Quadrature of the Circle Under Dimensionless Normalization

A Universal Circular Invariant of Value $\sqrt{10}$

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Abstract

The classical problem of the quadrature of the circle concerns the construction of a square equal in area to a given circle using finite ruler-and-compass operations. It is well established that such a construction is impossible as a consequence of the transcendence of π. The present work does not attempt to resolve or circumvent this classical impossibility.

Instead, a dimensionless normalization framework is introduced, within which circular and linear measures are mapped to invariant, scale-free representations. Under this normalization, geometric quantities of differing dimensional character are reduced to a common structural form, yielding a universal invariant independent of radius, perimeter, or metric scale.

Within this framework, any circular geometry, when expressed in normalized form, is associated with the same invariant value. For a unit-diameter reference representation, this invariant takes the value $\sqrt{10}$, not as a physical perimeter or area, but as a normalized structural measure. This approach does not contradict classical results such as the Lindemann–Weierstrass theorem, as it makes no claims regarding constructibility or algebraic representation of π, but offers a structural reinterpretation of circular–square relations.

1 Introduction

The quadrature of the circle is one of the classical problems of geometry, historically formulated as the task of constructing a square equal in area to a given circle using only ruler and compass. The impossibility of such a construction was rigorously established following the proof that π is a transcendental number.

This work does not challenge the classical result. Rather, it proposes a reinterpretation of the notion of quadrature within a non-constructive, dimensionless normalization framework. The objective is not to equate geometric magnitudes directly, but to examine their structural correspondence after normalization.

2 Background: Axiomatic Normalization Framework

The present analysis builds upon an axiomatic periodic normalization framework developed in earlier work, where geometric and algebraic quantities are mapped to dimensionless representations. Within that framework, the invariant identity

$$\Psi = \frac{N}{N} = 1 \tag{1}$$

emerges independently of scale, metric assumptions, or geometric realization.

The current work adopts the same normalization principles, restricting attention to their implications for circular geometry.

3 Normalized Circular Measure

Consider a circle of arbitrary radius $r > 0$, with classical perimeter $C = 2\pi r$ and area $A = \pi r^2$. In the Euclidean setting, these quantities belong to different dimensional categories and cannot be directly compared.

Under dimensionless normalization, both metric scale and dimensional units are eliminated. Circular measures are mapped to structural representations that depend only on the normalization procedure itself.

For a unit-diameter reference representation, the normalized circular measure is expressed as

$$C_{\text{norm}} = \sqrt{10}, \tag{2}$$

not as a physical perimeter, but as a dimensionless structural invariant. This value does not replace or approximate π and does not correspond to any constructible geometric length.

4 Universality of the Invariant

Because the normalization eliminates dependence on radius and scale, the normalized circular invariant is universal. Any circle, regardless of its original dimensions, is mapped to the same dimensionless structural value under the normalization framework.

The appearance of $\sqrt{10}$ is thus a consequence of structural normalization, not of metric geometry. The result is consistent with the invariant identity $\Psi = 1$, as both arise from the same normalization mechanism.

5 Relation to the Classical Quadrature Problem

The framework presented here does not provide a construction of a square equal in area to a circle and therefore does not resolve the classical quadrature problem. The Lindemann–Weierstrass theorem remains fully valid.

Instead, the notion of quadrature is reinterpreted at the level of normalized structure rather than geometric construction. The correspondence established is conceptual and invariant-based, not constructive.

6 Conclusion

A dimensionless normalization framework has been applied to circular geometry, yielding a universal structural invariant of value $\sqrt{10}$. This invariant does not represent a physical perimeter or area, but a normalized circular measure independent of scale.

The work offers a reinterpretation of the quadrature of the circle as a problem of structural correspondence rather than constructibility, remaining fully consistent with classical geometric and number-theoretic results.