/\partial t)\psi + \psi^*(\partial\psi/\partial t)$$

Substituting the Schrödinger equation (and its complex conjugate), the potential terms $V\cdot\psi^*\psi$ cancel (since V is real), leaving:

$$\partial\rho/\partial t = (i\hbar/2m)(\psi\nabla^2\psi^* - \psi^*\nabla^2\psi) = -\nabla\cdot j$$

where the probability current is $j = (i\hbar/2m)(\psi\nabla\psi^* - \psi^*\nabla\psi)$.

This is the continuity equation:

$$\boldsymbol{\partial\rho/\partial t + \nabla\cdot j = 0}$$

**Conclusion:** The total identity content $\int\rho\, dx$ is conserved over time. The 'imaginary' rotation i in the dynamics is precisely what allows the system to oscillate (change phase) without losing norm.

## 5. Interpretation of the 'i' (Layer 2)

This derivation demystifies the imaginary unit i. It is not a mathematical abstraction; it is the Layer 2 Rotation Operator (Connerty, 2026a).

**In the Heat Equation ($\partial\square \propto \nabla^2$):** The flow is parallel to the curvature. The system moves 'downhill' to flatten the curve.

**In Schrödinger ($\partial\square \propto -i\nabla^2$):** The flow is perpendicular to the curvature (rotated 90° by i).

The system acts like a gyroscope. When a force (gradient tension) is applied, the system doesn't fall over (dissipate); it precesses (rotates phase).

This explains the stability of bound states (atoms). An electron in a potential well V(x) experiences a strong gradient pulling it inward.

A **dissipative system** (Heat) would spiral in and collapse.

A **unitary system** (Schrödinger) converts that inward 'pull' into 'spin.' The electron settles into a standing wave where the spatial tension ($\nabla^2$) is perfectly matched by a constant phase rotation frequency ($E/\hbar$).

## 6. The Resolution Geometry Trilogy

This paper completes the trilogy of scaffold dynamics. We can now see that three distinct domains of science are describing the exact same geometric process—tension minimization—differing only in their ledger type and constraint.

| Domain | Paper | Ledger | Constraint | Dynamics |
|---|---|---|---|---|
| Finance | Black-Scholes | Value V (Scalar) | No-Arbitrage | Diffusion + Drift |
| Thermodynamics | Heat Equation | Mass u (Scalar) | Conservation | Pure Diffusion |
| Quantum Mechanics | Schrödinger | Phase ψ (Complex) | Unitary (Norm) | Wave Propagation |

Physics is not a collection of arbitrary laws. It is the inevitable behavior of a constrained scaffold managing different types of receipts.

If the receipt is **value**, you get Black-Scholes.

If the receipt is **stuff**, you get the Heat Equation.

If the receipt is **phase**, you get Schrödinger.

## 7. Conclusion

The Schrödinger equation is a theorem of Resolution Geometry. It describes the only way a phase-bearing ledger can manage spatial tension without violating the unitary constraint of probability conservation.

It converts the cost of 'being here' (localization/curvature) into the action of 'spinning' (frequency/energy). The result is a universe that vibrates rather than freezes.

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*The ledger has phase.*

*The phase has tension.*

*The tension drives rotation.*

*The rotation is the wave.*

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## Acknowledgments

This derivation emerged through collaborative development with AI systems (Claude, GPT, Gemini), identifying the Hamiltonian flow structure of the Schrödinger action as the geometric counterpart to the gradient flow of the Heat equation.

## References

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