Operational Time, Conformal Projection, and the Interpretation of Cosmological Expansion

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Abstract

We explore a conceptual interpretation of cosmological expansion motivated by the Clock Universality No-Go Theorem. Assuming that causal ordering—not elapsed proper time—is the invariant structure shared by cosmological observables, we examine how inequivalent operational time parametrizations can induce apparent expansion when null-propagated signals are interpreted using matter-adapted clocks. Without modifying General Relativity or introducing new dynamics, we show how a conformal projection between operational time classes provides a consistent reinterpretation of redshift, the Hubble constant, and their observed bifurcations. This framework is explicitly non-unique and is presented as an interpretive realization rather than a derived necessity.

1 Introduction and Motivation

It is known that the interpretation of cosmological expansion—as usually understood at the present time—when applied to measurements of the Hubble constant H_0, leads to a bifurcation which does not appear to be inherent in the phenomena. Take, for example, the reciprocal relationship between distance-ladder measurements and geometric probes. The observable phenomenon here depends only on the physical processes used to define time and distance, whereas the customary view draws no distinction between the two cases in which either stellar clocks or null-propagated signals are employed. For if stellar processes are used to define operational time, there arises in the cosmological inference a Hubble constant with a certain definite value, producing measurements near $H_0 \approx 73$ km/s/Mpc. But if null-propagated signals are used, no such value arises from the same underlying metric structure. In the geometric measurements, however, we find a different inferred expansion rate, to which in itself there is no corresponding metric growth, but which gives rise—assuming equality of the underlying spacetime geometry in the two cases discussed—to cosmological parameters of the same functional form as those produced by the stellar clocks in the former case.

Examples of this sort, together with the unsuccessful attempts to discover any systematic calibration error or early-universe physics capable of explaining the observed bifurcation, suggest that the phenomena of cosmological expansion as well as of local physics possess no properties corresponding to the idea of universal clock time. They suggest rather that, as has already been shown in the Clock Universality No-Go Theorem, the same laws of General Relativity and local Lorentz invariance will be valid for all frames of reference, but the operational interpretation of proper time may differ when different physical processes are employed to define it.

We will raise this conjecture (the purport of which will hereafter be called the "Principle of Operational Time Dependence") to the status of an interpretive framework, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that causal ordering is always preserved in empty space with a definite structure which is independent of the state of motion of the emitting body or the operational clock used to measure it. These two postulates suffice for the attainment of a simple and consistent interpretation of cosmological expansion based on General Relativity for local physics. The introduction of a "universal cosmic time" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely synchronized clock" provided with special properties, nor assign a unique time-parameter to a point of the spacetime in which cosmological processes take place.

The interpretation to be developed is based—like all cosmological inference—on the kinematics of physical processes, since the assertions of any such interpretation have to do with the relationships between clocks, rulers, and cosmological observables. Insufficient consideration of this circumstance lies at the root of the difficulties which the interpretation of cosmological expansion at present encounters.

Scope of the Present Work

This paper explores *how* expansion might be interpreted if clock universality fails, not *whether* it does. The framework presented here is:

1. **Interpretive, not deductive**: We do not derive the necessity of operational time dependence from first principles, but rather show that such a framework provides a consistent interpretation of observed phenomena.
2. **No new dynamics**: General Relativity remains unmodified. The Einstein field equations, metric propagation, and local Lorentz invariance are preserved.
3. **No numerical prediction**: This framework does not predict the magnitude of the observed H_0 bifurcation. It provides a structural interpretation of how such a bifurcation could arise, but does not claim uniqueness or exclusivity.

2 Causal Ordering as the Primary Invariant

2.1 Definition of Event Ordering

Let us take a system of events in spacetime in which the equations of General Relativity hold good. In order to render our presentation more precise and to distinguish this system of events verbally from others which will be introduced hereafter, we call it the "causal system."

If a material point is at rest relatively to this system of events, its position can be defined relatively thereto by the employment of rigid standards of measurement and the methods of Riemannian geometry, and can be expressed in coordinates adapted to the metric structure.

If we wish to describe the evolution of a physical process, we give the values of its coordinates as functions of a parameter. Now we must bear carefully in mind that a mathematical description of this kind has no physical meaning unless we are quite clear as to what we understand by "time." We have to take into account that all our judgments in which time plays a part are always judgments of event ordering. If, for instance, I say, "That supernova exploded at redshift $z = 0.5$," I mean something like this: "The ordering of the explosion event relative to the observation event, as determined by the null geodesic connecting them, and the operational clock reading at the observation, are related in a certain way."

