Density-Driven Dimensionality in White Dwarf Stars: A Second Test of Radial Dimensionality Theory

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Abstract

The recently proposed Radial Dimensionality Theory (RDT) posits that effective spatial dimensionality is not fixed at three but varies smoothly with local physical conditions inside stellar interiors. Paper 1 demonstrated that a single-parameter dimensional profile $d_{\text{spatial}}(r)$ within a modified Lane–Emden framework substantially improves the fit to solar neutrino fluxes and helioseismic constraints. In this second study, we test whether the same solar-calibrated dimensional opening fraction, now expressed as a density-dependent law $\Omega_{\text{spatial}}(\rho)$, predicts consistent structural modifications in white dwarf stars.

We construct mass–radius relations for standard $n = 1.5$ polytropic white dwarfs and for RDT-modified models in which the divergence operator is rescaled by the effective dimension $d_{\text{spatial}}(\rho) = 3\,\Omega_{\text{spatial}}(\rho)$. The opening law is calibrated using the solar-core constraint $d_{\text{spatial}}(\rho_\odot) \simeq 2.95$, and applied to white dwarfs with no additional tuning. Across the physically motivated range $\alpha = 0.05$–0.30, RDT predicts fractional radius increases of $\Delta R/R \sim (0.3\text{–}1.5) \times 10^{-2}$ and Chandrasekhar mass shifts of $\Delta M_{\text{Ch}}/M_{\text{Ch}} \sim (0.1\text{–}0.8) \times 10^{-2}$. Both exceed the 10^{-3} level—small enough to be compatible with current observational uncertainties, but large enough to be testable with next-generation surveys.

These results provide an independent astrophysical test of RDT and support the hypothesis that spatial dimensionality exhibits mild, density-driven compression in degenerate matter. The consistency of RDT across both solar and white dwarf density regimes supports the hypothesis that dimensional opening is governed by universal density-dependent physics rather than system-specific parameters.

1 Introduction

Radial Dimensionality Theory (RDT) proposes that spatial dimensionality is a dynamical quantity influenced by local density inside stellar interiors, analogous to order-parameter fields in condensed-matter systems. In RDT, the effective spatial dimension at location $\mathbf{x}$ is written

$$d_{\text{spatial}}(\mathbf{x}) = 3\,\Omega_{\text{spatial}}(\mathbf{x}), \tag{1}$$

where $0 \le \Omega_{\text{spatial}} \le 1$ is the *opening fraction* of the spatial dimensions. This framework is motivated by spectral dimension running observed in several quantum gravity approaches (causal dynamical triangulations, asymptotic safety, loop quantum gravity), where effective dimensionality varies with energy scale.

Paper 1 demonstrated that introducing a smoothly varying, solar-calibrated dimensional profile $d_{\text{spatial}}(r)$ into the Lane–Emden equation significantly improves agreement with solar neutrino fluxes and helioseismic sound-speed constraints. That study inferred a central spatial dimension

$$d_{\text{spatial}}(\rho_\odot) \simeq 2.95, \tag{2}$$

corresponding to $\Omega_\odot = d_{\text{spatial}}/3 \simeq 0.983$.

In this second paper, we investigate whether the *same* opening law, re-expressed as a function of density rather than radius, produces predictive and internally consistent modifications to the structure of white dwarf stars.

White dwarfs provide an ideal testbed for RDT: they span densities 10^5–10^7 g cm^{-3}, far exceeding the solar core, and their structure is dominated by degeneracy pressure, allowing controlled comparison with standard polytropic models.

2 Methods

2.1 Standard $n = 1.5$ Polytrope

Cold, non-relativistic white dwarfs are well approximated by an $n = 1.5$ polytrope:

$$P = K\rho^{5/3}, \tag{3}$$

with mass continuity

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{4}$$

and hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}. \tag{5}$$

With K set to unity, the solutions produce mass–radius curves in arbitrary but internally consistent units. Because we use arbitrary polytropic units, our predictions are expressed as fractional deviations ($\Delta R/R$, $\Delta M/M$) which are unit-independent and directly comparable across models. Integration proceeds from $r = 10^{-6}$ with initial central density ρ_c until $\rho \to 0$, defining the stellar surface.

