

# Thermodynamic Constraints on Measurement Events: A Boundary Framework for Classical Information

Moses Rahnama

Mina Analytics, New York, United States

(\*moses@minaanalytics.com)

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We develop a thermodynamic framework for quantum measurement and black hole information. The framework rests on three results: (1) a conditional record-formation heat bound with a three-stage measurement taxonomy and explicit operational conditions, validated by a differential microcalorimetry protocol and Lindblad master equation simulation<sup>21</sup>; (2) a proof that the Born rule  $P = |\psi|^2$  is the unique phase-independent probability rule consistent with  $L^2$  normalization, using one thermodynamic axiom and three structural axioms<sup>22</sup>; and (3) the exact Schwarzschild identity  $T_H S_{BH} = \frac{1}{2} M c^2$  (Smarr relation) and a complete information and energy accounting framework where measurement *creates* classical information and black holes *render classical records operationally inaccessible* to exterior observers, verified dynamically across the full evaporation trajectory and the Kerr-Newman parameter space<sup>23</sup>. Falsifiable predictions include measurement calorimetry and Hawking-Landauer verification.

## I. INTRODUCTION

The starting point of this work is a single observation: Landauer's principle<sup>1</sup> establishes that any logically irreversible operation erasing one bit of information must dissipate at least  $k_B T \ln 2$  of heat. We propose that the formation of an objective classical record from a quantum superposition is such an operation: stabilizing a classical record eliminates quantum alternatives and exports entropy to the environment.

From this single principle we develop three results. **First**, we formulate a conditional record-formation heat bound under explicit operational conditions (C1 to C6), with a three-stage measurement taxonomy separating reversible premeasurement, irreversible record formation, and memory reset<sup>21</sup>. **Second**, we show that the Born rule  $P = |\psi|^2$  is the unique probability rule compatible with phase-independent record formation in  $L^2$  Hilbert space<sup>22</sup>. **Third**, we organize the Smarr identity  $T_H S_{BH} = \frac{1}{2} M c^2$  into an information and energy accounting framework for black hole thermodynamics, positioning black holes as the complement to measurement in the record-formation regime<sup>23</sup>.

### Relation to existing work

Recent research has explored connections between thermodynamics, information theory, and quantum foundations. Cort  s and Liddle<sup>2</sup> showed that Hawking evaporation saturates the Landauer bound. Latune and Elouard<sup>3</sup> analyzed the thermodynamic cost of measurement apparatus. Mohammady and Buscemi<sup>4</sup> identified a thermodynamic trilemma for efficient measurements. Herrera<sup>5</sup> applied Landauer's principle to problems in general relativity. Bera *et al.*<sup>6</sup> derive thermodynamics from information conservation. Cabello *et al.*<sup>7</sup> use Landauer's principle to constrain interpretations of quantum mechanics.

Our framework differs from and extends this work in several ways:

- **Record-formation bound:** We formulate a conditional

heat bound for the specific stage of irreversible record formation, with explicit operational conditions (C1 to C6) that address critiques of over-application of Landauer's principle<sup>8</sup>.

- **Born rule from thermodynamics:** We show the Born rule is the unique phase-independent probability rule consistent with  $L^2$  normalization, using one thermodynamic axiom and three structural axioms<sup>22</sup>.
- **Black hole accounting:** We identify  $T_H S_{BH} = \frac{1}{2} M c^2$  as the Smarr relation forced by holographic scaling ( $S \propto M^2$ ), and use it as an accounting device<sup>23</sup>.
- **Thermodynamic complement:** Measurement and black holes are structurally complementary: both involve irreversible operations at the Landauer scale, one creating classical records and the other rendering them operationally inaccessible<sup>23</sup>.

### Structure of the paper

Section II develops the thermodynamic measurement criterion (Paper A). Section III presents the Born rule derivation (Paper B). Section IV draws an interpretive connection between these results and the quantum-relativity relationship. Section V develops the black hole thermodynamic accounting (Paper C). Section VI offers an information-theoretic reading of time dilation. Section VII collects falsifiable predictions. Section VIII discusses limitations and open problems.

## II. MEASUREMENT AS A THERMODYNAMIC BOUNDARY

This section summarizes the results of Ref.<sup>21</sup>. Full derivations, the Lindblad master equation simulation, and the complete experimental protocol are in the companion paper.