It might appear possible to overcome all the difficulties attending the definition of "time" by substituting "the reading of my atomic clock" for "time." And in fact such a definition is satisfactory when we are concerned with defining a time exclusively for the place where the clock is located; but it is no longer satisfactory when we have to connect in time series of events occurring at different places, or—what comes to the same thing—to evaluate the times of events occurring at places remote from the clock.

We might, of course, content ourselves with time values determined by an observer stationed together with the clock at the origin of the coordinates, and coordinating the corresponding positions of the clock hands with light signals, given out by every event to be timed, and reaching him through empty space. But this coordination has the disadvantage that it is not independent of the operational definition of the clock, as we know from the Clock Universality No-Go Theorem.

We arrive at a much more practical determination along the following line of thought.

If at the point A of space there is a clock, an observer at A can determine the time values of events in the immediate proximity of A by finding the positions of the hands which are simultaneous with these events. If there is at the point B of space another clock in all respects resembling the one at A, it is possible for an observer at B to determine the time values of events in the immediate neighbourhood of B. But it is not possible without further assumption to compare, in respect of time, an event at A with an event at B. We have so far defined only an "A time" and a "B time." We have not defined a common "time" for A and B, for the latter cannot be defined at all unless we establish by definition that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A.

However, we can define a common **ordering** for events at A and B without requiring a common time parameter. This ordering is provided by the causal structure of spacetime itself. Two events are ordered if and only if there exists a causal curve connecting them. This ordering is independent of any particular clock or operational time parameter.

Definition (Causal Ordering): Two events E_1 and E_2 in spacetime are causally ordered if there exists a causal curve (timelike or null) connecting them. If such a curve exists, we say $E_1 \prec E_2$ (read: E_1 precedes E_2) if E_1 lies in the causal past of E_2.

This definition of ordering is free from contradictions, and possible for any number of events; and the following relations are universally valid:—

1. If event E_1 precedes event E_2, then E_2 does not precede E_1 (antisymmetry).
2. If event E_1 precedes event E_2 and event E_2 precedes event E_3, then event E_1 precedes event E_3 (transitivity).

Thus with the help of certain imaginary physical experiments we have settled what is to be understood by causal ordering of events located at different places, and have evidently obtained a definition of "precedence," or "causal order," which is independent of any operational clock.

2.2 On the Relativity of Elapsed Time and Operational Parameters

The following reflections are based on the principle of causal ordering and on the principle of the constancy of causal structure. These two principles we define as follows:—

1. The causal ordering of events is not affected, whether these events be referred to one or the other of two different operational time parametrizations.

2. Any null geodesic moves in spacetime with a definite causal structure, whether the geodesic be parametrized by one operational clock or another. Hence

$$\text{ordering} = \text{causal structure}$$

where causal structure is to be taken in the sense of the definition in § 1.

Let there be given a physical process; and let its duration be Δt_1 as measured by a clock which is also bound to that process. We now imagine the process occurring along a worldline in spacetime, and that a uniform parametrization with parameter λ along the null geodesic connecting emission and observation is then imparted to the process. We now inquire as to the duration of the process, and imagine its duration to be ascertained by the following two operations:—

(a) The observer moves together with the given clock and the process to be measured, and measures the duration of the process directly by superposing the clock, in just the same way as if all were at rest.

(b) By means of null geodesics set up in spacetime and parametrized in accordance with § 1, the observer ascertains at what values of the affine parameter λ the beginning and end of the process are located. The difference between these two parameter values, measured by the affine parametrization already employed, is also a duration which may be designated "the duration of the process."

In accordance with the principle of causal ordering the duration to be discovered by the operation (a)—we will call it "the duration of the process in the matter-adapted system"—must be equal to the duration Δt_1 measured by the process-bound clock.

The duration to be discovered by the operation (b) we will call "the duration of the process in the null-adapted system." This we shall determine on the basis of our two principles, and we shall find that it differs from Δt_1.

Current cosmology tacitly assumes that the durations determined by these two operations are precisely equal, or in other words, that a physical process at a given causal ordering may in temporal respects be perfectly represented by the same process measured using any operational clock.

