Cosmos Automaton: A Deterministic Fractal Automaton Generating Primes

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Abstract: This paper introduces the "Cosmos Automaton" (CA), a deterministic system that generates prime numbers through iterative morphisms rather than arithmetic primality testing. Formalized as a non-stationary S-adic substitution system, the CA generates its own directive sequence endogenously through an internal feedback loop. Analysis of the incidence matrix M_p reveals a recursive growth factor of p-2 for twin prime templates, matching OEIS A059861 and the combinatorial core of the Hardy-Littlewood k-tuple conjecture. The model defines a linear Stability Zone that ensures pointwise convergence to a unique aperiodic limit-word, forming a Cantor-like set of measure zero termed "Heeren Dust." This framework provides a rigorous algorithmic bridge between automata theory and the structural emergence of primes.

Keywords: Prime numbers; Sieve of Eratosthenes; S-adic systems; Substitution morphisms; Symbolic dynamics; Fractal dimension; Twin Prime Conjecture; Deterministic automata.

1 Introduction

This work treats prime generation as an automaton-theoretic and algorithmic process rather than as a purely arithmetic predicate. We introduce the Cosmos Automaton, a deterministic fractal automaton that generates primes via automaton operations, without checking on divisibility. The CA is a formal implementation of a non-stationary S-adic system, providing a generative framework to analyze the structural emergence of the prime sequence.

1.1 Content

3. Algorithms of Cosmos Automaton and Eratosthenes Sieve
4. Foundation of Primality
5. Cosmos Automaton Sieves versus Eratosthenes Sieve
6. Gaps and Consecutive Primes
7. Stability Zone
8. Experiment “Frozen Window”
9. Fractal Dimension

2 Methodology

2.1 Alphabet of Symbols

The Natural Numbers are indexed by the following letters:

- P for prime,
- M (multiple) for a composite, and
- L (live) for all Natural Numbers (prime, composite or 1).
- ONE means the number 1 which is neither a composite nor a prime.

2.2 Construction plan

The Cosmos Automaton (CA) has two registers and one tape (Figure 2.2_1).

- Register N has one square and contains the step-number n.
- Register BP has one square and contains the *encoding* of step-number n.
- Tape CP starts with one square and grows new squares. The first square is the leftmost square and is denoted CP[1]. The last square is the rightmost square and is denoted CP[last]. The tape starts with CP having only one square thus $CP_1[1] = CP_1[last]$. The index is the number contained in N. New squares are only *added to the right end* of the tape, thus to CP[last]. CP contains a *periodic* word of symbols. Periodic in this context means strictly periodic (exactly repeating with no drift) from the start, like a rolling stamp.

The symbols / word are written with angle brackets.

2.3 Size of tape CP

The CA is designed to elucidate the distribution of prime numbers. It is NOT meant to produce large or large numbers of primes. This is because the size of CP (figure 8_1) increases with prime primorial (p_n#). p_n# is the least common multiple of all prime numbers ≤ step-number n. Cosmos Automaton is important in its *theoretical* operation.

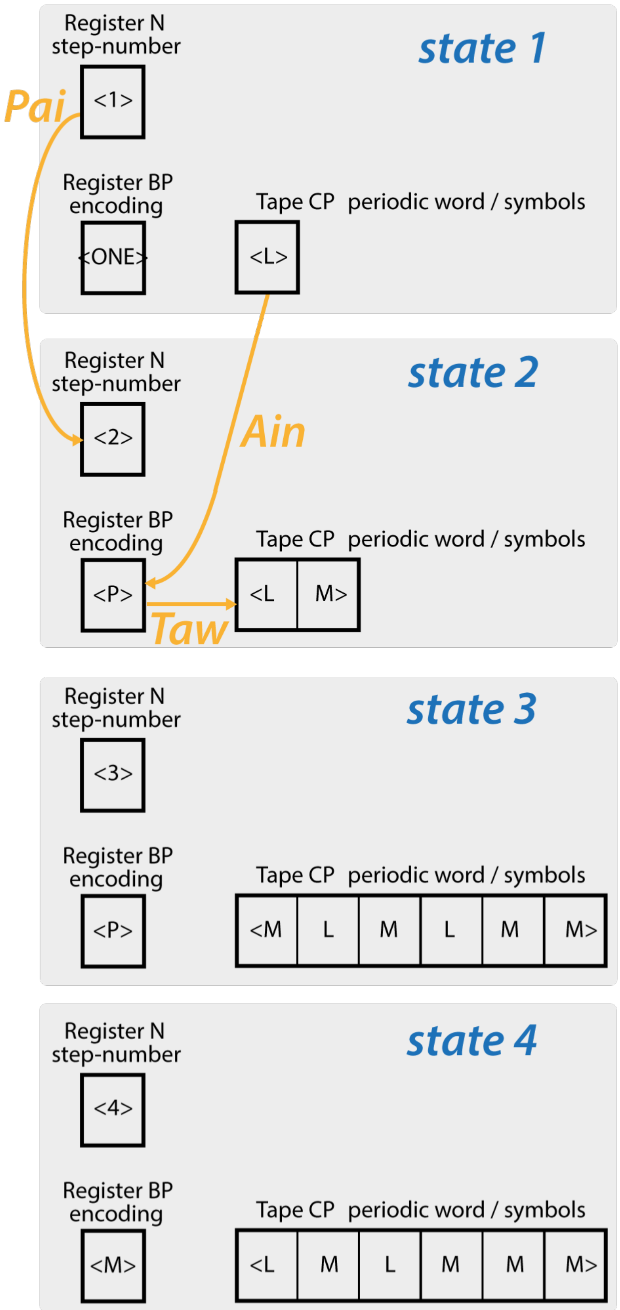

Figure 2.2_1 Dynamic development of Cosmos Automaton

2.4 Operation

The operations were named after Hebrew letters (Pai, Ain, Taw, …) for their brevity and aesthetic appeal. The *sequence* of operations is Pai, Ain, and Taw. And Taw consists of three fractal procedures in the sequence Resch, Bet, and Mem.

- **Pai**: Pai is a *plus 1 operation* for register N. There is no input into the automaton of the natural numbers as a set. We follow Peano (1889) [1]. We start with the number 1 in register N and the successors are calculated by the plus 1 operation:

$$N_n = N_{n-1} + 1 \tag{1}$$

Definition 1

The Cosmos Automaton starts at $N_n = 1$ and does $N_n = N_{n-1} + 1$, therefore the mapping between the states of register N and the natural numbers is a bijection (identity mapping), establishing a structural isomorphism between the automaton's progression and the set of natural numbers $\mathbb{N}$.

