# IV. QUANTITATIVE FORECASTS

We now derive the observable cosmological consequences of the Plastic Spacetime Fossil (PSF) mechanism. By integrating the memory equation (Eq. 9) with the astrophysical source history (Eq. 16), we obtain the redshift evolution of the dark energy density and its effective equation of state.

## A. The Plasticity Sign Law

In standard General Relativity, the equation-of-state parameter ($w$) for a single canonical scalar field is bounded from below by ($w \geq -1$), corresponding to quintessence. The fossil sector, however, is an open system that exchanges energy with the strong-field source sector. Consequently, the effective equation of state ($w_{\text{eff}}$), inferred from the background expansion history, is determined by the time evolution of the energy density rather than by the scalar Lagrangian alone.

From the Friedmann equations, the effective equation of state for any fluid component with energy density ($\rho_f$) is defined through the continuity equation,

$$\dot{\rho}_f + 3H(1+w_{\text{eff}})\rho_f = 0 \quad \Longrightarrow \quad w_{\text{eff}} = -1 - \frac{\dot{\rho}_f}{3H\rho_f}. \tag{19}$$

For the saturating fossil potential

$$\rho_f(\chi) = \rho_\Lambda \left(1 - e^{-\chi/\chi_*}\right),$$

the time derivative is

$$\dot{\rho}_f = \frac{d\rho_f}{d\chi}\dot{\chi} = \frac{\rho_\Lambda}{\chi_*} e^{-\chi/\chi_*}\dot{\chi}. \tag{20}$$

Substituting the overdamped memory equation,

$$\dot{\chi} = S(t) - \frac{\chi}{\tau},$$

into Eq. (20) and then into Eq. (19), we obtain the exact **Plasticity Sign Law**,

$$w_{\text{eff}}(z) = -1 - \frac{\Upsilon(z)}{3H(z)} \left[S(z) - \frac{\chi(z)}{\tau}\right], \tag{21}$$

where

$$\Upsilon(z) \equiv \frac{\rho_\Lambda}{\rho_f \chi_*} e^{-\chi/\chi_*}$$

is a strictly positive, dimensionless saturation factor.

This relation dictates the phenomenology of dark energy entirely through the competition between fossil creation and relaxation:

- **Phantom phase** ($S > \chi/\tau \Rightarrow w_{\text{eff}} < -1$). When the universe is dominated by yield-violating events—during epochs of high merger activity—the fossil field accumulates ($\dot{\chi} > 0$). The dark energy density increases with time, mimicking a phantom fluid. Crucially, this is a *growth-driven phantom* phase that does not involve negative kinetic energy and therefore avoids ghost instabilities.
- **Quintessence phase** ($S < \chi/\tau \Rightarrow w_{\text{eff}} > -1$). When the source rate drops below the relaxation rate, the fossil field decays ($\dot{\chi} < 0$). The energy density decreases, producing behavior analogous to a thawing quintessence field.
- **The crossing** ($S = \chi/\tau \Rightarrow w_{\text{eff}} = -1$). The phantom divide is crossed precisely when fossil production and relaxation are in balance.

## B. Numerical Results: The Forecasts

We numerically integrate the coupled Friedmann equation and memory equation using the LVK-motivated source history (Eq. 18) with ($\kappa = 2.9$). We explore the parameter range

$$\tau \in [0.1\, H_0^{-1},\ 10\, H_0^{-1}],$$

fixing the normalization ($S_{\rm norm}$) such that ($\Omega_{\rm DE} \simeq 0.7$) at the present epoch.

### 1. Buildup of Dark Energy (Fig. 1)

The source history ($S(z)$) peaks at ($z \simeq 2$), coincident with the peak of cosmic star formation and compact-object merger activity. The fossil field ($\chi(z)$) acts as an integrator, accumulating memory rapidly during the interval ($1 \lesssim z \lesssim 3$). As the merger rate declines toward ($z = 0$), the field enters the saturation regime.

Unlike ΛCDM, where $\rho_{\rm DE}$ is constant by construction, the PSF model predicts an energy density that rises sharply at ($z \gtrsim 1$) and asymptotes to a constant only at late times.

### 2. Equation-of-State Evolution (Fig. 2)

Figure 2 shows the evolution of $w_{\rm eff}(z)$. In agreement with the Plasticity Sign Law, we find:

- At high redshift ($z \gtrsim 1.5$), where $S(z)$ is maximal, the equation of state is strongly phantom ($w_{\rm eff} \simeq -1.1$ to $-1.2$), driven by rapid fossil accumulation.
- At intermediate redshifts ($z \lesssim 1$), as the source term declines, the system evolves toward the phantom divide.
- At low redshift, a crossing of $w = -1$ occurs. For memory times comparable to the Hubble time ($\tau \sim H_0^{-1}$), this crossing lies in the observable window ($0 < z < 0.5$).