We imagine further that at the two events E_1 and E_2 of the process, clocks are placed which synchronize with the clocks of the matter-adapted system, that is to say that their indications correspond at any instant to the "time of the matter-adapted system" at the places where they happen to be. These clocks are therefore "synchronous in the matter-adapted system."

We imagine further that with each clock there is a moving observer, and that these observers apply to both clocks the criterion established in § 1 for the ordering of two events. Let a null signal depart from E_1 at the affine parameter λ_1, let it be received at E_2 at the affine parameter λ_2, and reach E_1 again at the affine parameter λ_1'. Taking into consideration the principle of the constancy of causal structure we find that

$$|\lambda_2 - \lambda_1| = \frac{r_{12}}{c}$$

and

$$|\lambda_1' - \lambda_2| = \frac{r_{12}}{c}$$

where r_{12} denotes the proper distance between E_1 and E_2—measured in the matter-adapted system. Observers moving with the matter-adapted clocks would thus find that the two clocks were synchronized according to their operational definition, while observers using null-adapted parametrization would declare the clocks to measure different elapsed times for the same causal interval.

So we see that we cannot attach any absolute signification to the concept of elapsed time, but that two events which, viewed from a matter-adapted system of parametrization, are separated by a certain duration, can no longer be looked upon as separated by the same duration when envisaged from a null-adapted system which is in relative motion or employs a different operational definition.

2.3 Operational Time Classes

Let us in spacetime take two classes of physical processes, i.e. two classes, each of which defines an operational time parameter through its internal evolution. Let the causal structure of the two classes coincide, and their metric structure be the same. Let each class be provided with a physical clock and a number of processes, and let the two clocks, and likewise all the processes of the two classes, be in all respects consistent with the same underlying metric.

Now to one of the two classes (matter-adapted) let an operational time parameter τ_m be assigned through the internal phase evolution of the matter fields, and let this parameter be communicated to the clocks and processes of that class. To any causal ordering in spacetime there then will correspond a definite value of the operational time parameter, and from reasons of consistency we are entitled to assume that the parametrization may be such that the causal structure is preserved.

We now imagine spacetime to be measured from the matter-adapted system by means of the matter-adapted clocks, and also from the null-adapted system by means of the null-adapted affine parameter; and that we thus obtain the parameters τ_m and λ respectively. Further, let the parameter τ_m of the matter-adapted system be determined for all events thereof at which there are clocks by means of light signals in the manner indicated in § 1; similarly let the parameter λ of the null-adapted system be determined for all events of the null-adapted system at which there are processes at rest relatively to that system by applying the method, given in § 1, of null geodesics between the events at which the latter processes are located.

To any system of values x, y, z, t, which completely defines the place and time of an event in the matter-adapted system, there belongs a system of values $\xi, \eta, \zeta, \lambda$, determining that event relatively to the null-adapted system, and our task is now to find the system of equations connecting these quantities.

Definition (Operational Time Class): A physical process σ defines an operational time parameter t_σ if it induces a stable total ordering on events consistent with causal structure. Specifically:

1. There exists a coarse-graining (a partition of the process into equivalence classes of events that are indistinguishable at the chosen scale) under which relational events are monotonically orderable.
2. The ordering is stable against small perturbations (i.e., small variations in initial conditions or environmental parameters do not alter the event ordering).
3. The ordering is consistent with the causal structure of spacetime (i.e., if event E_1 causally precedes event E_2, then $t_\sigma(E_1) < t_\sigma(E_2)$).

Examples of operational time classes include:

- **Null propagation** (λ): The affine parameter along null geodesics provides an ordering parameter that is universal in the sense that it is defined by the metric structure alone.
- **Matter-bound clocks** (τ_m): The internal phase evolution of matter fields (nuclear decay, stellar pulsation, atomic transitions) provides an ordering parameter that is tied to the specific physical process.

No assumption is made regarding entropy, arrow-of-time, or thermodynamic properties; only stable orderability consistent with causal structure is required.

3 Conformal Ambiguity in Operational Time

3.1 Process-Dependent Parametrization

In the first place it is clear that the relationship between different operational time parameters must be well-defined on account of the properties of consistency which we attribute to causal structure.

If we place $\tau_m = \tau_m(\lambda)$, it is clear that a process at rest in the matter-adapted system must have a system of values τ_m independent of the null-adapted parameter λ in the local limit. We first define the relationship between τ_m and λ as a function of the process σ. To do this we have to express in equations that τ_m is nothing else than the summary of the data of clocks at rest in the matter-adapted system, which have been synchronized according to the rule given in § 1.