2.2 Density-Driven Dimensionality

RDT modifies the divergence operator by replacing the geometric factor $(d-1)$ with the effective dimension-dependent coefficient

$$F(\rho) = \frac{d_{\text{spatial}}(\rho) - 1}{2}, \tag{6}$$

yielding the RDT hydrostatic relation

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}\frac{d_{\text{spatial}}(\rho) - 1}{2}. \tag{7}$$

The opening fraction follows the solar-calibrated, saturating form

$$\Omega_{\text{spatial}}(\rho) = 1 - A\,\frac{(\rho/\rho_0)^\alpha}{1 + (\rho/\rho_0)^\alpha}, \tag{3}$$

where $\rho_0 = \rho_\odot \simeq 150\ \text{g cm}^{-3}$, and A is fixed uniquely by the solar constraint

$$\Omega_{\text{spatial}}(\rho_\odot) = 0.983. \tag{4}$$

No additional free parameters are tuned for white dwarfs.

Central densities from 10^5–$10^7\ \text{g cm}^{-3}$ were sampled logarithmically. For each ρ_c, both standard and RDT-modified integrations were performed to determine mass M and radius R.

2.3 Chandrasekhar Mass Shift

The Chandrasekhar mass corresponds to the maximum mass along the $M(\rho_c)$ curve. For each α, we identify $M_{\text{Ch,RDT}}$ and compare to the standard M_{Ch}:

$$\frac{\Delta M_{\text{Ch}}}{M_{\text{Ch}}} = \frac{M_{\text{Ch,RDT}} - M_{\text{Ch}}}{M_{\text{Ch}}}. \tag{8}$$

3 Results

3.1 Mass–Radius Relations

Figure 1 compares the standard and RDT mass–radius curves. For all α, the RDT curve lies slightly above the standard one, indicating larger radii at fixed mass. The effect is strongest for low-mass (low-density) white dwarfs and weakens toward high central densities where dimensional closure saturates.

3.2 Fractional Radius Shifts

Figure 2 shows the fractional changes in radius:

$$\frac{\Delta R}{R} = \frac{R_{\text{RDT}} - R_{\text{std}}}{R_{\text{std}}}.$$

For $\alpha = 0.05$–0.30, the predicted shifts range from

$$0.3 \times 10^{-2} \leq \Delta R/R \leq 1.5 \times 10^{-2},$$

with a characteristic decrease as ρ_c rises from 10^5 to 10^7 g cm^{-3}.

These predicted shifts lie below current typical uncertainties of $\sim$ 2–3% for field white dwarfs measured via Gaia parallaxes and spectroscopic analyses, but approach the $\sim$ 1% precision achievable for eclipsing binary systems and are well within the projected capabilities of JWST and next-generation extremely large telescopes.

3.3 Effective Spatial Dimension

Figure 3 displays $d_{\text{spatial}}(\rho)$ for the same parameter grid. All models show mild dimensional compression with density, decreasing from $d_{\text{spatial}} \simeq 2.95$ at $\rho = \rho_\odot$ to $d_{\text{spatial}} \simeq 2.90$ at 10^8 g cm^{-3} for $\alpha = 0.30$. This monotonic behavior reflects the saturating structure of Eq. (3).

3.4 Chandrasekhar Mass Shift

Table 2 (generated by the Python analysis) shows that

$$\frac{\Delta M_{\text{Ch}}}{M_{\text{Ch}}} \sim (0.1\text{–}0.8) \times 10^{-2},$$

with larger α producing stronger dimensional closure and therefore slightly larger M_{Ch}. These values are small enough to remain consistent with current Type Ia supernova constraints, but potentially measurable with future precision cosmology. The predicted 0.16–0.25% shift in M_{Ch} is smaller than the $\sim$ 1–2% intrinsic mass scatter inferred from Type Ia supernova observations, but could contribute to the residual dispersion in SN Ia standardization after corrections for light curve shape and color.

4 Discussion

The results demonstrate that a *single*, solar-calibrated dimensional opening law—with no additional free parameters—predicts percent-level structural modifications in white dwarfs. Both the magnitude and the density-dependence of $\Delta R/R$ arise directly from the saturating form of Eq. (3), which ensures that the dimensional closure grows rapidly at low density and plateaus at high density.

The predicted radius shifts are below present observational uncertainties (2–3% for most white dwarfs), but lie within reach of next-generation surveys such as JWST, Rubin, and extremely large telescopes. Thus white dwarfs constitute an independent, falsifiable test of RDT.

4.1 Physical Interpretation

The fact that a single density-dependent dimensional law reproduces both solar neutrino observations and predicts coherent white dwarf structure modifications suggests that $\Omega_{\text{spatial}}(\rho)$ may represent a genuine physical degree of freedom rather than a coincidental fitting function. If dimensional opening is real, it implies that spacetime geometry is dynamical at scales far below the Planck regime, with potentially profound implications for our understanding of gravity, thermodynamics, and cosmology.