## C. The Crossing Map and Falsifiability

The redshift of the phantom crossing, $z_\times$, constitutes a robust observational fingerprint of the theory. It is not a tunable fitting parameter but is structurally determined by the lag between the peak of the astrophysical source history ($z_{\rm peak} \simeq 2$) and the memory timescale ($\tau$).

Figure 3 (robustness heatmap) illustrates the dependence of $z_\times$ on $\tau$ and on the width of the source history:

- **Fast relaxation** ($\tau \ll H_0^{-1}$): The fossil field closely tracks the source. The crossing occurs early, near the peak of merger activity ($z_\times \sim 1.5$).
- **Slow relaxation** ($\tau \gtrsim H_0^{-1}$): The system exhibits long memory. Fossil accumulation persists, pushing the crossing to late times ($z_\times < 0.3$) or into the future.

This defines a clear **falsifiability window**. Detection of a phantom-to-quintessence crossing at $z \sim 0.1$–$0.5$ would strongly favor a memory time ($\tau \sim H_0^{-1}$). Conversely, an equation of state consistent with $w(z) = -1$ at the percent level across all redshifts would imply $\tau \to \infty$—the limit in which plasticity vanishes and spacetime remains purely elastic—thereby ruling out the PSF mechanism as the dominant driver of cosmic acceleration.

# V. STABILITY AND GHOST ANALYSIS

A central concern for any dark energy model predicting $w < -1$ is the potential appearance of ghost instabilities (negative kinetic-energy states) or gradient instabilities (imaginary sound speeds). We demonstrate explicitly that the PSF framework is free of these pathologies. The apparent "phantom" behavior arises as an **effective phenomenon** due to the open-system nature of the fossil sector, rather than from a violation of the Null Energy Condition (NEC) at the level of the fundamental action.

## A. Linear Stability of the Scalar Sector

The stability properties of the theory are determined by the action in Eq. (1), prior to taking the overdamped limit. We consider linear perturbations of the scalar field,

$$\chi(t, \vec{x}) = \bar{\chi}(t) + \delta\chi(t, \vec{x}),$$

together with metric perturbations.

The scalar Lagrangian density is canonical,

$$\mathcal{L}_\chi = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \chi \nabla_\nu \chi - V(\chi). \tag{22}$$

Expanding the action to second order in perturbations, the kinetic term for scalar fluctuations is proportional to the coefficient of $(\dot{\delta\chi}^2)$. For the metric signature $((-, +, +, +))$, this coefficient is $(+1/2)$. As a result, the Hamiltonian is bounded from below, and the theory is manifestly ghost-free.

The sound speed of scalar perturbations is defined as the ratio of pressure perturbations to energy-density perturbations in the rest frame. For a minimally coupled canonical scalar field,

$$c_s^2 \equiv \frac{\delta p}{\delta \rho} = 1. \tag{23}$$

Since $c_s^2 > 0$, the scalar sector is stable against gradient instabilities on sub-horizon scales. Unlike k-essence or clustering dark energy models—which often require $c_s^2 \ll 1$ to fit observations—the PSF framework retains $c_s^2 = 1$, ensuring that dark energy remains smooth on cluster scales and consistent with observational constraints on structure growth.

## B. The "Phantom Without Ghosts" Mechanism

Standard no-go theorems assert that a single scalar field cannot realize $w < -1$ without violating the NEC and introducing ghosts. The PSF model evades this conclusion because it is fundamentally an **effective open system**.

The inferred equation of state,

$$w_{\text{eff}} = -1 - \frac{\dot{\rho}_f}{3H\rho_f},$$

depends on the time evolution of the background energy density. In a closed system, $\dot{\rho}$ is determined solely by cosmological expansion. In the PSF framework, however, the density evolution includes an explicit source term,

$$\dot{\rho}_f = -3H(\rho_f + p_f) + \mathcal{T}, \tag{24}$$

where $\mathcal{T} \propto S(t)$ represents energy injection from the strong-field source sector.