- **Ain**: Ain is the encoding in register BP. The encoding is defined by:

$$BP_n = Ain_n = \begin{Bmatrix} <P> if\ CP_{n-1}[1] = <L> \\ <M> if\ CP_{n-1}[1] = <M> \end{Bmatrix} \tag{2}$$

Design Principle 1

Without the traditional definition of primeness "a number that has no other divisor than 1 and itself" we construct primeness as a result of CA operations.

- **Taw**: Taw consists of three fractal procedures on tape CP:

$$CP_n = Taw_n = \begin{Bmatrix} Mem_n(Bet_n(Resch_n(CP_{n-1})))\, if\ BP = <P> \\ Resch_n\ (CP_{n-1})\, if\ BP = <M> \end{Bmatrix} \tag{3}$$

- **Resch**: Resch means remove the first square with symbol CP[1] and append that at the right end of the tape, thus the symbol becomes CP[last].

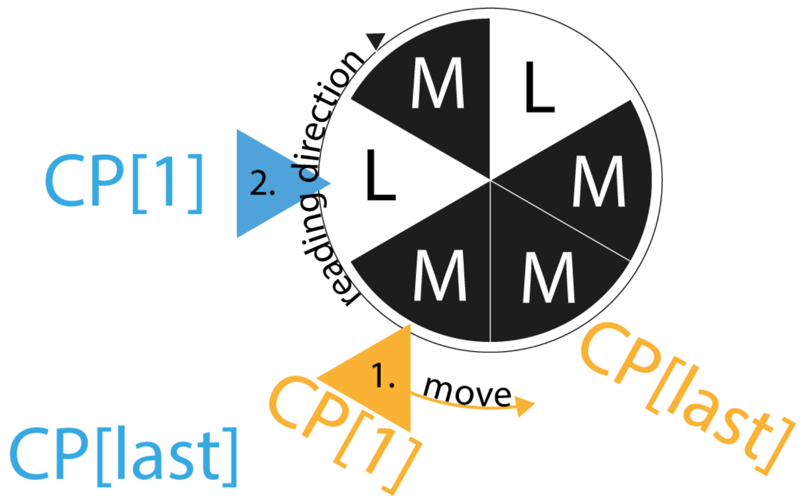

Figure 2.4_1 CP is periodical and can be displayed as a circle

Invariance Property 1

The shift of Resch keeps the symbols of CP in lockstep with N changed by Pai.

- **Bet**: Bet means copying the full tape CP and appending squares with symbols n – 1 times to the right end of the tape. Thus the tape CP is always of width prime primorial (p_n#).

Invariance Property 2

Operation Bet ensures that the structure (gaps, lumps) of the L symbols is always distributed self-similarly (fractal) over CP (left, middle, right). By construction plan, the word in CP is periodic, thus instead of displaying just one period <LM>, we can also write: (LM LM LM … ∞). Thus doing Bet just changes the periodicity but not the positions of L's and M's: (LMLMLM LMLMLM LMLMLM … ∞).

- **Mem**: Mem strides over tape CP with size n and turns any L's hit to M. By construction plan, the word in CP is periodic and it acts as a rolling stamp. Turning multiples of p in the rolling stamp from L to M turns all composites of p to M.

3 Algorithms of Cosmos Automaton and Eratosthenes Sieve

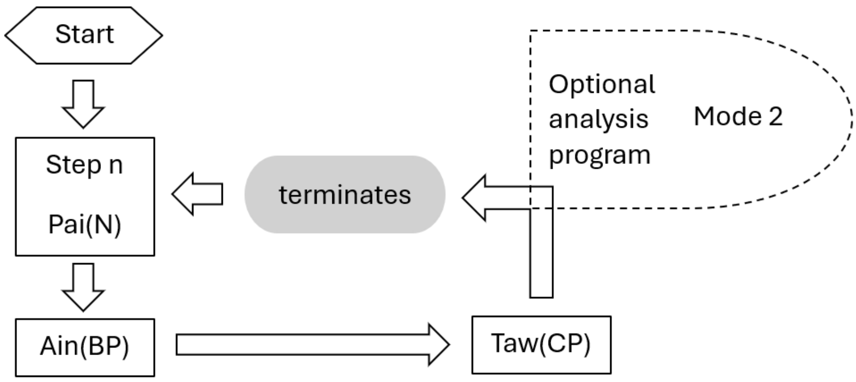

Figure 3_1 Cosmos Automaton Algorithm

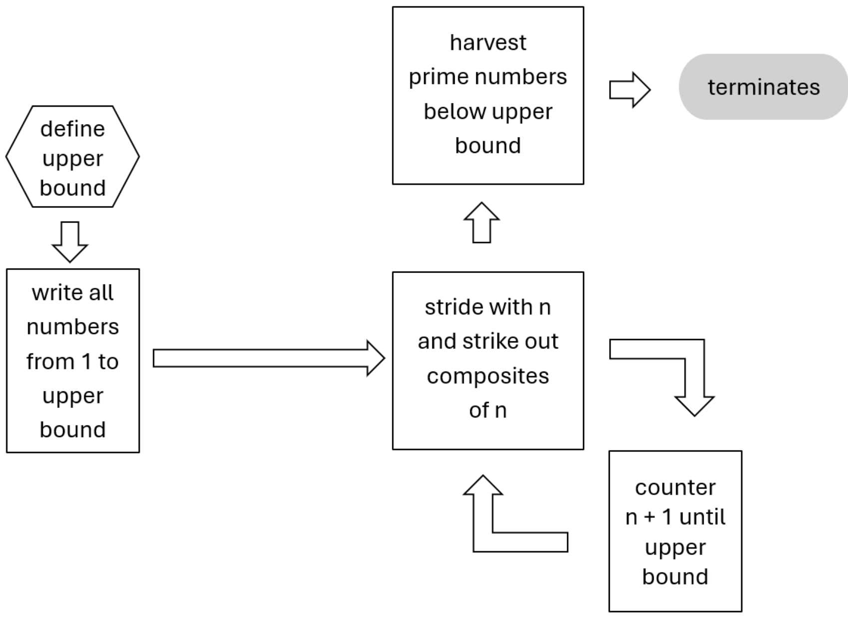

Figure 3_2 Eratosthenes Sieve Algorithm

3.1 Differences of the Algorithms (computer science theory and programming)

The Eratosthenes Sieve ES and the Cosmos Automaton CA are two different computational models. Neither is flawed, they have different characteristics. In the infinite limit (n → infinity), the CA and the ES reach a state of set-theoretic and topological identity. Both systems converge in the product topology to the same unique characteristic word of the primes. This isomorphism ensures that for every integer k, the final state as prime (L) or composite (M) is identical in both models.