### A. Three-stage taxonomy

We decompose measurement into three physically distinct stages:

1. **Stage 1: Premeasurement.** Unitary system-pointer correlation. Reversible. No Landauer cost.
2. **Stage 2: Record Formation.** Irreversible stabilization via environmental coupling. Landauer cost paid here.
3. **Stage 3: Memory Reset.** Erasure for apparatus reuse. Standard Landauer erasure cost.

This taxonomy resolves apparent contradictions: “reversible measurements” address Stage 1, while Landauer bounds constrain Stage 2 when an objective record is actually created.

### B. Record-formation heat bound

Under explicit operational conditions (C1 to C6), the mean record-formation heat obeys:

$$\langle Q_{\text{rec}} \rangle \geq k_B T \ln 2 \cdot I(X; Y), \quad (1)$$

where  $I(X; Y)$  is the classical mutual information between prepared state  $X$  and recorded outcome  $Y$ .

### C. Operational conditions (C1 to C6)

The bound is asserted as a *conditional* statement:

- C1:** A thermal bath at temperature  $T$  is present during record formation.
- C2:** A classical register  $Y$  is actually created (decohered pointer with redundant environmental encoding).
- C3:** The apparatus is cyclic over Stage 2 (or its entropy change is accounted).
- C4:** Irreversibility arises from bath coupling on the experiment timescale (no global reversal including the bath).
- C5:** No unaccounted work reservoir supplies free energy (if work is supplied, it must enter the inequality explicitly).
- C6:**  $I(X; Y)$  is the realized classical mutual information for the chosen measurement strength.

The bound does *not* claim that all measurement interactions must dissipate  $k_B T \ln 2$  per bit. If only Stage 1 occurs (premeasurement followed by coherent reversal), no objective record exists and no Landauer-type record cost is required. Likewise, the cost can be deferred by storing the pointer in coherent quantum memory, or shifted into fresh-memory entropy or supplied work.

### D. Concrete model: where the cost is paid

An explicit spin- $\frac{1}{2}$  + pointer + bath model locates the Landauer cost precisely. The system in superposition  $|\psi_S\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$  is coupled unitarily to a pointer (Stage 1, reversible), then coupled to a thermal bath that creates  $N \gg 1$  redundant environmental copies of the outcome (Stage 2, irreversible). The record is identified with the moment when the environmental overlap drops below a decoherence criterion:

$$\langle E_{\uparrow}^{(N)} | E_{\downarrow}^{(N)} \rangle < \epsilon_{\text{crit}} \ll 1. \quad (2)$$

At this point phase information is irreversibly lost, and the entropy exported to the bath gives heat

$$Q \geq T \cdot H(|\alpha|^2) \cdot k_B \ln 2. \quad (3)$$

A Lindblad master equation simulation validates the bound in a circuit-QED architecture with parameters matching the proposed experiment<sup>21</sup>. All computable numerical values are generated by the accompanying simulation pipeline (54 checks, all pass).

### E. Compatibility with Many-Worlds

In the Many-Worlds Interpretation, all branches exist and no collapse occurs. Our framework is compatible with MWI and suggests a thermodynamic reading: whether or not all branches “exist,” only one branch is accessible to any given classical observer. In this reading, the record-formation event is the moment when branches become *thermodynamically inaccessible* to each other. The Landauer cost is the price of this inaccessibility: the energy required to create sufficient decoherence that branches cannot interfere.

The interpretive claim is that *branching is not free*: it costs at least  $k_B T \ln 2$  per bit of branch distinction, because creating branch-distinguishing records is a record-formation event subject to the bound of Eq. (1). The record-formation event marks the point at which this thermodynamic cost has been paid, making interference between branches physically impossible. Developing this into a full model of branching costs within MWI (connecting to Carroll and Singh’s branch-counting formalism, for instance) is future work.

## III. THE BORN RULE FROM RECORD-FORMATION CONSTRAINTS

This section summarizes the results of Ref.<sup>22</sup>. The full standalone derivation, including the  $L^p$  counterfactual analysis and 88-check numerical verification, is in the companion paper.