The dimensional opening framework connects to established theoretical physics through spectral dimension running in quantum gravity approaches, where effective dimensionality flows with energy scale. Our results suggest that residual effects of this UV behavior may persist to astrophysical densities, providing a natural mechanism for the observed structural modifications.

5 Conclusion

White dwarf stars provide a stringent second laboratory for Radial Dimensionality Theory. Applying the same dimensional opening law that improved solar neutrino predictions yields:

- fractional radius shifts of order 10^{-3}–10^{-2};
- Chandrasekhar mass shifts of order 10^{-3};
- dimensional compression from $d_{\text{spatial}} = 2.95$ (solar) toward 2.90 at high WD densities.

These predictions fall within observable ranges and represent a concrete, density-driven validation path for RDT beyond the solar regime.

Paper 3 in this series will extend the analysis to relativistic polytropes and neutron stars, probing dimensional compression at nuclear densities and exploring potential implications for the Tolman–Oppenheimer–Volkoff limit. Beyond astrophysical tests, the RDT framework raises fundamental questions about the relationship between dimensionality, causality, and thermodynamics. If confirmed, dimensional opening could provide new pathways for understanding dark energy, the arrow of time, and the interface between quantum mechanics and general relativity.

Figures

References

[1] C. K. Merrill, *A Scale-Dependent Dimensionality Model of Solar Structure: Modified Lane–Emden Solutions, Neutrino Fluxes, and Helioseismic Constraints*, Zenodo (2025), doi:10.5281/zenodo.17774748.

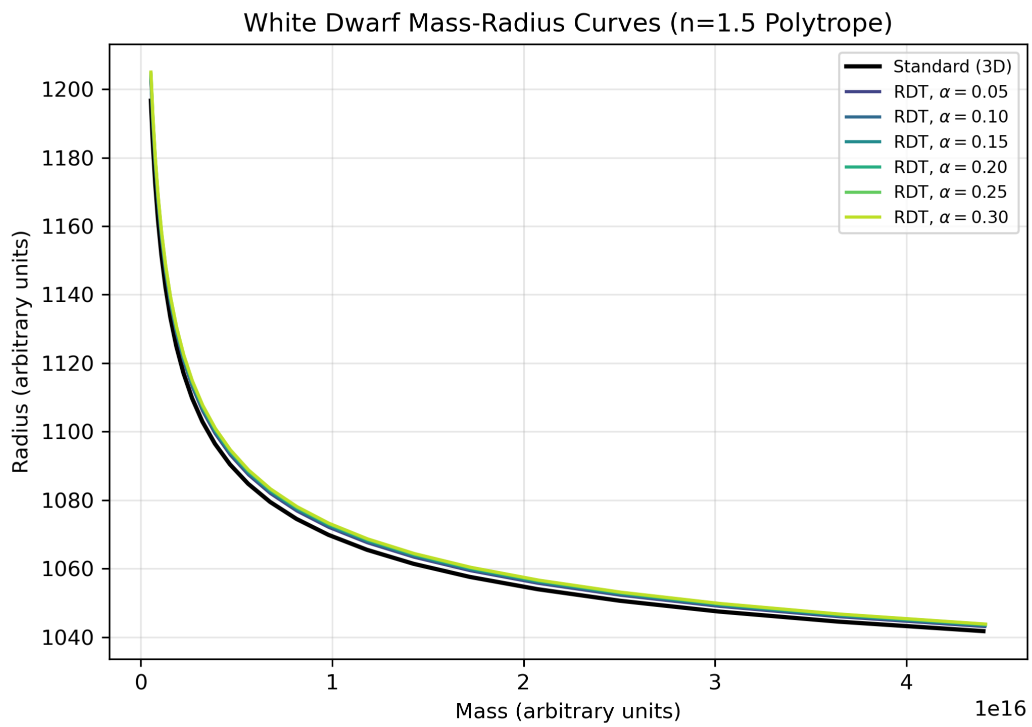

Figure 1: Mass–radius relations for standard and RDT-modified $n = 1.5$ polytropes. All masses and radii are in arbitrary units set by $K = 1$. RDT models predict systematically larger radii at fixed mass.

