

## Designing Problem-Based Situations in Teaching The Topic “Sequences – Arithmetic Progressions – Geometric Progressions” to Develop Mathematical Problem-Solving Competence for Grade 11th Students

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**ABSTRACT:** In the context of general education reform oriented toward the development of learners’ qualities and competencies, fostering mathematical problem-solving competence for students has become an essential requirement. This paper investigates the design and organization of problem-based situations in teaching the topic “Sequences – Arithmetic Progressions – Geometric Progressions” with the aim of developing mathematical problem-solving competence for Grade 11 students in accordance with the 2018 General Education Curriculum. Based on an analysis of theoretical perspectives on problem-based situations and mathematical problem-solving competence, the study clarifies the objectives and characteristics of the topic within the Grade 11 mathematics curriculum. A qualitative research approach is employed through document analysis, literature review, and examination of the current mathematics curriculum. The research proposes a process for designing and organizing problem-based situations in teaching sequences, arithmetic progressions, and geometric progressions, accompanied by a specific illustrative example. The findings provide practical references to support high school mathematics teachers in innovating teaching methods toward a competency-based approach.

**KEYWORDS:** Mathematical problem solving; Problem-based teaching; Sequences; Arithmetic and geometric progressions; Grade 11 mathematics

### I. INTRODUCTION

The demands placed on contemporary education go beyond the mere transmission of knowledge and extend to the training of human resources capable of adapting to rapid changes in society and the labor market. This requires learners to develop autonomy, responsibility, and the ability to analyze and solve complex real-life problems.

The General Education Curriculum for Mathematics identifies mathematical problem-solving competence as one of the five core subject-specific competencies that need to be developed in students. This highlights the importance of creating learning situations and opportunities that foster this competence, particularly at the upper secondary level. Accordingly, developing students’ mathematical problem-solving competence is considered an urgent task, contributing to equipping learners with effective learning methods, modes of thinking, and independent and creative problem-solving strategies through mathematics learning.

The topic “Sequences – Arithmetic Progressions – Geometric Progressions,” allocated seven periods in the Grade 11 Mathematics curriculum, plays a significant role in upper secondary mathematics education. Although the basic concepts are not unfamiliar to students, the topic involves increasingly advanced content that requires flexibility, accuracy, and rapid reasoning. Moreover, applications of these concepts in real-life contexts are often abstract in nature. Therefore, this topic offers considerable potential for developing students’ mathematical problem-solving competence.

International assessments such as PISA conducted by the OECD emphasize the importance of problem-solving competence in learning. According to the PISA 2018 Results, mathematical literacy is defined as an individual’s capacity to formulate, employ, and interpret mathematics in various contexts, enabling informed judgments and decisions in real life.

Several international studies recommend connecting sequences and progressions to real-world models such as population growth, finance, and biology to help students understand that exponential or linear growth is not merely a formula but a process reflecting real phenomena.

In Vietnam, Nguyen Canh Toan (1996) emphasized that problem solving represents the highest expression of intellectual activity and serves both as a goal and a means for developing creative mathematical thinking. Nguyen Ba Kim (2000) argued that to foster students’ thinking capacity, teaching must place learners in problem-based situations that encourage exploration and multiple

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solution strategies. Other studies have also highlighted the effectiveness of problem-based teaching in developing independent thinking and problem-solving competence.

However, in practice, teaching the topic of sequences and progressions often focuses primarily on theoretical exposition and procedural exercises, while systematic and well-grounded problem-based situations aimed at competence development remain limited. From this reality, the present study proposes a process for designing problem-based situations in teaching this topic to develop mathematical problem-solving competence for Grade 11 students, thereby enhancing the effectiveness of mathematics teaching under the 2018 curriculum.

## II. RESEARCH METHODS

This study adopts a qualitative research approach, focusing on reviewing and analyzing relevant scientific literature and theoretical foundations related to developing mathematical problem-solving competence in high school mathematics teaching. In addition, the Grade 11 Mathematics Curriculum is examined to design illustrative problem-based situations.

## III. RESEARCH RESULTS

### 3. Research Results

#### 3.1. Mathematical Problem-Solving Competence

##### 3.1.1. Problem-Based Situations

According to Nguyen Ba Kim, a situation is understood as a complex system involving a subject and an object. When the subject does not yet know at least one element of the object, the situation becomes a problem situation. A task is considered a problem if the subject does not have an available algorithm to find the unknown element.

##### 3.1.2. Mathematical Problem-Solving Competence

According to Nguyen Huy Thao and colleagues (2024), mathematical problem-solving competence is defined as the learner’s ability to mobilize previously acquired mathematical experiences, knowledge, and skills to deal with a specific mathematical problem.