The Eratosthenes Sieve is defined as a finite truncation process up to a bound N. By contrast, the Cosmos Automaton is defined as a step wise process with no global bound. The CA was developed to formalize the sifting process as a dynamical system, enabling the analysis of its spectral properties.

3.2 Optional analysis program

The CA is a recorder in mode 1 and has got a mode 2 with an optional analysis program. This will be used for an experiment.

3.3 First five runs of the CA

The Cosmos Automaton starts with an initial state and then inflates tape CP. CP is locally finite but globally unbounded. It represents a *potential infinity*: at no point is the tape actually infinite, yet its growth from run to run never ceases.

N_{start} = <1>
BP_{start} = <ONE>
CP_{start} = <L>

N = Pai(N) = <2>
BP = Ain(BP) = <P>
CP = Taw(CP) = <LM>

N = Pai(N) = <3>
BP = Ain(BP) = <P>
CP = Taw(CP) = <MLMLMM>

N = Pai(N) = <4>
BP = Ain(BP) = <M>
CP = Taw(CP) = <LMLMMM>

N = Pai(N) = <5>
BP = Ain(BP) = <P>
CP = Taw(CP) = <MLMMMLMLMMMLMLMMMLMMMMMLMLMMMM>

4 Foundation of Primality

The CA starts with a starting state and operates by Pai, Ain, Taw (Resch, Bet, Mem). That is it. There are *no external definitions* of primeness. Yet, we can express the tape CP with equations and get the traditional definition of primeness as a result.

The Cosmos Automaton operates on a strict *feedback loop*: The structural state of the pattern tape CP determines the qualitative property of the number N, while the number value and its qualitative property BP conversely control the modulation and geometric expansion of the structure CP (figure 2.2_1). This self-referential cycle drives the emergent complexity of prime distribution.

The symbols ONE, P and M denote qualitative properties and therefore three sets of numbers grow from the operation of the Cosmos Automaton, without prior knowledge of composite or primeness.

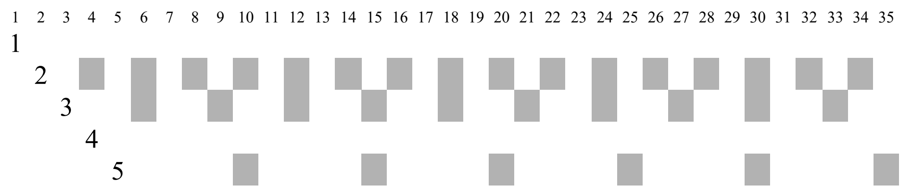

Figure 4_1 CP as periodic pulse trains, gray = M = periodic pulse

A periodic pulse train is a sequence of periodic pulses (figure 4_1). Period (T) is the time from the start of one pulse to the start of the next one. When periodic pulse trains t_1 and t_2 have constructive interference, then *period* T_{total} is the *product* of:

$$T_{total}=T_1 \cdot T_2 \tag{4}$$

The *pattern* is the *sum* of pattern 1 and pattern 2 (figure 4_2).

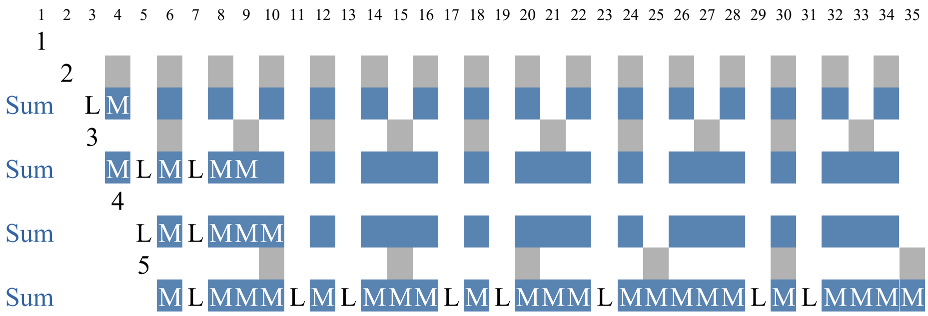

Figure 4_2 CP with constructive interference = blue

It is important to know, that we *do not* put any prior knowledge of prime numbers into the analysis of periodic pulse trains of the CA.

n	CP	encodes
1	<L>	{2,3, …, ∞}
2	<LM>	{3,4, …, ∞}
3	<MLMLMM>	{4,5,6,7,8,9, …, ∞}
4	<LMLMMM>	{5,6,7,8,9,10, …, ∞}

Figure 4_3 Output from the Cosmos Automaton

We needed to find equations that have the same output as the CA. We found the traditional definition of divisibility:

$$CP_1 = \langle \mathrm{L} \rangle \tag{5}$$

$$CP_{\mathrm{n>1}} = \sum_{k=n+1}^{n\#+n} f_{(k \bmod n)} \vee \prod_{j=2}^{n-1} g_{(k \bmod j)} \tag{6}$$

$$with \begin{Bmatrix} hit = M \; if \; f \vee g = 0 \\ no\,hit = L \; otherwise \end{Bmatrix} \tag{7}$$

n		k	f	j	g	$CP_{n>1}$	pattern
2	$\sum_{k=3}^{4} f_{(k\,mod\,n)} \vee \prod_{j=2}^{1} g_{(k\,mod\,j)}$	3	1			L	<LM>
		4	0			M	
3	$\sum_{k=4}^{9} f_{(k\,mod\,n)} \vee \prod_{j=2}^{2} g_{(k\,mod\,j)}$	4	1	2	0	M	<MLMLMM>
		5	2	2	1	L	
		6	0	2	0	M	
		7	1	2	1	L	
		8	2	2	0	M	
		9	0	2	1	M	
4	$\sum_{k=5}^{10} f_{(k\,mod\,n)} \vee \prod_{j=2}^{3} g_{(k\,mod\,j)}$	5	1	2	1	L	<LMLMMM>
				3	2		
		6	2	2	0	M	
				3	1		
		7	3	2	1	L	
				3	1		
		8	0	2	0	M	
				3	2		
		9	1	2	1	M	
				3	0		
		10	2	2	0	M	
				3	1		

Figure 4_4 Calculation of equations 6 and 7

5 Cosmos Automaton Sieves versus Eratosthenes Sieve

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Figure 5_1 Eratosthenes Sieve, gray = composite, white = prime