From the emission event of a process σ let a null signal be emitted at the operational time τ_0 along a null geodesic parametrized by λ, and at the operational time τ_1 be received at the observation event. We then must have a relationship between τ_0, τ_1, and the affine parameter values λ_0, λ_1 corresponding to these events.

The central relation of this framework is:

$$\frac{d\tau_m}{d\lambda} = J(\sigma) > 0,$$

where $J(\sigma)$ is a positive, dimensionless function that encodes the operational normalization between the matter-adapted proper time and the null-adapted affine parameter. This function depends on the physical process σ that defines the matter-adapted time, but is independent of the specific emission or observation events to first order in local variations.

Properties of $J(\sigma)$:

1. **Positivity**: $J(\sigma) > 0$ ensures that the ordering is preserved (i.e., if $\lambda_1 < \lambda_2$, then $\tau_m(\lambda_1) < \tau_m(\lambda_2)$).
2. **Dimensionless**: Since both τ_m and λ have dimensions of time (or length in natural units), their ratio is dimensionless.
3. **Process-dependent**: Different physical processes (nuclear decay, stellar pulsation, atomic transitions) may define different values of $J(\sigma)$, reflecting the fact that their internal evolution rates need not be identical when measured against null propagation.
4. **No dynamics implied**: The function $J(\sigma)$ encodes only the operational mapping between time parameters. It does not enter into the Einstein field equations or modify gravitational dynamics.

3.2 Effective Conformal Metrics

If operational time is used to normalize the four-velocity of matter fields, we obtain an effective description in terms of conformally related metrics.

Consider a matter field with four-velocity u_m^μ normalized using matter-adapted proper time:

$$u_m^\mu = \frac{dx^\mu}{d\tau_m}, \quad g_{\mu\nu} u_m^\mu u_m^\nu = -1.$$

If we instead attempt to describe the same physical situation using the null-adapted affine parameter λ, we would define:

$$\tilde{u}^\mu = \frac{dx^\mu}{d\lambda} = \frac{dx^\mu}{d\tau_m}\frac{d\tau_m}{d\lambda} = u_m^\mu J(\sigma).$$

The normalization condition becomes:

$$g_{\mu\nu}\tilde{u}^\mu \tilde{u}^\nu = J^2(\sigma) g_{\mu\nu} u_m^\mu u_m^\nu = -J^2(\sigma).$$

To maintain the standard normalization $g_{\mu\nu}\tilde{u}^\mu \tilde{u}^\nu = -1$, we must introduce an effective conformal metric:

$$g_{\mu\nu}^{(m)} = \Omega^2(\sigma) g_{\mu\nu}, \quad \Omega(\sigma) = J(\sigma).$$

Properties of the Conformal Transformation:

1. **Causal structure preserved**: Since $\Omega(\sigma) > 0$, the conformal transformation preserves the causal structure. Timelike curves remain timelike, null curves remain null, and spacelike curves remain spacelike.

2. **Volume growth becomes frame-dependent**: The volume element transforms as:

   $$\sqrt{-g^{(m)}} = \Omega^4(\sigma)\sqrt{-g} = J^4(\sigma)\sqrt{-g}.$$

   This means that the inferred expansion rate depends on which operational time class is used to define the volume element.

3. **Expansion rate becomes operational**: The Hubble parameter inferred from matter-adapted clocks becomes:

   $$H_{\text{matter}} = H_{\text{null}} \cdot J(\sigma),$$

   where H_{null} is the expansion rate inferred from null-adapted signals. This provides a natural interpretation of the observed H_0 bifurcation: different operational time classes yield different inferred expansion rates, even though the underlying metric structure is the same.

3.3 Symmetry versus Asymmetry in Operational Parametrization

The persistent impasse in cosmological inference stems from an unexamined adherence to temporal symmetry—the assumption that the proper time τ of a matter-bound observer and the affine parameter λ of a null-propagated signal are operationally identical across all scales. In the standard FLRW framework, this symmetry is treated as an ontological necessity, forcing any observed discrepancy in signal frequency to be interpreted as a kinematic recession or a geometric expansion of the metric itself.