From a similar perspective, Phan Van Ly and Trinh Thi The (2023) also emphasize the application of existing experience, knowledge, and skills to solve mathematical problems, while highlighting the requirement for learners to demonstrate a positive attitude during the problem-solving process.

Mai Thi Thu Huyen and Dinh Thanh Tuan (2022) affirm that mathematical problem-solving competence is manifested in the ability to solve a mathematical problem effectively based on the use of available knowledge, experience, and skills.

From a structural viewpoint, Nguyen Ngoc Ha and Nguyen Van Thai Binh (2020) argue that mathematical problem-solving competence is a combination of competencies expressed through skills in learning activities aimed at effectively accomplishing mathematical tasks. They also point out that mathematics provides favorable conditions for developing this competence through concept acquisition, theorem proving, and especially problem-solving activities [10].

Rajadurai Rajkumar and Ganapathi Hema (2019) broaden the scope by linking mathematical problem-solving competence to real-life contexts. According to these authors, the competence includes the ability to handle real-world problems, transform problem-solving approaches through cognition and technology, and apply cognitive skills such as reasoning and logical thinking to find solutions.

Niss (2003) approaches this competence as the ability to pose and solve various types of mathematical problems across different content domains, as well as the ability to critically analyze and evaluate one’s own solutions and those of others.

Based on the perspectives above, this study adopts the definition of mathematical problem-solving competence in accordance with the 2018 Mathematics General Education Curriculum issued by the Ministry of Education and Training. Accordingly, mathematical problem-solving competence is understood as an individual’s ability to effectively employ cognitive and action processes, while mobilizing factors such as attitudes, motivation, and emotions to solve problem situations in mathematics for which no standard procedures or conventional solutions are readily available.

In this study, mathematical problem-solving competence for high school students through teaching the topic “Sequences – Arithmetic Progressions – Geometric Progressions” is identified and specified as follows:

**Table 1. Manifestations of Mathematical Problem-Solving Competence in the Topic “Sequences – Arithmetic Progressions – Geometric Progressions”**

Core components	Manifestations of mathematical problem-solving competence in the topic “Sequences – Arithmetic Progressions – Geometric Progressions”
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Identifying and recognizing the problem to be addressed using mathematical tools and methods	<input type="checkbox"/> Accurately identifying given data: first term, common difference $d$ , common ratio $q$ , general term, and sum. <input type="checkbox"/> Recognizing whether the problem involves an arithmetic progression or a geometric progression. <input type="checkbox"/> Restating the problem using diagrams, tables, or clear verbal descriptions. <input type="checkbox"/> Clearly determining the required quantities (terms, sum, number of terms $n$ , or solutions of equations).
Selecting and proposing appropriate approaches and methods to solve the problem	<input type="checkbox"/> Choosing suitable formulas (general term and sum formulas for arithmetic and geometric progressions). <input type="checkbox"/> Proposing methods such as formula-based modeling, step-by-step tabulation, or backward reasoning. <input type="checkbox"/> Explaining the rationale for selecting a method (efficiency, accuracy, ease of verification). <input type="checkbox"/> Proposing alternative solutions when the initial approach is ineffective.
Mobilizing and applying appropriate mathematical knowledge and skills (including tools and algorithms) to solve the given problem	<input type="checkbox"/> Proficiently applying formulas for general terms and sums of arithmetic and geometric progressions. <input type="checkbox"/> Accurately performing algebraic transformations, including solving equations and inequalities and problems involving logarithms. <input type="checkbox"/> Connecting to related knowledge (limits, logarithms, quadratic equations) when necessary. <input type="checkbox"/> Using supporting tools (calculators, spreadsheets).
Examining and evaluating the reasonableness of the chosen solution and generalizing it to similar problems	<input type="checkbox"/> Verifying results by substitution or testing neighboring values ( $n - 1$ , $n + 1$ ). <input type="checkbox"/> Comparing multiple solution methods to confirm correctness. <input type="checkbox"/> Commenting on the practicality and reasonableness of the results (e.g., number of seats per row, amount of savings). <input type="checkbox"/> Proposing variations or generalizations to similar situations.

### 3.1.3. Role of Mathematical Problem-Solving Competence

Mathematical problem-solving competence plays a particularly important role in mathematics teaching, not only by contributing to the development of higher-order thinking skills but also by helping students connect mathematical knowledge with real-life contexts, enhancing learning motivation, and fostering essential qualities required in the modern era.