	BP			
	P	L	M	CP
N	2	3	4	head
		5	6	
		7	8	
		9	10	
		11	12	
		13	14	
		15	16	
		17	18	
		19	20	
		21	22	
		23	24	
		25	26	
		27	28	
		29	30	
		31	32	
		33	34	
		35	36	
		etc. → ∞		
		width=2		

	BP							
	P	M	L	M	L	M	M	CP
N	3	4	5	6	7	8	9	head
		10	11	12	13	14	15	
		16	17	18	19	20	21	
		22	23	24	25	26	27	
		28	29	30	31	32	33	
		34	35	36	37	38	39	
		40	41	42	43	44	45	
		46	47	48	49	50	51	
		52	53	54	55	56	57	
		58	59	60	61	62	63	
		64	65	66	67	68	69	
		70	71	72	73	74	75	
		76	77	78	79	80	81	
		82	83	84	85	86	87	
		88	89	90	91	92	93	
		94	95	96	97	98	99	
		100	101	102	103	104	105	
				etc. → ∞				
				width=6				

	BP							
	P	L	M	L	M	M	M	CP
N	4	5	6	7	8	9	10	head
		11	12	13	14	15	16	
		17	18	19	20	21	22	
		23	24	25	26	27	28	
		29	30	31	32	33	34	
		35	36	37	38	39	40	
		41	42	43	44	45	46	
		47	48	49	50	51	52	
		53	54	55	56	57	58	
		59	60	61	62	63	64	
		65	66	67	68	69	70	
		71	72	73	74	75	76	
		77	78	79	80	81	82	
		83	84	85	86	87	88	
		89	90	91	92	93	94	
		95	96	97	98	99	100	
		101	102	103	104	105	106	
				etc. → ∞				
				width=6				

	BP																															
	P	M	L	M	M	M	L	M	L	M	M	M	L	M	L	M	M	M	L	M	M	M	M	M	L	M	L	M	M	M	M	CP
N	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	head
		36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	
		66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	
		96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	
																etc. → ∞																
																width=30																

Figure 5_2 CA sieves, gray = composite, white = prime candidates

The Eratosthenes Sieve has been known for about 2300 years. It is attributed to Eratosthenes of Cyrene (c. 276–194 BC), as documented by Dickson [2]. Here is an example from number 2 to 120 (figure 5_1).

The Cosmos Automaton produces at every step one sieve (figure 5_2). Every CA Sieve consists of vertical columns, which can be described by linear equations. The *M-columns* are composite numbers only, the *L-columns* contain both, composite and prime numbers. Therefore L-columns are *envelopes* to prime numbers (figure 5_3) and L's are thus *prime candidates* until they reach CP[1] and are encoded in BP.

$$f_{(x)} = width \cdot x + head\,number \quad with\, x \in \mathbb{N}_0 \tag{8}$$

$f_{(x)} = 2x + 3$	$f_{(x)} = 30x + 11$	$f_{(x)} = 30x + 7$
$f_{(x)} = 6x + 5$	$f_{(x)} = 30x + 17$	$f_{(x)} = 30x + 13$
$f_{(x)} = 6x + 7$	$f_{(x)} = 30x + 23$	$f_{(x)} = 30x + 19$
	$f_{(x)} = 30x + 29$	$f_{(x)} = 30x + 31$

Figure 5_3 Envelope equations of CA Sieves L-columns

Design Principle 2

The L-columns of the Cosmos Automaton correspond to arithmetic progressions of the form f(x) = width · x + head (equation 8). According to Dirichlet's Theorem on Arithmetic Progressions (1837), such a sequence contains infinitely many primes, provided that the width and the head number are coprime [3].

6 Gaps and Consecutive Primes

6.1 Letters 'a', 'b', 'c', and 'd'

At step-number n = 4 we find this pattern in CP <LMLMMM>. We use this and define four letters:

a:= <LMLMMM> (9)

b:= <LMMMMM> (10)

c:= <MMLMMM> (11)

d:= <MMMMMM> (12)

This fundamental block of width 6 defines the structure of step n = 4 and for all subsequent steps of CP, thus n ≥ 4. How can letter 'a' change? As Taw acts on the symbols L and M, it also acts on the letters. In subsequent steps because the step size is larger n > 4, than the two L of letter 'a' are apart (they are 2 symbols apart) only one L of letter 'a' can be changed to M within one step (figure 6.1_1).

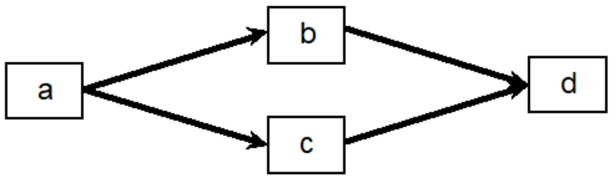

Figure 6.1_1 Transition rules of the four letters

6.2 Gaps

Letter 'd' and clusters of letter 'd' are prime gaps. As the CA is run again and again, all prime gaps in CP are multiplied (Bet) and become part of the next CA Sieve. Thus any prime gap that was ever discovered will appear repeatedly to infinity and can only become larger.

6.3 Three Consecutive Primes

It holds true that no matter in which sequence the four letters are arranged, it is impossible to build the pattern <LMLML> of three consecutive primes beyond the known triple 3, 5, and 7.

6.4 Twin Primes (NOT meant as a proof of the Twin Prime Conjecture)

In the CA symbolic framework, only letter 'a' carries the twin prime pattern <LML>. Thus only letter 'a' can produce actual twin primes in subsequent steps. We now address an exciting question: Can our Cosmos Automaton ever eliminate all twin prime templates (thus letter 'a')? We demonstrate that the automaton's recursive growth prevents this extinction through a mechanism we term the *Hydra Effect*. In contrast to the antic story of the Hydra, here the letter 'a' multiplies first (Bet) and afterwards letter 'a' is cut down (Mem), meaning 'a' transitions either to letter 'b' or 'c'.