However, if we adopt a skeptical stance toward this "Universal Clock", we recognize that this symmetry is likely an artifact of our local, high-density environment. In reality, the "rhythm" of matter-bound processes (baryonic clocks) and the causal ordering of empty space (null geodesics) constitute inequivalent operational classes. The observed "expansion" of the universe and the resulting H_0 bifurcation are thus revealed not as dynamic phenomena, but as the accumulated projection error of an asymmetric relationship.

By forcing an asymmetric reality into a symmetric mathematical mold, we are necessitated to invent dark energy and other "fudge factors" to reconcile the drift. When this symmetry is discarded, the Hubble tension ceases to be a crisis of measurement and becomes a predictable consequence of the mismatch between the clocks used to define the scale of the cosmos.

4 Redshift as an Integrated Projection Effect

4.1 Standard Redshift Recalled

In General Relativity, the observed redshift of a photon is given by:

$$1 + z = \frac{(k^\mu u_\mu)_{\text{emit}}}{(k^\mu u_\mu)_{\text{obs}}},$$

where k^μ is the photon's four-momentum (tangent to a null geodesic), and u^μ is the four-velocity of the emitting or observing matter.

This expression is process-independent in standard cosmology because it is assumed that all matter fields share the same operational identification of proper time $d\tau$. That is, the normalization $g_{\mu\nu}u^\mu u^\nu = -1$ is taken to define the same proper time parameter for all processes.

However, as established in the Clock Universality No-Go Theorem, this assumption leads to an inconsistency when different physical processes are used to measure H_0. The observed bifurcation reveals that the operational identification of $d\tau$ is not universal.

4.2 Redshift Under Operational Mismatch

We now examine how redshift is interpreted when there is a mismatch between the operational time classes used to define k^μ and u^μ.

Consider a matter-adapted emission process (e.g., a stellar source), where the emitter's four-velocity is normalized using τ_m:

$$u^\mu_{\text{emit}} = \frac{dx^\mu}{d\tau_m}, \quad g_{\mu\nu}u^\mu_{\text{emit}}u^\nu_{\text{emit}} = -1.$$

The photon propagates along a null geodesic with affine parameter λ, so:

$$k^\mu = \frac{dx^\mu}{d\lambda}, \quad g_{\mu\nu}k^\mu k^\nu = 0.$$

At the emission event, a naive calculation of the contraction would give:

$$(k^\mu u_\mu)_{\text{emit}} = g_{\mu\nu}k^\mu u^\nu_{\text{emit}} = g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\tau_m}.$$

Using the relationship $d\tau_m = J(\sigma)d\lambda$, this becomes:

$$(k^\mu u_\mu)_{\text{emit}} = J(\sigma)g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0,$$

since k^μ is null. This suggests we must be more careful about the operational interpretation.

The key insight is that redshift measurements implicitly assume a consistent operational time class. When we write $1 + z = (k^\mu u_\mu)_{\text{emit}}/(k^\mu u_\mu)_{\text{obs}}$, we are assuming that both k^μ and u^μ are defined using the same operational time parameter. If k^μ is ordered by λ (null-adapted) and u^μ is normalized by τ_m (matter-adapted), then the contraction scales with the Jacobian. More precisely, if we attempt to interpret a null-propagated signal using matter-adapted clocks, the effective redshift becomes:

$$1 + z_{\text{effective}} = \frac{(k^\mu u_\mu)_{\text{emit}}}{(k^\mu u_\mu)_{\text{obs}}} \cdot \frac{J(\sigma_{\text{emit}})}{J(\sigma_{\text{obs}})}.$$

For cosmological sources, if $J(\sigma)$ varies slowly along the null geodesic, we can write:

$$z(\lambda) = \int_0^\lambda \frac{d}{d\lambda'} \ln J(\sigma(\lambda'))\, d\lambda',$$

where the integral is taken along the null geodesic from emission to observation.

Physical Interpretation:

Redshift arises from cumulative projection between operational time classes, not necessarily from kinematic recession. The standard interpretation of redshift as a Doppler effect or metric expansion effect assumes clock universality. When this assumption fails, redshift encodes the accumulated mismatch between the null-adapted ordering parameter and the matter-adapted operational time.

This does not mean that metric expansion is absent, but rather that the *inferred* expansion rate depends on which operational time class is used to measure it. The same underlying metric structure can yield different inferred expansion rates when different clocks are employed.

5 Reinterpreting the Hubble Constant

5.1 Expansion Rate as a Derived Quantity

The Hubble constant H_0 is not a primitive quantity in General Relativity; it is an inferred scalar that depends on the operational definition of time and distance.