The role of mathematical problem-solving competence includes:

- Contributing to the formation and development of higher-order mathematical thinking;
- Helping students apply mathematical knowledge to real-life situations;
- Stimulating learning interest and fostering positive learning motivation;
- Developing essential learning skills and cultivating important learner qualities;
- Aligning with the orientation of competency-based education in the new General Education Curriculum.

## 3.2. Content of the Topic “Sequences – Arithmetic Progressions – Geometric Progressions” in the Vietnamese Mathematics Curriculum

### 3.2.1. Objectives

Through learning the topic “Sequences – Arithmetic Progressions – Geometric Progressions,” students are expected to achieve the following objectives:

- Master fundamental concepts of sequences, arithmetic progressions, and geometric progressions.
- Understand and apply formulas for determining general terms and the sums of initial terms of progressions.

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- Apply acquired knowledge to solve mathematical problems and handle situations involving real-life contexts.
- Form and develop mathematical problem-solving competence through problem identification, selection of appropriate methods, and evaluation of results.

### 3.2.2. Structure and Content

The topic “Sequences – Arithmetic Progressions – Geometric Progressions” is presented in Chapter II of the Grade 11 Mathematics textbook (Volume 1). The content primarily focuses on concept formation, formula establishment, and application practice. The lessons are arranged from general to specific, enabling students to gradually approach and master the knowledge.

**Table 1.3. Content of the Topic “Sequences – Arithmetic Progressions – Geometric Progressions”**

Lesson	Periods	Knowledge and Skills
Lesson 5. Sequences	2	<ul style="list-style-type: none"> <li>- Definition of a sequence and terms of a sequence.</li> <li>- Methods for defining sequences by formulas or by verbal description.</li> <li>- Identifying the formation rules of sequences through specific examples.</li> <li>- Developing skills in determining terms of a sequence in basic problem types.</li> </ul>
Lesson 6. Arithmetic Progressions	2	<ul style="list-style-type: none"> <li>- Definition of an arithmetic progression and common difference.</li> <li>- Applying learned formulas to solve mathematical problems and some problems related to real-life contexts.</li> </ul>
Lesson 7. Geometric Progressions	2	<ul style="list-style-type: none"> <li>- Definition of a geometric progression and common ratio.</li> <li>- Formula for the general term of a geometric progression.</li> <li>- Formula for the sum of the first <math>n</math> terms of a geometric progression.</li> <li>- Applying the theory of geometric progressions to solve problems related to growth, interest rates, decay, and similar phenomena.</li> </ul>
Exercises at the end of Chapter II	1	<ul style="list-style-type: none"> <li>- Consolidating definitions and formulas related to sequences, arithmetic progressions, and geometric progressions.</li> <li>- Integrating knowledge to solve problems with diverse content.</li> <li>- Developing skills in selecting and applying appropriate solution methods.</li> </ul>

From the analysis above, the content of the topic is designed with streamlined theoretical knowledge, emphasizing concept formation, formulas, and application skills. This structure provides favorable conditions for teachers to design and implement learning activities that encourage students to identify problems and actively seek solutions. However, the textbook lessons mainly focus on direct application of formulas; therefore, teachers need to proactively design additional problem-based situations to promote student engagement and develop mathematical problem-solving competence.

### 3.3. Process of Designing Problem-Based Situations in Teaching the Topic “Sequences – Arithmetic Progressions – Geometric Progressions” Toward Developing Mathematical Problem-Solving Competence

#### 3.3.1. Design and Implementation Process

Designing problem-based situations not only enhances the effectiveness of mathematical knowledge acquisition but also serves as an important means of developing students’ problem-solving competence. For the topic “Sequences – Arithmetic Progressions – Geometric Progressions,” organizing learning activities through practical and cognitively stimulating situations helps students become more interested in the content, actively identify connections between mathematical knowledge and real-life contexts, and thereby develop core competence components such as problem identification, tool selection, mathematical modeling, problem solving, and solution evaluation.

#### Step 1: Identifying lesson objectives and core problems

The topic possesses characteristics such as high generalizability, clear developmental patterns, and strong connections to real-life phenomena, particularly in finance, technology, and biology. Teachers may select content such as additive or multiplicative patterns in sequences, general term formulas, sum calculations, or growth and decay patterns as starting points for designing learning situations.

Examples:

- Periodic saving situations (related to arithmetic progressions);
- Bacterial cell division situations (related to geometric progressions);
- Gift distribution situations with increasing quantities (related to general sequences).

#### Step 2: Selecting and designing appropriate problem-based situations

Teachers must ensure that the designed situations:

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- Create cognitive conflict and motivate exploration of new knowledge;
- Involve real-life contexts appropriate to students’ level and experience;
- Closely align with the core content of the lesson;
- Can be implemented as active, autonomous, and collaborative learning activities.