Population Dynamics of Letter 'a'

Let $G_{\text{n (a)}}$ be the number (growth) of letter 'a' existing in the pattern CP_n. Be step n' the last prime step and step n the current prime step with new prime $n = p_n$. Then the Cosmos Automaton performs Taw: First Resch, then:

- **Growth** (Bet): The intermediate population of letter 'a' becomes:

$$G_{temp}=G_{\text{n (a)}}\cdot p_n \tag{13}$$

- **Elimination** (Mem): We must determine how many of these p_n clones are destroyed. For any specific letter 'a' at position x: The clones are located at

$$x+w, x+2w, \ldots \quad (where\, w=p_{\text{n'}}\#) \tag{14}$$

The condition for the new prime p_n to eliminate an L at relative position k is given by the congruence:

$$x+k\cdot w\equiv 0 (mod\ p_n) \tag{15}$$

Since p_n is a new prime, it is coprime to the previous CPn' width $p_{\text{n'}}\#$:

$$w(gcd(w, p_{\text{n'}})=1) \tag{16}$$

According to the Chinese Remainder Theorem, this linear congruence has exactly *one solution* for k in the range $[0, p_n - 1]$.

This implies a strict geometric rule, the "*left*" L of the letter 'a' is hit exactly *once* and the "*right*" L of the letter 'a' is hit exactly *once*. Since the distance between two L's of one twin is 2 and step size is ≥ 5, a single stride cannot hit both simultaneously.

Therefore, for every single template a from the previous step, *exactly 2 clones are destroyed* (converted to letters b and c) and $p_n - 2$ clones survive as 'a'.

The population of letters 'a' evolves according to the relation:

$$G_{\text{n (a)}}=|a|_n=|a|_{\text{n'}}\cdot(p_n-2) \tag{17}$$

We call this the *Hydra Equation*. It leads to a geometric growth of letter 'a' and therefore of twin prime candidates.

n	Growth	Elimination	total	CA count
4	1			
5	5	2	3	3
7	21	6	15	15
11	165	30	135	135
13	1755	270	1485	1485
17	25245	2970	22275	22275

Figure 6.4_1 Hydra calculation versus CA count of letter a

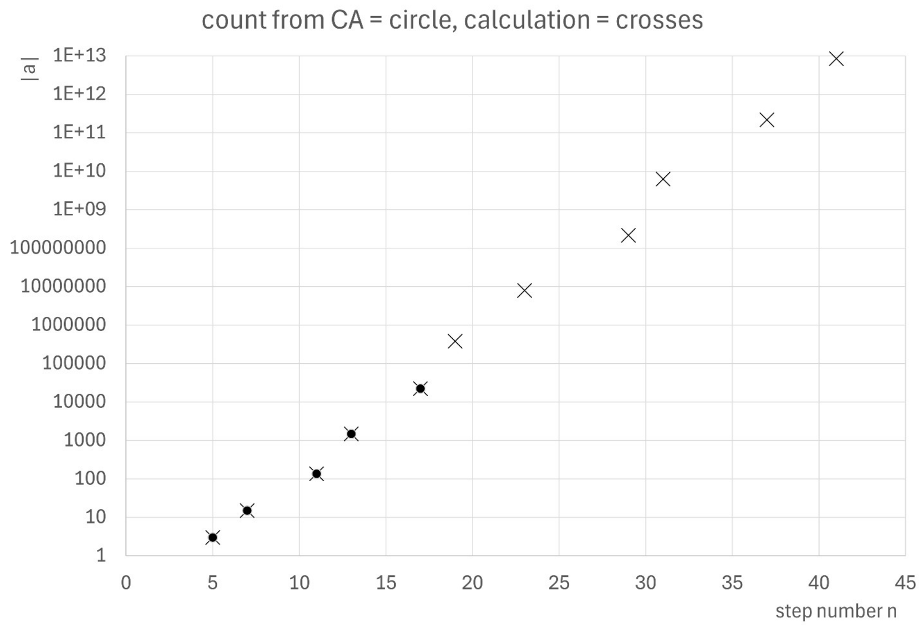

Figure 6.4_2 Hydra calculation versus CA count of letter a

This population dynamic is identical to the sequence A059861 in the OEIS, which describes the number of differences of size 2 in a reduced residue system modulo $p_n\#$. The alignment of our automaton's output with A059861 confirms that the Cosmos Automaton's "*Hydra Effect*" is a structural manifestation of the Hardy-Littlewood k-tuple conjecture logic.

6.5 Morphism in Symbolic Dynamics

The alphabet $\sum = \{a, b, c, d\}$ (18)

	Becomes a	Becomes b	Becomes c	Becomes d
a	$p-2$	1	1	0
b	0	$p-1$	0	1
c	0	0	$p-1$	1
d	0	0	0	p

Figure 6.5_1 Transition Matrix M_p

The substitution rules for the alphabet Σ are derived from the transition matrix M_p and are defined by the morphism σ_p as follows:

$$\sigma_p(a) = a^{p-2}bc \tag{19}$$

$$\sigma_p(b) = b^{p-1}d \tag{20}$$

$$\sigma_p(c) = c^{p-1}d \tag{21}$$

$$\sigma_p(d) = d^p \tag{22}$$

The population vector at step n' is

$$v_{n'} = (|a|, |b|, |c|, |d|) \tag{23}$$

The population at the next prime step n is obtained by

$$v_n = v_{n'} \cdot M_p \tag{24}$$

This matrix is an upper triangular matrix. The eigenvalues are the entries on the main diagonal.

$$M_p = \begin{pmatrix} p-2 & 1 & 1 & 0 \\ 0 & p-1 & 0 & 1 \\ 0 & 0 & p-1 & 1 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{25}$$

6.6 Normalized Transition Matrix and Relative Density

We analyze the density of the templates and normalize the transition matrix by dividing it by the growth factor p. The yields the Stochastic Matrix W_p, which describes the probability of the letters surviving relative to the expanding tape length.

$$W_p = \frac{1}{p} \cdot M_p = \begin{pmatrix} \frac{p-2}{p} & \frac{1}{p} & \frac{1}{p} & 0 \\ 0 & \frac{p-1}{p} & 0 & \frac{1}{p} \\ 0 & 0 & \frac{p-1}{p} & \frac{1}{p} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{26}$$

The relative density of letter d tends toward 1 (100%). This reflects the Prime Number Theorem: Almost all numbers are composites as $n \to \infty$.

The letters 'b' and 'c' decay slowly. The letter 'a' decays twice as fast, but the factor is strictly positive.

The density of twin prime templates at step n is given by the product of the eigenvalues of all previous steps:

$$Density(a)_n \propto \prod_{k=1}^{n} \left(1 - \frac{2}{p_k}\right) \tag{27}$$

This product diverges to 0 very slowly, proving that while letter 'a' becomes rare (density $\to$ 0), it persists structurally.

The matrix proves that the "deletion" of letter 'a' is never total. It is noteworthy that the transition probabilities for letters 'b' and 'c' are identical in W_p. The system does not favor one side over the other.