In standard cosmology, H_0 is defined through the relation:

$$z \approx H_0 D/c \quad (z \ll 1),$$

where z is redshift and D is distance. Both quantities are inferred using physical processes that act as clocks and rulers. The universality of H_0 therefore presupposes that these processes define a common notion of time.

However, if different physical processes define inequivalent operational time parameters, then the inferred H_0 will depend on which processes are used. Specifically:

- **Matter-adapted measurements** (distance ladder, stellar clocks): These use baryonic processes (Cepheids, Type Ia supernovae) to define both time and distance. The inferred expansion rate is:

$$H_{\text{matter}} = H_{\text{intrinsic}} \cdot J(\sigma_{\text{matter}}).$$

- **Null-adapted measurements** (geometric probes, CMB, BAO): These use null-propagated signals to define the ordering, and infer expansion from geometric relationships. The inferred expansion rate is:

  $$H_{\text{null}} = H_{\text{intrinsic}} \cdot J(\sigma_{\text{null}}) = H_{\text{intrinsic}},$$

  since null propagation defines $J(\sigma_{\text{null}}) = 1$ by construction.

The observed bifurcation $H_{\text{matter}} \approx 73$ km/s/Mpc versus $H_{\text{null}} \approx 67$ km/s/Mpc is then naturally interpreted as:

$$\frac{H_{\text{matter}}}{H_{\text{null}}} = J(\sigma_{\text{matter}}) \approx \frac{73}{67} \approx 1.09.$$

This provides a structural interpretation of the bifurcation without requiring new physics or modification of General Relativity.

5.2 Bifurcation as Differential Coupling

The observed bifurcation in H_0 measurements can be understood as arising from differential coupling between matter-adapted and null-adapted operational time classes.

Matter–matter paths: When both emission and observation use matter-adapted clocks (e.g., distance ladder measurements), the operational time mismatch accumulates along the matter worldlines. The inferred expansion rate reflects the cumulative effect of $J(\sigma_{\text{matter}})$ along these paths.

Null–matter paths: When emission uses null-propagated signals but observation uses matter-adapted clocks (e.g., CMB observations with local atomic clocks), the operational time mismatch accumulates along the null geodesic. The inferred expansion rate reflects the integrated effect of $J(\sigma)$ along the photon path.

The differential accumulation of conformal mismatch between these two types of paths naturally produces different inferred expansion rates, even when the underlying metric structure is the same.

Explicit Disclaimer:

This framework does not predict the magnitude of observed bifurcations. It provides a structural interpretation of *how* such bifurcations could arise, but does not claim uniqueness or exclusivity. Other interpretations (systematic errors, new physics, modified gravity) remain possible. The value of this framework lies in its consistency with General Relativity and its ability to explain the bifurcation without invoking new dynamics.

6 Consistency With Local Physics

6.1 Why Laboratory Clocks Agree

It is a well-established empirical fact that different types of clocks (atomic, nuclear, gravitational) agree to high precision in laboratory settings. This agreement is often cited as evidence for clock universality. However, this fact is also consistent with the operational time framework presented here.

Shared Operational Environment: In laboratory settings, all clocks operate in the same local environment. They are subject to the same gravitational field, the same electromagnetic environment, and the same matter density. Under these conditions, the effective Jacobian $J(\sigma) \to 1$ for all processes, because they all share the same dominant ordering class.

More precisely, if all clocks are matter-bound and operate in the same local environment, then:

$$J(\sigma_1) \approx J(\sigma_2) \approx 1$$

for any two processes σ_1 and σ_2 in that environment. The relative difference between different clock types is unobservable because they all share the same operational normalization.

Epistemic Locking: The agreement of laboratory clocks can be understood as "epistemic locking"—all clocks belong to the same dominant process environment, leading to internal renormalization. The Jacobian $J(\sigma)$ is only observable when comparing clocks from different ordering classes (e.g., matter-adapted versus null-adapted), not when comparing clocks within the same class.

This explains why local null results (e.g., GPS, laboratory tests of Lorentz invariance) do not conflict with the operational time framework. These tests compare clocks within the same operational class, where $J(\sigma) \approx 1$ by construction.

6.2 Compatibility With GR and Lorentz Invariance

The operational time framework is fully compatible with General Relativity and local Lorentz invariance.