### A. Physical setting

Quantum amplitudes  $\alpha_i = |\alpha_i|e^{i\phi_i}$  contain both magnitude and phase. The phase enables interference:

$$|\alpha_1 e^{i\phi_1} + \alpha_2 e^{i\phi_2}|^2 = |\alpha_1|^2 + |\alpha_2|^2 + 2|\alpha_1||\alpha_2|\cos(\phi_1 - \phi_2) \quad (4)$$

A classical measurement record cannot exhibit interference. Creating such a record requires that phase information is not retained, an operation with Landauer cost  $k_B T \ln 2$  per bit.

### B. Uniqueness theorem

**Scope note: why  $L^2$ ?** The proof assumes  $L^2$  normalization, which is fixed by unitarity. If quantum mechanics used  $L^1$  normalization, the proof would give  $n = 1$ ; if  $L^4$ , it would give  $n = 4$ . The probability rule must match the norm. The physical content is in phase independence: thermodynamics determines *which* axiom constrains the map, and the Hilbert space structure determines *which power* satisfies that constraint.

**Theorem 1** (Born Rule from Thermodynamic Constraints). *Let  $f : \mathbb{C} \rightarrow \mathbb{R}^+$  satisfy:*

1. **Normalization:**  $\sum_i f(\alpha_i) = 1$  for any normalized state
2. **Phase independence:**  $f(\alpha e^{i\phi}) = f(\alpha)$  for all  $\phi$
3. **Interference consistency:** consistent with observed interference before measurement
4. **Tensor product factorization:**  $f(\alpha \otimes \beta)$  factorizes for independent systems
5. **Continuity:**  $f$  is continuous in  $\alpha$

Then  $f(\alpha) = |\alpha|^2$  is the unique such function.

*Proof.* Phase independence implies  $f(\alpha) = g(|\alpha|)$ . Tensor product factorization gives  $g(r_1 r_2) = g(r_1)g(r_2)$ , which with continuity implies  $g(r) = r^n$  for some  $n > 0$ . Normalization on the  $L^2$  unit sphere requires  $\sum_i |\alpha_i|^n = 1$  whenever  $\sum_i |\alpha_i|^2 = 1$ . The only power for which the  $L^2$  unit sphere is also an  $L^n$  unit sphere is  $n = 2$ .  $\square$

### C. Why not linear?

The need for squaring is rooted in a distinction between classical and quantum notions of “opposite.” In classical probability, opposites are *complementary*:  $P(A) + P(\neg A) = 1$ . In quantum mechanics, opposites are *annihilating*:  $\psi + (-\psi) = 0$ .

The linear rule  $f(\alpha) = |\alpha|$  fails on two independent grounds: normalization failure (with  $L^2$  normalization,  $\sum_i |\alpha_i| \neq 1$  in general) and interference failure (a linear rule gives  $|\psi + (-\psi)| = 2|\psi| \neq 0$  when the amplitudes cancel).

### D. Connection to Gleason’s theorem

Our result is a different uniqueness theorem from a different axiom set than Gleason’s<sup>19</sup>:

- **Gleason:** The Born rule is the unique *non-contextual* measure on projective Hilbert space (dimension  $\geq 3$ ).
- **This work:** The Born rule is the unique *phase-independent* probability rule consistent with  $L^2$  normalization (all dimensions, including dimension 2).

The axioms differ. Gleason requires non-contextuality and  $\dim(\mathcal{H}) \geq 3$ ; we replace non-contextuality with phase independence (a thermodynamic constraint) and tensor-product factorization (a structural constraint), which together work in all dimensions including  $\dim(\mathcal{H}) = 2$ .

### E. $L^p$ counterfactual analysis

The uniqueness of  $n = 2$  can be understood through a counterfactual: what if quantum mechanics used a different norm? If states were normalized on the  $L^1$  sphere ( $\sum_i |\alpha_i| = 1$ ), the same proof would give  $n = 1$  (linear probabilities). If  $L^4$  ( $\sum_i |\alpha_i|^4 = 1$ ), it would give  $n = 4$ . The probability rule must match the norm. The physical content is that  $L^2$  is fixed by unitarity, and phase independence then forces  $n = 2$ . Neither alone is sufficient. The supplementary simulation (Ref.<sup>22</sup>) verifies this by constructing explicit states on the  $L^1$  and  $L^4$  unit spheres and confirming that only  $n = 2$  satisfies all constraints on the  $L^2$  sphere (88 checks, all pass).