6.7 Parable

In our parable CP is like train tracks. The first piece of track is taken away and is added to the end. Therefore *the track moves with speed one*. Then they are being build in sections, that are copies of the track system already there. Along the train tracks there are stations. Small stations (single L) and large hubs (twin L). Whenever the small stations where build, one is shut down. This shut down station may be anywhere (CRT). Whenever the large hubs were build, two are shut down. These shut down stations may be anywhere (CRT). Some parts of the tracks have no stations, there is nothing to be shut down. Are there zones that are protected from shut downs?

7 Stability Zone

The Stability Zone (SZ) develops (figure 7_1), because each Mem does a first stride of size n (indicated in yellow) and therefore all L's inside the SZ (indicated in blue) are stable survivors, that will become prime numbers in future Ain.

<u>Invariance Property 3</u>

Since the Cosmos Automaton operates sequentially (strictly monotonic: 1, 2, …, n)

- Mem for prime < n have already been executed in a previous step.
- A prime is considered "generated", when it was processed by BP. No L can be processed more than once by BP.

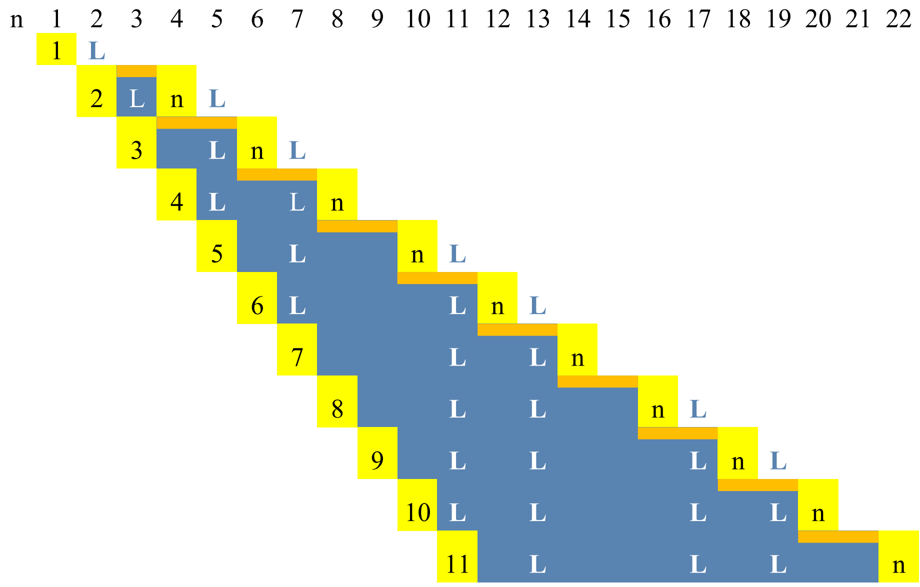

Figure 7_1 Stability Zone in blue with L's as stable survivors, advance of the SZ in orange

The SZ is defined as the interval:

$$SZ=[n+1,2n-1] \tag{28}$$

The Stability Zone advances from n = 1 to n = 2 one square (indicated in orange). As this is the start, there was no Taw (thus **no Resch**) before. **Pai** produces a +1 stride => +1 square.

<u>Invariance Property 4</u>

The Stability Zone advances from n = 2 to n = 3 *two squares* (indicated in orange). **Resch** produced +1 square. **Pai** produces a +1 stride => +1 square. This continues by Design of the Cosmos Automaton for all n ≥ 2.

Invariance Property 4 is essential, because without an advance of two squares no L's – with the exception of n = 2 – could ever enter the Stability Zone.

8 Experiment "Frozen Window"

To empirically verify the persistence of letters up to high step-numbers in the SZ, we encountered the limitation of primorial growth in CP. The full symbolic tape for n = 250000, thus 250000# would exceed the number of atoms in the universe. However, to analyze the Stability Zone, it is not necessary to store the entire CP tape. One only needs a pattern segment large enough to cover the interval [n + 1, 2n – 1].

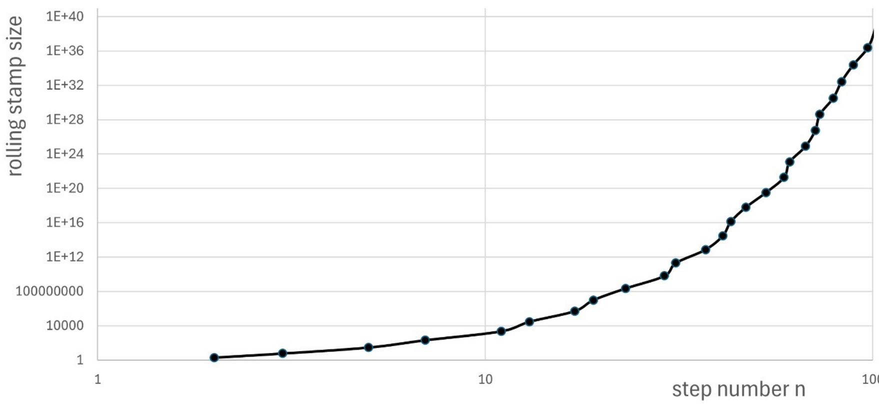

Figure 8_1 Size of CP versus n

For our target simulation of n = 250000 we need a window size of 2n = 500000. At step n = 18 the pattern-width is 510510 > 500000. This provides a sufficient size.

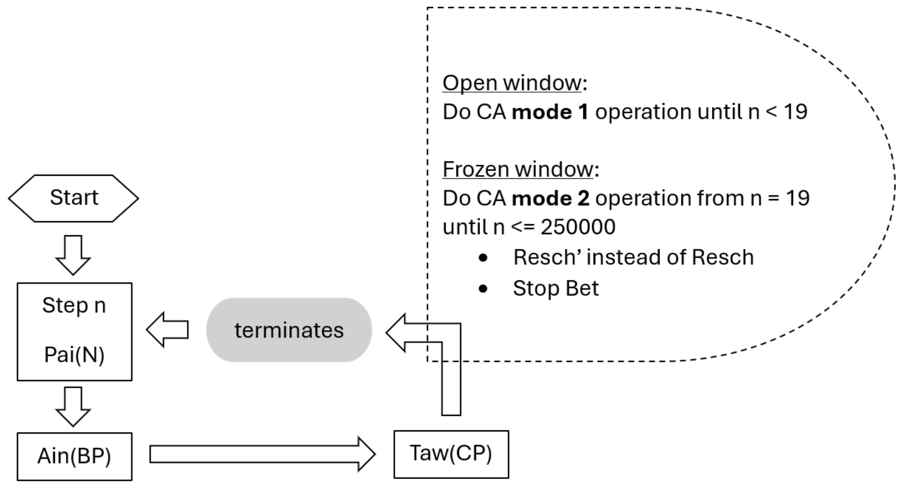

Figure 8_2 Pseudocode for mode 2 analysis "Frozen Window"