Null Structure Untouched: The causal structure of spacetime, defined by null geodesics, is preserved. The conformal transformation $g^{(m)}_{\mu\nu} = J^2(\sigma) g_{\mu\nu}$ preserves the causal structure because $J(\sigma) > 0$. Timelike, null, and spacelike curves remain so under this transformation.

Local Frames Preserved: In any local inertial frame, the metric reduces to the Minkowski form, and all operational time parameters agree (i.e., $J(\sigma) \to 1$ locally). This ensures that local Lorentz invariance is preserved, and that the equivalence principle is not violated.

No Modification of Einstein Equations: The Einstein field equations remain unchanged:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

The operational time framework does not modify gravitational dynamics; it only affects the *interpretation* of observables derived from different physical processes.

Redshift Invariance: The standard GR expression for redshift $1+z = (k^\mu u_\mu)_{\mathrm{emit}}/(k^\mu u_\mu)_{\mathrm{obs}}$ remains valid. However, it is only process-independent if all matter fields share the same operational identification of $d\tau$. When this identification fails, the inferred redshift depends on which operational time class is used, but the mathematical form of the expression remains the same.

This is not a coordinate redefinition or GR modification, but an error in operational identification. The framework identifies a categorical error in the operational interpretation of u^μ when observables derived from inequivalent physical processes are combined without accounting for the transformation Jacobian.

7 Alternative Interpretations and Limitations

7.1 Non-Uniqueness

The conformal projection framework presented here is not unique. Other realizations of clock non-universality are possible, and the conformal projection is only one consistent interpretation. For example, one could imagine process-dependent metric modifications, emergent time proposals, or relational cosmology formulations. However, the value of the conformal projection framework lies in its consistency with General Relativity (it does not modify gravitational dynamics), its mechanism-agnostic nature (it does not require specifying the microscopic origin of $J(\sigma)$), and its falsifiability (it makes specific predictions about the convergence of different measurement methods).

It is important to emphasize that this framework is **interpretive, not deductive**. It shows how the observed bifurcation *could* arise, but does not prove that it *must* arise in this way.

7.2 Scope Limitations

The operational time framework is deliberately limited in scope. It does not attempt to explain dark energy, structure formation, or other cosmological phenomena beyond the H_0 bifurcation. The Einstein field equations remain unchanged; the framework only affects the interpretation of observables, not the dynamics of gravity. Moreover, it does not predict the magnitude of the observed bifurcation—the specific value of $J(\sigma_{\text{matter}}) \approx 1.09$ is not derived from first principles. These limitations are intentional: the framework is designed to be minimal, explaining the observed bifurcation with the fewest possible assumptions.

7.3 Observational Ambiguity

Multiple interpretations may fit the same observational data. The operational time framework is not the only possible explanation for the H_0 bifurcation; other interpretations include systematic errors, new physics (modified gravity, extra dimensions), or unknown early-universe physics. The framework does not claim to exclude these alternatives. Rather, it provides a consistent interpretation that avoids invoking systematic errors (which would need to be simultaneously present across multiple independent measurements) or new physics beyond General Relativity. The framework's value lies in its **internal consistency** and **falsifiability**, not in its uniqueness.

8 Falsifiability and Observational Handles

8.1 Convergence Tests

The operational time framework makes specific, falsifiable predictions about the convergence of different measurement methods.

Case 1: Matter-based convergence

If the framework is correct, all matter-adapted measurements of H_0 should converge to the same value (modulo statistical and systematic errors), because they all use the same operational time class. This prediction is consistent with current observations, where distance-ladder measurements cluster near $H_0 \approx 73$ km/s/Mpc.

Case 2: Null-based convergence

Similarly, all null-adapted measurements of H_0 should converge to the same value, because they all use null propagation to define the ordering parameter. This prediction is also consistent with current observations, where geometric probes, gravitational-wave standard sirens, and early-universe measurements cluster near $H_0 \approx 67$ km/s/Mpc.

Case 3: Cross-class divergence

The framework predicts that matter-adapted and null-adapted measurements will *not* converge, because they use different operational time classes. This prediction is confirmed by the observed bifurcation.

Failure criterion: If matter-adapted measurements begin to converge with null-adapted measurements (or vice versa), the framework would be falsified. This would indicate that clock universality holds, or that some other mechanism is responsible for the bifurcation.

8.2 Process-Sensitive Cosmological Clocks

The framework predicts that different physical processes used as cosmological clocks should yield different inferred expansion rates, with the difference proportional to their respective Jacobians $J(\sigma)$.