### F. Information content of phase

For a two-phase preparation with  $\Delta\phi \in \{0, \pi\}$  at maximal contrast, the phase determines whether interference is constructive or destructive. This encodes 1 bit:

$$I_{\text{before}}(X; Y) = H(Y) - H(Y|X) = 1 \text{ bit} \quad (5)$$

After measurement,  $I(X; Y) = 0$ : the outcome is determined by  $|\alpha_i|^2$  alone, independent of phase. Rendering this phase information inaccessible costs at least  $k_B T \ln 2$  per bit.

## IV. THE QUANTUM-RELATIVITY RELATIONSHIP

The three results above suggest a structural observation about the relationship between quantum mechanics and general relativity.

Quantum mechanics describes the space of *possibilities*: the amplitudes, phases, and interference patterns that characterize what *could* happen next. General relativity describes the geometry of *actualities*: the spacetime structure determined by the mass-energy distribution of what *has* happened.

The record-formation event (Sec. II) is the boundary between these two descriptions. Before record formation, the

system is in superposition (quantum description applies). After record formation, a definite classical outcome exists and contributes to the stress-energy content of spacetime (relativistic description applies). A single record-formation event updates the boundary:

- It selects one outcome from the superposition (via Born probabilities, Sec. III)
- It creates irreversible classical information at a space-time location
- The accumulated classical information determines geometry

In this sense, quantum mechanics and general relativity are not two theories awaiting unification but two descriptions of the same physical process, separated by record formation. This is an interpretive observation, not a derived result; it follows from taking the three companion papers together but does not constitute an independent prediction beyond what those papers already contain.

## V. BLACK HOLES AND THE THERMODYNAMIC ACCOUNTING

This section summarizes the results of Ref.<sup>23</sup>. The full standalone analysis, including counterfactual evaporation dynamics, greybody source-channel arguments, Kerr/RN parameter sweeps, and a 54-check numerical consistency pipeline, is in the companion paper.

### A. The Smarr identity

For a Schwarzschild black hole of mass  $M$ :

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}, \quad (6)$$

$$S_{BH} = \frac{4\pi k_B G M^2}{\hbar c}. \quad (7)$$

Multiplying yields the Smarr relation<sup>9</sup>:

$$\boxed{T_H S_{BH} = \frac{1}{2} M c^2} \quad (8)$$

The factor of  $\frac{1}{2}$  is forced by the holographic scaling  $S \propto M^2$  (area law). By Euler's theorem, if entropy scaled as  $M^k$ , the Smarr relation would give  $E = k T_H S_{BH}$  and the fraction would be  $1/k$ :

Scaling	Smarr relation	Fraction
$S \propto E^1$ (linear)	$E = T_H S_{BH}$	1
$S \propto E^2$ (area/holographic)	$E = 2 T_H S_{BH}$	$\frac{1}{2}$
$S \propto E^3$ (volume)	$E = 3 T_H S_{BH}$	$\frac{1}{3}$
$S \propto E^4$	$E = 4 T_H S_{BH}$	$\frac{1}{4}$

We emphasize that the “information/field-cost split” is an interpretive accounting device for organizing the Smarr identity, not an ontological claim about a localizable gravitational energy density.

### B. Entropy dictionary

To avoid conflation between different entropy concepts:

- $S_{BH} = k_B c^3 A / (4\hbar G)$ : the **Bekenstein-Hawking entropy**. This is a coarse-grained thermodynamic entropy counting macroscopically indistinguishable microstates consistent with  $(M, J, Q)$ .
- $S(\rho_{\text{rad}})$ : the **fine-grained von Neumann entropy** of the Hawking radiation state. This is the quantity tracked by the Page curve<sup>10</sup> and computed by the island formula<sup>11</sup>.

Our accounting uses  $S_{BH}$  (coarse-grained). The Landauer saturation result of Cort s and Liddle<sup>2</sup> relates  $dE$  to  $dS_{BH}$ .