Resch: Remove the first square CP[1] and append it to the end of CP

Resch': Remove the first square CP[1]

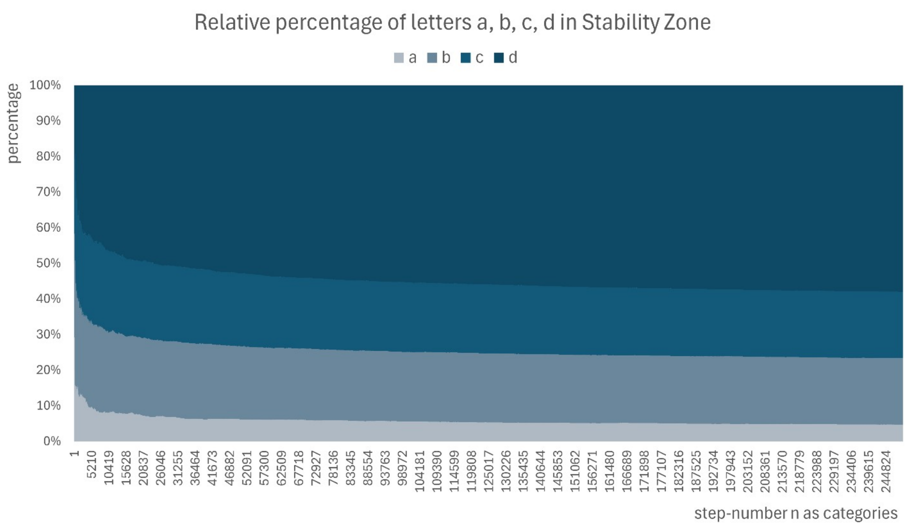

Figure 8_3 Percentage of letters

The result is a counter-intuitive perspective: While twin primes are known to be less frequent overall at higher numbers, their amount increases within the ever-enlarging Stability Zone.