Testable prediction: If we could construct a cosmological clock based on a different physical process (e.g., gravitational-wave inspiral, rather than stellar pulsation), it should yield a different inferred H_0, unless $J(\sigma_{\text{new}}) = J(\sigma_{\text{matter}})$ or $J(\sigma_{\text{new}}) = 1$.

This prediction is difficult to test directly, because most cosmological clocks rely on similar physical processes (nuclear or stellar). However, gravitational-wave standard sirens provide a different operational time class (based on the inspiral dynamics rather than electromagnetic emission), and they do show convergence with null-adapted measurements, consistent with the framework.

8.3 Internal Consistency Tests

The framework predicts internal consistency within each operational time class, but inconsistency between classes.

Redshift ladder consistency: Within the distance-ladder method, different rungs (Cepheids, Type Ia supernovae, etc.) should be internally consistent, because they all use matter-adapted clocks. This is observed.

Geometric probe consistency: Different geometric probes (strong lensing, gravitational waves, CMB, BAO) should be internally consistent, because they all use null-adapted or geometric ordering. This is also observed.

Cross-ladder inconsistency: When combining distance-ladder measurements with geometric probes, inconsistencies should arise due to the operational time mismatch. This is the observed bifurcation.

Failure criterion: If internal consistency tests within a single operational time class fail, or if cross-class measurements begin to converge, the framework would be falsified.

9 Conclusion: Limits of Clock Universality

Cosmological expansion is commonly interpreted under the assumption that all physically realizable clocks instantiate a single, universal notion of elapsed time. In this paper we have argued that this assumption is stronger than warranted by either General Relativity or cosmological observation. By treating **causal ordering** as the invariant structure shared by all observables, and allowing for **operational asymmetry** in how that ordering is parametrized, several persistent inconsistencies in cosmological inference admit a coherent reinterpretation.

Within this framework, the observed bifurcation in Hubble constant measurements does not require new dynamical components or modifications of gravitational theory. Instead, it reflects a mismatch between operational time parameters associated with distinct classes of physical processes. The discrepancy arises when observables that are fundamentally null-adapted are combined with inference pipelines normalized to matter-based clocks, without accounting for the resulting conformal projection.

Importantly, this work does not claim to replace the standard expansion paradigm, nor to eliminate the empirical successes of ΛCDM. Rather, it identifies a structural ambiguity in the interpretation of redshift-based observables that is normally hidden by the assumption of clock universality. The persistence of the Hubble bifurcation suggests that this ambiguity may be observationally relevant.

The task suggested by this analysis is therefore limited and concrete: to determine whether different operational classes of clocks admit consistent mappings to a shared causal ordering, and whether any residual Jacobians are empirically measurable. If future observations enforce convergence across all methods, the framework presented here is falsified. If not, cosmological inference will require a more explicit accounting of how time is operationally defined, rather than assuming universality by default.

A Relation to Other Frameworks

The frameworks referenced below are mentioned solely to delimit what the operational time interpretation **does not claim**. No equivalence, reduction, shared ontology, or dynamical overlap is implied.

A.1 Conformal Gravity

Conformal gravity modifies the gravitational action via metric conformal invariance (Mannheim, 2006). The operational time framework does not modify the action, field equations, or spacetime symmetries. The factor $J(\sigma)$ is process-dependent and inference-level, not a geometric or universal conformal degree of freedom.

A.2 Emergent Time

Emergent time proposals posit a microscopic origin of time. The operational time framework is agnostic to the fundamental status of time and requires only that different physical processes define inequivalent but causally consistent operational parametrizations.

A.3 Relational Cosmology

Both frameworks emphasize relational ordering over absolute time. However, the operational time framework leaves gravitational dynamics unchanged and targets a specific empirical issue: the interpretation of cosmological expansion and the H_0 bifurcation.

A.4 Entropic Gravity

Entropic gravity derives gravity from thermodynamic principles and targets dark matter phenomenology (Verlinde, 2011). The operational time framework treats General Relativity as exact, is non-thermodynamic, and addresses inference-level inconsistencies in cosmological clocks rather than gravitational dynamics.

Boundary Condition: The operational time framework is an inference-level reinterpretation within unmodified General Relativity. It introduces no new dynamics, no microscopic degrees of freedom, and claims no equivalence to the frameworks above.

References

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