### C. Landauer saturation

The first law  $dE = T_H dS_{BH}$  implies that each bit of  $S_{BH}$  lost releases energy  $k_B T_H \ln 2$ , the Landauer minimum. This saturation depends on identifying  $T_H$  as the relevant temperature; for astrophysical black holes in the CMB, net evaporation does not occur until the environment cools below  $T_H$ .

*a. Source saturation vs. channel transmission.* Greybody factors  $\Gamma(\omega)$  modify the Hawking spectrum but do not break Landauer saturation. The first law is a statement about the *source process* at the horizon. Greybody factors describe the *transmission channel*. Modes that fail to escape are re-absorbed, so the net ratio  $dE_{\text{net}}/dS_{\text{net}} = T_H$  remains exact. Greybody factors modify the evaporation *rate* but not the *ratio*<sup>15,16</sup>.

Recent work by Song<sup>17</sup> formalizes a related decomposition:  $\delta A/4G = \delta \langle K \rangle + \ln 2 \cdot \delta N_c$ , connecting the accounting split to Einstein's equations from local entropy balance in the spirit of Jacobson<sup>14</sup>.

### D. The accounting complement to measurement

In the record-formation framework of Ref.<sup>21</sup>, creating a classical record costs at least  $k_B T \ln 2$  per bit under C1 to C6. When matter carrying classical records falls into a black hole, the specific bit patterns become inaccessible to exterior observers. Many distinct classical histories are mapped to the same  $(M, J, Q)$ . This many-to-one character is what makes the process analogous to Landauer erasure, not merely cryptographic inaccessibility. As the black hole evaporates, energy is returned as Hawking radiation at temperature  $T_H$ , with  $k_B T_H \ln 2$  per bit.

$$\begin{array}{ccc} \text{Superposition} & \xrightarrow{\text{record formation}} & \text{Classical Record} \\ & \searrow \text{black hole} & \\ & & \text{Hawking Radiation} \end{array} \quad (9)$$

This is a *global energy-accounting balance*, not a thermodynamic cycle that could do work.

*a. Illustrative analogy.* The accounting complement is analogous to a physical transaction. Before an exchange, ownership is ambiguous; at the moment of exchange, ownership becomes definite. The transaction requires physical presence (system and environment), an irreversible action (record creation), a cost ( $k_B T \ln 2$  per bit), and a record (classical information written to the environment). The black-hole scaling identity  $T_H S_{BH} = \frac{1}{2} M c^2$  can be read as an accounting reflection of this transaction structure: the horizon-entropy term behaves like an information-cost ledger equal to half the global mass-energy (for Schwarzschild).

### E. Extensions to Kerr and Reissner-Nordström

The generalized Smarr relation<sup>9</sup> for Kerr-Newman black holes gives:

$$T_H S_{BH} = \frac{1}{2} (M c^2 - 2 \Omega_H J - \Phi_H Q) \quad (10)$$

As  $a/M \rightarrow 1$  (Kerr) or  $Q/M \rightarrow 1$  (RN),  $T_H \rightarrow 0$  and  $T_H S_{BH} \rightarrow 0$ . The equal-halves interpretation is Schwarzschild-specific.

### F. Compatibility with unitarity and the Page curve

Our framework distinguishes coarse-grained classical records (operationally inaccessible after absorption) from fine-grained quantum information (potentially preserved in radiation correlations). The Page curve<sup>10</sup> tracks  $S(\rho_{\text{rad}})$ : it increases before the Page time, then turns over via the island formula<sup>11</sup>, returning to zero at complete evaporation. Our accounting tracks  $S_{BH}$  (monotonically decreasing). These agree before the Page time and diverge after it: our framework says “classical records continue to become operationally inaccessible”; the Page curve says “quantum information is becoming accessible in radiation correlations.” Both are correct; they track different quantities. The Page entropy formula is the large- $N$  limit of the PSSY model<sup>18</sup>.

### G. Analog gravity: a proposed estimator

Analog Hawking radiation has been observed in BEC acoustic horizons<sup>12</sup>. We propose the Landauer Ratio:

$$R_L = \frac{1}{\ln 2} \cdot \frac{\int P_{\text{rad}}(t) dt}{\int T_{H,\text{eff}}(t) \dot{S}_{\text{ent}}(t) dt} \quad (11)$$

$R_L = 1$  indicates Landauer saturation;  $R_L > 1$  indicates dissipation above the bound. Steinhauer’s experiments ( $T_H \approx 1.2$  nK) provide the inputs<sup>12,13</sup>.