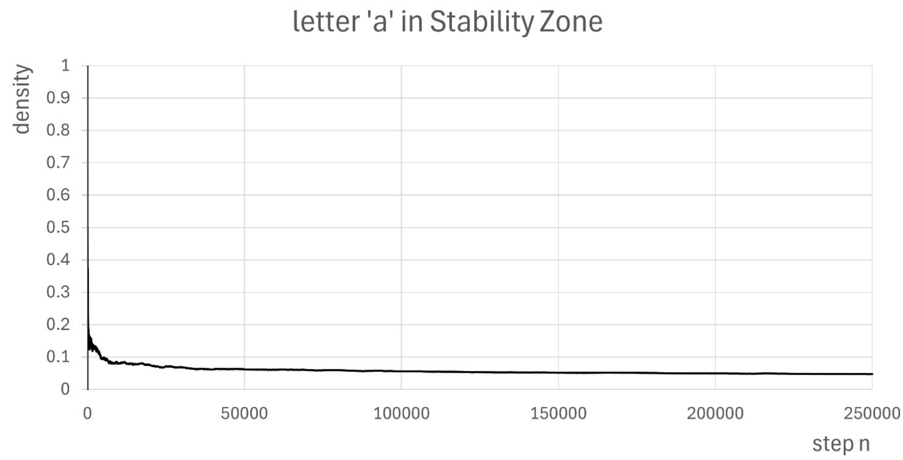

Figure 8_3 Density of letter 'a' in SZ

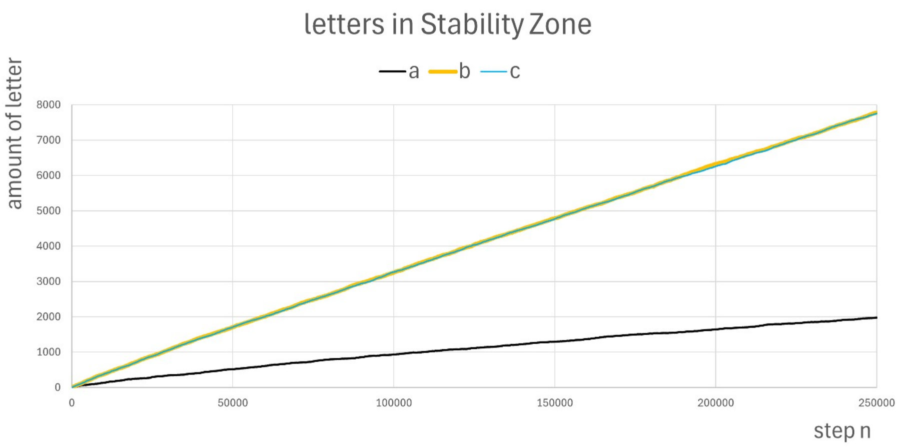

Figure 8_4 Amount of letters in SZ

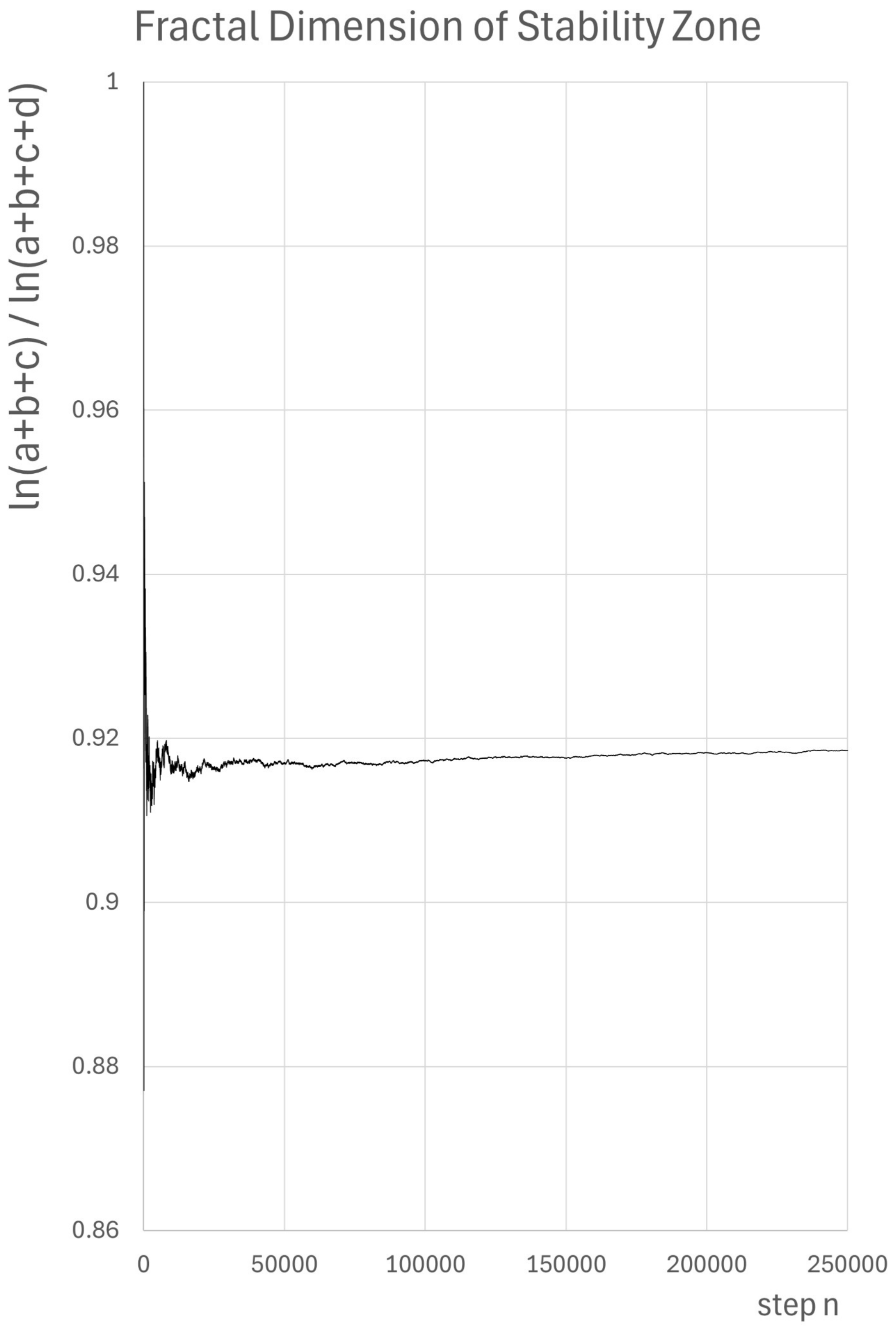

Figure 8_5 Fractal dimension of letters in SZ

9 Fractal Dimension

Spectral Derivation of the Fractal Dimension

The eigenvalues of the transition matrix (figure 6.5_1) allow us to calculate the theoretical fractal dimension of the generated set analytically. Following the definition of the similarity dimension D, where the scaling factor is p and the replication factor for all L symbols is the eigenvalue $\lambda = p - 1$, we obtain:

$$D_{all} = \frac{\ln(\lambda)}{\ln(p)} = \frac{\ln(p-1)}{\ln(p)} \tag{29}$$

For small primes, this dimension reflects the strong “dust-like” filtering. As p→∞, the dimension converges:

$$\lim_{p \to \infty} \frac{\ln(p-1)}{\ln(p)} = 1 \quad with \quad \frac{\ln(p-1)}{\ln(p)} < 1 \,\forall\, p \tag{30}$$

The limes is 1, but the actual values for D_{all} are just below 1.

The limit set of L-symbols produced by the Cosmos Automaton as n approaches infinity forms a specific topological object (figure 9_1), which we term "Heeren Dust." Mathematically, this set is a non-stationary S-adic Cantor set. While its global Lebesgue measure is zero according to the divergence of the prime harmonic series, its structural complexity is preserved through the iterative morphisms. Heeren Dust represents the crystalline framework of the prime distribution, where the local similarity dimension D < 1 characterizes the scaling of the gaps at each finite stage of the sieve's evolution.

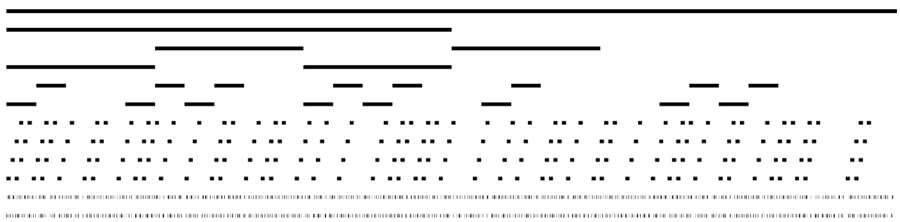

Figure 9_1 Heeren Dust

10 Conclusion

The Cosmos Automaton (CA) provides a rigorous symbolic framework for the Sieve of Eratosthenes, transitioning the problem from arithmetic subtraction to the language of non-stationary S-adic dynamical systems. As formalized in the work of

Pytheas Fogg [4], the CA operates as a sequence of substitution morphisms σ_p over a finite alphabet, where the directive sequence is endogenously generated by the system's own feedback loop. Our analysis of the incidence matrix Mp demonstrates that the symbolic architecture of the primes is governed by specific spectral properties—most notably the growth eigenvalue λ = p-2 for twin prime templates. This "Hydra Effect" provides a deterministic, constructive derivation of the combinatorial factors central to the Hardy-Littlewood k-tuple conjecture.

Following the framework of Lapidus [5] regarding fractal strings and the geometry of the number line, we have shown that the sifting process generates a Cantor-like set, or "Heeren Dust," with a local similarity dimension D = ln(p-1)/ln(p). While the topological dimension converges to unity in the infinite limit, the persistent fractal scaling at any finite step characterizes the non-stationary nature of the distribution. The introduction of the Stability Zone provides a topological guarantee for the pointwise convergence of the automaton towards the characteristic word of the primes.

Ultimately, the CA formalizes the Twin Prime Conjecture not as a statistical probability, but as a problem of coordinate-survival within a shifting S-adic frame. The question of whether the template 'a' <LMLMMM> meets the Stability Zone infinitely often is thus reduced to the measure-theoretic properties of the limit word. The Cosmos Automaton serves as a new observatory for the structural emergence of order from the interference of infinite pulse trains, providing a bridge between automata theory and the deepest mysteries of number theory.

11 Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the author used free of charge versions of Copilot, Mistral Le Chat, Perplexity, ChatGPT, Gemini, and Grok in order to get feedback, find wording and as a teacher. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

From 2007 to 2024 ideas were developed by the author. In 2025 the author turned to AI. All figures where created by the author. The programming was done by the author in Java.

12 References

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