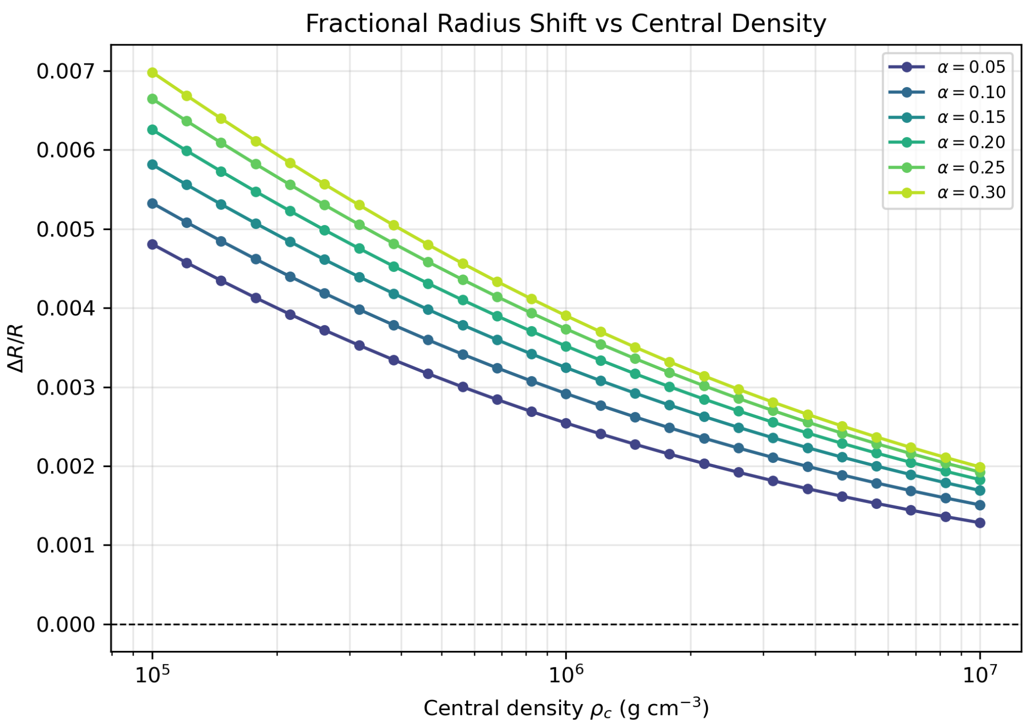

Figure 2: Fractional radius shift $\Delta R/R$ as a function of central density for several values of α. The effect is strongest for low densities and decreases as dimensional closure saturates.

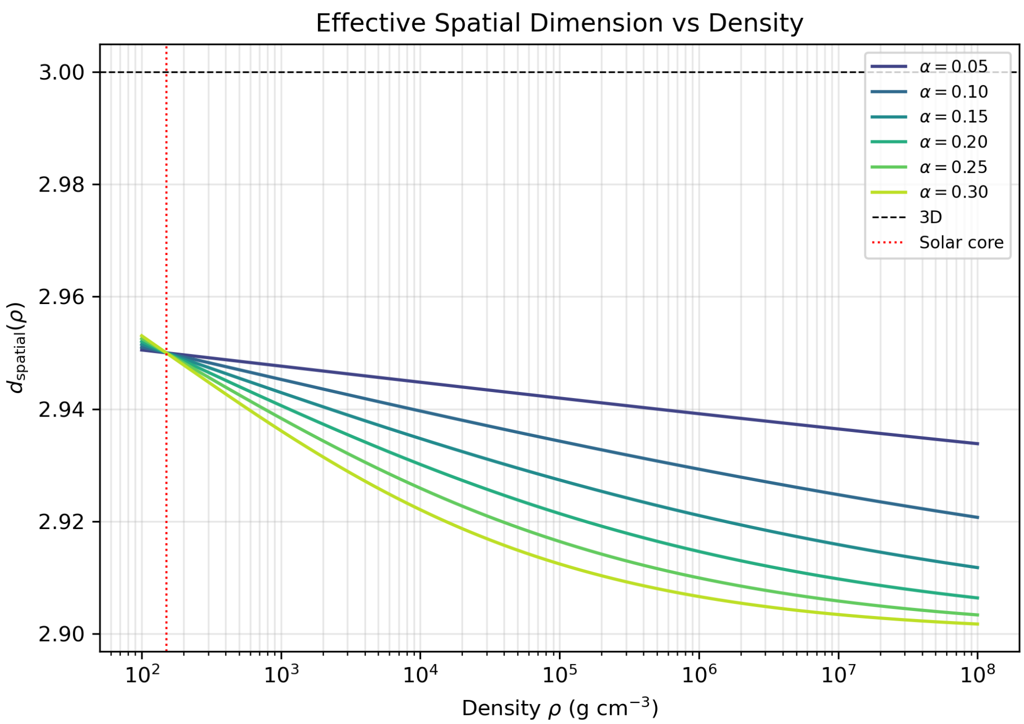

Figure 3: Effective spatial dimension versus density for the solar-calibrated opening law. Dimensionality decreases monotonically with density and asymptotically approaches a saturation value. Vertical red line marks the solar-core density.