### H. Scope: derived vs. interpretive

**Derived or standard** (not novel): first law ( $dE = T_H dS_{BH}$ ), Smarr relation<sup>9</sup>, Landauer saturation<sup>2</sup>, Page curve<sup>10,11</sup>.

**Interpretive** (proposed bookkeeping): “entropy-energy term vs. complement” accounting split, “measurement pays, black hole returns” language,  $R_L$  as analog estimator.

## VI. INTERPRETIVE EXTENSION: TIME AS INFORMATION PROCESSING

The following is an interpretive restatement of special-relativistic time dilation in the language of the framework. It does not derive new physics; it provides a consistent information-theoretic reading of the Lorentz factor.

If record-formation events are physical computations, then the speed of light  $c$  sets the maximum rate at which these computations can propagate. A system at rest devotes all of its information-processing capacity to internal evolution (temporal computation). A system moving at velocity  $v$  divides its capacity between spatial propagation and temporal evolution. The total capacity satisfies a Pythagorean constraint:

$$I_{\text{tot}}^2 = I_{\text{space}}^2 + I_{\text{time}}^2 \quad (12)$$

Setting  $I_{\text{tot}} = c$  and  $I_{\text{space}} = v$ :

$$I_{\text{time}} = c \sqrt{1 - v^2/c^2} = c/\gamma \quad (13)$$

This is proportional to  $d\tau/dt$ , the proper time per coordinate time. The Lorentz factor  $\gamma$  emerges as a capacity trade-off: at  $v = c$ , all capacity is spatial and time stops; at  $v = 0$ , all capacity is temporal.

This is a restatement of special relativity, not a derivation. Its value is that it provides a natural language for time dilation within the record-formation framework: proper time measures the computational work available for internal processes. Whether this interpretive language leads to testable consequences beyond standard relativity is an open question.

## VII. PREDICTIONS

### A. Measurement calorimetry

**Prediction 1:** A Landauer-scale heat signature appears at *record formation* (Stage 2), not during reversible premeasurement (Stage 1). Ref.<sup>21</sup> designs a circuit-QED differential microcalorimetry experiment with four key controls: ground-state baseline, measurement-strength scaling, reversal-delay timing sweep, and prior-variation at fixed hardware. Sensitivity analysis shows detection is feasible with  $N \sim \text{few} \times 10^9$  ON/OFF pairs at 10 mK for SNR  $\sim 10$ .

The bound is **falsified** if the residual  $\Delta Q - k_B T \ln 2 \cdot I(X; Y)$  falls statistically below zero at any tested operating point, after full uncertainty propagation.

## B. Heat-probability correlation

**Prediction 2:** For measurements creating different amounts of classical information, the heat dissipated should scale with the Shannon entropy of the outcome distribution:  $\langle Q \rangle \gtrsim k_B T \ln 2 H(P)$ . This is a direct consequence of the record-formation bound (Eq. 1) and is testable via the measurement-strength sweep in Ref.<sup>21</sup>.

## C. Weak measurement interpolation

**Prediction 3:** For weak measurements with partial phase information loss, the effective probability should interpolate between quantum interference and classical mixture:  $P_{\text{eff}} = (1 - \varepsilon)|\psi_1 + \psi_2|^2 + \varepsilon(|\psi_1|^2 + |\psi_2|^2)$ , where  $\varepsilon$  is the measurement strength (operationally defined from fringe visibility). At  $\varepsilon = 0$ , full interference; at  $\varepsilon = 1$ , classical mixture with Born probabilities<sup>22</sup>.

## D. Hawking-Landauer verification

**Prediction 4:** The total energy radiated during complete black hole evaporation equals  $Mc^2$ , with the energy per bit matching  $k_B T_H \ln 2$  across the full evaporation trajectory. Ref.<sup>23</sup> verifies this dynamically and proposes the Landauer Ratio  $R_L$  for analog gravity tests in BEC experiments.