During the fossil-creation phase ($S > \chi/\tau$), the injection term dominates, leading to $\dot{\rho}_f > 0$. An observer interpreting this evolution under the assumption of energy conservation would infer an effective equation of state

$$w_{\text{eff}} \simeq -1 - \frac{\mathcal{T}}{3H\rho_f} < -1. \tag{25}$$

Crucially, this phantom-like behavior does **not** originate from negative kinetic energy—the scalar field remains canonical and stable—but instead reflects energy transfer between sectors. This mechanism is physically analogous to bulk viscosity or reheating, where effective equations of state can transiently violate standard bounds due to entropy production or external energy exchange. The PSF framework therefore provides a consistent and stable realization of phantom crossing without ghosts.

### C. Tensor Sector and Gravitational Waves

Modifications to scalar dynamics can sometimes propagate into the tensor sector, leading to deviations in the gravitational-wave propagation speed ($c_{\text{GW}} \neq c$). In the PSF framework, this does not occur.

The gravitational sector is governed strictly by the Einstein–Hilbert action, and the scalar field is minimally coupled to the metric. In particular, there are no derivative couplings of the form $G^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi$ or other higher-curvature operators affecting tensor dynamics.

As a result, the equation of motion for tensor perturbations ($h_{ij}$) remains

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0. \tag{26}$$

This dispersion relation implies a propagation speed

$$c_{\text{GW}} = 1$$

(in units where $c = 1$). The model is therefore automatically consistent with stringent observational bounds on the gravitational-wave speed, such as those inferred from binary neutron star mergers.

## VI. DISCUSSION AND OBSERVATIONAL TESTS

The Plastic Spacetime Fossil framework provides a distinct physical narrative for cosmic acceleration. In this picture, acceleration is not an intrinsic property of the vacuum state, but a historical consequence of the universe's violent astrophysical evolution. Unlike standard quintessence models—which fit $w(z)$ by tuning a potential $V(\phi)$—or (K)-essence models—which tune a kinetic function—the PSF model is structurally constrained by the **Plasticity Sign Law** (Eq. 21). This rigidity sharply limits the allowed phenomenology and enables decisive falsification tests.

### A. The Crossing Fingerprint

The most robust prediction of the theory is a generic evolution of the dark energy equation of state from a phantom-like regime ($w < -1$) at high redshift ($z \gtrsim 1$) to a quintessence-like regime ($w > -1$) at late times, provided the relaxation timescale is comparable to the Hubble time ($\tau \sim H_0^{-1}$).

Current Stage IV cosmological surveys will tightly constrain the time dependence of dark energy, often parameterized as

$$w(a) = w_0 + w_a(1 - a).$$

– **Consistency.** The PSF model naturally occupies the region $w_0 \gtrsim -1$ and $w_a < 0$, corresponding to a thawing evolution from a phantom phase. This quadrant is currently compatible with mild tensions observed in combined supernova and baryon acoustic oscillation datasets.

– **Falsifiability.** If future precision measurements establish that $w(z) = -1$ to within $\sim 1\%$ accuracy over the full range $0 < z < 2$, the PSF mechanism would be effectively ruled out as the primary driver of cosmic acceleration. While the formal limit $\tau \to \infty$ reproduces a pure cosmological constant (perfect memory), this limit corresponds to purely elastic General Relativity and renders the plasticity hypothesis observationally irrelevant.

## B. The Multi-Messenger Consistency Test

The most distinctive application of the PSF framework is the rigid coupling between the dark energy source term $S(z)$ and the compact-object merger rate $\mathcal{R}_{\rm GW}(z)$. In standard cosmology, the evolution of dark energy and the history of black hole mergers are independent. In the PSF framework, they are causally linked:

$$\rho_{\rm DE}(z) \propto \int_{-\infty}^{t} \mathcal{R}_{\rm GW}(t')\, e^{-(t-t')/\tau}\, dt'. \tag{27}$$

This relation implies a **multi-messenger consistency condition**. Any revision to the inferred merger history—such as changes in the peak redshift $z_p$ or in the high-redshift slope of $\mathcal{R}_{\rm GW}(z)$—must induce a corresponding shift in the predicted evolution of $w(z)$.

For example, if future gravitational-wave observations were to show that the binary black hole merger rate peaks at significantly lower redshift ($z_p \sim 0.5$ instead of $z_p \sim 2$), the PSF model would necessarily shift the phantom crossing redshift $z_\times$ to later times. This creates a testing capability unavailable to conventional scalar-field models: astrophysical data can be used to generate an external prior on the cosmological expansion history. A mismatch between the merger-rate–derived source history and the expansion history inferred from supernovae would directly falsify the model.

## C. Perturbations and Large-Scale Structure

Although the background evolution of the PSF model mimics phantom and quintessence phases, its perturbative behavior clearly distinguishes it from clustering dark energy scenarios.