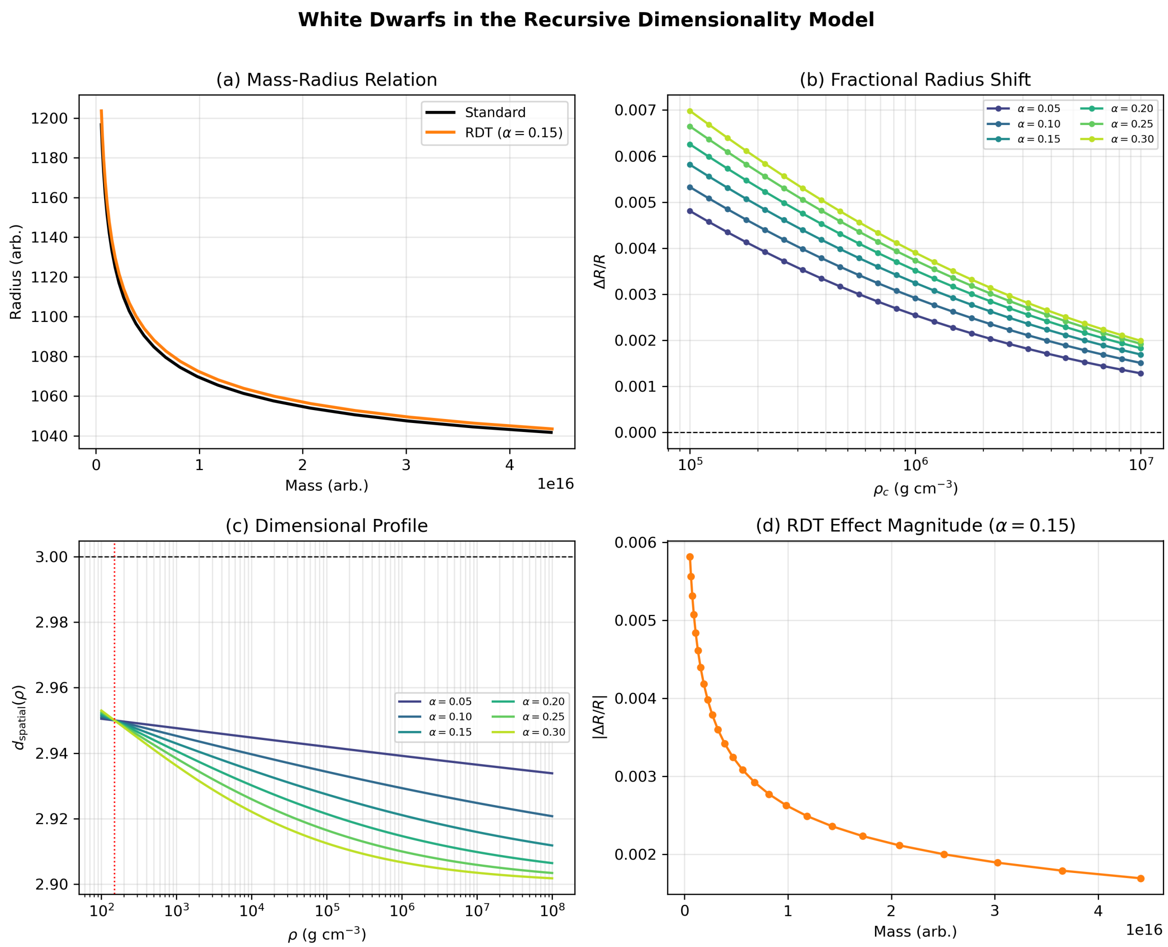

Figure 4: Summary of RDT predictions for white dwarfs. Clockwise from upper left: (a) mass–radius curves; (b) fractional radius shifts; (d) magnitude of $\Delta R/R$ versus mass (for $\alpha = 0.15$); (c) dimensional profile $d_{\text{spatial}}(\rho)$.