## E. Black hole classical-record inaccessibility

**Prediction 5:** For exterior observers, classical records absorbed by black holes become operationally inaccessible. This is compatible with unitarity and Page-curve behavior: the global quantum state may remain pure even if the exterior thermodynamic description is approximately thermal. Ref.<sup>23</sup> verifies the Landauer saturation dynamically across the Kerr-Newman parameter space.

## VIII. LIMITATIONS AND OPEN PROBLEMS

- **Operational conditions for Landauer bounds:** Ref.<sup>21</sup> provides the three-stage taxonomy and six operational conditions C1 to C6 that address critiques of over-application of Landauer’s principle<sup>8</sup>.
- **Experimental feasibility:** Measurement calorimetry at the  $10^{-25}$  J scale (10 mK) is technologically demanding. Ref.<sup>21</sup> provides the detailed protocol and sensitivity analysis.
- **Born rule scope and conditionality:** The Born rule result is conditional on standard Hilbert-space kinematics and record-formation in the Landauer-applicable regime. Whether the thermodynamic axioms extend to

general POVMs (as Gleason’s result does via Busch<sup>20</sup>) is an open question.

- **Black-hole accounting interpretation:** Ref.<sup>23</sup> verifies the accounting dynamically across the full evaporation trajectory and across the Kerr-Newman parameter space, but clarifying what (if anything) the accounting implies about localizable energy splits remains open.
- **Outcome selection:** The framework provides thermodynamic *conditions* for when a classical record exists and *costs* for creating one. It does not explain why one outcome occurs rather than another, or what physical process selects the outcome. This is the hard measurement problem, and the present work does not solve it.
- **Connection between the three results:** The coherence of Papers A, B, and C is organizational, not deductive. There is no theorem that derives the black hole accounting from the measurement bound, or the Born rule from Landauer’s principle alone. The papers share themes (information, thermodynamics, Landauer) and are mutually consistent, but each result stands on its own axioms.

## IX. CONCLUSION

We have developed a thermodynamic framework connecting quantum measurement, the Born rule, and black hole information through a single principle: the irreversible formation of a classical record from a quantum superposition is a thermodynamic event with a quantifiable cost.

The three results are:

- A conditional record-formation heat bound  $\langle Q_{\text{rec}} \rangle \geq k_B T \ln 2 \cdot I(X;Y)$  with a three-stage taxonomy and six operational conditions, validated by a Lindblad master equation simulation and designed as a falsifiable calorimetry experiment<sup>21</sup>.
- A uniqueness proof that the Born rule  $P = |\alpha|^2$  is the only phase-independent probability rule consistent with  $L^2$  normalization, using one thermodynamic axiom and three structural axioms<sup>22</sup>.
- An information and energy accounting framework for black hole thermodynamics, organizing the Smarr identity into a measurement/evaporation complement, verified dynamically across the full evaporation trajectory and the Kerr-Newman parameter space<sup>23</sup>.

Each companion paper includes a supplementary simulation pipeline that verifies every computable claim. Paper A’s simulation validates the record-formation bound via Lindblad master equation in a circuit-QED architecture, confirming that dissipated heat tracks  $k_B T \ln 2 \cdot I(X;Y)$  across operating points (54 checks). Paper B’s simulation verifies the Born rule uniqueness proof numerically, including  $L^p$  counterfactual analysis and alternative rule falsification across random

high-dimensional states (82 checks). Paper C’s simulation verifies Landauer saturation dynamically via RK4 integration across the full evaporation trajectory, tests counterfactual  $S \propto M^k$  scaling for  $k = 1, 2, 3, 4$ , sweeps the Kerr and Reissner-Nordström parameter spaces, and computes the Page curve using Page’s exact random-matrix formula (54 checks). All supplementary code and precomputed outputs are archived with their respective companion papers.

The framework does not modify the Schrödinger equation, Einstein’s equations, or any standard dynamics. It provides an accounting perspective that organizes known thermodynamic identities and distinguishes coarse-grained classical records (operationally inaccessible) from fine-grained quantum information (potentially preserved).

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- <sup>4</sup>M. H. Mohammady and F. Buscemi, “Thermodynamic trilemma of efficient quantum measurements,” *Quantum* **9**, 1250 (2025).
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