As shown in Sec. V, the scalar sector is canonical and therefore has a sound speed

$$c_s^2 = 1.$$

This implies that the fossil field possesses a sound horizon comparable to the particle horizon.

– **No clustering on cluster scales.** Unlike cold dark matter ($c_s^2 \approx 0$), the fossil energy density does not cluster on galaxy or cluster scales ($k \gg H_0$). It remains a smooth background component.

– **Integrated Sachs–Wolfe effect.** The primary perturbative signature arises from the time variation of $w_{\rm eff}(z)$. This induces a late-time decay of the gravitational potential, generating an Integrated Sachs–Wolfe (ISW) signal in the cosmic microwave background. Because the PSF framework predicts a specific transition profile for $w(z)$, cross-correlations between the CMB and large-scale structure provide a complementary observational probe.

### D. Resolution of the Coincidence Problem

The standard ΛCDM model suffers from the well-known "Why now?" coincidence problem: why are $\rho_\Lambda$ and $\rho_m$ comparable only at the present epoch? The PSF framework replaces this coincidence with a causal delay mechanism.

In this model, dark energy becomes dynamically relevant only after the universe has experienced a prolonged era of structure formation. Star formation and compact-object production peak at $z \sim 2$, while cosmic acceleration emerges later, around $z \sim 0.5$, following a delay set by the memory timescale $\tau$.

If $\tau \sim H_0^{-1}$, the onset of cosmic acceleration is no longer an accident of initial conditions, but a natural response of spacetime to the maturation of the universe's stellar and black hole population.

# VII. CONCLUSION

The nature of cosmic acceleration is commonly framed as a choice between introducing a new fundamental substance—dark energy—or modifying gravity on infrared scales. In this work, we have proposed a third possibility: that cosmic acceleration is a macroscopic manifestation of the **memory of spacetime itself**—a fossil record of the universe's history of strong-field curvature.

We introduced the **Plastic Spacetime Fossil (PSF)** framework, an effective field theory in which General Relativity describes spacetime as an elastic medium only up to a critical covariant yield threshold ($K > K_c$). When this threshold is exceeded—during stellar collapse, black hole formation, and compact-object mergers—spacetime undergoes plastic deformation, generating a residual scalar memory field ($\chi$). We showed that the energy density associated with this accumulated memory, $\rho_f(\chi)$, naturally saturates to a constant value, providing a dynamical resolution of the cosmological constant fine-tuning problem.

### Key Theoretical Results

- **Stability.** We demonstrated that the phantom-like behavior ($w_{\text{eff}} < -1$) predicted by the model does not require negative kinetic energy or violation of the Null Energy Condition at the level of the fundamental action. Instead, it emerges effectively from the open-system thermodynamics of the fossil sector, driven by energy injection from strong-field astrophysical sources. The underlying scalar theory is canonical and ghost-free, with sound speed $c_s^2 = 1$, ensuring stability against gradient and clustering instabilities.
- **The Plasticity Sign Law.** We derived a rigid phenomenological relation,

$$w_{\text{eff}}(z) + 1 \propto -\left[S(z) - \frac{\chi}{\tau}\right],$$

  which fixes the evolution of the equation of state through the competition between fossil creation and memory relaxation. This structure forbids arbitrary choices of $w(z)$ and generically predicts a phantom-to-quintessence transition.

### Key Observational Predictions

The most distinctive feature of the PSF framework is the **Multi-Messenger Consistency Condition**. Unlike conventional dark energy models—where the expansion history is decoupled from astrophysical processes—the PSF model links the evolution of dark energy directly to the cosmic history of compact-object mergers. We showed that if the memory timescale is of order the Hubble time ($\tau \sim H_0^{-1}$), the model predicts a crossing of the phantom divide ($w = -1$) at a redshift $z_\times$ that is constrained by the peak of the gravitational-wave merger rate. This renders the mechanism directly testable with combined cosmological and gravitational-wave data.

## Future Directions

This work serves as **Paper I**, establishing the macroscopic mechanism and its cosmological consequences. Future studies will investigate the microphysical origin of the yield threshold $K_c$ within candidate quantum gravity frameworks—for example, as a phase transition in loop quantum gravity or string compactifications—and will perform a full Markov Chain Monte Carlo analysis using current SNe Ia (Pantheon+), BAO, and CMB datasets to place quantitative constraints on the memory timescale ($\tau$).

In summary, the Plastic Spacetime Fossil hypothesis reframes dark energy from a static vacuum anomaly into a dynamic geometric echo of the universe's violent past. Cosmic acceleration becomes historically contingent, theoretically stable, and—most importantly—falsifiable by the next generation of multi-messenger observations.