After selecting suitable situations, teachers should design detailed classroom activities that create opportunities for students to actively participate, explore, and experience learning. Possible implementation forms include: (i) Real-life situations simulated through images or videos;

(ii) Verbal descriptions of real-life situations; (iii) Experiential situations through games or group activities.

### **Step 3: Guiding students in problem solving**

After presenting the situation, teachers organize students to work individually or in small groups to analyze data, identify the underlying mathematical problem, construct a mathematical model, and solve the problem. During this process, teachers may use guiding questions such as:

- What sequence is formed in this situation?
- What is the pattern of the sequence?
- Which formula can be used to solve the problem?
- Are there multiple solution methods? Which one is more concise?

Student discussion, argumentation, and presentation not only reinforce knowledge but also foster mathematical communication competence and critical thinking.

### **Step 4: Evaluation and adjustment**

After students complete the problem-solving process, teachers guide them to generalize patterns and establish related mathematical formulas. This stage involves abstracting from specific situations to systematic mathematical knowledge. Teachers also help students reflect on the problem-solving process, evaluate results, and compare different solution approaches.

#### **3.3.2. Illustrative Example**

### **A verbally described problem-based situation for the topic of Arithmetic Progressions**

#### **Step 1: Identifying lesson objectives and core problems**

Objectives:

- Recognize a constant increase pattern in a real-life situation;
- Form the concept of an arithmetic progression and determine its common difference.

Competencies:

- Mathematical communication competence: verbally expressing patterns and participating in group discussions;
- Mathematical problem-solving competence: analyzing given data and identifying sequence patterns;
- Self-directed learning competence: independently reasoning and completing assigned tasks;
- Mathematical reasoning competence: identifying and explaining relationships among terms.

Core problems:

- The constant growth pattern of a quantity;
- The relationship between a real-life situation and an arithmetic progression.

#### **Step 2: Selecting and designing the problem-based situation**

Teaching situation (verbal description):

In a cinema, the first row has 10 seats. Each subsequent row has 2 more seats than the previous one.

Guiding questions:

1. How does the number of seats change between consecutive rows?
2. Does this increase remain constant across rows?
3. Can this change be represented by a sequence?

#### **Step 3: Guiding students in problem solving**

- Teacher assigns learning tasks:
  - Students work in groups of 3–4.
  - Record the number of seats in the first rows and identify the pattern.
- Students perform tasks:
  - List: 10, 12, 14, 16, ...
  - Identify the pattern: each term increases by a constant amount.
- Reporting and discussion:
  - Group representatives present results.
  - Teacher guides students toward the concept of an arithmetic progression.

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- Conclusion:
  - Students recognize the constant increase pattern.
  - Students develop the need to generalize using arithmetic progression knowledge.

### Step 4: Evaluation and adjustment

- Teacher evaluates students’ ability to identify patterns.
- Adjusts the increment to suit student levels.
- Extends the situation by changing the initial number of seats.

## IV. CONCLUSIONS

In the context of fundamental and comprehensive educational reform oriented toward the development of learners’ qualities and competencies, placing emphasis on the formation and development of mathematical problem-solving competence for upper secondary school students is an inevitable requirement. The topic “Sequences – Arithmetic Progressions – Geometric Progressions” in the Grade 11 Mathematics curriculum, characterized by clear patterns, high potential for generalization, and numerous real-life applications, provides favorable conditions for organizing teaching activities in this direction. The study has contributed to clarifying the theoretical foundations of problem-based situations and mathematical problem-solving competence, while also specifying the manifestations of this competence within the scope of the selected topic.

Based on an analysis of curriculum content and current teaching practice, the paper proposes a process for designing and organizing problem-based situations consisting of fundamental steps, including identifying learning objectives, selecting appropriate situations, guiding students in problem solving, and evaluating and adjusting learning outcomes. This process not only ensures alignment with mathematical content but also creates opportunities for students to actively and creatively experience the processes of problem identification, analysis, and solution. The illustrative example in teaching arithmetic progressions demonstrates that problem-based situations grounded in real-life contexts can effectively stimulate learning interest, help students gain a deeper understanding of the nature of mathematical knowledge, and foster essential mathematical thinking skills.

The research findings have practical significance in supporting high school mathematics teachers in designing teaching activities oriented toward competency development, thereby contributing to improving the effectiveness of teaching the topic “Sequences – Arithmetic Progressions – Geometric Progressions.” In the future, further empirical pedagogical studies are needed to verify the effectiveness of the proposed process in diverse teaching contexts.

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